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The principle of the excluded third and the Bolzano-Weierstrass lemma

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One proposition may be true, another false; but a positive judgement necessitates knowledge that is not always within our reach. We may have to wait ten years to decide the truth-value (i. e. the truth or falsehood) of the proposition: "Mr. F. will die within ten years". Thus the truth-value of the proposition is undecided at present, but will be decided in at most ten years. When shall we be able to decide the truth-value of the proposition: "There are greater craters on the back of the moon than on its front"? The doctrine that every proposition is either true or false (the principle of the excluded third) must, therefore, have a proviso added to it: that our knowledge suffices to decide the truth-value of the proposition.

In my note "A Failure of the Bolzano-Weierstrass Lemma"[1], I have shown that the lemma is formulated too generally. The lemma runs as follows. If a given interval contains an unlimited sequence of numbers (the primary sequence), we can find within the interval at least one arbitrarily small interval containing an unlimited sequence of numbers from the primary sequence. The proof, which is based on the principle of the excluded third without the above reservation, presupposes that we know whether a sub-interval, e. g. half of the given interval, contains a limited or unlimited sequence of numbers from the primary sequence. If we do not know this, the proof breaks down. An instance of such a lack of knowledge about a sequence of numbers is given in the note referred to above. The lemma is, therefore, valid solely when we have sufficient knowledge about the distribution of numbers within the given interval.

[1] Arkiv för Matematik, Astronomi och Fysik, Band 34 B, n:o 11.

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