A class of II_1 factors without property P but with zero second cohomology

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If $\mathfrak{A}$ is a von Neumann algebra, or indeed any Banach algebra, $\mathcal{H}^2(\mathfrak{A}, \mathfrak{A})$ is the quotient of the space of continuous bilinear maps $S : \mathfrak{A} \times \mathfrak{A} \to \mathfrak{A}$ such that

$$\delta S(a, b, c) \equiv aS(b, c) - S(ab, c) + S(a, bc) - S(a, b)c = 0 \quad (a, b, c \in \mathfrak{A})$$

by the subspace of those maps of the form

$$S(a, b) = aR(b) - R(ab) + R(a)b = (\delta R)(a, b)$$

for some continuous linear map $R : \mathfrak{A} \to \mathfrak{A}$. The background to the present paper is the three papers [6], [7] and [5], in which it is shown that $\mathcal{H}^2(\mathfrak{A}, \mathfrak{A}) = 0$ for type I von Neumann algebras and for hyperfinite von Neumann algebras. In this paper we construct some non hyperfinite II_1 factors which have this property. Besides the three papers above we shall also use ideas from [4].

LEMMA 1. *Let G be a group of permutations of a set X, $x_0 \in X$ and $H = \{g : g \in G, gx_0 = x_0\}$. Suppose H is amenable and G is 3-fold transitive on X. Then $\mathcal{H}^1(G, \ell^\infty(X)/\mathbf{C}\,\mathbf{1}) = 0$.*

$\ell^\infty(X)$ is the space of bounded functions on X and if $f \in \ell^\infty(X)$, $g \in G$ we define gf by $(gf)(x) = f(g^{-1}x)$ $(x \in X)$. $\mathbf{C}\,\mathbf{1}$ is the set of constant functions in $\ell^\infty(X)$ and is closed under multiplication by elements of G so that if $F \in \ell^\infty(X)/\mathbf{C}\,\mathbf{1}$, gF is well defined. Saying $\mathcal{H}^1(G, \ell^\infty(X)/\mathbf{C}\,\mathbf{1}) = 0$ means that whenever Φ is a map $G \to \ell^\infty(X)/\mathbf{C}\,\mathbf{1}$ with

$$\|\Phi(g)\| \le K \qquad g \in G$$
$$\Phi(gg') = \Phi(g) + g\Phi(g') \qquad g, g' \in G,$$

that is, if Φ is a bounded crossed homomorphism, then there is $F \in \ell^\infty(X)/\mathbf{C}\,\mathbf{1}$ with

$$\Phi(g) = gF - F \quad g \in G,$$

that is Φ is a principal crossed homomorphism. Saying that G is 3-*fold transitive* means that if $\{x_1, x_2, x_3\}$, $\{x_1', x_2', x_3'\}$ are two sets of 3 distinct points of X then there is $g \in G$ with $gx_i = x_i'$ $i = 1, 2, 3$. Amenability of groups is discussed in [3, §17].

Proof. If X is finite the result is a consequence of [4, Theorem 3.4]. Accordingly we assume X has at least 3 points. Let Φ be a bounded crossed homomorphism as above. As H is amenable and as $\ell^\infty(X)/\mathbf{C}\,\mathbf{1}$ is the dual of $\ell_0 = \{a;\ a \in \ell^1(X),\ (a, \mathbf{1}) = 0\}$ and $F \mapsto gF$ is the adjoint of the map $a \mapsto ag$ where $ag(x) = a(gx)$ on ℓ_0, there is F in $\ell^\infty(X)/\mathbf{C}\,\mathbf{1}$ with $\Phi(h) = hF - F$ for all h in H [4, Theorem 2.5]. Replacing Φ by $\Psi: g \mapsto \Phi(g) - gF + F$ we see that Ψ is a bounded crossed homomorphism and Ψ is principal if and only if Φ is. Ψ is zero on H so $\Psi(gh) = \Psi(g)$, $g \in G$, $h \in H$. Thus Ψ is constant on the left cosets of H, which are in one to one correspondence with the points of X, and so can be considered as a function Ψ' on X which is zero at x_0. If $\ell^0 = \{f:\ f \in \ell^\infty(X),\ f(x_0) = 0\}$ then the quotient map q onto $\ell^\infty(X)/\mathbf{C}\,\mathbf{1}$ is one to one on ℓ^0 and if we define

$$(g \circ f)(x) = f(g^{-1}x) - f(g^{-1}x_0) \quad f \in \ell^0, \quad g \in G, \quad x \in X$$

then $g \circ f \in \ell^0$ and $q(g \circ f) = gq(f)$. Thus we can assume that Ψ' takes values in ℓ^0 rather than $\ell^\infty(X)/\mathbf{C}\,\mathbf{1}$ and we have

$$\Psi'(gg') = \Psi'(g) + g \circ \Psi'(g').$$

Let $\Theta(x, y) = (\Psi'(x))(y)$ $(x, y \in X)$. Then Θ is a bounded complex valued function on $X \times X$ which is zero if either variable is x_0. The crossed homomorphism property for Ψ shows

$$\Theta(x, g^{-1}y) - \Theta(x, g^{-1}x_0) - \Theta(gx, y) + \Theta(gx_0, y) = 0 \quad g \in G, x, y \in X.$$

If $x, y \in X \setminus \{x_0\}$ then there is $g \in G$ with $gx_0 = x_0$, $gx = y$ and the above equation yields $\Theta(x, x) = \Theta(y, y)$. If $gx_0 = x_0, z = g^{-1}y$ we get $\Theta(x, z) = \Theta(gx, gz)$ so that, because G is 3-fold transitive on X, Θ is constant off the diagonal of $(X \setminus \{x_0\}) \times (X \setminus \{x_0\})$. If α is the value of Θ on the diagonal and β the value off the diagonal then writing $g^{-1}y = z$ and choosing g with $gx = x_0$ we have

$$\Theta(x, z) - \Theta(x, x) + \Theta(gx_0, gz) = 0$$

so that if x, x_0, z are distinct then $\alpha - \beta + \alpha = 0$. Defining $\varphi(x) = -\beta$ if $x \neq x_0$, $\varphi(x_0) = 0$ we easily check that $\Theta(gx_0, y) = \varphi(g^{-1}y) - \varphi(g^{-1}x_0) - \varphi(y)$ for all $g \in G$, $y \in X$. $\varphi \in \ell^0$ and this equation can be rewritten $\Psi'(gx_0) = g \circ \varphi - \varphi$ from which we see $\Psi(g) = gq(\varphi) - q(\varphi)$.

THEOREM 2. *Let (Z, ν) be a locally compact, σ-compact measure space and G a group of homeomorphisms of Z such that $\nu \circ g$ is absolutely continuous with respect* *to ν for all g in G. Let K be an amenable normal subgroup of G and H an amenable subgroup containing K. We suppose that*

(i) $\mathcal{A} \cap \mathcal{A} U_g = \{0\}$ *if* $g \neq e$

(ii) K *is ergodic on* Z

(iii) G *is 3-fold transitive on the left coset space* G/H

where the notation is that of [1, p. 134]. *Let* $\mathcal{B}$ *be the von Neumann algebra constructed from* Z, ν, G [1, p. 133—135]. *Then* $\mathcal{H}^2(\mathcal{B}, \mathcal{B}) = 0$.

Proof. We shall use the notation of [1, Ch. 1, §9], in particular that of Ex. 1 p. 137. Let $\mathfrak{A}_0$ be the norm closed *subalgebra of $\mathcal{L}(\mathfrak{H})$ generated by the operators $\{\Phi(T)\,;\, T \in \mathcal{A}\}$, $\{\tilde{U}_k\,;\, k \in K\}$, $\mathfrak{A}$ the weak closure of $\mathfrak{A}_0$, $\mathfrak{B}_0$ the norm closed algebra generated by $\{\Phi'(T)\,;\, T \in \mathcal{A}\}$ and $\{\tilde{U}'_h\,;\, h \in H\}$ and $\mathfrak{B}$ the weak closure of $\mathfrak{B}_0$. It is easy to see that the subgroup $\mathcal{U}$ of the unitary group of $\mathfrak{A}_0$ generated by the $\{\Phi(U)\,;\, U^{-1} = U^* \in \mathcal{A}\}$ and $\{\tilde{U}_k\,;\, k \in K\}$ is an extension of the abelian group $\{\Phi(U) : U^{-1} = U^* \in \mathcal{A}\}$ by a group isomorphic with K and so $\mathcal{U}$ is amenable [3, Theorems 17.5 and 17.14]. Thus $\mathfrak{A}_0$ is strongly amenable [4, Proposition 7.8]. Similarly $\mathfrak{B}_0$ is strongly amenable. If M is a translation invariant mean on $\mathcal{U}$ then defining

$$(PX\xi, \eta) = \underset{U \in \mathcal{U}}{M}\ (U^*XU\xi, \eta) \quad \xi, \eta \in \tilde{\mathfrak{H}},\ \ X \in \mathcal{L}(\tilde{\mathfrak{H}}),$$

where the right hand side indicates the value of M at the function $U \mapsto (U^*XU\xi, \eta)$, we define a projection $P : \mathcal{L}(\tilde{\mathfrak{H}}) \to \mathfrak{A}'_0 = \mathfrak{A}'$ with $P(XB) = P(X)B, P(BX) = BP(X)$ for $B \in \mathfrak{A}', X \in \mathcal{L}(\tilde{\mathfrak{H}})$. There is a similar projection Q onto $\mathfrak{B}'$.

By [5, Lemma 5.4] to show $\mathcal{H}^2(\mathcal{B}, \mathcal{B}) = 0$ it is enough to show that if $S : \mathcal{B} \times \mathcal{B} \to \mathcal{B}$ is separately ultraweakly continuous, $\delta S = 0$ and $S(a, b) = 0$ if either a or b lies in $\mathfrak{A}_0$ (and so too if a or b lies in $\mathfrak{A}$) then $S = \delta R_0$ for some norm continuous map $R_0 : \mathcal{B} \to \mathcal{B}$. Using [7, Theorem 2.4] we see that there is a norm continuous map $R : \mathcal{B} \to \mathcal{L}(\tilde{\mathfrak{H}})$ with $S = \delta R$ and by [5, Lemma 5.5] with $\mathcal{M} = \mathcal{L}(\tilde{\mathfrak{H}})$ we can take R to be ultraweakly continuous. As $R(ab) = aR(b) + R(a)b$ for all a in $\mathfrak{A}$ using the definition of amenable algebra [4, §5] we see that there is $x \in \mathcal{L}(\tilde{\mathfrak{H}})$ with $R(a) = ax - xa$ for all a in $\mathfrak{A}_0$ and so, by ultraweak continuity, for all a in $\mathfrak{A}$. Replacing R by $a \mapsto R(a) - ax + xa$ if necessary we can assume R is zero on $\mathfrak{A}$. Replacing R by QR if necessary we can assume in addition that R maps $\mathcal{B}$ into $\mathfrak{B}'$. We have $0 = S(a, b) = aR(b) - R(ab)$ $a \in \mathfrak{A}, b \in \mathcal{B}$. Similarly $R(ba) = R(b)a$ $a \in \mathfrak{A}$, $b \in \mathcal{B}$.

The set of generators of $\mathfrak{A}_0$ is mapped onto itself under the automorphism $X \mapsto \tilde{U}_g^* X \tilde{U}_g$ of $\mathcal{L}(\tilde{\mathfrak{H}})$ so $\tilde{U}_g^* \mathfrak{A} \tilde{U}_g = \mathfrak{A}$ for all g in G. Hence $R(\tilde{U}_g)\tilde{U}_g^* A \tilde{U}_g =$

$= R(A\tilde{U}_g) = AR(\tilde{U}_g)$ for all $A \in \mathfrak{A}$, $g \in G$, so that $R(\tilde{U}_g)\tilde{U}_g^* \in \mathfrak{A}'$. Also $R(\tilde{U}_g)U_g^* \in \mathfrak{B}'$ because $R(\tilde{U}_g)$ and $\tilde{U}_g$ are.

$\tilde{\mathfrak{H}}$ is a direct sum of copies of $\mathfrak{H}$ so any element L of $\mathcal{L}(\tilde{\mathfrak{H}})$ can be represented as a $G \times G$ matrix with entries from $\mathcal{L}(\mathfrak{H})$. We shall investigate the special form this matrix takes when $L \in \mathfrak{A}' \cap \mathfrak{B}'$. As $L\Phi(T) = \Phi(T)L$ we have $L_{s,u}T = TL_{s,u}$ for all T in $\mathcal{A}$, s, u in G. As $\mathcal{A}$ is maximal abelian this shows $L_{s,u} \in \mathcal{A}$. A similar calculation starting from $L\Phi'(T) = \Phi'(T)L$ shows $L_{s,u} \in U_s \mathcal{A} U_u^* = \mathcal{A} U_{su^{-1}}$ so $L_{s,u} \in \mathcal{A} \cap \mathcal{A} U_{su^{-1}} = \{0\}$ if $s \neq u$. Thus $L_{s,u} = \delta_{s,u} Y_s$ where for each s in G, $Y_s \in \mathcal{A}$. The equation $L\tilde{U}_k = \tilde{U}_k L$ shows $Y_{ku} = U_k Y_u U_k^*$ for $k \in K$, $u \in G$. The equation $L\tilde{U}_h' = \tilde{U}_h' L$ shows $Y_{uh} = Y_u$ for all $u \in G$, $h \in H$. Thus $Y_u = U_k^* Y_{ku} U_k = U_k^* Y_{uu^{-1}ku} U_k = U_k^* Y_u U_k$, $u \in G$, $k \in K$. As K is ergodic on Z this implies $Y_u = y_u I_{\mathfrak{H}}$ for some $y_u \in \mathbf{C}$. Thus if $L \in \mathfrak{A}' \cap \mathfrak{B}'$ then

$$L_{s,t} = \delta_{s,t} y_t I_{\mathfrak{H}}$$

for some complex valued function y on G which is constant on the left cosets of H. Clearly y is bounded. Writing JL for y we see that J is a linear isometry of $\mathfrak{A}' \cap \mathfrak{B}'$ onto $\ell^\infty(X)$ where X is the space of left cosets of H in G. Moreover $\tilde{U}_g L \tilde{U}_g^* \in \mathfrak{A}' \cap \mathfrak{B}'$ and $J(\tilde{U}_g L \tilde{U}_g^*) = gJL$ where the product of $g \in G$, $JL \in \ell^\infty(X)$ is as defined in Lemma 1. Another calculation shows that $J(\mathcal{B} \cap \mathfrak{A}') = \mathbf{C}\,\mathbf{1}$.

Put $\Phi_0(g) = J(R(\tilde{U}_g)\tilde{U}_g^*)$. The equation $S(\tilde{U}_g, \tilde{U}_{g'}) = \delta R(\tilde{U}_g, \tilde{U}_{g'})$ where $S(\tilde{U}_g, \tilde{U}_{g'})\tilde{U}_{g'}^*\tilde{U}_g^* \in \mathcal{B}$ and $R(\tilde{U}_{g''})U_{g''}^* \in \mathfrak{A}' \cap \mathfrak{B}'$ for all $g'' \in G$ shows that $\delta R(\tilde{U}_g, \tilde{U}_{g'})\tilde{U}_{g'}^*\tilde{U}_g^* \in \mathcal{B} \cap \mathfrak{A}'$ from which we see $g\Phi_0(g') - \Phi(gg') + \Phi(g) \in \mathbf{C}\,\mathbf{1}$. Thus $q\Phi_0$ is a bounded crossed homomorphism from G into $\ell^\infty(X)/\mathbf{C}\,\mathbf{1}$. Let $z \in \ell^\infty(X)$ with $q\Phi_0(g) = gq(z) - q(z)$ (using the Lemma) and let $L_0 \in \mathfrak{A}' \cap \mathfrak{B}'$ with $JL_0 = z$. We have

$$J(R(\tilde{U}_g)\tilde{U}_g^* - \tilde{U}_g L_0 \tilde{U}_g^* + L_0) \in \mathbf{C}\,\mathbf{1}$$

so that $R(\tilde{U}_g)\tilde{U}_g^* - \tilde{U}_g L_0 \tilde{U}_g^* + L_0 \in \mathbf{C}\, I_{\tilde{\mathfrak{H}}} \subset \mathcal{B}$. Thus defining $R_0(B) = R(B) - (BL_0 - L_0 B)$ for all B in $\mathcal{B}$ we see that R_0 is an ultraweakly continuous map from $\mathcal{B}$ into $\mathfrak{B}'$.

Because $L_0 \in \mathfrak{A}'$ and $R(AB) = AR(B)$, $R(BA) = R(B)A$ and $R(A) = 0$ if $A \in \mathfrak{A}$, $B \in \mathcal{B}$, R_0 has the same properties. In addition if $g \in G$ then $R_0(\tilde{U}_g) = (R(\tilde{U}_g)\tilde{U}_g^* - \tilde{U}_g L_0 \tilde{U}_g^* + L_0)\tilde{U}_g \in \mathcal{B}$. Thus if $T \in \mathcal{A}$, $g \in G$ then $R_0(\Phi(T)\tilde{U}_g) = \Phi(T)R_0(\tilde{U}_g) \in \mathcal{B}$. As R_0 is ultraweakly continuous and the ultraweakly closed linear span of the $\Phi(T)\tilde{U}_g$ is $\mathcal{B}$ we see that $R_0(\mathcal{B}) \subseteq \mathcal{B}$. For all B_1, B_2 in $\mathcal{B}$ we have

$$S(B_1, B_2) = B_1 R(B_2) - R(B_1 B_2) + R(B_1)B_2 = B_1 R_0(B_2) - R_0(B_1 B_2) + R_0(B_1)B_2.$$

Thus to provide our example we have only to show that the hypotheses can be satisfied in some situation in which $\mathcal{B}$ is a type II_1 factor without property P [8, Definition 1]. To facilitate this we simplify condition * [1, p. 135].

LEMMA 3. *Let Z be a locally compact σ-compact metrizable space, ν a positive Radon measure on Z, s a homeomorphism of Z. Then the following condition * is satisfied if and only if* $\nu(\{z : z \in Z,\ sz = z\}) = 0$.

(*) *For each measurable set Z' in Z with $\nu(Z') \neq 0$ there is a measurable subset Z'' of Z' with $\nu(Z'') \neq 0$ and $Z'' \cap sZ'' = \emptyset$.*

Proof. If $F = \{z : z \in Z,\ z = sz\}$ has $\nu(F) = 0$, $Z' \subseteq Z$, $\nu(Z') > 0$ and d is a metric on Z compatable with the topology then

$$Z_n = \{z : z \in Z',\ \ d(z, sz) > n^{-1}\}$$

defines a monotonic increasing sequence of measurable subsets of Z with union $Z \setminus F$ where $\nu(Z \setminus F) = \nu(Z) > 0$. Thus for some $n, \nu(Z_n) > 0$. Taking a compact subset K of Z_n with $\nu(K) > 0$ and a ball B centre z_0 of radius $(2n)^{-1}$ with $\nu(B \cap K) > 0$ we put $Z'' = B \cap K$. Then if $z \in Z''$ we have $d(z_0, sz) \geq$ $\geq d(z, sz) - d(z_0, z) > (2n)^{-1}$ showing $sz \notin Z''$. The converse is obvious.

Example 4. In Theorem 2 let $Z = \mathbf{Z}_2^{\mathbf{Q}^2}$, that is the product of a countable number of copies of the group of integers mod 2, the factors being indexed by pairs of rational numbers, with the usual product topology and let ν be Haar measure on Z with $\nu(Z) = 1$. Thus Z is a compact metrizable group. Let $Z_0 = \{z : z \in Z,\ z_p = 0$ for all but a finite number of $p \in \mathbf{Q}^2\}$ and let K be the set of all mappings of Z onto itself of the form $z \mapsto z + z_0$. K is then an abelian group of homeomorphisms preserving ν and, in particular, is amenable [3, Theorem 17.5]. If $F \in L^\infty(\nu)$ has $F(kz) = F(z)$ for almost all z in Z for each $k \in K$ then we have $\int f(z - z_0)F(z)d\nu(z) = \int f(z)F(z)d\nu(z)$ $(f \in C(Z),\ z_0 \in Z_0)$. As $y \mapsto \int f(z - y)F(z)d\nu(z)$ is continuous and Z_0 is dense in Z this shows $F\nu$ is an invariant integral on $C(Z)$ and hence F is a constant. Thus K is ergodic on Z.

For any one to one map α of $\mathbf{Q}^2$ onto itself $\alpha' : z \mapsto \{z_{\alpha(p)}\,; p \in \mathbf{Q}^2\}$ is an automorphism of the topological group Z and so is a homeomorphism preserving ν. G is the group of homeomorphisms of Z generated by K and the α' with $\alpha \in \mathrm{SL}(2, \mathbf{Q})$ where the matrix group acts on $\mathbf{Q}^2$ in the usual way. Every element of G can be written $k\alpha'$, $k \in K$, $\alpha \in \mathrm{SL}(2, \mathbf{Q})$ in exactly one way. If $\alpha \in \mathrm{SL}(2, \mathbf{Q})$, and α is not the identity then α has an infinite number of non fixed points in its action on $\mathbf{Q}^2$ so we can find an infinite subset E of $\mathbf{Q}^2$ with $E \cap \alpha(E) = \emptyset$. If $k \in K$ is the homeomorphism $z \mapsto y + z$ then the equation $k\alpha' z = z$ is equivalent to the system $z_p = z_{\alpha(p)} + y_p$ $(p \in \mathbf{Q}^2)$ so that if E_0 is a subset of E containing exactly n elements the set of fixed points of $k\alpha'$ is a subset of

$$\{z : z \in Z,\ z_p = z_{\alpha(p)} + y_p,\ p \in E_0\}$$

where this latter set has ν measure 2^{-n}. Thus the set of fixed points of $k\alpha'$ has measure zero. If $k \in K$ then k has no fixed points unless k is the identity. Thus by Lemma 3 we see that G satisfies condition *. G is ergodic on Z because K is. Thus [1, p. 135] condition (i) of Theorem 3 holds.

Let H be the subgroup of G containing K and those homeomorphisms α' where $\alpha \in \Gamma_1 = \{\alpha : \alpha \in \mathrm{SL}(2, \mathbf{Q}),\ \alpha_{21} = 0\}$ and H_1 the group generated by K and the α' with

$$\alpha \in \Gamma_2 = \{\alpha : \alpha \in \mathrm{SL}(2, \mathbf{Q}),\ \ \alpha_{11} = 1, \alpha_{21} = 0\}.$$

Γ_2 is normal in Γ_1 and H_1 and H are the inverse images of Γ_2 and Γ_1 under the isomorphism $\varkappa : k\alpha' \mapsto \alpha$ of G/K onto $\mathrm{SL}(2, \mathbf{Q})$ so that K is normal in H_1, H_1 is normal in H and K, H_1/K and H/H_1 are abelian. Thus [3, Theorems 17.5 and 17.14] H is amenable. In the usual way $\mathrm{SL}(2, \mathbf{Q})$ acts on the rational projective line and Γ_1 is the subgroup leaving $(0, 1)$ fixed. Under $\varkappa$ the action of G on G/H is mapped onto this action and it is well known that the action of $\mathrm{SL}(2, \mathbf{Q})$ on the projective line is 3-fold transitive. Thus condition (iii) of Theorem 2 is satisfied. By [1, p. 135] $\mathcal{B}$ is a II_1 factor.

To complete our example we copy the argument in [8, Lemma 7] to show that $\mathcal{B}$ does not have property P. As G contains a group isomorphic with $\mathrm{SL}(2, \mathbf{Z})$ which in turn contains a free group on two generators ([2, p. 26]), G is not amenable [3, Theorem 17.16]. However $\mathcal{B}$ is spatially isomorphic with $\mathcal{B}'$ [1, p. 137, Ex. 1] so that if $\mathcal{B}$ has property P so has $\mathcal{B}'$ and in this case there is a state τ on $\mathcal{L}(\tilde{\mathfrak{H}})$ with $\tau(U^*AU) = \tau(A)$ whenever $A \in \mathcal{L}(\tilde{\mathfrak{H}})$ and U is unitary in $\mathcal{B}'$ [8, Corollary 6]. If $F \in \ell^\infty(G)$ and A_F is the element of $\mathcal{L}(\tilde{\mathfrak{H}})$ defined by $(A_F)_{s,t} = F(s)\mathrm{I}_{\mathfrak{H}}$ if $s = t$, $(A_F)_{s,t} = 0$ otherwise, then denote $\tau(A_F)$ by $M(F)$. M is then a state on $\ell^\infty(G)$ and $M(gF) = \tau(A_{gF}) = \tau(\tilde{U}'_g A_F \tilde{U}'^*_g) = \tau(A_F) = M(F)$ so that M is an invariant mean for $\ell^\infty(G)$.

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