Correction to "Estimates and solvability"

Nils Dencker

The original article appeared in *Arkiv för matematik* 37 (1999), 221–243.

In the final typesetting, the end of the proof of Theorem 2.4 disappeared. Thus, the following lines should be inserted before Remark 6.3 on p. 242.

*End of the proof of Theorem 2.4.* Thus, by Lemma 6.2 we find that $(\text{Re } a)^w \geq c_0^w$ almost everywhere for some uniformly bounded and real $c_0 \in L^\infty(\mathbb{R}, S(h^2, g))$. Put $A = a_{0}^w = (a - c_0)^w$, this only adds $ic_0^w b^w \in \text{Op } S(h, g)$ to $R_0$, then we obtain that $\text{Re } A \geq 0$. When $\text{Re } a \geq c > 0$, we may replace $a$ with $a - c$ and do the same construction to obtain $\text{Re } A = \text{Re } (a - c_0)^w \geq c$. This gives the preparation (6.11) of $P$ in the case when $\text{Im } r + \frac{1}{2} \{\text{Im } a, b\} \in S(h, g)$.

In the general case, we conjugate with $E^w$, where

$$(6.21) 
E = \exp \left( - \int_0^t \text{Im } \{r + \frac{1}{2} \{a, b\} \} \, ds \right) \in S(1, g)$$

is real valued. We find $E^{-1} \in S(1, g)$ and $E^w (E^{-1})^w \cong (E^{-1})^w E^w \cong 1$ modulo an operator in $\text{Op } S(h^2, g)$ with $L^2$ operator norm bounded by $C_0 T$ when $|t| \leq T$. In fact, we find $(E - 1)/T \in S(1, g)$ uniformly when $|t| \leq T \leq C$. Thus, $E^w$ is invertible for small enough $T$, with an $L^2$ bounded inverse. We find that

$$(D_t + i f^w + r_0^w) E^w \cong E^w P$$

modulo $\text{Op } S(h, g)$, where

$$r_0 = \text{Re } r - \frac{1}{2} \{\text{Im } a, b\} - \{f, E\} E^{-1} \in S(1, g)$$

satisfies the condition that

$$\text{Im } (r_0 + \frac{1}{2} \{a, b\}) \equiv 0.$$
Since $E^w BE^w \cong (b E^2)^w$ modulo $L^2$ bounded operators, we obtain the result with $b_0 = b E^2 \in S(h^{-1}, g)$ by applying the estimate (6.17) on $E^w u$ for small enough $T$, with $P = D_t + i f^w + r_0^w$. This completes the proof of Theorem 2.4. $\Box$

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Nils Dencker
Department of Mathematics
Lund University
Box 118
SE-221 00 Lund
Sweden
email: dencker@maths.lth.se