

Correction to

The geometry and structure of isotropy irreducible homogeneous spaces

by

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Professors McKenzie Wang and Wolfgang Ziller pointed out to me that Theorem 4.1 omits the spaces $\mathbf{Sp}(n)/\mathbf{Sp}(1) \times \mathbf{SO}(n)$ and $\mathbf{SO}(4n)/\mathbf{Sp}(1) \times \mathbf{Sp}(n)$, which are isotropy irreducible for $n > 1$. The gap in the proof is in the argument of Case 2 on page 69, where it is assumed that the representation η_1 is nontrivial, which is the case only for $p_1 > 1$. Since $\mathbf{Sp}(2)/\mathbf{Sp}(1) \times \mathbf{SO}(2) = \mathbf{Sp}(2)/\mathbf{U}(2)$, which is hermitian symmetric, the correct statement is:

4.1. THEOREM. *The only simply connected nonsymmetric coset spaces G/K of compact connected Lie groups, where (a) G acts effectively, (b) G is a classical group, (c) $\text{rank}(G) > \text{rank}(K)$, (d) K is not simple, and (e) K acts \mathbf{R} -irreducibly on the tangent space, are the following:*

(1) $\mathbf{SU}(pq)/\mathbf{SU}(p) \times \mathbf{SU}(q)$, $p > 1$, $q > 1$, $pq > 4$. Here $G = \mathbf{SU}(pq)/\mathbf{Z}_m$ where $m = \text{lcm}(p, q)$, $K = \{\mathbf{SU}(p)/\mathbf{Z}_p\} \times \{\mathbf{SU}(q)/\mathbf{Z}_q\}$, and the isotropy representation is the tensor product of the adjoint representations of $\mathbf{SU}(p)$ and $\mathbf{SU}(q)$.

(2) $\mathbf{Sp}(n)/\mathbf{Sp}(1) \times \mathbf{SO}(n)$, $n > 2$. Here $G = \mathbf{Sp}(n)/\mathbf{Z}_2$. If n is even then $K = \{\mathbf{Sp}(1)/\mathbf{Z}_2\} \times \{\mathbf{SO}(n)/\mathbf{Z}_2\}$ and the isotropy representation is given by

$$\overset{2}{\circ} \otimes (\overset{2}{\circ} \otimes \overset{2}{\circ}) \quad \text{if } n = 4,$$

by

$$\overset{2}{\circ} \otimes \overset{2}{\circ} - \circ - \dots - \circ \begin{array}{l} \circ \\ \circ \end{array} \quad \text{if } n > 4.$$

If n is odd then $K = \{\mathbf{Sp}(1)/\mathbf{Z}_2\} \times \mathbf{SO}(n)$ and the isotropy representation is given by

$$\overset{2}{\circ} \otimes \overset{2}{\circ} - \circ - \dots - \circ = \bullet.$$

(3) $\mathbf{SO}(4n)/\mathbf{Sp}(1) \times \mathbf{Sp}(n)$, $n > 1$. Here $G = \mathbf{SO}(4n)/\mathbf{Z}_2$, $K = \{\mathbf{Sp}(1)/\mathbf{Z}_2\} \times \{\mathbf{Sp}(n)/\mathbf{Z}_2\}$, and the isotropy representation is given by

$$\overset{2}{\circ} \otimes \bullet = \overset{1}{\circ} \quad \text{if } n = 2,$$

by

$$\overset{2}{\circ} \otimes \bullet - \bullet - \dots - \bullet = \overset{1}{\circ} \quad \text{if } n > 2.$$

The table on pages 107–110 should, accordingly, be modified by insertion of two families of spaces, corresponding to items 2 and 3 of the corrected Theorem 4.1. The table information which is not contained in the statement of Theorem 4.1 above, is:

G/K	Z	$N_G(K)/ZK$	$K \subset G$	Conditions
$\mathbf{Sp}(n)/\mathbf{Sp}(1) \times \mathbf{SO}(n)$	{1}	{1}	$\overset{1}{\circ} \otimes \overset{1}{\circ} - \dots - \circ = \bullet$	n odd, $n > 2$
	{1}	\mathbf{S}_3	$\overset{1}{\circ} \otimes (\overset{1}{\circ} \otimes \overset{1}{\circ})$	$n = 4$
	{1}	\mathbf{Z}_2	$\overset{1}{\circ} \otimes \overset{1}{\circ} - \dots - \circ \begin{matrix} \circ \\ \circ \end{matrix}$	n even, $n > 4$
$\mathbf{SO}(4n)/\mathbf{Sp}(1) \times \mathbf{Sp}(n)$	{1}	{1}	$\overset{1}{\circ} \otimes \bullet - \dots - \bullet = \overset{1}{\circ}$	$n > 1$

There is no change in the rest of the paper. In particular, Sections 13 and 14 are unchanged because, in the additional cases noted here, the isotropy representation is absolutely irreducible so there is no invariant almost complex structure, and its $\mathbf{Sp}(1)$ factor acts by a 3-dimensional representation so there is no invariant quaternionic structure.

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