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Correction to "Uniformization of Kähler manifolds with vanishing Bochner tensor"

by

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In [2], we discussed the uniformization of Kähler manifolds with vanishing Bochner tensor, called Bochner–Kähler or Bochner-flat manifolds. The resulting uniformization theorem, Theorem A, was used to classify compact Bochner-flat Kähler manifolds. In Theorem A, we claimed that every Bochner-flat Kähler manifold is uniformized by one of four types of Hermitian symmetric space. Subsequent work by R. Bryant [1] has revealed that this statement is false and that there are many Bochner-flat Kähler manifolds which are not locally symmetric. In the compact case, however, the statement of our classification result turned out to be correct, and a proof was given in [1].

In order to explain the error and to indicate how it might be corrected, let us recall the argument. The main idea, due to Webster [3], is to observe that over any simply-connected domain U in a Kähler $2n$-manifold M, there is a CR structure on $p: U\times\mathbf{R}\to M$ whose contact form ω satisfies $d\omega=p^*\Omega$, where Ω is the Kähler form of M. The contact distribution $\ker\omega$ is transverse to the fibres of p and is equipped with the lift of the complex structure on TM. There is a natural fibrewise $\mathbf{R}$-action by CR automorphisms.

If M is Bochner-flat, it follows from [3] that this CR structure is spherical, and therefore there is a developing map $\mathrm{dev}: U\times\mathbf{R}\to S^{2n+1}$, together with an induced group homomorphism $\varrho: \mathbf{R}\to \mathrm{PU}(n+1,1)$ into the group of CR automorphisms of S^{2n+1}. This pair is uniquely determined up to CR automorphism. We let G denote the closure of $\varrho(\mathbf{R})$ and X the complement of its fixed-point set in S^{2n+1}: since the natural fibrewise $\mathbf{R}$-action is free and dev is an immersion, it follows that $\mathrm{dev}(U\times\mathbf{R})\subset X$.

If we have a good open cover U_α of M, we have developing pairs $(\mathrm{dev}_\alpha, \varrho_\alpha)$ related by CR automorphisms on pairwise intersections, and (assuming that M is connected), after conjugating by CR automorphisms, we can suppose that $\varrho := \varrho_\alpha$ coincide for all α, hence also G and X are independent of α.

A uniformization result would then follow by passing to the quotient of X by $\varrho(\mathbf{R})$. In [2, §3], we implicitly assumed that the quotient geometry was determined uniquely by X (also we omitted the case $X = S^{2n+1} - \{0, \infty\}$), and so we did not take full account of the possible conjugacy classes of $\varrho(\mathbf{R})$. Furthermore in the case that $\varrho(\mathbf{R})$ is not closed, the quotient of X by $\varrho(\mathbf{R})$ is not a manifold, so we need to work with local quotients. When M is compact, the non-closed case does not occur: establishing this would lead to an alternative proof of the classification theorem to the one provided by [1].

However, since the analysis of the non-closed case requires some additional work, we will present the details elsewhere.

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