

A novel approach on protein classification

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Abstract

Protein universe is a complex system with critical problem of protein evolution to be analyzed. Early studies have used geometric distances and polygenetic-trees to solve this problem. However, the traditional methods are bivariate, whose taxonomy classification relies on bivariate branching. This is not sufficient to describe the complex nature of protein universe. Therefore, we propose a novel approach on multivariate protein classification. The new method bases on the theory of information and network, can be used to analyze multivariate relationships of proteins. The new method is alignment-free and have wide-applications to both sequences and 3D structures. We demonstrate the new method on six protein examples, results show that the new method is efficient and can potentially be used for future protein classifications.

Keywords: Protein classification, information, network, sequence, evolution.

Introduction

The protein universe is diverse and has long been a mysterious entity and essential underpinning in biology [9, 21]. In protein universe, the protein sequences can be branched into families of fine hierarchy. Many researchers have spared their efforts to develop methods for future classification of unknown-lineage proteins [5, 6, 9, 12, 15, 19-21]. Early studies have used geometric methods in combination of protein biological nature to explore the universe of proteins. Prevalent idea is to use amino acid sequence homology to calculate their biological distances and draw polygenetic-trees according to the distances. They believe that sequence homology is highly related to protein relationships [6]. The polygenetic-trees show bivariate branching of protein lineages. Typical methods of this kind are the natural vector [21, 24, 25], protein map [19,20], K-string dictionary [22] and Yau-Hausdorff distance [15].

These early methods represent protein relationships in a bivariate manner. However, these methods may not be sufficient to describe the comprehensive nature of protein

34 universe, in that protein relationships may not be only bivariate. In other words, in a big
 35 family of species, a parent species may not necessarily be evolved into exactly two
 36 children species, and one species may not necessarily has only one sister or brother
 37 species. In fact, like all other natural systems [14, 17], one protein may have
 38 multivariate connections to more than one other proteins. To reveal a more natural
 39 picture of protein evolution, one needs to globally survey the multivariate relationships
 40 of proteins. In this paper, we use networks [11] to model the space of proteins, in which
 41 each protein is a node, we aim to use network tools to analyze the global relationships
 42 of the protein nodes [14, 16, 17]. The theory of information and network provides
 43 ready tools to analyze the model of protein universe, where we aim to use property of
 44 networks to draw new global picture of protein universe.

45 The paper is divided into five parts. This section is an introduction to the study. In the
 46 next section, we describe the materials and methods of the new method. The third
 47 section describes six examples to demonstrate the application and efficiency of the
 48 method, where we present pictures on protein taxonomy classifications. The fourth and
 49 fifth sections are the discussion and conclusion to this paper, where we discuss and
 50 conclude the efficiency and properties of the new method.

51

52 **Materials and methods**

53 We combine information and network theories to develop a new approach in identifying
 54 global protein relationships. Protein amino acid sequence can be viewed as discrete time
 55 series, where the amino acid order is time and the species of the 20 amino acid are states.
 56 To start with, we map the amino acid sequence to integer sequence with states from 1 to
 57 20. Since information theoretic measures are independent of the label of states [22],
 58 using different labels will not change the result. The discrete time series of integers are
 59 taken as inputs to the new global connectivity method.

60 **The maximum mutual information rates**

61 Before using the information theoretic measure, we first map the amino acid sequences
 62 into discrete time series. Each amino acid $a \in \{A, C, D, E, F, G, H, I, K, L, M, N, P, Q, R, S, T, V, W, Y\}$ is
 63 uniquely mapped to an integer $b \in \{1, 2, 3, 4, \dots, 20\}$. All protein sequences are transformed
 64 to sequences of discrete integers. For each pair of the integer sequences $x_1 x_2 \dots x_M$ and
 65 $y_1 y_2 \dots y_N$ (M and N denote the length of protein), without loss of generality, assuming $M \leq$
 66 N , we pick the length N segment for the longer sequence M , and calculate the mutual
 67 information [2, 4, 16, 26] between $x_1 x_2 \dots x_M$ and $y_i y_{i+1} \dots y_{i+M-1}$, for $1 \leq i \leq N - M + 1$,

$$68 \quad I(X;Y_i) = \sum_{x \in S_x, y \in S_y} p(x_n = x, y_{n+i-1} = y) \log \frac{p(x_n = x, y_{n+i-1} = y)}{p(x_n = x)p(y_{n+i-1} = y)} \frac{1}{2}, \quad (1)$$

69 where X is the shorter sequence and Y_i is the length M segment of the longer sequences,
 70 S_x and S_y denote the state sets of the sequence X and Y respectively, which are subsets of
 71 positive integers with elements from 1 to 20.

72 Mutual information rate describes the mutual relationship between two proteins.
 73 Shifting i from 1 to $N - M + 1$, we obtain a sequence of mutual information rates denoted
 74 as I_1, \dots, I_{N-M+1} . Here we extract the maximum mutual information rates between X and Y ,

$$75 \quad I_{\max,xy} = \sum_{1 \leq i \leq N-M+1} I(X;Y_i) \quad (2)$$

76 We set the maximum mutual information rate $I_{\max,XY}$ as the (X,Y) elements of the
 77 adjacency matrix. Note that the mutual information rates are symmetric such that the
 78 adjacency elements

$$79 \quad a_{XY} = a_{YX} = I_{\max,XY} = I_{\max,YX} \quad (3)$$

80 We use the symmetric maximum mutual information rates as elements of the adjacency
 81 matrix to construct simple undirected protein network [11].

82 Without loss of generality, assume there are K protein sequences in a set. The
 83 adjacency matrix is a $K \times K$ symmetric matrix. The maximum mutual information rate for
 84 sequences X and Y is reached for some $1 \leq k \leq N - M + 1$:

$$85 \quad I_{\max,XY} = I(X;Y_k). \quad (4)$$

86 Knowing that the mutual information rate is up bounded by entropy [2,4,26]:

$$87 \quad I(X;Y) \leq \min\{H(X), H(Y)\}, \quad (5)$$

88 where $H(X) = -\sum_{x \in S_x} p(x_n = x) \log p(x_n = x)$ and $H(Y) = -\sum_{y \in S_y} p(y_n = y) \log p(y_n = y)$, thus

$$89 \quad I_{\max,XY} = I(X;Y_k) \leq \min\{H(X), H(Y_k)\}. \quad (6)$$

90 Additionally, the entropy is non-decreasing as state number increases, we have

$$91 \quad H(Y_k) \leq H(Y), \quad (7)$$

92 because the number of states in Y is no less than the number of states in Y_k . Then, we
 93 have

$$94 \quad I_{\max,XY} \leq \min\{H(X), H(Y)\}. \quad (8)$$

95 To make fair threshold, we normalize the adjacency matrix in terms of the entropies.
 96 Denote the sequences as X_1, X_2, \dots, X_K , elements of the adjacency matrix now become:

$$a_{ij} = a_{X_i, X_j} = \frac{I_{max, X_i X_j}}{\max_k H(X_k)}, \quad (9)$$

97

98 all elements of the new adjacency matrix are bounded between 0 and 1.

99 **Connect component**

100 Connect component is a basic concept in network theory, which is a good method for
 101 clustering. To unveil the evolutionary relationship among proteins, we set up a threshold
 102 to filter the corrected adjacency matrix. Elements below the threshold are set to zero,
 103 with the rest elements are unchanged. The threshold is defined as constant multiple of
 104 the maximum adjacency element, more specific, denote

$$105 \quad T_c = c \cdot \max_{i,j} a_{ij} \quad (10)$$

106 as the threshold at multiplicity c , where the multiplicity c takes uniform distributed
 107 values from 0.1 to 1, with interval of 0.01. For each multiplicity c , we filter the corrected
 108 adjacency matrix A , the connect components of the protein network are the sets of
 109 proteins whose adjacency elements are all non-vanishing, the inclusion of any other new
 110 proteins in the set will break the law (i.e. introduce vanishing adjacency elements). In
 111 other words, connect component is a subset of all vertices in a network preserving the
 112 connection criterion. The criterion requires that each member of the subset has at least
 113 one path connecting to any other member of the subset, where the path is the joint of
 114 links that are connected end to end. The connect components are maximum, because no
 115 other vertex in the network can be added to the subset while preserving this property
 116 [11]. Connect components of undirected networks are called weakly connect
 117 components, to distinguish from the strongly connect components of directed networks.
 118 Nodes in one connect component are highly related to each other.

119 The members of the connect components are all mutually connected by at least one
 120 path in the network, no matter the length of the path. By nature of the undirected
 121 networks, in a network of n nodes, the lengths of such paths should be no longer than
 122 $n-1$. In matrix form, there exists a path from node j to node i of length m ($\leq n$) if and only
 123 if the (i,j) -th element in the power m adjacency matrix A^m is positive [11]. To identify the
 124 connect components, we need to find all such paths from length 1 to length $n-1$. In
 125 matrix notation, denote the sum of the 1 to $n-1$ power adjacency matrix as

126

$$A_{sum} = \sum_{i=1, \dots, n-1} A^i$$

127 via reversible matrix transformations, the matrix A_{sum} can be written in block diagonal
128 form as

$$A_{sum} = \begin{pmatrix} D_1 & 0 & \cdots \\ 0 & D_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

129

130 if the network has more than one component. This implies that the non-zero elements of
131 the sum matrix are confined to square blocks along the diagonal of the matrix, with all
132 other elements being zero.

133 Changing the multiplicity of the threshold, we can see the variations of the connect
134 components. As the multiplicity c varies from 1 to 0.1, the connect components of higher
135 threshold bond together to form larger components at lower threshold. The components
136 of higher T_c value indicate stronger mutual relations among the members of the
137 components, whereas the components of lower T_c value implies weaker mutual relations
138 among the member of the components.

139 By varying the threshold, we can draw a graph of sets with inclusion and exclusion of
140 the connect components (sets) at different thresholds. We take the advantages of these
141 set relations to inspect the evolutionary relations among proteins. For each multiplicity
142 threshold, the connect component is drawn as a set enclosing all its members (proteins)
143 and labeled with the multiplicity c of the threshold. The components of lower thresholds
144 may contain the components of higher thresholds, in that the thresholding condition is
145 looser when the threshold is lower. We put the sets of higher thresholds into the sets of
146 lower thresholds if the connect components of the latter contain the connect component
147 of the former. The graph of sets representing the connect components is a fine approach
148 to represent the sequence relations among proteins. The hierarchy of protein evolution
149 can be well delineated by the connectivity of the network.

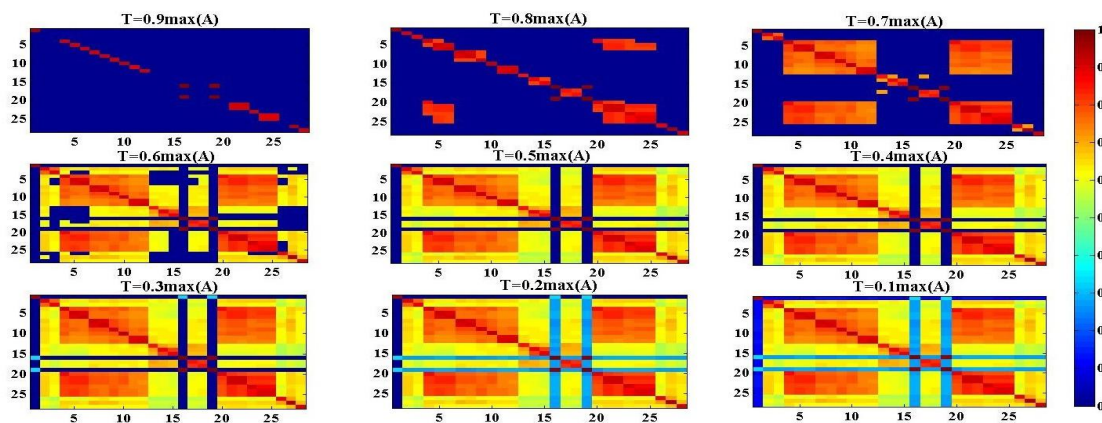
150 **Results**

151 We use six protein data sets to illustrate the method. In each protein data set, we draw a
152 graph of sets representing the connect components of the protein network at different
153 thresholds. For each connect component, all members in the component are mutually
154 connected at the given threshold. We take advantages of the changes of the connect
155 components on the varying thresholds to demonstrate evolutionary relationships of the
156 proteins. Note that the protein networks are undirected, given the symmetric
157 characterization of the maximum mutual information rates.

158 Mitochondrial proteins of 28 mammal species

159 In the first example, we analyze the data set of 28 mitochondrial proteins formerly used
 160 by [1, 7, 18, 20]. This dataset consists of 28 proteins encoded by the mitochondrial
 161 genome of 28 different mammal species. Each of the 28 protein sequences is
 162 concatenated from 10 proteins (COI, COIII, COII, Cyt-b, ND1, ATPase 6, ND4, ND5, ND6,
 163 ND2) encoded by the same strand of the mitochondrial genome [1, 7, 18, 20]. Among the
 164 13 protein-coding mitochondrial genes, the 3 shortest genes (ATPase 8, ND3, and ND4L)
 165 are excluded, and the 10 proteins (COI, COIII, COII, Cyt-b, ND1, ATPase 6, ND4, ND5, ND6,
 166 ND2) are coded by the 10 genes left. The 28 mammal species and their Genbank
 167 accession number are namely, the hedgehog (GenBank accession number X88898),
 168 mouse (J01420), rat (X14848), cat (U20753), gray seal (X72004), harbor seal (X63726),
 169 horse (X79547), donkey (X97337), rhinoceros (X97336), cow (V00654), fin whale
 170 (X61145), blue whale (X72204), gibbon (X99256), Sumatran orangutan (X97707),
 171 Bornean orangutan (D38115), gorilla (X93347), pygmy chimpanzee (D38116),
 172 chimpanzee (D38113), and human (X93334), tiger (EF551003), dog (U96639), wolf
 173 (EU442884), black bear (DQ402478), brown bear (AF303110), polar bear (AF303111),
 174 opossum (Z29573), wallaroo (Y10524), and platypus (X83427).

Color-mapped adjacency matrix of the 28 mammal species filtered by different thresholds



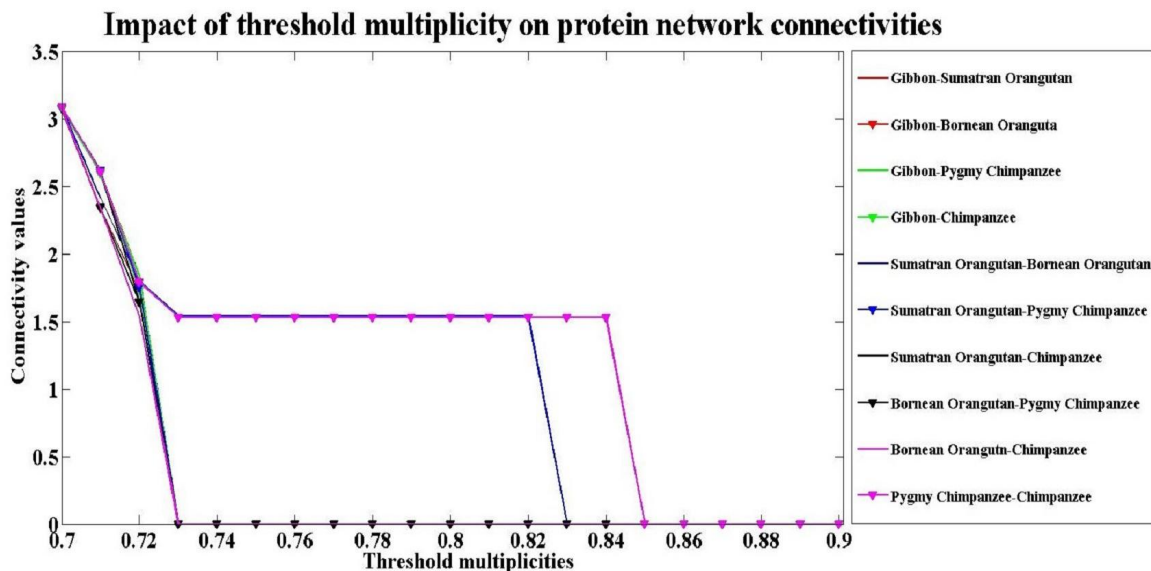
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176 **Fig 1. Color-mapped adjacency matrix of the 28 mammal species filtered by different**
 177 **thresholds. This figure shows the color-map of the filtered adjacency matrices for the 28**
 178 **mammal species. The multiplicity of the threshold ($T_c = c \cdot A_{max}$) is varied from $c = 0.9$ to $c = 0.1$.**
 179 **The elements of the adjacency matrix below the thresholds are filtered to zero, while the other**
 180 **elements remain unchanged. The adjacency elements are mapped to colors ranging from cold**
 181 **(dark blue, minimum) to warm (dark red, maximum) as shown in the color-bar.**

182 The color-map of the adjacency matrix filtered by the different thresholds are shown
183 in Fig 1. In this figure, the whole adjacency matrix after filtering is mapped to colors as
184 indicated by the color-bar. The color is ranged from cold (blue) to warm (red), indicating
185 the causality values from minimum to maximum. Two nodes are connected if the
186 corresponding adjacency element is positive, which is indicated by a bright color in the
187 color-map. We can see that higher threshold filters out more connections (less bright
188 areas in the color-maps), which leaves out fewer nodes to be connected. Decrease the
189 threshold, more nodes become connected. The detailed impact of the threshold variation
190 can be seen from the case study of 5 primate species (Gibbon, Sumatran Orangutan,
191 Bornean Orangutan, Pygmy Chimpanzee and Chimpanzee) as shown in Fig 2. In this
192 figure, the clustering of the primate species can be seen by the positiveness of the
193 connectivity values. The Pygmy chimpanzee and the Chimpanzee are first clustered at
194 multiplicity $c = 0.84$, then the Sumatran Orangutan and the Bornean Orangutan are
195 clustered at the multiplicity of $c = 0.82$, after which the cluster of the two chimpanzees
196 and the cluster of the two orangutans are grouped together along with the Gibbon at a
197 lower threshold with multiplicity $c = 0.72$. The connect component is enlarged as the
198 threshold multiplicity decreases.

199 The classification result is shown in Fig 3. In this figure, the proteins of the 28
200 mammal species are classified by the contour of connect components at different
201 threshold multiplicity. The higher the multiplicity implies stronger mutual relations
202 among the members of the components. Each member protein of the component is
203 represented by their animal species, e.g. the mitochondrial protein of the hedgehog is
204 represented by the name of hedgehog in the figure. This figure shows that the mammal
205 species of Carnivora are first classified according to their families: Phocidae (Gray seal
206 and Harbor seal, $c = 0.9$), Canidae (dog and wolf, $c = 0.9$), Ursidae (brown bear, polar bear,
207 and black bear, $c = 0.89$), Felidae (cat and tiger, $c = 0.88$). The Carnivora families
208 (Phocidae, Canidar, Ursidae) are grouped into a larger cluster at $c = 0.85$, which is later
209 joined by another Carnivora family i.e. the family of Felidae, along with the two families
210 (Equidae: horse and donkey, $c = 0.9$, and Rhinocerotidae: rhinoceros) of Perissodactyla
211 order and one species (cow) of Artiodactyla order, at the multiplicity of $c = 0.82$. The
212 Infra-class of Marsupialia (Opossum and Wallaroo, $c = 0.73$) and the order of Cetacea
213 (Fin whale and Blue whale, $c = 0.9$), another species of Artiodactyla order (Platypus), and
214 the order of Rodentia (Mouse and Rat, $c = 0.79$), are added into the original group level
215 by level. As to the primates, the Ponginae subfamily (Sumatran orangutan and Bornean
216 orangutan, $c = 0.82$), the Homininae subfamily (Pygmy chimpanzee and Chimpanzee, $c =$
217 0.84) of the Hominidae family are first grouped with the species (Gibbon) of the
218 Hylobatidae family at $c = 0.72$, then this group of primates first joins the big group of
219 mixed Carnivora, Perissodactyla, Artiodactyla, Marsupialia, Cetacea, and Rodents species.
220 The network of the proteins of 28 mammal species is entirely formed, once the mixed

221 large group is joined by another small mixed group of two primates: Hominoidea family
 222 (Gorilla and Human, $c = 1$) and one Eulipotyphla (Hedgehog) at $c = 0.3$.



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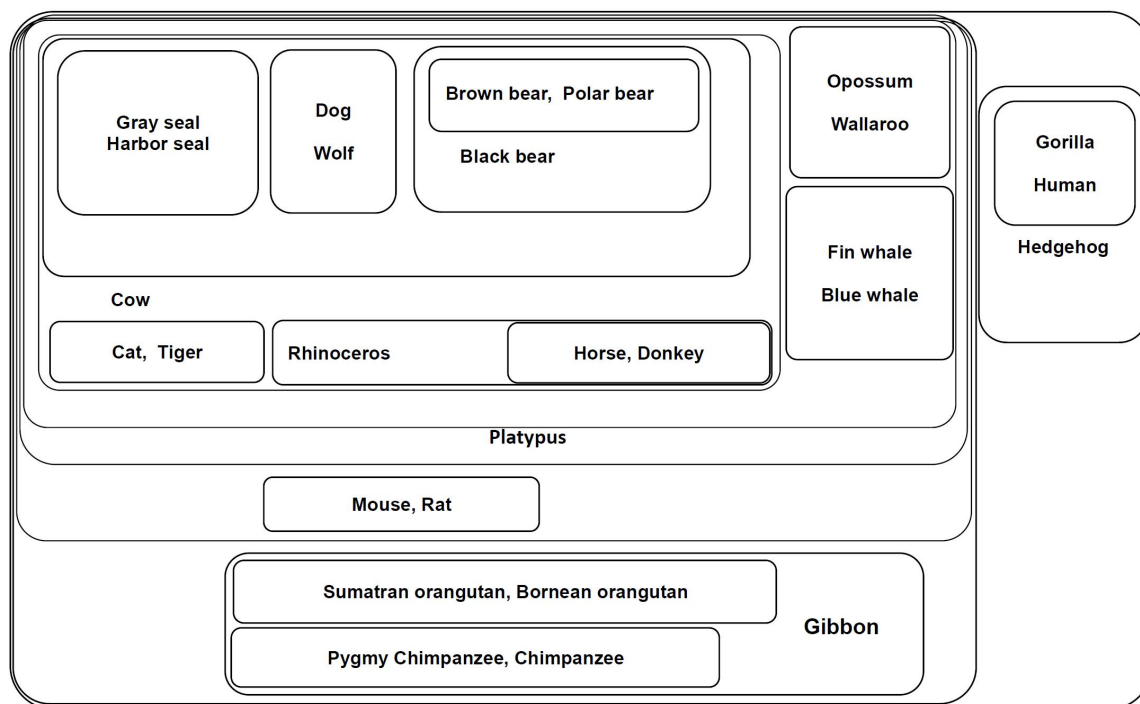
224 **Fig 2. Impact of the threshold multiplicity on protein network connectivity.** This figure shows
 225 the connectivity values varied against the threshold multiplicities ($c = 0.7, 0.71, \dots, 0.9$). The
 226 connectivity values are the mean value of the roots of the powered adjacency matrix

227 $\bar{a}(i, j) = \frac{\sum_{n=1,2,\dots,27} a_{n,ij}^{1/n}}{27}$, where the nominator is 27 because the entire network is consisted of 28

228 nodes so the maximum length of a path is 27, to get the connectivity values, we need to
 229 account all paths from length 1 to length 27, and get their square root averages from the 27
 230 powered adjacency matrix, $a_{n,ij}$ is the ij -th element of the power n adjacency matrix A^n . The
 231 positiveness of $\bar{a}(i, j)$ indicates the existence of a connection between node i and node j in the
 232 protein network. This graph shows the connectivity values among the mitochondrial proteins
 233 of Gibbon, Sumatran Orangutan, Bornean Orangutan, Pygmy Chimpanzee and Chimpanzee,
 234 over the threshold multiplicity between $c = 0.7$ and 0.9 .

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Fig 3. Component graph of mitochondrial proteins of 28 mammal species. This figure shows the graph of connect components of 28 mammal species at different thresholds. Each set represents a connect component whose members are mutually connected at a certain threshold ($T_c = c \cdot A_{max}$, $c \in [0,1]$). Components of higher thresholds are included by components of lower threshold.

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This result sheds light to the global relations among the 28 mammal species. In contrast to the bivariate branching of the polygenetic-trees [1, 7, 12, 18] and [20], the classification is more universal, indicating the parallel mutual connections among the different Carnivora families. This result brings a more natural explanation to the evolution compared to the conventional bivariate branching, because the relations among the different species may not necessarily be pairwise, i.e. it is insufficient to say that one species is close to only one other species in the universe.

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Mitochondrial proteins of 35 mammal species

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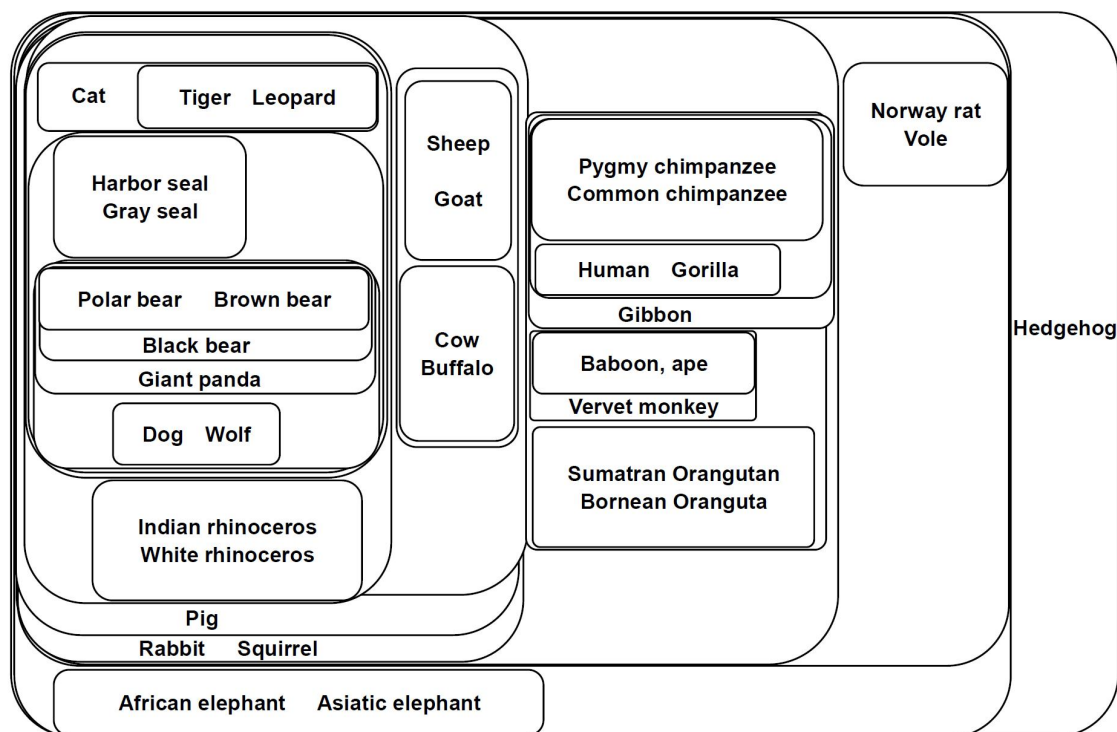
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The second data set consists of 35 proteins of NADH dehydrogenase encoded by the mitochondrial genes from 35 different mammal species [23]. GenBank accession numbers of the 35 mammal genes [23] are human (V00662), pygmy chimpanzee (D38116), common chimpanzee (D38113), gorilla (D38114), gibbon (X99256), baboon (Y18001), vervet monkey (AY863426), ape (NC 002764), Bornean orangutan (D38115), Sumatran orangutan (NC 002083), cat (U20753), dog (U96639), pig (AJ002189), sheep

257 (AF010406), goat (AF533441), cow (V00654), buffalo (AY488491), wolf (EU442884),
 258 tiger (EF551003), leopard (EF551002), Indian rhinoceros (X97336), white rhinoceros
 259 (Y07726), harbor seal (X63726), gray seal (X72004), African elephant (AJ224821),
 260 Asiatic elephant (DQ316068), black bear (DQ402478), brown bear (AF303110), polar
 261 bear (AF303111), giant panda (EF212882), rabbit (AJ001588), hedgehog (X88898),
 262 Norway rat (X14848), vole (AF348082), squirrel (AJ238588).

263 The evolutionary relationship of the species are shown in Fig 4. In this figure, the
 264 species of different families are clearly classified into separate groups according to their
 265 mammal orders (Carnivora, Artiodactyla, Perissodactyla, Lagomorpha, Rodentia,
 266 Proboscidea, Primate, and Eulipotyphla). It is identified in the figure that the Carnivora is
 267 the core of the networks, which are closely surrounded by the species of the
 268 Perissodactyla order and the Artiodactyla order, the components of the three orders are
 269 then closely connected to the species of Lagomorpha order and the Proboscidea order.
 270 The Primate species are in the next order level that are fully connected to the core, which
 271 is followed by the Rodentia order and the Eulipotyphla order. This hierarchical relations
 272 are similar to those found by moment vectors on mitochondrial genes [23], except for
 273 the difference that the cousins or non-brother peers in moment vector analysis now
 274 become mutually related at certain levels by the multivariate nature of the new method.



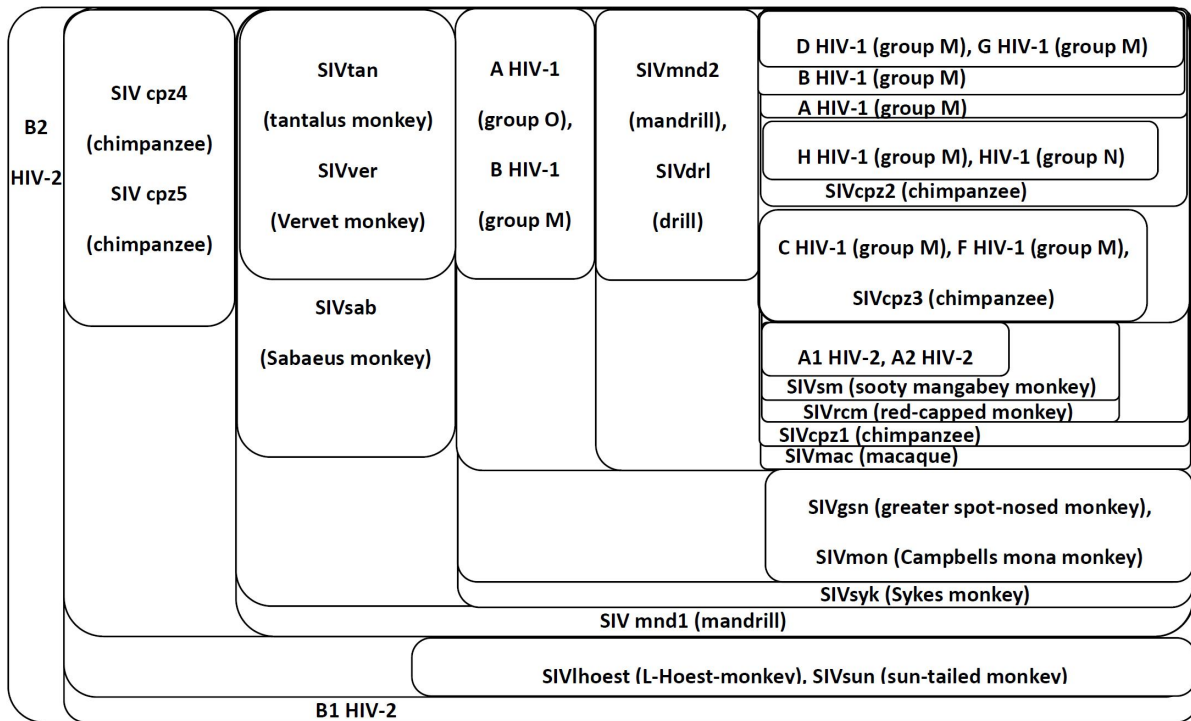
276 **Fig 4. Component graph of mitochondrial proteins of 35 mammal species. This figure**
277 **shows the graph of connect components of 35 mammal species at different thresholds. Each**
278 **set represents a connect component whose members are mutually connected at a certain**
279 **threshold ($T_c = c \cdot A_{max}$, $c \in [0,1]$). Components of higher thresholds are included by**
280 **components of lower threshold.**

281 Inside each different order, the mammal species are also well-classified. The
282 Carnivora is classified into families of Phocidae (Gray seal and Harbor seal, $c = 0.98$),
283 Ursidae (brown bear, polar bear, black bear and Giant panda $c = 0.94$), Canidae (dog and
284 wolf, $c = 1$), and Felidae (cat and tiger, leopard, $c = 0.94$). The only one family
285 Rhinocerotidae (Indian rhinoceros and White rhinoceros, $c = 0.93$) of Perissodactyla
286 order is inter-connected to the families of Carnivora. The Artiodactyla is divided into two
287 groups of the same family (Bovidae) but different subfamilies: Bovinae (Cow and Buffalo,
288 $c = 0.97$) and Caprinae (Sheep and Goat, $c = 0.96$), along with one species of the Suidae
289 family (Pig). Aside from the other two Rodentia families: Muridae (Norway rat) and
290 Cricetidae (Vole), the one species of Sciuridae family (squirrel) affiliated to the Rodentia
291 order is close to the Carnivora, Perissodactyla, Artiodactyla, Lagomorpha (Leporidae
292 family: rabbit), and Proboscidea (Elephantidae family: African elephant and Asiatic
293 elephant, $c = 0.98$). The Primate is also classified into different families: Hominidae
294 (Ponginae: Sumatran orangutan and Bornean orangutan, $c = 0.8$), and Homininae (Pygmy
295 chimpanzee, Common chimpanzee, Gorilla and Human, $c = 0.85$), Hylobatidae (Gibbon),
296 the Cercopithecidae (baboon, Vervet monkey) and Hominidae (ape). The species of the
297 Cercopithecidae and Hominidae families are interconnected. The hedgehog of the
298 Erinaceidae family of the Eulipotyphla order is the furthest species to the others in the
299 protein network.

320 Parakeet (P21668.1), Chicken (P02112.2), Zebra (P67824.1), Wolf (P60526.1), Cod
321 (O13077.2), Turtle (P13274.1), Langur (P02032.1).

322 The classification results using our new method are given in Fig 5. In this figure, we
323 can see that the fish (Actinopterygii), avian, mammal and reptile are reasonably
324 classified. The four big clusters are similar to those branches identified in the polygenetic
325 tree by K-string dictionary method [22], with a few exceptions in the species sub-classes.
326 The distinction between our results and the results of the other methods, is mainly due
327 to the differences of the method orientations, in which our method pay more attention to
328 the global connectivity rather than bivariate branching of the proteins.

329 In this result, the aves, reptiles, mammal, fish (Actinopterygii) are well-classified. All
330 Aves species are clustered together, with Anatidae family (Duck, Mallard, Swan, $c = 1$) of
331 the Anseriformes order as the core, all the rest aves species are enclosed around. The
332 species of mammal class are categorized into clusters of different animal orders (Primate,
333 Rodentia, Cetacea, Carnivora, Artiodactyla, Perissodactyla, Proboscidea, Chiroptera),
334 where Primate species (Cercopithecidae family: Grivet, Langur, the Hominidae family:
335 Human, Chimpanzee, Gorilla, and the Hylobatidae family: Gibbon) are the closest species
336 to the Carnivora species (Ursidae family: Lesser panda, Giant panda, Canidae family:
337 Coyote, Dog, Wolf, Red fox). The Bat of the Chiroptera Order is mixed with the Primates
338 and the Carnivoras. In another part of the mammals, the Artiodactyla order (Bovidae
339 family: Bison, Buffalo, Sheep, Hippopotamidae family: Hippopotamus), the Cetacea order
340 (Whale, Dolphin), one species of the Perissodactyla order (Hinocerotidae family:
341 Rhinoceros), and the Proboscidea order are closely connected. On a weaker threshold
342 level ($c = 0.58$), the main mammal orders: Primates, Carnivoras, Artiodactylas, Cetaceas,
343 and Proboscideas, as well as Perissodactyla order (Hinocerotidae family: Rhinoceros,
344 Equidae family: Horse, Zebra) are all mutually connected. At a lower threshold ($c = 0.5$),
345 the class of Aves, Mammals, Reptilia (Turtle, Tortoise), and a new joined species in the
346 Artiodactyla order (Suidae family: Pig), as well as two other mammal species in the
347 Rodentia order (Marmot, Rat), are all mutually connected. The fishes: Actinopterygii
348 class (Dragonfish, Cod, Goldfish, Salmon, and the Catfish) and the Chondrichthyes class
349 (Shark) are the last components joining the whole network, where the Chondrichthyes
350 class (Shark) is the farthest class to all other animal species.



351

352 **Fig 6. Component graph of proteins encoded by HIV virus. This figure shows the connect**
 353 **components of HIV virus proteins at different thresholds. Each set represents a connect**
 354 **component whose members are mutually connected at a certain threshold ($T_c = c \cdot A_{max}$,**
 355 **$c \in [0,1]$). Components of higher thresholds are included by components of lower threshold.**

356 HIV proteins

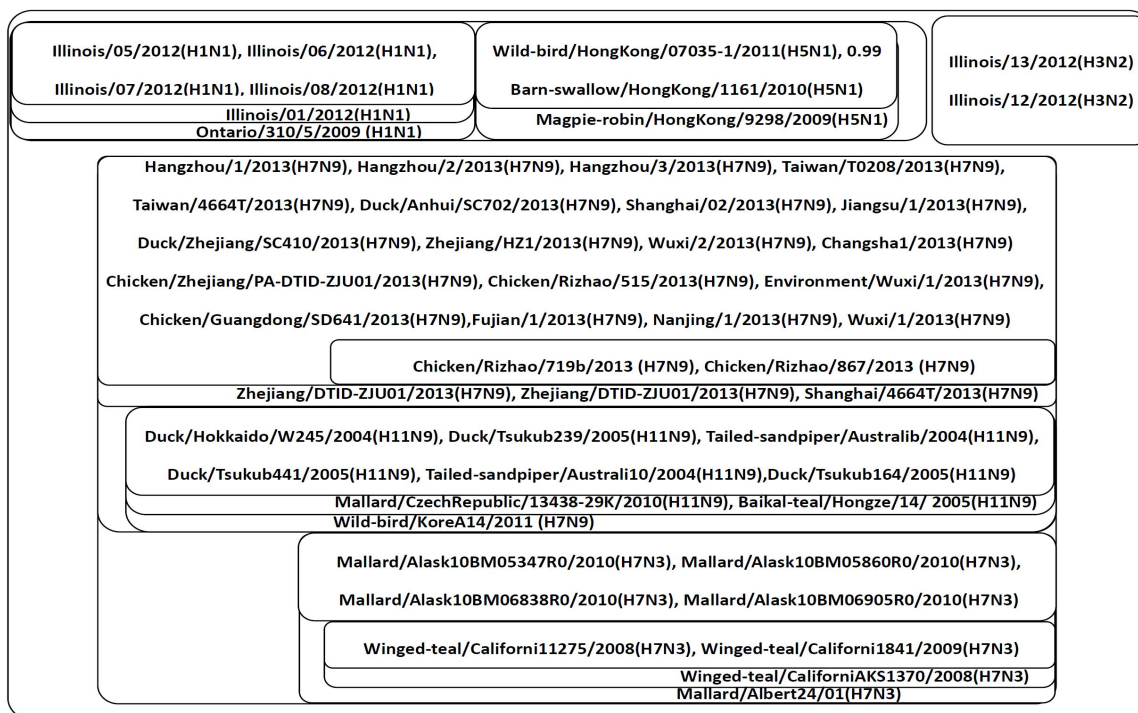
357 Human immunodeficiency virus (HIV) is a lenti virus that can lead to acquired immune
 358 deficiency syndrome (AIDS [13, 23]). To develop the anti-HIV drugs and vaccines, the
 359 research into the origins and evolution of this virus becomes very important. Rambaut et.
 360 al. [13] used maximum likelihood method to reconstruct the phylogenetic tree of the
 361 primate lenti viruses including HIV-1, HIV-2, and the simian immunodeficiency viruses
 362 (SIVs). It discovers that the two HIV viruses are related to different SIVs and therefore
 363 have different evolutionary origins. Here, we used the same dataset as they used to
 364 examine the global connections among proteins. The dataset consists of 33 protein
 365 sequences encoded by the DNA sequences of the 33 HIV and SIV viruses. The RNA
 366 genomes are transformed into DNA sequences (change U by T) before downloaded from
 367 the GenBank. The subtypes of HIV-1, HIV-2 and SIV viruses [13, 23] and their primate
 368 hosts and GenBank accession numbers are listed as follows: HIV-1, group M: A
 369 (AF004885); B (A04321); C (AF443079); D (K03454); F (AY173957); G (AY772535); H
 370 (AF190127); group N (DQ017382); group O: A (AY169802); B (AY169803); HIV-2: A1

371 (AF082339); A2 (M30502); B1 (L07625); B2 (X61240); SIV chimpanzee (Pan
 372 troglodytes troglodytes): SIVcpz1 (AY169968), SIVcpz2 (AJ271369), SIVcpz3
 373 (DQ373063); SIV chimpanzee (Pan troglodytes schweinfurthii): SIVcpz4 (DQ374657),
 374 SIVcpz5 (DQ374658); SIVdrl, drill (AY159321); SIVgsn, greater spot-nosed monkey
 375 (AF468659); SIVlhoest, L' Hoest monkey (AF188114); SIVmac, macaque (D01065);
 376 SIVmnd1, mandrill (M27470), SIVmnd2, mandrill (AY159322); SIVmon, Campbells mona
 377 monkey (AY340701); SIVrcm, red-capped monkey (AF382829); SIVsab, Sabaeus monkey
 378 (U04005); SIVsm, sooty mangabey monkey (U72748); SIVsun, sun-tailed monkey
 379 (AF131870); SIVsyk, Sykes' monkey (L06042); SIVtan, tantalus monkey (U58991);
 380 SIVver, vervet monkey (M29975).

381 The evolutionary relationship interpreted by our method is shown in Fig 6. The
 382 classifications of our analysis are quite different from those found by moment vectors
 383 [23], but are similar to those found by the maximum likelihood method [13]. In our
 384 analysis, the different types of the HIV-1 and HIV-2 proteins have different lineages to
 385 the SIV of the primates. From the global connections, the group M proteins of HIV-1 virus
 386 are closest to the SIV proteins of chimpanzee (Pan troglodytes troglodytes). The HIV-2 A
 387 and HIV-2 B are separate. The HIV-2 A proteins are most closely related to the SIVsm
 388 (sooty mangabey monkey), SIVrcm (red-capped monkey), SIV chimpanzee (Pan
 389 troglodytes troglodytes) and the SIVmac (macaque), whereas HIV-2 B is closer to SIV
 390 chimpanzee (Pan troglodytes schweinfurthii), and SIVlhoest (L' Hoest monkey) and
 391 SIVsun (sun-tailed monkey). Separate clusters of HIV-2 A and of the group M of HIV-1,
 392 each along with some SIVs, are first enclosed into a larger connect component, then the
 393 other SIVs are connected to the joined group of the HIV-1 group M and HIV-2 A, when the
 394 threshold multiplicity decreases. The proteins of HIV-2 B is farthest to the proteins of
 395 HIV-1 and HIV-2 A, where HIV-2 B joins the HIV-1 and HIV-2 A at the lowest threshold.

396 **Influenza A virus**

397 Influenza A virus is a kind of negative-sense, single-stranded, segmented RNA viruses.
 398 Here, we use the dataset of 52 proteins encoded by the genes of 52 different influenza A
 399 virus [15]. These proteins are characterized by three factors: the virus subtypes, the
 400 geographical location of the occurrence and the host of the influenza A virus. The virus
 401 subtypes are labeled by the combination of an H number for the type of hemagglutinin
 402 and an N number for the type of neuraminidase. Our dataset is made up of six virus
 403 subtypes: H7N3, H11N9, H1N1, H7N9, H3N2, H5N1 [15].

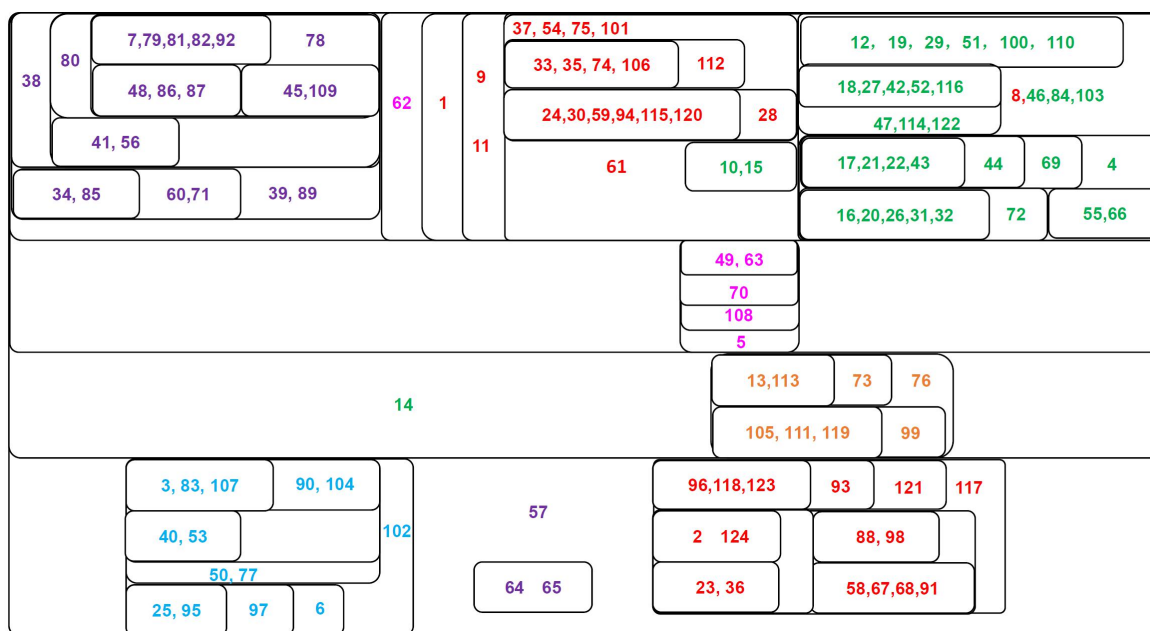


404

405 **Fig 7. Component graph of proteins encoded by Influenza A virus. This figure shows the**
 406 **connect components at different thresholds for proteins encoded by Influenza A virus genes.**
 407 **Each set represents a connect component whose members are mutually connected at a certain**
 408 **threshold ($T_c = c \cdot A_{max}$, $c \in [0,1]$). Components of higher thresholds are included by**
 409 **components of lower threshold.**

410 The classification of influenza A virus is shown in Fig 7. In this figure, the influenza A
 411 virus subtypes are well-classified, and within each subtype, the proteins are classified
 412 in terms of their host and geographic locations. For instance, the proteins of the H7N3
 413 virus with the host of Mallard in Alaska are grouped together, and are separated to the
 414 group of H7N3 proteins with the host of winged-teal in California.

415 Within each subtype of the influenza A viruses, the proteins are grouped first
 416 according to the N number for the type of neuraminidase, and then the H number of the
 417 type of hemagglutinin. The proteins of H1N1 virus are closely grouped with the proteins
 418 of H5N1 virus, while the proteins of H7N9 virus are closely classified with the proteins of
 419 H11N9 virus. The evolutionary hierarchy of Influenza A virus is clearly shown in this
 420 classification. The connection core is formed by the proteins of H7N9 and H11N9, which
 421 is joined by the proteins of H7N3. The enlarged core is finally joined by the union of
 422 H3N2 proteins and the union group of H1N1 and H5N1.



423

424 **Fig 8. Component graph of protein Kinase C families.** This figure shows the connect
 425 **components of the PKC (protein kinase C) at different thresholds.** Each set represents a
 426 **connect component whose members are mutually connected at certain threshold** ($T_c = c \cdot$
 427 A_{max} , $c \in [0,1]$). **Components of higher thresholds are included by components of lower**
 428 **threshold.** Each member of the 124 proteins is correspond to a unique index number between
 429 **1 and 124 as referenced in [21].** For presentation convenience, we only labeled the 124 unique
 430 **index number to represent the 124 proteins.** Proteins of different PKC subfamilies are labeled
 431 **by different colors: aPKC (blue), cPKC (green), nPKC (red), PKC1 (purple), PRK (pink), PKCmu**
 432 **(orange).** Description of the 124 PKCs can be found in supplementary materials of [21].

433 Protein kinase C

434 In the sixth example, we analyzed the protein kinase C families. Protein kinase C, in
 435 abbreviation the PKC, is a family of enzymes involved in controlling the function of other
 436 proteins through the phosphorylation of hydroxyl groups of serine and threonine amino
 437 acid residues on these proteins [21]. The entire PKC family can be divided into six
 438 subfamilies: cPKC, nPKC, aPKC, PKC μ , PKC1 and PRK. There are 124 protein sequences in
 439 total. The classification results of the PKC families are shown in Fig 8. In this figure, the
 440 six subfamilies of PKC are clearly clustered into separate groups: PKC1 (upper left block,
 441 purple), nPKC (Upper middle and the bottum right blocks, red), cPKC (upper right block,
 442 green), PRK (the center block, pink), PKCmu (the below center block, orange), aPKC
 443 (bottum left block, blue). All elements in each block are from the same PKC subfamilies.

444 Inside each block, the protein members are classified according to their NCBI
445 descriptions. The nPKCs are divided into η (33, 35, 74, 106, 112), (24, 30, 59, 94, 115,
446 120, 28), δ (1, 2, 124, 23, 36, 88, 98, 58, 67, 68, 91), Serine\Threonine (9, 11), θ (96, 118,
447 123, 93, 121). The cPKCs are divided into subgroups of the γ (18, 27, 42, 52, 116, 47, 114,
448 122), α (16, 20, 26, 31, 32, 72), β (17, 21, 22, 43, 44, 69). The aPKCs are classified into ι (3,
449 83, 107, 90, 104, 40, 53) and ζ (25, 95, 97, 6). The PKCmu proteins are divided into
450 nuPKCmu (13, 113, 73) and muPKCmu (105, 111, 119, 99). The classification results are
451 similar to those found by natural vectors [21].

452 Clear hierarchy of the PKC families can be seen from this figure. The PKC1 (purple),
453 cPKC (green) and the η , ϵ and Serine\Threonine sub-classes of the nPKC (red) are all
454 parallel connected. The PRK (pink) is connected to the above at a lower threshold, which
455 is followed by PKCmu (orange) and aPKC (blue). The δ and θ sub-classes of nPKC (red) is
456 the farthest group to the core.

457 Discussion

458 Protein universe is a complex system can be modeled as an undirected network with
459 evolutionary relations as interactions. In this paper, we described a global connectivity
460 method to identify multivariate evolutionary relationship among proteins. This method
461 bases on information and network theories is powerful. It takes advantages of the
462 distribution of amino acids and use maximum mutual information rates to detect
463 alignment-free mutual relationships among proteins. In analysis, protein universe is
464 modeled as a protein network, where protein sequences as nodes and their relations as
465 links, the evolutionary relationships of the proteins are identified by connect
466 components of the network.

467 The key point and innovation of our method is that it considers the protein
468 evolutionary relations as multivariate. Each taxon may have more than one sisters or
469 brothers, i.e. their parents may have one, two or more than two children. Traditional
470 protein classification methods inspect the protein relations pairwise, which limited the
471 protein classification in a bivariate view. In contrast, our method examine the global
472 relationships of proteins, the lineage of one species may be inherited by more than one
473 sub-lineages. Our method explain reasonable multivariate evolutionary relationships
474 among proteins of existing datasets, even though the true evolutionary hierarchy of some
475 of the species are still controversial indeed. Compared to earlier polygenetic-tree
476 representations, this method introduces brand-new ideas in protein evolutionary
477 classification.

478 The classification process relies on the division of connect components and the
479 variation of adjacency thresholds. The threshold of adjacency matrix acts as the cut-off to
480 protein network connections. It cuts weak connections below the threshold, while
481 keeping strong connections equal or above the threshold. By varying the threshold, one
482 is able to classify proteins by examining the inclusion\exclusion of connect components.

483 Results of the mitochondrial proteins show that the animal species are classified first
484 by their biological families, then their orders, and classes. The connection strength
485 decreases as their biological similarity decreases. Animal species of the same family are
486 strongly related, the relations or connections are weakened when their families in the
487 same order differ, and are again weakened if their biological orders differ. Sometimes,
488 the species in the same class, are cross related with different orders or families, which
489 may be because their species lie on the same level of evolution. For instance, the species
490 of Artiodactyla and Perissodactyla are not only intra-connected, but also inter-connected,
491 and they are more closely related to Carnivora rather than Primates. Primates are
492 comparatively far away than the other mammals, reflected by their weaker connections
493 to the rest mammal orders. Rodentia is also a bit far away to other mammals, whose
494 inter-connections to the other mammal species are weaker. Hedgehog is found close to
495 the Hominoidea family (Gorilla and Human) of Primate in the 28 mammal analysis,
496 which shows a consistency to [8] for the convergence of Hedgehog to Primates. Analysis
497 of beta-globins shows that the mammals are firstly connected to Aves, then Reptilia, and
498 finally the fishes (Actinopterygii class and the Chondrichthyes class), the fish is the
499 farthest class to the mammals particularly the Primates.

500 We also found that the HIV-1 and HIV-2 proteins have close-connections to different
501 SIVs. The HIV-1 group M is comparatively closer to HIV-2 A, and far away to HIV-2 B,
502 where the HIV-1 group M is closest to the SIV chimpanzee (*Pan troglodytes troglodytes*),
503 while HIV-2 A is also closer to the SIV chimpanzee (*Pan troglodytes troglodytes*) along
504 with some other SIVs. The HIV-2 B is farthest to HIV-1 and HIV-2 A, but it is closer to the
505 SIV chimpanzee (*Pan troglodytes schweinfurthii*) and some other SIVs. The results of
506 Influenza A virus indicate that variations of the Influenza A virus are first gathered
507 according to their neuraminidase types i.e. the N number, and then their hemagglutinin
508 types, i.e. the H numbers. The classification results of Influenza A virus are much better
509 by using our new method than by using Yau-Hausdorff distance [15]. For the same
510 dataset, Yau-Hausdorff distance doesn't give a clear classification for the Influenza A
511 virus. Our results of the PKC families are similar to those found by natural vectors [21],
512 but we demonstrate more on the universal relationships of the six PKC subfamilies.

513 By taking advantages of connect components, our global connectivity method
514 provides a universal view on the multivariate-connections among proteins. The new

515 method is alignment-free, because mutual information rate only depends on the
516 probability distribution of amino acids. The new method can also be used on protein 3D
517 structure, which is done simply by replacing the discrete map of amino acid to real
518 valued coordinates. However, we do not analyze protein 3D structures, because the
519 datasets are only sequences. If we have 3D structure of proteins, the classification results
520 may be improved. Our new method has advantage over other methods, because
521 traditional methods such as K-string dictionary [22], protein map [20] and the natural
522 vectors [21, 24, 25] can apply only to sequences or only to 3D structures (Yau-Hausdorff
523 distance [15]), none of them can apply to both.

524 **Conclusion**

525 We have described a new method on protein classification. This new method innovates
526 multivariate evolutionary relationships among proteins. In contrast to conventional
527 methods, our new method is able to infer multivariate relationships among proteins, and
528 is alignment-free that purely depend on probability distribution of amino acids. The new
529 method can have wide-applications that it can be used to analyze both amino acid
530 sequences and their 3D structures. This is an advantage of our method over traditional
531 approaches, where old methods such as K-string dictionary, protein map, natural vector
532 can only analyze on sequence rather than structure, and Yau-Hausdorff distance can only
533 analyze 3D structures rather than sequences. The new method can help improve our
534 understanding on complexity of protein universe from global connectivity perspective
535 and is an efficient tool for future protein classification analysis.

536 **Conflict of interest**

537 The authors declare no conflict of interest of any nature or kind in any product, service
538 and/or company that could be construed as influencing the position presented in, or the
539 review of, the manuscript entitled.

540 **Ethics Statement**

541 N/A.

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