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Total reflection and cloaking by zero index metamaterials loaded with rectangular dielectric defects

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In this work, we investigate wave transmission property through a zero index metamaterial (ZIM) waveguide embedded with rectangular dielectric defects. We show that total reflection and total transmission (cloaking) can be achieved by adjusting the geometric sizes and/or permittivities of the defects. Our work provides another possibility of manipulating wave propagation through ZIM in addition to the widely studied dielectric defects with cylindrical geometries. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4804201]

Since the construction of the first metamaterial with negative refractive index in 2000,[1] the study of metamaterials has been a very hot research topic across different disciplinaries due to many potentially revolutionary applications in areas such as perfect lens, invisibility cloaks, and subwavelength imaging devices. Details can be found in some recent monographs (e.g., Refs. 2 and 3), and references therein. The most studied metamaterials are the so-called double negative index media, whose permittivity ϵ and permeability μ are negative over some common frequencies. Recently, scientists found that zero index metamaterials (ZIMs), whose permittivity and permeability are simultaneously or individually near zero, also have many exciting anomalous properties[4] as double negative index metamaterials. For example, Ziolkowski[5] showed that a matched ZIM slab can be used to transform curved wave fronts into planar ones. Zhu *et al.*[6] demonstrated that the radiation from a point source embedded in a system of ZIM with gain inserts can be effectively amplified. Several scientists have also shown that ZIMs can be used to control electromagnetic (EM) wave transmission. It is shown that EM waves can be perfectly bended and transmitted by using ZIMs,[7,8] the radiation of a line current surrounded by a ZIM shell can be greatly enhanced or completely suppressed,[9] and ZIMs can be used to squeeze electromagnetic energy.[10–12] Total transmission (which results in cloaking[13,14]) and total reflection (which results in wave blocking) can also be realized with ZIMs embedded with proper defects.[15–19] For ZIMs embedded with dielectric defects, so far only cylindrical geometry has been utilized to achieve such intriguing transmission properties.[17–19]

In this paper, we investigate how perfect transmission or reflection of EM waves can occur by adjusting the size or the material parameters of the embedded rectangular dielectric object in a ZIM. Theoretical analysis is provided to guide the design of dielectric defects, and numerical simulations are then carried out to justify our theory. Our finding opens the possibilities of embedding rectangular defects in ZIMs to block wave or conceal objects.

Consider an EM wave incident from the left into a waveguide structure illustrated in Fig. 1. The ZIM region is a metamaterial, whose relative permittivity and permeability are described by the so-called Drude model[3,15]

$$\epsilon_1 = \mu_1 = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)},$$

where ω is the excitation angular frequency, ω_p denotes the plasma frequency, and Γ denotes the loss parameter. On both sides of the ZIM region are the free space regions 0 and 3. The defects in region 2, which is embedded in ZIM, are composed of N rectangles with permittivity and permeability of $\epsilon_{2,k}$ and $\mu_{2,k}, k = 1, 2, \cdots, N$, respectively. Assuming $\exp(-i\omega t)$ time harmonic factor, the EM wave in each region satisfies the Maxwell's equations: For any $m = 0,1,2,3$,

$$\boldsymbol{H}_m = \frac{1}{i\omega\mu_0\mu_m}\nabla \times \boldsymbol{E}_m, \quad \boldsymbol{E}_m = \frac{i}{\omega\epsilon_0\epsilon_m}\nabla \times \boldsymbol{H}_m. \quad (1)$$

We can simplify Eq. (1) to the vector Helmholtz equation

$$(\partial_{xx} + \partial_{yy} + k_0^2\epsilon_m\mu_m)\boldsymbol{U}_m = 0, \quad (2)$$

where $\boldsymbol{U}_m = \boldsymbol{E}_m$ or $\boldsymbol{H}_m$, and $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the wave vector in free space.

Assume that the walls of the waveguide are made of perfect electric conductor, and the waveguide supports the fundamental transverse magnetic mode, i.e., $\boldsymbol{H}$ field is polarized in the z direction. Suppose the incident wave propagates to the right along the x direction, we can write the $\boldsymbol{H}$ field in region 0 as a sum of the incident and reflected waves which satisfy Eq. (2)

$$\boldsymbol{H}_0 = \hat{z}H_0[e^{ik_0(x+d)} + \mathcal{R}e^{-ik_0(x+d)}], \quad (3)$$

where H_0 denotes the amplitude of the incident field, and $\mathcal{R}$ is the reflection coefficient.

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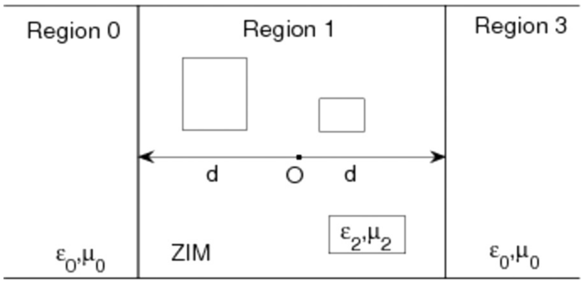

FIG. 1. The schematic description of the waveguide structure with vacuum (regions 0 and 3), ZIM (region 1), and an arbitrary number of rectangular defects.

By Eq. (1), we obtain the corresponding electric field in region 0 as

$$\boldsymbol{E}_0 = \hat{\boldsymbol{y}}\eta_0 H_0[e^{ik_0(x+d)} - \mathcal{R}e^{-ik_0(x+d)}], \tag{4}$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is free space impedance.

Similarly, we can obtain the $\boldsymbol{H}$ field and in $\boldsymbol{E}$ field region 3 as

$$\boldsymbol{H}_3 = \hat{z}H_0\mathcal{T}e^{ik_0(x-d)}, \quad \boldsymbol{E}_3 = \hat{\boldsymbol{y}}\eta_0 H_0\mathcal{T}e^{ik_0(x-d)}, \tag{5}$$

where $\mathcal{T}$ is the transmission coefficient.

In region 1 (the ZIM region), when the frequency $\omega \approx \omega_p$, $Re(\epsilon_1)$ is almost zero (assume very small loss $\Gamma \approx 0$). Hence to guarantee a finite $\boldsymbol{E}_1$, by Eq. (1), $\nabla \times \boldsymbol{H}_1$ must be zero, which leads to $\nabla H_1^z = 0$, i.e., the magnetic field in ZIM region is a constant denoted as H_1. Then by using Eqs. (3) and (5), and the tangential continuity of the magnetic field at interfaces $x = -d$ and $x = d$, we have

$$(\mathcal{R}+1)H_0 = H_1, \quad \mathcal{T}H_0 = H_1, \tag{6}$$

which leads to $\mathcal{R} + 1 = \mathcal{T}$.

Applying Faraday-Maxwell law and Stokes' theorem, we can show that the transmission coefficient is given by[18]

$$\mathcal{T} = \frac{1}{1 - (1/2wH_1)\sum_{k=1}^{N}\oint_{\partial S_{2,k}} \boldsymbol{E}_{2,k}\cdot dl_k}, \tag{7}$$

where w is the width of the waveguide, and $\partial S_{2,k}$ denotes the boundary (oriented clockwise) of each defect.

To evaluate $\mathcal{T}$, we need to find the electric field $\boldsymbol{E}_{2,k}$, which can be obtained from Eq. (1) once we know the magnetic field $\boldsymbol{H}_{2,k}$ in each defect region. Note that $\boldsymbol{H}_{2,k}$ satisfies Eq. (2) with Dirichlet boundary condition $\boldsymbol{H}_{2,k}\cdot\hat{\boldsymbol{t}} = \boldsymbol{H}_1$, where $\hat{\boldsymbol{t}}$ is the unit tangential vector along the boundary of each defect region.

Using the method of separation of variables, we can obtain the solution $\boldsymbol{H}_{2,k}$ on a rectangular defect region $[0,a]\times[0,b]$ with permittivity ϵ_2 and permeability μ_2

$$\boldsymbol{H}_{2,k}(x,y) = \hat{z}\left[H_1 + \sum_{n,m\geq 1} C_{n,m}\phi_{n,m}(x,y)\right], \tag{8}$$

where the coefficient

$$C_{n,m} = \frac{-4k_2^2 H_1}{nm(k_2^2 - \lambda_{n,m})\pi^2}[1-(-1)^n][1-(-1)^m]. \tag{9}$$

Here, we denote $k_2^2 = \omega_0^2\epsilon_2\mu_2$, $\lambda_{n,m} = (\frac{n\pi}{a})^2 + (\frac{m\pi}{b})^2$, and $\phi_{n,m}(x,y) = \sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$.

From Eqs. (1) and (8), we have

$$\boldsymbol{E}_{2,k}(x,y) = \frac{i}{\omega\epsilon_0\epsilon_2}\left[\hat{\boldsymbol{x}}\sum_{n,m\geq 1} C_{n,m}\frac{\partial\phi_{n,m}}{\partial y} - \hat{\boldsymbol{y}}\sum_{n,m\geq 1} C_{n,m}\frac{\partial\phi_{n,m}}{\partial x}\right]. \tag{10}$$

From Eqs. (7) and (10), we see that if any $C_{n,m} \approx \infty$, then $\mathcal{T} \approx 0$, in which case total reflection happens. To have this, one possible solution is that both n and m are odd integers, and $k_2^2 = \lambda_{n,m}$, which leads to

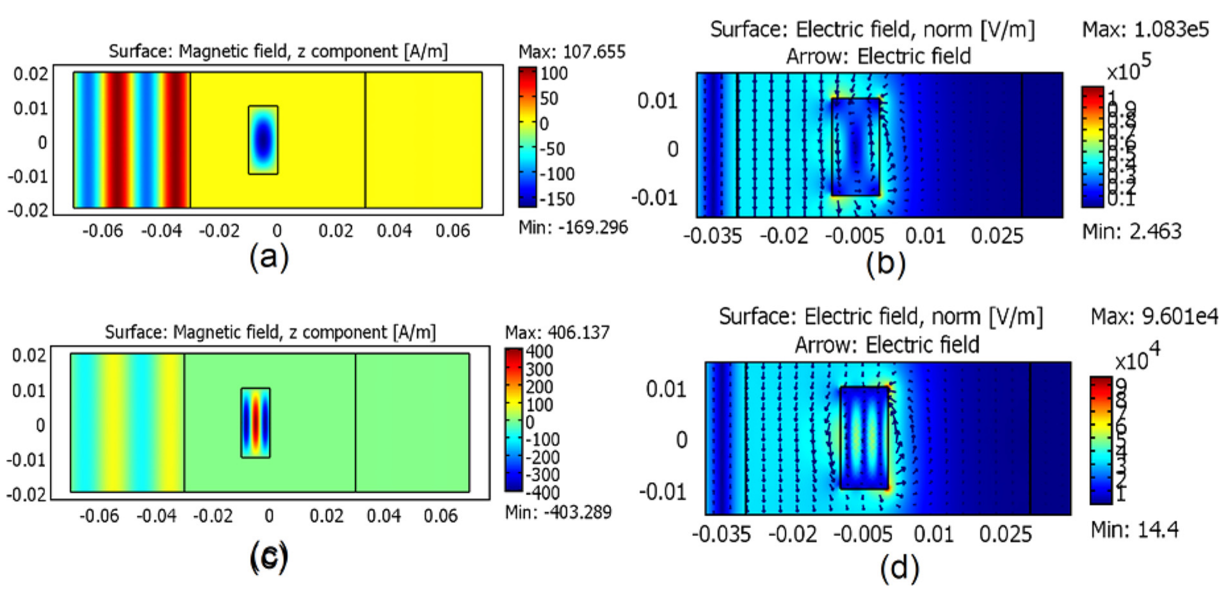

FIG. 2. One rectangular defect case with $m=1$, $n=1$ (top row), and $m=1$, $n=3$ (bottom row): (left column) whole H_z field; (right column) zoomed-in electric fields.

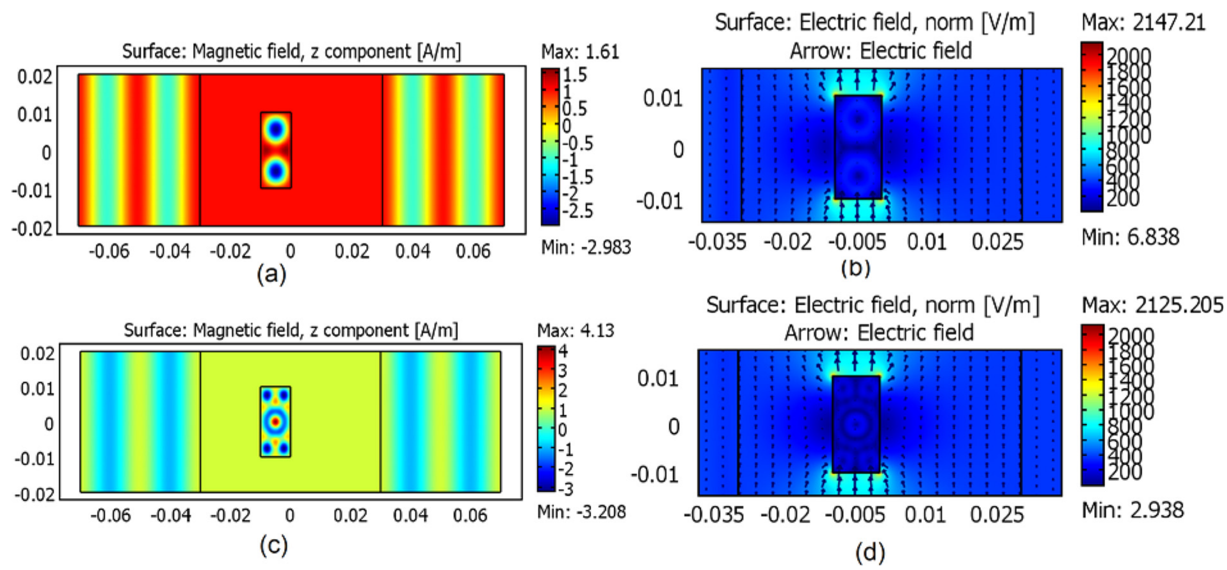

FIG. 3. One rectangular defect case with $m=1$, $n=2$ (top row) and $m=1$, $n=4$ (bottom row): (left column) whole H_z field; (right column) zoomed-in electric fields.

$$\epsilon_2 = \left[\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right] \Big/ (k_0^2\mu_2). \tag{11}$$

To justify the analysis, in the following, we carry out extensive numerical simulations using the commercial finite element package COMSOL. We assume that the computational domain is $[-0.07, 0.07]m \times [-0.02, 0.02]m$, the ZIM region is $[-0.03, 0.03]m \times [-0.02, 0.02]m$, the excitation plane wave has a frequency of $f = 15$ GHz and all defects are non-magnetic (i.e., $\mu_{2,k} = 1$). All our computations are based on triangular elements with the quadratic basis function.

First, we consider the simple case with one rectangular defect region $[-0.01, 0]m \times [-0.01, 0.01]m$, i.e., we have $a = 0.01$, $b = 0.02$. Substituting this into Eq. (11) with $k_0 = \omega\sqrt{\epsilon_0\mu_0} = 2\pi f/c = 100\pi$, we have

$$\epsilon_2 = n^2 + \left(\frac{m}{2}\right)^2. \tag{12}$$

Substituting $m=1$, $n=1$ and $m=1$, $n=3$ into Eq. (12) yields $\epsilon_2 = 1.25$ and $\epsilon_2 = 9.25$, in both cases, we observed total reflection as expected. The contour plot of the whole H_z field and the zoomed-in electric fields (obtained with 3680 and 14 720 elements for $n=1$ and $n=3$, respectively) are presented in Fig. 2. We would like to remark that in the total reflection case, the $\boldsymbol{H}_2$ field is mainly dominated by the mode $\phi_{n,m}(x,y)$, which corresponds to the infinitely large $C_{n,m}$. This fact is shown clearly in Fig. 2.

From Eq. (10) and some lengthy calculation, we obtain the line integrals along the boundaries of the rectangular defect

$$\oint_{left} + \oint_{right} = \frac{-i}{w\epsilon_0\epsilon_2} \sum_{n,m\geq 1} C_{n,m}\frac{nb}{ma}[1-(-1)^m][1-(-1)^n],$$
$$\oint_{top} + \oint_{bottom} = \frac{-i}{w\epsilon_0\epsilon_2} \sum_{n,m\geq 1} C_{n,m}\frac{ma}{nb}[1-(-1)^m][1-(-1)^n],$$

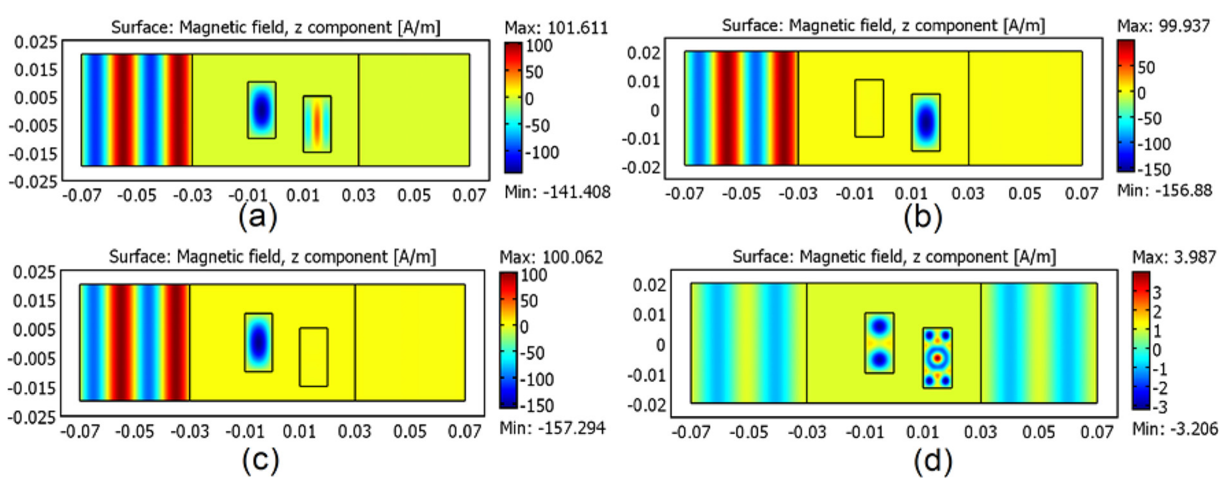

FIG. 4. Total reflection and transmission obtained with two rectangular defects: (top left) the H_z field with $\epsilon_L = 1.25$ (reflection defect) and $\epsilon_R = 9.25$ (reflection defect); (top right) the H_z field with $\epsilon_L = 4.25$ (transmission defect) and $\epsilon_R = 1.25$ (reflection defect); (bottom left) the H_z field with $\epsilon_L = 1.25$ (reflection defect) and $\epsilon_R = 4.25$ (transmission defect); (bottom right) the H_z field with $\epsilon_L = 4.25$ (transmission defect) and $\epsilon_R = 16.25$ (transmission defect).

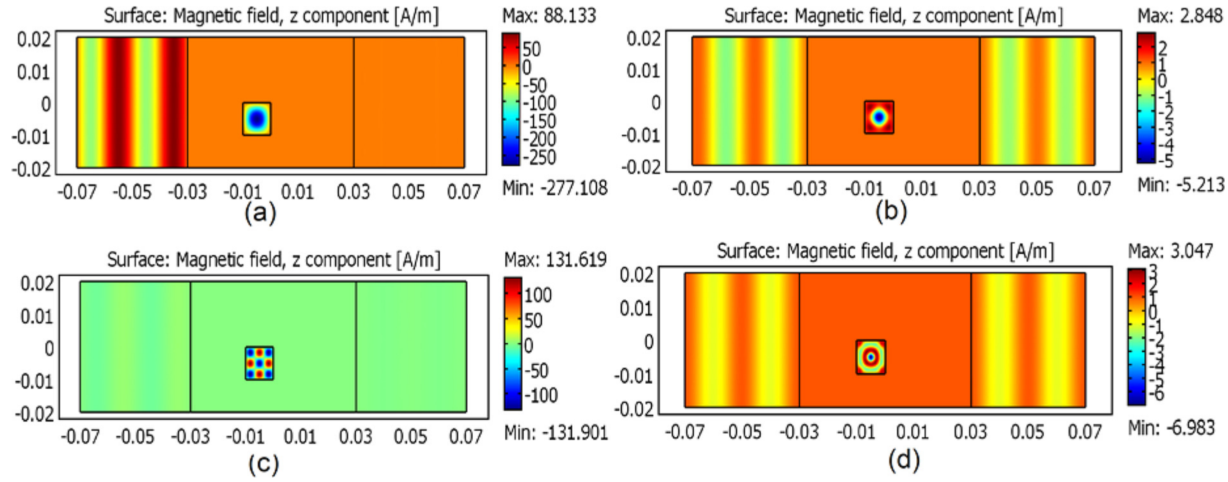

FIG. 5. Total reflection and transmission obtained with one square defect: (top left) the H_z field with $m = n = 1$ (i.e., $\epsilon_2 = 2$); (top right) the H_z field with $m = n = 2$ (i.e., $\epsilon_2 = 8$); (bottom left) the H_z field with $m = n = 3$ (i.e., $\epsilon_2 = 18$); (bottom right) the H_z field with $m = n = 4$ (i.e., $\epsilon_2 = 32$).

from which and Eq. (7), we see that when either n or m is even with finite $C_{n,m}$, we have $\mathcal{T} \approx 1$, in which case we have a total transmission. Hence we can obtain invisibility cloaking using such rectangular defects. Cloaking examples with $m = 1$, $n = 2$ and $m = 1$, $n = 4$ are presented in Fig. 3, where the size of the defect is identical to the one shown in Fig. 2, but the dielectric constants are changed to 4.25 and 16.25, respectively. In Fig. 3, we presented the whole H_z field and the zoomed-in electric fields. We like to remark that in the total transmission case, the $\boldsymbol{H}_2$ field is composed of multiple modes, and not dominated by a single mode $\phi_{n,m}(x, y)$, since none of $C_{n,m} \approx \infty$ in this case.

We would like to point out that while all the defects presented earlier are of the same size, it is indeed very flexible to have different sizes as long as the conditions for the total transmission/reflection are satisfied. Moreover, from Eq. (7), total reflection and cloaking can still be obtained with multiple embedded rectangular defects. If any of the defects results in total reflection, then total reflection will always happen regardless of whether the rest defects will induce total transmission or total reflection. However, to achieve total transmission with multiple defects, all the embedded defects must cause total transmission. Results in Fig. 4 show that total reflection and cloaking can be achieved with two defects. Note that the positions of the non-overlapping defects are arbitrary.

We like to mention that the above results hold true for the square defect, which is a special case of the rectangular defect and often used. In Fig. 5, we present some total reflection and cloaking obtained with one square defect, in which case we choose $a = b = 0.01$, which leads to $\epsilon_2 = n^2 + m^2$. To avoid non-uniqueness of the solution H_2 easily caused due to the symmetry of squares, we choose $n = m$ in Fig. 5.

In conclusion, we have demonstrated how to achieve total reflection and total transmission of an EM wave by embedding rectangular (including square) dielectric defects in the zero index metamaterial. Extensive numerical simulations confirm our theory. Our work provides more choices for blocking wave or concealing objects by embedding rectangular defects in ZIMs. In this work, we consider simple dielectric material which is non-metallic. If the defects are made of metal or a material with negative permittivity, similar phenomena can be achieved under certain circumstances.

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