Application of Secondary fitting to Grey forecasting model in the development tendency predication of Yellow River's water quality

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Abstract

The classical GM (1, 1) Grey forecasting model has some defects like the low precision. So the classical model always needs to be improved based on the characteristics of the raw data. This article researched the various statistical data of Yellow River in the last 10 years, including the total flow, the volume of sewage and the percentage of various types of water quality, and found the new model, which was improved though two-order fitting, be more suitable for the prediction of the Yellow River water quality trends. The author processed the initial data by softening factor and improved the classical GM (1,1) Grey model though two-order fitting, then established a grey forecasting model about the total flow, the volume of sewage and the percentage of various types of water quality:

$$A^{(0)}(k+1) = A^{(1)}(k+1) - A^{(1)}(k) = M * (e^{-ak} - e^{-a(k-1)}) .$$

The article made a reasonable prediction of the future development trend of Yellow River water quality pollution by using the model above. The results are as follows: in the next few years，the volume of sewage will be reduced and the percentage of high-quality water will rise slowly. It means that the water quality of Yellow River is improving. However, water quality recovers very slow and will not be radically improved if we do not take effective control measures. In order to speed up the improvement of water quality，we should strengthen pollution control. Suppose our goal is to make the ⅣandⅤtype of water of Yellow River to be controlled in 10% and there is no inferior Ⅴ type water，the list of treated sewage each year during the period 2009 to 2018 will be as follows: (39.142,35.617,34.436,33.25,32.062,30.872,29.682,28.493,27.306,26.123) (Unit:100 millions tons) and the total domestic sewage will be 15779 million tons. The test result shows the improved model has high precision and the simulation results have good practical.

Finally, this article presented some effective and feasible ideas and recommendations for water pollution prevention and control of Yellow River according to the current situation of Yellow river water quality slow recovery.

Key words: Water Quality of Yellow River, Grey Forecasting Model, Softening Factor, Two-order Fitting

1、Preface

The Yellow River is the mother river of China, which gave birth to the splendid civilization of China for five thousand years. The total area is 795,000 square kilometers.(including the internal flow area of 42,000 square kilometers).As the cradle of China ancient civilization, the Yellow River has made an indelible contribution for China and the world culture.

However, with the impact of human activities on the intensification of the natural environment and the rapid development of industry and agriculture of the Yellow River the Yellow River water quality has declined significantly, particularly since the reform and opening. It is seriously affecting people's production and living within the catchments and national economy. Although it makes people be aware the serious consequences of water quality and take a series of measures to protect the mother river, so far, the Yellow River water pollution problem is still serious. From the 80s of last century, the total discharge of sewage annual emissions from 2 per year million tons to 4 billion tons more sharply, almost doubled to the last century. The annual chemical oxygen emissions demand is 1.4 million tons and the ammonia emission is thirteen forty thousand tons per year. Respectively, over the Yellow River water environmental capacity of 1 / 3 and 2.5 times. The proportion of five categories and poor water quality is higher, many accidents have occurred caused by deterioration of water quality. Especially in the Yellow River and Huaihe River region, water resources are very scarce. The proportion of Ⅴ type water and inferior Ⅴ type water is as large as 40 to 50 percent, which has aggravated the water supply and demand. According to the Ministry of Water Resources, the current water shortage cities is more than 300 and the affected population is 1 million or more. In rural areas 3 million people are drinking unsafe water, which about 1.9 million people drinking excessive levels of harmful substances, more than 6300 million people drink high fluorine water,200 million drink high-arsenic water and more than 3800 million people drink brackish water. The issue of Excessive levels of harmful substances in drinking water is very prominent and the trend is increasing. It affects and threatens people’s water safety and physical health. In general, in some areas and watersheds, water pollution trend has been showing extending from the tributaries to the mainstream, spreading from urban to rural, from surface to underground infiltration, from land to sea.

So, what will Yellow River water quality change in the future? Is worsening or improve? What we should do is how to control the discharge of the Yellow River basin and take Measures to protect the Mother River.

This article will analyze the past decade (1999 ~ 2008) development of the Yellow River water quality changes, using the gray theory system to predict the future of the Yellow River water quality trends, giving the Yellow River water discharge limits in the future, making the appropriate comments and suggestions , providing the basis for the Yellow River water pollution control.

2、Analyzing of water quality in Yellow River for past decade(1999~2008)

According to the Yellow River water resources newspaper ,during the period

1999 to 2008 the Yellow River of the total quantity of water, total water consumption, into the sea, the total flow (Note: The total flow is the total quantity of water, total water consumption is into the sea), emissions in Table 2-1.

Table 2-1 Statistics of the Yellow Water flows、consumption and emissions from 1999 to 2008

Statistics of the Yellow Water flows and emissions from 1999 to 2008					
Year	Total quantity water （100 millions m^3）	Total consumption （100 millions m^3）	Into the sea （100 millions m^3）	Total flow （100 millions m^3）	Emission （100 millions t）
1999	516.82	392.74	61.69	971.25	41.98
2000	480.68	365.89	41.74	888.31	42.22
2001	474.55	361.79	40.89	877.23	41.35
2002	494.93	382.23	34.62	911.78	41.28
2003	429.12	336.45	189.6	955.17	41.46
2004	444.75	342.3	196.18	983.23	42.65
2005	465.01	361.75	204.08	1030.84	43.53
2006	512.1	401.73	186.7	1100.53	42.63
2007	484.88	379.78	199.8	1064.46	42.86
2008	490.95	383.54	141.6	1016.09	40.06

Chart 2-1 and 2-2 Changes of the Yellow Water flows and emissions from 1999 to 2008

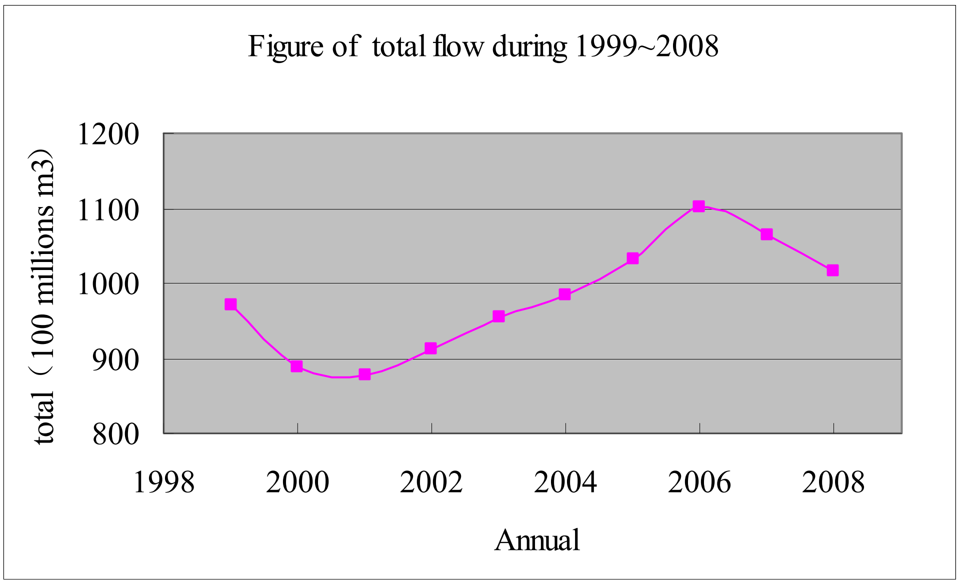

Figure 2-1 Changes of the Yellow Water flows from 1999 to 2008

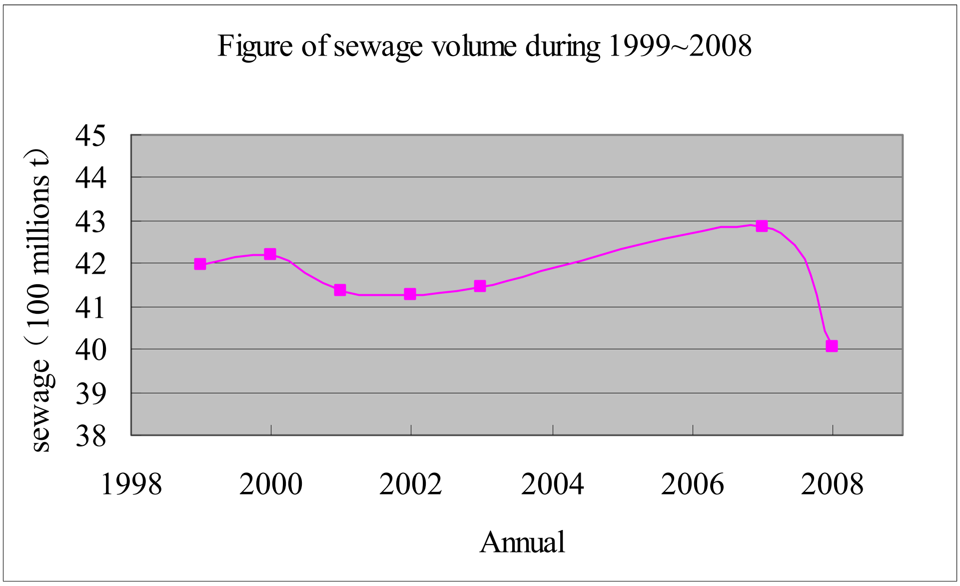

Figure 2-2 Changes of the Yellow Water emissions from 1999 to 2008

In Table 2-2 listing the Yellow River Class Ⅰ ~ Ⅱ, Ⅲ, Ⅳ, Ⅴ and worse Grade Ⅴ sections account for the proportion of all those involved in statistical sections from 1999 to 2008. The annual variation is shown in Figure 2-3

Table 2-2 the percentage of various types of water during 1999 to 2008

Statistics of the Yellow Water quality percentage from 1999 to 2008（%）					
Annual	Class Ⅰ～Ⅱ	Class Ⅲ	Class Ⅳ	Class Ⅴ	Worse Grade Ⅴ
1999	7.4	32.1	26.1	10.5	23.9
2000	3	35.7	20.1	17.2	24
2001	1.5	30.2	26.4	16.5	25.4
2002	5.1	14.3	17.6	15.8	47.2
2003	7.9	13.5	22.3	13.7	42.6
2004	16.9	10.8	19.3	9.6	43.4
2005	20.3	19.7	23.9	4.9	31.1
2006	11.9	29.7	15.4	11.9	31.1
2007	16.1	27.5	15.8	6.9	33.7
2008	17.8	21.4	13.5	10.5	36.8

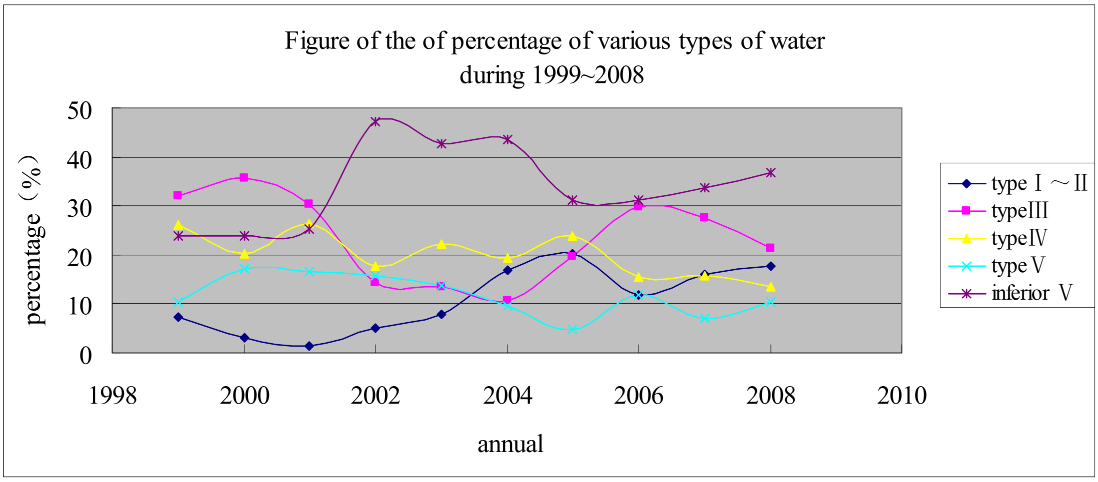

Figure 2-3 percentages of various types of water during 1999 to 2008

Figure 2-3 Changes of the Yellow Water quality percentage from 1999 to 2008

From Table 2-1 and Figure 2-1and 2-2 can be seen that the total flow of the Yellow River has remained fluctuated about 100 billions m^3 over the past years and the annual amount of sewage has fluctuated around the 42 tons. However, from the data analysis, the changes of Yellow River flow and changes in emissions has no significant. We can not find any disciplinarian about pollution.

From Table 2-2 and Figure 2-3 can be seen：

1、Over the past years, sections of the Yellow River water quality is worse than the proportion of Class Ⅲ section.

2、The proportion of inferior class Ⅴ section has always higher.

3、There was no significant change in the proportion of Various sections, can not reflect changes development of the Yellow River water quality in the future.

Just from the above data, in recent years the Yellow River emissions remained at high level. The water pollution is more serious and poor quality sections account for high proportion. There is no obvious disciplinarian. We can not find out the future of the Yellow River water quality trends, which meet the gray theory. Therefore, we consider the modified gray prediction model for the future development of the Yellow River water quality trends to predict and base on forecast results pose suggest protect programs for Yellow water quality.

3. Grey Forecasting Model Analysis and Improvement

3.1 The classical GM (1,1) Grey forecasting model

The traditional modeling process is as follows:

The raw data is recorded as $A^{(0)} = (A^{(0)}(1), A^{(0)}(2), \cdots, A^{(0)}(i))$

$$A^{(1)}(k) = \sum_{i=\beta}^{k} A^{(0)}(i) \qquad k = \beta, \cdots, n \qquad \text{------------------ (1)}$$

$A^{(1)}$ is named 1-AGO sequence of original sequence.

$z^{(1)}$ is weighted average value of $A^{(1)}$,

so $z^{(1)}(k) = \alpha A^{(1)}(k) + (1-\alpha)A^{(1)}(k-1)(k = 2,3,\cdots,i)$. α is certain parameters and generally α=0.5 . $z^{(1)}$ should be recorded as: $z^{(1)} = (z^{(1)}(2), z^{(1)}(3),\cdots, z^{(1)}(i))$. If we take the k(k=1,2,…i) as continuous variable (t) and take the sequence $A_i^{(1)}$ as a function on t, so $A_i^{(1)} = A_i^{(1)}(t)$.

Then we can create a differential equation model as following:

$$\frac{dA_1^{(1)}}{dt} + aA_1^{(1)} = b \quad \text{---------------------------------- (2)}$$

Because $A^{(1)}(k) - A^{(1)}(k-1) = A^{(0)}(k)$ and $A^{(0)}(k)$ is grey derivative and $z^{(1)}(k)$ is background value, so the grey differential equation of the equation (2) can be turned into the follow equation:

$$A^{(0)}(k) + az^{(1)}(k) = b(k = 2,3,\cdots,i)$$

It's matrix form should be:

$$Y^{(0)} = B*(a,b)^T$$

Where

$$Y^{(0)} = (A^{(0)}(2), A^{(0)}(3),\cdots, A^{(0)}(i))^T, B = \begin{pmatrix} -z^{(1)}(2) & -z^{(1)}(3)\cdots & -z^{(1)}(i) \\ 1 & 1 \quad \cdots & 1 \end{pmatrix}^T$$

The parameter values estimated through least square method should be:

$$(a,b)^T = (B^T * B)^{-1} * B^T * Y^{(0)}$$

Then we can get the special solution of the equation (2) as follow:

$$A^{(1)}(t+1) = (A^{(0)}(1) - \frac{b}{a}) * e^{-at} + \frac{b}{a}$$

So:

$$A^{(0)}(k+1) = A^{(1)}(k+1) - A^{(1)}(k)$$
$$= (A^{(0)}(1) - \frac{b}{a}) * (e^{-ak} - e^{-a(k-1)}) \quad \text{-------------------- -------------- (3)}$$

Finally, we can get the simulated values and predicted values.

3.2 The improvement process of $GM(1,1)$ --two-order fitting

Calculation error of the traditional model is always large and the model cannot meet the requirements. The checking results show that it do not apply the forecast of the amount of the Yellow River sewage and water quality data. However, the improved forecasting model, which has been improved though two-order fitting, has very high accuracy.

The modeling process of two-order fitting forecasting model is as following:

Record the model originated from(the first simulation as:

$$A^{(1)}(k+1)=Me^{-ak}+N \text{ ---------------------------------- } (4)$$

Then

$$A^{(1)}(1)=Me^{0}+N$$
$$A^{(1)}(2)=Me^{-a}+N$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$A^{(1)}(n)=Me^{-a(n-1)}+N$$

Its matrix form should be：

$$A^{(1)}=G\begin{pmatrix} M \\ N \end{pmatrix}$$

Where：

$$A^{(1)}=(A^{(1)}(1),A^{(1)}(2),\cdots,A^{(1)}(n))^{T}$$

$$G=\begin{pmatrix} e^{0} & 1 \\ e^{-a} & 1 \\ \vdots & \vdots \\ e^{-a(n-1)} & 1 \end{pmatrix} \text{ ---------------------------------- } (5)$$

We can get follow equation through least square method：

$$\begin{pmatrix} M \\ N \end{pmatrix}=(G^{T}G)^{-1}G^{T}A^{(1)} \text{ ---------------------------------- } (6)$$

There will be a more accurate two-order fitting forecasting model：

$$A^{(1)}(k+1)=Me^{-ak}+N \text{ ---------------------------------- } (7)$$

Then：

$$\begin{aligned} A^{(0)}(k+1) &= A^{(1)}(k+1)-A^{(1)}(k) \\ &= M*(e^{-ak}-e^{-a(k-1)}) \end{aligned} \text{ ---------------------------------- } (8)$$

4. Modeling process and solution of two-order fitting model

4.1 Questions

We should achieve the following objectives through further analysis and research on the data:

(1).Forecast the total flow and the volume of sewage of the Yellow River in the next few years;

(2). Find out the development law of the water quality;

(3). Forecast the development trend of water pollution in 10 years if we do not take any control measures;

(4).In order to control the amount of Ⅳ and Ⅴ type water within 20% and there be no inferior Ⅴ type water, how much wastewater should be purified in 10 years;

(5). Practical suggestions and comments for solving the problem of water pollution of the Yellow River.

4.2 Analysis of the problem

According to the results in the chapter 2, the development law of various data was not obvious in the past few years and there will be many uncertainties. Grey theory is suitable in this case. So we can create grey forecasting model and forecast the development trend of various types of water and the total domestic sewage in 10 years. According to these predictions, we can calculate the volume of sewage should be purified in 10 years.

Finally, develop corresponding strategies from many aspects for pollution prevention.

4.3 Model

Create improved forecast model through the grey theory. Take the random quantities in the initial data as gray numbers and find out the law hidden inside the initial data.

4.4Solution of the model

4.4.1Data Preprocessing

The data in table 2-1 and table 2-2 are volatile and should be pretreated before use. In this paper, the author deals with the initial data by softening factor.

Record the original sequence as： $X = (x(1), x(2), \cdots, x(i))$.

Record the processed sequence as: $XD = (x(1)d, x(2)d, \cdots, x(i)d)$，

Namely,

$A^{(0)} = (A^{(0)}(1), A^{(0)}(2), \cdots, A^{(0)}(i))$ ·

Where

$$A^{(0)}(k) = \frac{1}{n-k+1}(x(k) + x(k+1) + \cdots + x(n)) \quad \text{---------------------------------} (9)$$

$$k = 1,2,\cdots n$$

The processed date is in table 4-1.

Table 4-1 processed date

annul	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
Total flow	979.9	996.9	1004	1010	1017	1024	1031	1038	1045	1052
Sewage volume	42	42	41.98	42.07	42.2	42.35	42.27	41.85	41.46	40.06
Type Ⅰ～Ⅱ	14.62	15.04	15.52	16	16.38	16.63	16.64	16.67	17.38	17.8
TypeⅢ	23.49	22.53	20.89	19.56	20.43	21.82	24.58	26.2	24.45	21.4
TypeⅣ	20.04	19.37	19.28	18.26	18.37	17.58	17.15	14.9	14.65	13.5
TypeⅤ	11.75	11.89	11.23	10.47	9.583	8.76	8.55	9.767	8.7	10.5
inferior Ⅴ type	33.92	35.03	36.41	37.99	36.45	35.22	33.18	33.87	35.25	36.8

4.4.2. Solution of the improved model

The functions in table 4-2 are quadratic functions of model fitting. According to these functions, we can get all kinds of forecast data in the table 4-3. Through checking the deviation of the results, we find out that the predicted value from the grey forecasting model is very close to the true value. Take the percentage of Ⅰ and Ⅱ type of water for example, the ratio of standard deviation (C) is 0.1884 and the small error probability (p) is 1, so the improved model can be considered to achieve the general requirements.

Table 4-2 Quadratic functions of model fitting

Projects	(M, N)	Quadratic functions
Total flow	(148969.041,-147989.1522)	148969.041exp(0.0066696t)+(-147989.1522)
Sewage volume	(-11150.1128,11191.4613)	-11150.1128exp(-0.0038275t)+(11191.4613)
Type Ⅰ～Ⅱ	(823.4814,-808.9393)	823.4814exp(0.018387t)+(-808.9393)
TypeⅢ	(1193.4506,-1170.2151)	1193.4506exp(0.017402t)+(-1170.2151)
TypeⅣ	(-478.0769,497.479)	-478.0769exp(-0.04336t)+(497.479)
TypeⅤ	(-392.9031,405.651)	-392.9031exp(-0.027926t)+(405.651)
inferior Ⅴ type	(-9633.7778,9668.1429)	-9633.7778exp(-0.003745t)+(9668.1429)

Table 4-3 Percentage of various types of water

Annual	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Total flow	1058.6	1065.6	1072.8	1079.9	1087.2	1094.4	1101.8	1109.1	1116.6	1124
Sewage volume	41.009	40.853	40.697	40.541	40.386	40.232	40.078	39.925	39.773	39.621
Type Ⅰ～Ⅱ	19.026	19.971	20.963	22.005	23.098	24.245	25.45	26.714	28.041	29.434
TypeⅢ	20.449	24.872	25.309	25.753	26.205	26.665	27.133	27.61	28.094	28.588
TypeⅣ	13.607	13.03	12.477	11.947	11.44	10.955	10.49	10.045	9.6188	9.2106
TypeⅤ	8.6179	8.3805	8.1497	7.9253	7.707	7.4948	7.2884	7.0877	6.8925	6.7027
inferior Ⅴ type	34.915	34.785	34.655	34.525	34.396	34.268	34.14	34.012	33.885	33.758

We can get the percentage of various types of water through normalizing the data in the table 4-3((i.e.: the percentage of one type of water / sum of the percentage of various types of water * 100%). The results are in the table 4-4.

Table 4-4 Percentage (normalized) of various types of water

Annual	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
Type Ⅰ～Ⅱ	19.693	19.766	20.643	21.54	22.458	23.396	24.353	25.329	26.322	27.331
TypeⅢ	21.165	24.617	24.922	25.21	25.48	25.732	25.965	26.178	26.372	26.545
TypeⅣ	14.084	12.896	12.286	11.695	11.124	10.571	10.038	9.5242	9.0291	8.5527
TypeⅤ	8.9198	8.2944	8.0251	7.758	7.4937	7.2324	6.9745	6.7202	6.4699	6.2239
inferior Ⅴ type	36.139	34.427	34.125	33.797	33.444	33.068	32.669	32.249	31.807	31.347

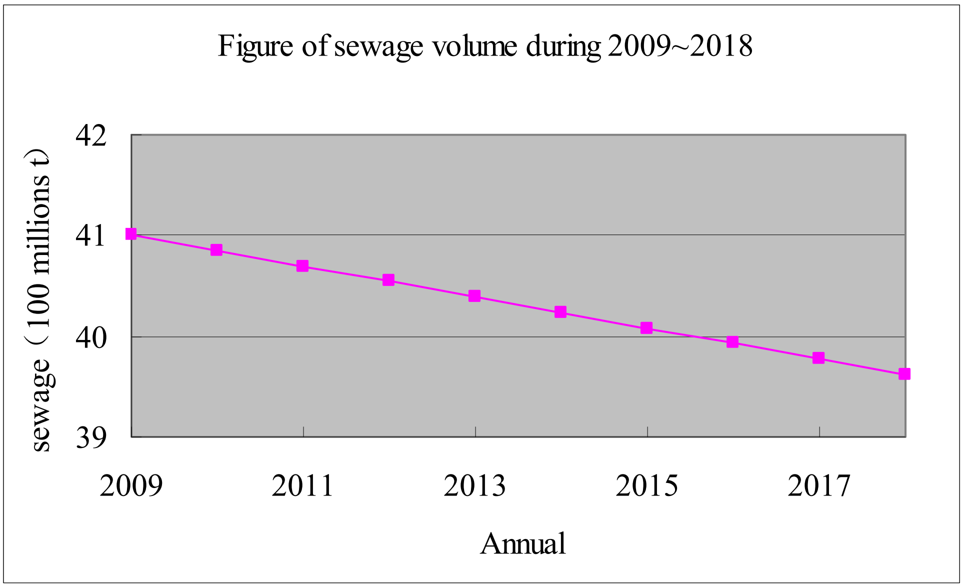

Figure 4-1 sewage volume during 2009~2018

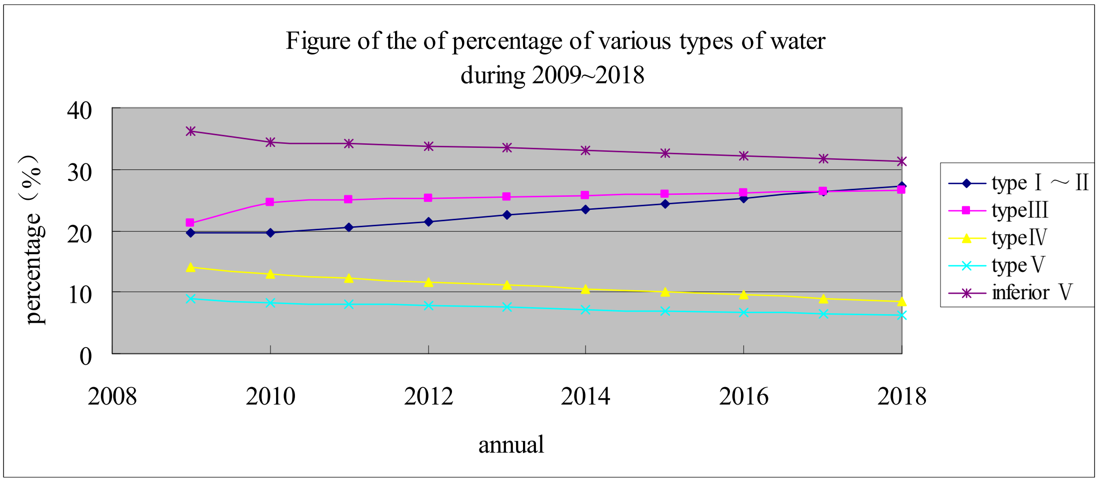

Figure 4-2 development trend of percentage of various types of water during 2009~2018

From table 4-3 and figure 4-1 can be seen, the next ten years the Yellow River flow basically unchanged, emissions have a faint decreased trend, about 0.4 per cent year by year.

From table 4-4 and figure 4-2 can be seen, the next ten years the Yellow River high quality water quality cross-section (Ⅰ ～ Ⅱ classes and Ⅲ class water quality cross-section) proportion is slowly rising trend, inferior quality cross-section (Ⅳ class, Ⅴ class and inferior Ⅴ class water quality cross-section) proportion is slow decline. This is consistent with emissions of diminishing rule.

But the emissions rate reduce sluggishly, it also extremely slow that high quality water quality cross-section the increment of the proportion. Ⅰ ～ Ⅱ classes and Ⅲ class water quality cross-section proportion in the next ten years still remained low. To 2018, Ⅰ ～ Ⅱ classes and Ⅲ class water quality cross-section of the total compared to just 53.9 and inferior quality cross-section proportion still remains high.

To 2018, Ⅳ class, Ⅴ classes and inferior Ⅴ class total ratio of water quality cross-section reach up to 46.1%. That if not adopt effective measures, the next decade the water won't have a fundamental change, the future of the Yellow River emissions control still very arduous task.

The above analysis results, the emissions of Yellow River tended to decrease in the next decade. The proportion of high quality cross-section will increased year by year, while the inferior quality will decline. This shows the water quality of Yellow River will improve for the strong management. But the overall of water is still far from being satisfactory. The Yellow River water quality restoration speed is slow. If do not take more effective measures, the water is still poor and pollution situation still very serious in the future.

5. The controlling model of the Yellow River sewage

In order to expedite recovery the Yellow River water's quality, it is necessary to control the annual discharge capacity and process the sewage. And then it is necessary to change the poor-quality water into the high quality water again. After that , drain away the water to the Yellow River. The aim is to increase the proportion of the high quality water plane of fracture(The water plane of fracture for class of Ⅰ、Ⅱand Ⅲ). Consequently, it would reduce the proportion of the poor-quality water plane of fracture(The water plane of fracture for class of Ⅳ、ⅤandⅤ).

Suppose that the component of water quality for sewage which to be discharge into the Yellow River can't be changed and sewage in various water quality in proportion is maintained the level of 2009.It wants to set goals for the percentage of Ⅳ and Ⅴ'class water to be controlled in 20% and no poor quality water of Ⅴ's class. Therefore, the mass of sewage to be processed in the i(i=1…10) year. And so the mass of sewage to be processed=(the percentage of Ⅳ's class+ the percentage of Ⅴ's class-20%+ the percentage of poor-quality water for the Ⅴ's class)*the discharge capacity. According to the above relation, it can calculate the per annum mass of sewage to be processed for 2005 to 2014 as in the table 5-1.

Table 5-1 the per annum mass of sewage to be processed for 2009 to 2018 (Unit：100 millions tons)

annual	2009	2010	2011	2012	2013
the mass of sewage to be processed	16.052	15.991	15.930	15.869	15.808
annual	2014	2015	2016	2017	2018
the mass of sewage to be processed	15.748	15.688	15.628	15.568	15.509

According to table5-1, we can calculate the total mass of sewage to be processed is 15779 of millions in the next 10 years (2009-2018).

6. The conclusion and proposal

According to the description above forecast analysis，we can conclude as the follows ：

（1）Over the next several years, discharge sewage will decrease somewhat and the proportion of the high quality water plane of fracture will increase slowly. Therefore, the quality of the Yellow River water is gradually improving.

（2）Over the next several years, the quality of the Yellow River water get well very slowly. According to current development trend, the quality of the Yellow River water will not receive the fundamental change.

（3）In order to make the quality of the Yellow River water improvement very quickly, it is necessary to increase managing the pollution disposal. If it wants to control the water of class Ⅳ and Ⅴ to be 20% and no poor-quality water of class Ⅴ,the sewage water needed to dispose is 15779 of millions ton in the next ten years.

To protect the water resources of the Yellow River, and then prevention and cure the Yellow River water pollution. Therefore to change the water quality of the Yellow River as soon as possible, this paper suggests:

（1）In the aspect of thought，strengthen the education of environmental and health consciousness for citizen, and then improve the citizen' consciousness of environmental protection.

（2）In the aspect of national policy，complete protection agency of the Yellow River's resources; draft specific management regulations of the Yellow River; bring about the institutionalize of the water resource in the Yellow River. At the same time, supervise and manage the water environmental quality. At last, prevent and cure the pollution of water.

（3）In the aspect of science, it establish sewage system and then reduce the content of the harmful substances in the sewage water. At the same time, it needs to introduce and develop industrial technology which without waste or less and which without waste or less waste actively. Based on the requirements of the water environment function and the capability of the water's dilute and self-purification, it can establish different types of protection zones and protect the resources of life water not to be polluted.

（4）In the aspect of ecological environment，it improves the proportion of the Yellow River basin and reduce the soil erosion in the both shores of the Yellow River. Therefore, there will be kept down the deterioration of ecological environment.

（5）It determines the extent of the water resource which society and environment can bear. At the same time, it plans water conservancy project layout reasonably and dispatch the mass of water resource. Therefore, the environment function of the Water resources may be sufficient play and use.

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