Understanding Flocking Dynamics in Nature
Section II: A New Model

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1 Introduction

Simulation of our previous model has shown that the model needed major structural improvement, as the simulation has revealed some problems, the most serious of them being the huge speed buildup. This article will describe the updated model, but it will not discuss what is new; rather, the entire model is re-introduced. Although the updated model has some major changes, the key principles of the model remain the same.

The main change in the updated model is that each agent has certain *ideal velocity* that it tries to reach, which is determined by its own perspective. This is different from the previous model where ideal velocity was added to the current velocity in the form of acceleration.

Note that imaginary unit will be denoted ι in this article. Also, units for time, length and angle will be second, meter and radian respectively.

2 Eyesight Function

We define the concept of "eyesight function" to describe how agents see things; this function indicates the "degree of importance" that an agent puts into its surroundings. Since the degree of importance can be thought as positive real number, we can think of eyesight function as a scalar field that assigns real number to every point on the plane.

2.1 How to Apply Standard Eyesight Function

However, note that same position can have different degrees of importance assigned, depending on position and orientation of the observer agent. Therefore, it is reasonable to first define "standard" eyesight function that accounts for observer agent at fixed position and orientation, and then apply it to arbitrary

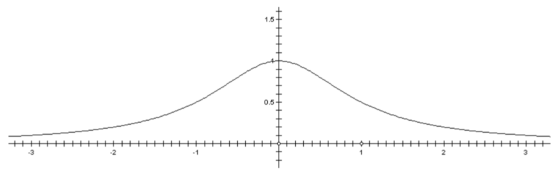

Figure 1: Plot of Distance Function

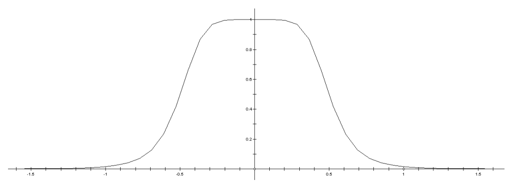

Figure 2: Plot of Angle Function

agent. That is, given the standard eyesight function $w : \mathbb{C} \to \mathbb{R}$ (w for 'weight') that describes degree of importance from the perspective of agent at origin pointing to $+x$ direction, the respective expression that agent i at position x_i and velocity v_i puts on arbitrary position x would be

$$w\left(\frac{x - x_i}{v_i/|v_i|}\right) \tag{1}$$

2.2 Modelling Standard Eyesight Function

Now that we know how to apply the standard eyesight function w to arbitrary agent, let us model the standard eyesight function. The function was modelled based on two principles: faraway objects are hard to see, and objects off one's direction of sight are hard to see too. Therefore if we are concerned with object at position $z(\in \mathbb{C})$, then we think about how large $|z|$ and $\arg(z)$ are, and combine the two factors to decide how clearly the observer can see the object at z.

Distance function is modelled such that as the object goes farther, the degree of importance decreases gradually:

$$\frac{1}{1 + |z|^2} \tag{2}$$

Angle function is modelled such that as the object goes more off-sight, the degree of importance decreases gradually, but this time with more clear boundary that indicates "off-sight region":

$$\frac{1}{1 + (\arg z)^6} \tag{3}$$

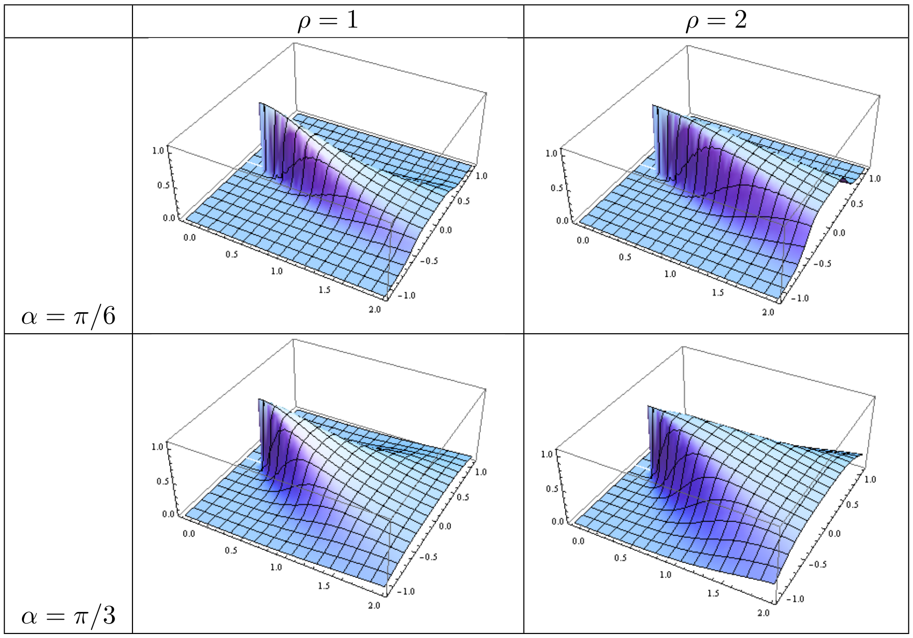

Figure 3: 3D Plots of Standard Eyesight Function w

Note that, however, some agents can see farther than others, and some agents see narrower than others. Such can be easily implemented by "strething" the Distance and Angle functions. Now combining everything, we have:

$$w(z) = \left(1 + \left|\frac{z}{\rho}\right|^2\right)^{-1} \left(1 + \left(\frac{\arg z}{\alpha/2}\right)^6\right)^{-1} \tag{4}$$

Here, ρ denotes how far the agent can see and α denotes how wide the agent can see.

2.3 Weighted Average

The concept of "average value" can be defined using eyesight function. When an agent determines certain average value about its surroundings, such as average velocity, it would rely on its eyesight to do so. Hence a reasonable way to model how agent i decides the average value of $p_j, j = 1, 2, \cdots, N, j \neq i$ would be to use the weighted mean:

$$M_i(p_j) = \left(\sum_{j=1, j\neq i}^{N} w\left(\frac{x_j - x_i}{v_i/|v_i|}\right) p_j\right) \Bigg/ \left(\sum_{j=1, j\neq i}^{N} w\left(\frac{x_j - x_i}{v_i/|v_i|}\right)\right) \tag{5}$$

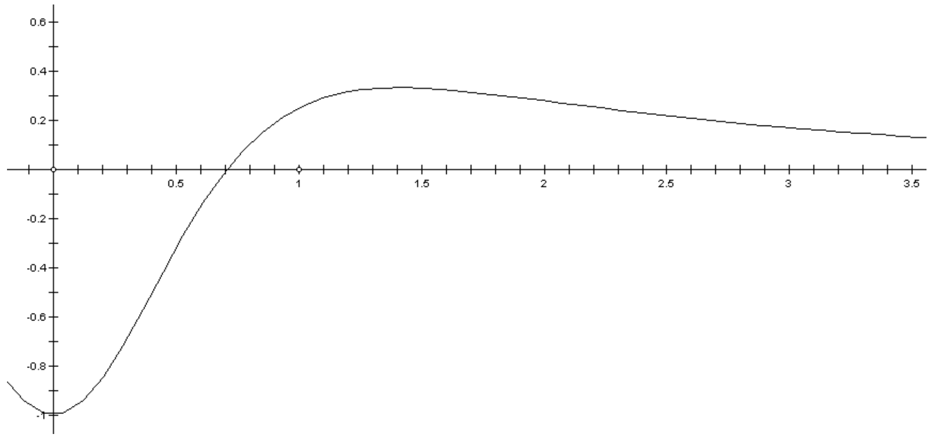

Figure 4: Plot of One-to-one Cohesion/Avoidance Strength

3 The Model

In our model, each agent will determine the "ideal velocity" from its own perspective, and then will accelerate accordingly. The ideal velocity will be determined according to the Reynold rules, which are cohesion, avoidance and alignment. However if the agent sees too few of other agents, it will start turning around to see where the others are. (This 'turnaround factor' was previously not introduced in other models because only with the concept of eyesight the turning around is necessary.)

3.1 Ideal Velocity

The ideal velocity is modelled such that it is primarily the average velocity of other agents (which accounts for alignment rule) but additional vector that accounts for cohesion/avoidance will be added onto it, so that if an agent is too far away or too close to others, it will accelerate or deccelerate accordingly.

One-to-one cohesion/avoidance 'strength' is modelled using the function $(2x^2-1)/(1+x^2)^2$. Here the unit vector to the direction $x_j - x_i$ is multiplied to produce the one-to-one interaction vector $\frac{x_j-x_i}{|x_j-x_i|}\frac{2|x_j-x_i|^2-1}{(1+|x_j-x_i|^2)^2}$ (in a strict sense, this is a complex number not vector but it can be thought as a vector.) Hence taking average of velocities and adding a constant multiple of average one-to-one cohesion/avoidance vectors, we obtain:

$$V_i = M_i(v_j) + kM_i\left(\frac{x_j-x_i}{|x_j-x_i|}\frac{2|x_j-x_i|^2-1}{(1+|x_j-x_i|^2)^2}\right) \tag{6}$$

as the ideal velocity. (the constant k will be determined accordingly after simulations)

3.2 Turnaround

It is natural that if a person in the crowd can't see enough of the others the person will start turning around out of doubt to look for others. Hence we will introduce $\tau_i \in \{-1, 0, +1\}$ to indicate whether agent i is turning, and if turning, which direction it is turning to. τ_i will change according to whether agent i is seeing enough of other agents. Number of agents that it sees will be given by

$$\sum_{j=1, j\neq i}^{N} w\left(\frac{x_j - x_i}{v_i/|v_i|}\right) \tag{7}$$

because agents off the eyesight can't be seen clearly. Meanwhile, let us define constant L, which indicates the minimal number of agents that need to be seen so as to 'comfort' the observer agent. Hence we have:

$$\tau_i = \begin{cases} 0 & \text{if } \sum_{j=1,j\neq i}^{N} w\left(\frac{x_j - x_i}{v_i/|v_i|}\right) \geq L \\ +1 \text{ or } -1 & \text{if } \sum_{j=1,j\neq i}^{N} w\left(\frac{x_j - x_i}{v_i/|v_i|}\right) < L \end{cases} \tag{8}$$

where whether $\tau = +1$ or -1 is decided randomly. In the simulation, nonzero value of τ stays the same as long as $\sum w\left(\frac{x_j - x_i}{v_i/|v_i|}\right) < L$, so that each agent will keep on turning to the same direction once it starts doing so.

3.3 How is the Ideal Velocity Reached?

We treat speed and orientation separately. That is,

$$v_i = s_i e^{\iota\theta_i} (v_i \in \mathbb{C}, \iota = \sqrt{-1}) \tag{9}$$

The agent will accelerate faster when the speed discrepancy between the current speed and ideal speed is larger. Hence it is reasonable to make $\frac{\mathrm{d}s_i}{\mathrm{d}t} = |V_i| - s_i$. However, even if the ideal speed is very large, it is reasonable that agents would have some maximum speed that it can move no faster than. Hence to account for maximum speed L_i of agent i, we multiply $1 - e^{L_i - s_i}$ and hence obtain

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = (|V_i| - s_i)(1 - e^{L_i - s_i})$$

(when $\tau \neq 0$) The choice of function $1 - e^{L_i - s_i}$ here was such that when $s_i \ll L_i$, $1 - e^{L_i - s_i} \to 1$, while as s_i approaches L_i more, $1 - e^{L_i - s_i}$ will grow negative very fast.

Meanwhile, L_i will follow a normal distribution with respective mean and standard deviations that depend on choice of the type of agent.

It is also reasonable to think that θ_i adjusts faster when the discrepancy between θ_i and ideal orientation $\arg V_i$ is larger. That is,

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \arg V_i - \theta_i$$

However this time, we do not have maximal angle or such and hence no 'moderation' is required. Summarizing the above, we have:

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} =(|V_i| - s_i)(1 - e^{L_i - s_i}) \tag{10}$$
$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} =\arg V_i - \theta_i \tag{11}$$

The above accounts only for the case of $\tau_i = 0$. The situation is different when $\tau_i \neq 0$, when agent wants to turn around and look for others. The agent will of course start slowing down, and it will start turning to the random direction of left or right. That is, when $\tau \neq 0$,

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} =- s_i \tag{12}$$
$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} =\tau_i T \tag{13}$$

(Constant T denotes how fast the agent turns.)

3.4 The Model

Combining the above results, we obtain the equations for the model:

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = \begin{cases} (|V_i| - s_i)(1 - e^{L_i - s_i}) & \text{if } \tau_i = 0 \\ -s_i & \text{if } \tau_i \neq 0 \end{cases} \tag{14}$$

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = \begin{cases} \arg V_i - \theta_i & \text{if } \tau_i = 0 \\ \tau_i T & \text{if } \tau_i \neq 0 \end{cases} \tag{15}$$

4 Simulation

Simulation is yet to be implemented, however the simulation will be shown in the presentation of the project.