Shock Profile For Gas Dynamics in Thermal Nonequilibrium

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1 Introduction

The motion of a gas in local thermodynamic equilibrium is governed by the compressible Euler equations. In Lagrangian coordinates, the equations for one dimensional flow read (cf. [1]):

$$\begin{cases} v_t - u_x = 0, \\ u_t + p_x = 0, \\ (e + \frac{u^2}{2})_t + (pu)_x = 0, \end{cases} \tag{1.1}$$

where where v, u, p and e are, respectively, the specific volume, velocity, pressure and internal energy of the gas. For an ideal gas

$$e = \frac{1}{\gamma - 1} pv, \tag{1.2}$$

where $\gamma > 1$ is the adiabatic constant. During rapid changes in the flow the internal energy e may lag behind the equilibrium value corresponding to the ambient pressure and density. The translational energy adjusts quickly, but the rotational and vibrational energy may take an order of magnitude longer. If we suppose that α of the degrees of freedom adjust instantaneously but a further α_r degrees of freedom take longer to relax, we may take (cf. [2]):

$$e = \frac{\alpha}{2} pv + q, \tag{1.3}$$

where q is the energy in the lagging degress of freedom. In equilibrium, q would have the value

$$q_{\text{equil}} = \frac{\alpha_f}{2} pv. \tag{1.4}$$

A simple overall equation to represent the relaxation is (in Lagrangian coordinates) :

$$q_t = -\frac{1}{\tau}(q - \frac{\alpha_f}{2} pv), \tag{1.5}$$

where $\tau > 0$ is the relaxation time. Therefore, in thermal nonequilibrium, we have the following system of equations to model the gas motion:

$$\begin{cases} v_t - u_x = 0, \\ u_t + p_x = 0, \\ (\frac{\alpha}{2}pv + q + \frac{u^2}{2})_t + (pu)_x = 0, \\ q_t = -\frac{1}{\tau}(q - \frac{\alpha_f}{2}pv). \end{cases} \tag{1.6}$$

If the relaxation time τ is taken to be so short that $q = \frac{\alpha_f}{2}pv$ is an adequate approximation to the last equation in (1.6), we have the following equilibrium theory:

$$\begin{cases} v_t - u_x = 0, \\ u_t + p_x = 0, \\ (\frac{\alpha+\alpha_f}{2}pv + \frac{u^2}{2})_t + (pu)_x = 0. \end{cases} \tag{1.7}$$

The three characteristic speeds for (1.7) are:

$$\lambda_1 = -\sqrt{(1 + \frac{2}{\alpha + \alpha_f})\frac{p}{v}}, \lambda_2 = 0, \lambda_3 = \sqrt{(1 + \frac{2}{\alpha + \alpha_f})\frac{p}{v}}.$$

For system (1.7), $((v_-, u_-, p_-), (v_+, u_+, p_+), \sigma)$ with two constant states (v_-, u_-, p_-) and (v_+, u_+, p_+) and speed σ is called a shock wave (cf. [1]) if the following Rankine-Hugoniot conditions

$$\begin{cases} -\sigma(v_+ - v_-) = (u_+ - u_-), \\ \sigma(u_+ - u_-) = (p_+ - p_-), \\ \sigma\{(\frac{\alpha+\alpha_f}{2}p_+v_+ + \frac{u_+^2}{2}) - (\frac{\alpha+\alpha_f}{2}p_-v_- + \frac{u_-^2}{2})\} = (p_+u_+ - p_-u_-), \end{cases} \tag{1.8}$$

hold, and some other entropy conditions hold, where v_-, v_+, p_-, p_+ are positive constants, u_- and u_+ are constants. A shock wave is called a 1-shock wave if

$$-\sqrt{(1 + \frac{2}{\alpha + \alpha_f})\frac{p_-}{v_-}} > \sigma > -\sqrt{(1 + \frac{2}{\alpha + \alpha_f})\frac{p_+}{v_+}}. \tag{1.9}$$

A shock wave is called a 3-shock wave if

$$\sqrt{(1 + \frac{2}{\alpha + \alpha_f})\frac{p_-}{v_-}} > \sigma > \sqrt{(1 + \frac{2}{\alpha + \alpha_f})\frac{p_+}{v_+}}. \tag{1.10}$$

In this paper, we consider a 3-shock wave, because a 1-shock wave can be handled by the same method. For a 3-shock wave, it follows from (1.8) and (1.10) that,

$$v_- < v_+, \ u_- > u_+, \ p_- > p_+. \tag{1.11}$$

A shock profile for the 3-shock wave $((v_-, u_-, p_-), (v_+, u_+, p_+), \sigma)$ is a traveling wave solution for system (1.6) of the form $(v,\ u,\ p, q)(\frac{x-\sigma t}{\tau})$ satisfying

$$(v,\ u,\ p, q)(\pm\infty) = (v_\pm,\ u_\pm,\ p_\pm, \frac{\alpha_f}{2} p_\pm v_\pm). \tag{1.12}$$

So we have

$$\begin{cases} -\sigma v' - u' = 0, \\ -\sigma u' + p' = 0, \\ -\sigma(\frac{\alpha}{2}pv + q + \frac{u^2}{2})' + (pu)' = 0, \\ -\sigma q' = -(q - \frac{\alpha_f}{2}pv), \end{cases} \tag{1.13}$$

where $' = \frac{d}{d\xi}$ and $\xi = \frac{x-\sigma t}{\tau}$.

In this paper, we are interested in the existence and properties of the shock profile. For general hyperbolic system with relaxation, the existence of the shock profile has been proved in [3] by using the center manifold method with the assumption that the shock strength is sufficiently small. In this paper, we find the sufficient and necessary condition, which is $\frac{p_-}{p_+} < 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}$, to ensure the existence of the shock profile. Moreover, we can calculate the shock profile solution in some explicit details. This is in sharp contrast to the abstract construction in [3]. Before we state our theorem, we introduce some notations. Let

$$\begin{cases} m = \sigma v_- + u_- = \sigma v_+ + u_+, \\ P = -\sigma u_- + p_- = -\sigma u_+ + p_+, \\ Q = -\sigma(\frac{\alpha+\alpha_f}{2}p_- v_- + \frac{u_-^2}{2}) + p_- u_- \\ \quad = -\sigma(\frac{\alpha+\alpha_f}{2}p_+ v_+ + \frac{u_+^2}{2}) + p_+ u_+, \end{cases} \tag{1.14}$$

$$f(v) = -\sigma^2(1+\alpha)v + (1+\frac{\alpha}{2})(\sigma^2 v_- + p_-) = -\sigma^2(1+\alpha)v + (1+\frac{\alpha}{2})(\sigma^2 v_+ + p_+). \tag{1.15}$$

Our theorem is the following:

Theorem 1 Suppose the two constant states (v_-, u_-, p_-), (v_-, u_-, p_-) and the speed σ satisfy the Rankine-Hugoniot conditions (1.8) and the Lax shock condition (1.10). Then,

1) If

$$\frac{p_-}{p_+} < 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}, \tag{1.16}$$

then there exists a solution to the problem (1.13) and (1.12).

2) If

$$\frac{p_-}{p_+} \geq 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}, \tag{1.17}$$

the problem (1.13) and (1.12) does not admit a smooth solution.
3) In case 1), i. e., if (1.16) holds, the solution of the problem (1.13) and (1.12) satisfying $v(0) = v_0$ for some constant v_0 satisfying $v_- < v_0 < v_+$ is giving by

$$\begin{aligned}
&2f(v_+)\left(\ln(v_+ - v) - \ln(v_+ - v_0)\right) - 2f(v_-)\left(\ln(v - v_-) - \ln(v_0 - v_-)\right) \\
&= -\sigma\xi(1 + \alpha + \alpha_f)(v_+ - v_-),
\end{aligned} \tag{1.18}$$

$$u(\xi) = m - \sigma v(\xi),\ p(\xi) = m\sigma + P - \sigma^2 v(\xi). \tag{1.19}$$

for $-\infty < \xi < +\infty$. For this solution, we have

$$v'(\xi) > 0,\ u'(\xi) < 0,\ p'(\xi) < 0, \tag{1.20}$$

for $-\infty < \xi < +\infty$, and

$$\begin{aligned}
&C_1 \exp\left(-\frac{1 + \alpha + \alpha_f}{2f(v_+)}\sigma(v_+ - v_-)\xi\right) \\
&\leq v_+ - v(\xi),\ v'(\xi) \\
&\leq C_2 \exp\left(-\frac{1 + \alpha + \alpha_f}{2f(v_+)}\sigma(v_+ - v_-)\xi\right)
\end{aligned} \tag{1.21}$$

for $\xi > 0$,

$$\begin{aligned}
&C_3 \exp\left(\frac{1 + \alpha + \alpha_f}{2f(v_-)}\sigma(v_+ - v_-)\xi\right) \\
&\leq v(\xi) - v_-,\ v'(\xi) \\
&\leq C_4 \exp\left(\frac{1 + \alpha + \alpha_f}{2f(v_-)}\sigma(v_+ - v_-)\xi\right)
\end{aligned} \tag{1.22}$$

for $\xi < 0$, where C_i $(i = 1, 2, 3, 4)$ are some positive constants. For $u(\xi)$ and $p(\xi)$, we have the similar estimates.

2 Proof of Theorem 1

In this section, we give a proof of the Theorem 1.

We integrate (1.13) to get

$$\begin{cases}
\sigma v + u = m, \\
-\sigma u + p = P, \\
-\sigma(\frac{\alpha}{2}pv + q + \frac{u^2}{2}) + (pu) = Q,
\end{cases} \tag{2.1}$$

where m, P and Q are given by (1.14) By the third equation of (2.1), we have:

$$q = -\left(\frac{\alpha}{2}pv + \frac{u^2}{2}\right) + \frac{pu - Q}{\sigma}, \tag{2.2}$$

Substituting (2.2) into the fourth equation of (1.13), using (2.1) and (2.2), we get

$$f(v)\frac{dv}{d\xi} = \frac{1}{\sigma}\left(\frac{\alpha + \alpha_f}{2}pv + \frac{u^2}{2} - \frac{pu - Q}{\sigma}\right), \tag{2.3}$$

where $f(v)$ is given by (1.15) So

$$f(v_-) = v_-\left(-\frac{\alpha}{2}\sigma^2 + (1 + \frac{\alpha}{2})\frac{p_-}{v_-}\right). \tag{2.4}$$

In view of (1.10), we have

$$\begin{aligned} & -\frac{\alpha}{2}\sigma^2 + (1 + \frac{\alpha}{2})\frac{p_-}{v_-} \\ & > -\frac{\alpha}{2}(1 + \frac{2}{\alpha + \alpha_f})\frac{p_-}{v_-} + (1 + \frac{\alpha}{2})\frac{p_-}{v_-} \\ & = \left(1 - \frac{\alpha}{\alpha + \alpha_f}\right)\frac{p_-}{v_-} > 0. \end{aligned} \tag{2.5}$$

Therefore

$$f(v_-) > 0.$$

By (1.15), we get

$$f(v_+) = v_+\left(-\frac{\alpha}{2}\sigma^2 + (1 + \frac{\alpha}{2})\frac{p_+}{v_+}\right). \tag{2.6}$$

Let

$$\bar{v} = \frac{1 + \frac{\alpha}{2}}{1 + \alpha}\frac{\sigma^2 v_+ + p_+}{\sigma^2}. \tag{2.7}$$

Then

$$f(\bar{v}) = 0. \tag{2.8}$$

So, if

$$v_+ < \bar{v}, \tag{2.9}$$

then

$$f(v_+) > 0. \tag{2.10}$$

In the next lemma, we will give a neat condition to ensure (2.10).

Lemma 2.1 1) If $\frac{p_-}{p_+} < 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}$, then $v_+ < \bar{v}$ and thus $f(v_+) > 0$.
2) If $\frac{p_-}{p_+} = 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}$, then $v_+ = \bar{v}$ and $f(v_+) = 0$.
3) If $\frac{p_-}{p_+} > 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}$, then $v_+ > \bar{v}$ and $f(v_+) < 0$.

Proof. First, we use (1.8) to show that

$$\frac{\alpha+\alpha_f}{2}(p_+v_+ - p_-v_-) = (v_- - v_+)\frac{p_+ + p_-}{2}. \tag{2.11}$$

In fact, by the third equation of (1.8), we have

$$\begin{aligned}
&\frac{\alpha+\alpha_f}{2}(p_+v_+ - p_-v_-) \\
&= \frac{1}{\sigma}(p_+u_+ - p_-u_-) - \frac{u_+^2 - u_-^2}{2} \\
&= \frac{1}{\sigma}(p_+u_+ - p_-u_-) - \frac{u_+ + u_-}{2}(u_+ - u_-).
\end{aligned} \tag{2.12}$$

By the second equation of (1.8), we have $(u_+ - u_-) = \frac{1}{\sigma}(p_+ - p_-)$. This, together with (2.12), implies that

$$\begin{aligned}
&\frac{\alpha+\alpha_f}{2}(p_+v_+ - p_-v_-) \\
&= \frac{1}{\sigma}\left(p_+u_+ - p_-u_-) - \frac{p_+ - p_-}{2}(u_+ + u_-)\right) \\
&= \frac{1}{\sigma}\left(\frac{1}{2}p_+u_+ - \frac{1}{2}p_-u_- - \frac{1}{2}p_+u_- + \frac{1}{2}p_-u_+\right) \\
&= \frac{1}{\sigma}(u_+ - u_-)(p_+ + p_-).
\end{aligned} \tag{2.13}$$

This proves (2.11). Divided by p_-v_+ both sides of (2.11), we get

$$\frac{\alpha+\alpha_f}{2}\left(\frac{p_+}{p_-} - \frac{v_-}{v_+}\right) = \left(\frac{v_-}{v_+} - 1\right)\frac{\frac{p_+}{p_-} + 1}{2}.$$

We solve for $\frac{v_-}{v_+}$ from this to get

$$\frac{v_-}{v_+} = \frac{(\alpha+\alpha_f)\frac{p_+}{p_-} + \frac{p_+}{p_-} + 1}{(\alpha+\alpha_f) + \frac{p_+}{p_-} + 1}. \tag{2.14}$$

It is easy to verify that $v_+ < \bar{v}$ is equivalent to

$$\sigma^2 < (1 + \frac{2}{\alpha})\frac{p_+}{v_+}. \tag{2.15}$$

From the first and second equations of (1.8), we know that

$$\sigma^2 = \frac{p_- - p_+}{v_+ - v_-} \tag{2.16}$$

So $v_+ < \bar{v}$ is equivalent to

$$\frac{p_- - p_+}{v_+ - v_-} < (1 + \frac{2}{\alpha})\frac{p_+}{v_+} \tag{2.17}$$

Now we use (2.14) to show (2.17) if (1.16) is true.

By (2.14), we have

$$\begin{aligned}
\frac{p_- - p_+}{v_+ - v_-} &= \frac{(\frac{p_-}{p_+}) - 1}{1 - (\frac{v_-}{v_+})}(\frac{p_+}{v_+}) \\
&= \frac{(\frac{p_-}{p_+}) - 1}{1 - \frac{(\alpha+\alpha_f)(\frac{p_+}{p_-})+(\frac{p_+}{p_-})+1}{(\alpha+\alpha_f)+(\frac{p_+}{p_-})+1}}(\frac{p_+}{v_+}) \\
&= \frac{(\frac{p_-}{p_+}) - 1}{(\alpha+\alpha_f)(1 - \frac{p_+}{p_-})}[(\alpha+\alpha_f) + \frac{p_+}{p_-} + 1](\frac{p_+}{v_+}) \\
&= \frac{(\frac{p_-}{p_+}) - 1}{1 - (\frac{p_+}{p_-})}(1 + \frac{(\frac{p_+}{p_-}) + 1}{\alpha+\alpha_f})(\frac{p_+}{v_+}) \\
&= \frac{(\frac{p_-}{p_+})(\frac{p_-}{p_+} - 1)}{(\frac{p_-}{p_+}) - 1}(1 + \frac{(\frac{p_+}{p_-}) + 1}{\alpha+\alpha_f})(\frac{p_+}{v_+}) \\
&= (\frac{p_-}{p_+})(1 + \frac{(\frac{p_+}{p_-}) + 1}{\alpha+\alpha_f})(\frac{p_+}{v_+}) \\
&= (\frac{p_-}{p_+} + \frac{1}{\alpha+\alpha_f}(1 + \frac{p_-}{p_+}))(\frac{p_+}{v_+}) \\
&= ((1 + \frac{1}{\alpha+\alpha_f})(\frac{p_-}{p_+}) + \frac{1}{\alpha+\alpha_f})(\frac{p_+}{v_+})
\end{aligned} \tag{2.18}$$

So, if (1.16) holds, then we have

$$\begin{aligned}
&(1 + \frac{1}{\alpha+\alpha_f})\frac{p_-}{p_+} + \frac{1}{\alpha+\alpha_f} \\
&< (1 + \frac{1}{\alpha+\alpha_f})(1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}) + \frac{1}{\alpha+\alpha_f} \\
&= 1 + \frac{1}{\alpha+\alpha_f}(2 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}) + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)} \\
&= 1 + \frac{1}{\alpha+\alpha_f}(\frac{2\alpha(1+\alpha+\alpha_f) + 2\alpha_f}{\alpha(1+\alpha+\alpha_f)}) + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)} \\
&= 1 + \frac{2}{\alpha+\alpha_f}\frac{(\alpha+\alpha_f)(1+\alpha)}{\alpha(1+\alpha+\alpha_f)} + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)} \\
&= 1 + \frac{2(1+\alpha)}{\alpha(1+\alpha+\alpha_f)} + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)} \\
&= 1 + \frac{2}{\alpha}
\end{aligned} \tag{2.19}$$

(2.17) follows from (2.18) and (2.19). This proves 1). 2) and 3) follow from the same arguments.

Proof of Theorem 1:

Let

$$G(v) = (\alpha + \alpha_f)pv + u^2 - \frac{2(pu - Q)}{\sigma} \tag{2.20}$$

It yields from (1.14) and (2.20) that

$$G(v) = (\alpha+\alpha_f)(m\sigma+P-\sigma^2 v)v+(m-\sigma v)^2-\frac{2}{\sigma}\left((m-\sigma v)(m\sigma+P-\sigma^2 v)-Q\right), \tag{2.21}$$

where m, P, Q are given in (1.14) So, $G(v)$ is a quadratic function of v. Moreover, by (1.14) and (1.15), we have

$$G(v_+) = G(v_-) = 0. \tag{2.22}$$

Therefore,

$$G(v) = -\beta(v - v_-)(v - v_+) \tag{2.23}$$

for some constant β. By comparing (2.22) with (2.21), we get $\beta = \sigma^2(1 + \alpha + \alpha_f)$ Hence,

$$G(v) = -\sigma^2(1 + \alpha + \alpha_f)(v - v_-)(v - v_+) \tag{2.24}$$

So

$$G(v) > 0 \tag{2.25}$$

as $v_- < v < v_+$.
In case 1), we choose a constant v_0 satisfying $v_- < v_0 < v_+$ and set $v(0) = v_0$. Then we have from (2.3) that

$$\int_{v_0}^{v} \frac{2f(v)}{G(v)} dv = \frac{1}{\sigma}\xi. \tag{2.26}$$

Also, by Lemma 2.1, and (2.24), we have, if

$$\frac{p_-}{p_+} < 1 + \frac{2\alpha_f}{\alpha(1 + \alpha + \alpha_f)},$$

then $\frac{f(v)}{G(v)} > 0$ for $v_- < v < v_+$, and

$$\int_{v_0}^{v_+} \frac{2f(v)}{G(v)} dv = +\infty, \tag{2.27}$$

$$\int_{v_0}^{v_-} \frac{2f(v)}{G(v)} dv = -\infty. \tag{2.28}$$

Therefore, the map: $\xi \to v(\xi)$ is a one-to-one and onto map from $(-\infty, +\infty)$ to (v_-, v_+). Moreover, it follows from (2.27) and (2.28) that

$$v(-\infty) = v_-, v(+\infty) = v_+.$$

This proves 1) in Theorem 1.

If

$$\frac{p_-}{p_+} \geq 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}, \tag{2.29}$$

by Lemma 2.1, we know that $v_- < \bar{v} \leq v_+$. In this case, we use the proof by contradiction to prove 2) in Theorem 1 as follows. Suppose that the problem (1.12) and (1.13) has a solution $v(\xi)$. Since in this case, $v_- < \bar{v} \leq v_+$, and $f'(v) < 0$, we have $f(v_-) > 0 \geq f(v_+)$. We may write (2.3) as

$$2\sigma f(v) \frac{dv}{d\xi} = G(v). \tag{2.30}$$

Since $v(-\infty) = v_-$ and $f(v) > 0$ for $v_- < v < \bar{v}$ and $G(v) > 0$ for $v_- < v < v_+$, we have $\frac{dv}{d\xi} > 0$ when $v_- < v < \bar{v} < v_+$. For a constant v_1 satisfying $v_- < v_1 < \bar{v}$, there exists $\xi_1 \in (-\infty, +\infty)$ such that $v(\xi_1) = v_1$. It follows from (2.30) that

$$\int_{v_1}^{v} \frac{2\sigma f(w)}{G(w)} dw = \xi - \xi_1. \tag{2.31}$$

By (1.15), we know that $f(v)$ is a linear function of v in the form

$$f(v) = -k(v - \bar{v}), \tag{2.32}$$

where $k = \sigma^2(1+\alpha)$. It follows from (2.23), and the fact that $v_- < v < \bar{v} \leq v_+$ (when (2.29) holds) that

$$\int_{v_1}^{\bar{v}} \frac{2\sigma f(w)}{G(w)} dw < +\infty. \tag{2.33}$$

We let

$$\bar{\xi} = \xi_1 + \int_{v_1}^{\bar{v}} \frac{2\sigma f(w)}{G(w)} dw.$$

Then by (2.30), we can conclude that, as $\xi \to \xi_1$, $v(\xi) \to \bar{v}$ and $\frac{dv(\xi)}{d\xi} \to +\infty$. This proves 2) in Theorem 1.

We can prove 3) in Theorem 1 as follows. We have already proved that if $\frac{p_-}{p_+} < 1 + \frac{2\alpha_f}{\alpha(1+\alpha+\alpha_f)}$, the the problem (1.13) and (1.12) has a soluion. In this case, $v'(\xi) > 0$ is an easy consequence of the above argument in 1). So $v_- < v(\xi) < v_+$ for $-\infty < \xi < +\infty$. Next, we prove (1.18). We may write (2.26) as, in view of (2.26) and (2.24)

$$\int_{v_0}^{v} \frac{2f(w)}{(w - v_-)(w - v_+)} dw = -\sigma(1 + \alpha + \alpha_f)\xi \tag{2.34}$$

It is easy to verify that, by noting that $f(w) = (1 + \frac{\alpha}{2})m - \sigma^2(1 + \alpha)w$ (see (1.15))

$$\frac{2f(w)}{(w - v_-)(w - v_+)} = \frac{-2f(v_-)}{(w - v_-)} \frac{1}{(v_+ - v_-)} + \frac{2f(v_+)}{(w - v_+)} \frac{1}{(v_+ - v_-)} \tag{2.35}$$

(1.18) then follows from (2.34) and (2.35). From (1.18), we can easily get the bounds for $v_+ - v(\xi)$ in (1.21). Similarly, we can get the bounds for $v(\xi) - v_-$ in (1.22). By (1.18), we have

$(\frac{2f(v_+)}{v_+ - v} + \frac{2f(v_-)}{v - v_-})v'(\xi) = -\sigma(1 + \alpha + \alpha_f)(v_+ - v_-)$

Therefore, the bounds for $v'(\xi)$ in (1.21) and (1.22) can be derived from the bounds of $v_+ - v(\xi)$ and $v(\xi) - v_-$ which we have just proved. This finishes the proof of Theorem 1.

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