最优交通拥堵费定价研究

The Optimal Pricing for Traffic Congestion

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Content

Abstract

This paper investigates how to efficiently allocate the scarce resources of transportation by using the price mechanism. High time cost consumer can have priority to access the transportation resources by paying the congestion tolls, which reduces the overall social welfare loss resulting from traffic congestion. Minimizing the overall social welfare loss, we build up an optimal pricing model for congestion tolls, considering factors including distributional randomness of individual time cost, differentiate transportation capacity, and variation of traffic flow during driving peak and off peak time. The numerical analysis employing data from Nanjing implies the significant positive impact of congestion tolls on social welfare. Furthermore, we show that significant influence of the price mechanism to the behavior of drivers as well as the driving speed of toll facilities.

Keywords: Congestion Tolls; Price Mechanism; Optimal Pricing; Truncated Normal Distribution

I. Introduction and Related Literatures

Traffic congestion, resulting from more lagging construction of urban infrastructures than rapid economic development, has increasingly become a serious social problem, and the way to minimize traffic congestion costs under constraints of scarce road resources has become a hot topic drawing the public's attention. This paper discusses the way to minimize social costs of traffic congestion through the price mechanism based upon a congestion charge system.

The traffic congestion charge system essentially is a price mechanism, broadly speaking, it contains all charges levied on road users for traffic congestion adjustment, including charges levied on tunnels, bridges, highways and other road infrastructures, as well as road use charges levied against specific regions and time periods. In the case of serious urban traffic congestion, the congestion charge is important for public regulators in traffic demand guidance and adjustment, and thus in realization of overall social welfare improvement.

Vehicles in traffic environment that lacks of congestion charge system may excessively occupy scarce traffic resources, so all road users may expend certain time on waiting and thus result in certain time costs, therefore a total of the individual time costs borne by all road users may constitute the total social costs, that is, the total social welfare losses caused by traffic congestion. As unit time cost of different road users varies, road users bearing a higher time cost will, despite of the same time period for traffic congestion, bear more time losses than those bearing a lower cost, resulting in higher total social welfare losses. Price mechanism based upon the congestion charge system can be used for road demand management. Without compromising fairness as much as possible, public regulators will effectively reduce the overall social welfare losses by allowing road users bearing a higher time cost to use road resources with priority after paying toll.

This paper discusses the optimal social congestion charges minimizing the total social welfare losses when unit time cost of different individuals varies. This paper has firstly constructed a road use model that individual time costs satisfy certain probability distribution, and derived a function of the total social welfare losses. Secondly, this paper respectively assumes distributions of individual time costs as uniform distribution and truncated normal distribution and, based upon the field research data coming from the Nanjing Yangtze River Tunnel and the Nanjing

Yangtze River Bridge, calculates and analyzes the optimal congestion charges by numerical analysis. It concludes that overall social welfare losses will be effectively reduced by allowing road users bearing a higher time cost to use road resources with priority after paying toll.

A variety of literatures have discussed on the traffic congestion and congestion charges. Professor Pigou from the University of Cambridge firstly proposed the theory of traffic congestion in 1920, and indicated that the demand for road traffic involved complicated traffic behaviors. On the basis of the research of Pigou (1920), Button and Verhoef (1998) constructed a static pricing model of congestion charges. Virkrey, Nobel laureate in economics, established a congestion model for bottleneck sections in 1969, afterwards, Braid (1989), Arnott (1993) and a plenty of researchers have developed this model into a dynamic pricing model of congestion. However, models established above may not apply to special total social welfare analysis, and a majority of literatures only take into account the condition of a single road, but ignore other alternatives for road users. This paper has constructed a pricing model of congestion charges based upon the total social welfare losses minimization, and taken into account the presence of multiple roads selective for road users. This paper has further taken into account the pricing rules of optimal congestion charges in presence of peak and off peak hours for traffic. In addition, This paper has also brought the uncertainty (caused by differences of individual time costs) into the model.

The structure of this paper is as follows: Part II constructs an optimal pricing model of congestion charges based upon total social welfare losses minimization; Part III makes data description, analyzes the optimal level of charges for the Nanjing Yangtze River Tunnel and the Nanjing Yangtze River Bridge under the assumption of uniform distribution and truncated normal distribution, and makes an in-depth discussion on price mechanism based upon the congestion charge system; Part IV concludes the paper.

II. Model

(I) Basic assumptions

It is assumed that road 1 and road 2 can be selected to move from place A to place B. Road capacity s_i of road i (i = 1,2) is defined as the maximum traffic flow loadable by road i per unit time. If we assume that the road capacity of road 1 is higher than that of road 2 (that is, $s_1 > s_1$) and, in view of the presence of peak and off peak hours for traffic actual traffic flow during peak hours as N^H, during off peak hours as N^L ($N^H > N^L$), and actual traffic flow of road i during the m time period as N_i^m ($m \in \{H, L\}$), then

$$N_1^m + N_2^m = N^m \tag{1.1}$$

If t_i^m indicates the time required to transport N_i^m vehicles with road i during the time period m, then

$$t_i^m = \frac{N_i^m}{s_i} \tag{1.2}$$

Let L_j be the unit time cost of No. j vehicle and satisfies the continuous probability distribution $F(\cdot)$ (j=1，2…) on $[0, \bar{L}]$....

We define the total social welfare losses caused by traffic congestion as the total time cost of all vehicles during the peak and off peak hours:

$$W = \sum_{m\in\{H,L\}} \sum_{j=1}^{N^m} \int_0^{\bar{L}} t^m(L_j) L_j dF(L_j) \tag{1.3}$$

where $t^m(L_j)$ indicates the travel time for No. j vehicle moving from place A to place B during time period m, for any $j \in \{1, 2, \cdots, N^m\}$, $t^m(L_j) \in \{t_1^m, t_2^m\}$.

(II) Travel road selection of vehicles and total social welfare with unavailability of the congestion charge system

Time required for vehicles to pass through road 1 and road 2 must be equal if the congestion charge system is unavailable. Because roads that can be passed through in a shorter travel time will attract more vehicles, until the time for travel on the two roads turns to be equal. Therefore, for any $m \in \{H, L\}$, by the following conditions:

$$\begin{cases} t_1^m = t_2^m \\ t_1^m = \dfrac{N_1^m}{s_1} \\ t_2^m = \dfrac{N_2^m}{s_2} \\ N_1^m + N_2^m = N^m \end{cases} \tag{2.1}$$

We may conclude:

$$t_1^m = t_2^m = \frac{N^m}{s_1 + s_2}, N_1^m = \frac{s_1}{s_1 + s_2} N^m, N_2^m = \frac{s_2}{s_1 + s_2} N^m \tag{2.2}$$

From the above equation, it is evident that enhancement of the road capacity will accelerate the transportation efficiency. If $N^H > N^L$, then $t_1^H > t_1^L$, it indicates that traffic flow at peak hours will be heavier. For any time period, if $s_1 > s_1$, then $N_1^m > N_2^m$, it indicates that more vehicles will take the initiative to drive on roads with relatively higher road capacity available.

Total social welfare losses are:

$$\begin{aligned} W &= N^H \int_0^{\bar{L}} t_1^H L_j dF(L_j) + N^L \int_0^{\bar{L}} t_1^L L_j dF(L_j) \\ &= \left[(N^H)^2 + (N^L)^2 \right] \frac{\int_0^{\bar{L}} L_j dF(L_j)}{s_1 + s_2} \end{aligned} \tag{2.3}$$

According to the above equation, the total social welfare losses will increase as the traffic flow N increases. Only enhancement of the road capacity s_1 and s_2 can reduce the total social welfare losses when the congestion charge system is unavailable.

(III) Travel road selection of vehicles and optimal level of charges under the congestion charge system

We assume that crossing fees are charged on road 1, if the toll for road 1 is set as $P > 0$, then, for any vehicle j, if and only if the total cost charged on road 1 (including time costs and tolls) is less than that of road 2 (only including the time cost), that is, $t_1^m L_j + P < t_2^m L_j$, road 1 will be selected by vehicle j; if and only if $t_1^m L_j + P \geq t_2^m L_j$, road 2 will be selected by vehicle j. Therefore, for any vehicle j,

$$t^m(L_j)=\begin{cases} t_1^m, & t_1^m L_j + P < t_2^m L_j \\ t_2^m, & t_1^m L_j + P \geq t_2^m L_j \end{cases} \tag{3.1}$$

For any toll P, L_0^m is available under the equilibrium state to enable

$$t_1^m L_0^m + P = t_2^m L_0^m \tag{3.2}$$

Then, vehicles j satisfying $L_j > L_0^m$ will choose road 1, while vehicles j satisfying $L_j \leq L_0^m$ will choose road 2. In that case, actual traffic flow of road 1 and road 2 during the time period m will respectively be:

$$N_1^m = N^m \Pr(L_j > L_0^m) \tag{3.3}$$

$$N_2^m = N^m - N_1^m \tag{3.4}$$

Given $F(L_j)$, we may conclude the following from(1.1), (1.2), (3.2) and (3.4):

$$L_0^m = L_0^m(P, N^m, s_1, s_2) \tag{3.5}$$

According to the L_0^m, we can further solve for t_1^m and t_2^m, and calculate the total social welfare losses when the congestion charge system is available[①]:

$$W = \sum_{m\in\{L,H\}} \left(N_1^m \int_{L_0^m}^{\bar{L}} t_1^m L_j dF(L_j) + N_2^m \int_0^{L_0^m} t_2^m L_j dF(L_j) \right) \tag{3.6}$$

Optimal level of congestion charge minimizes the total social welfare losses:

$$P^* = \arg\min W \tag{3.7}$$

The above equation is the optimal pricing function for congestion charges based upon minimization of the total social welfare losses.

In part III, we examine the optimal level of congestion charges when L_j is under uniform distribution and truncated normal distribution and, based upon the field research data made on the Nanjing Yangtze River Tunnel and the Nanjing Yangtze River Bridge, we calculate and analyze the optimal and specific congestion charge by numerical analysis.

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① It is noted that tolls payment will not be calculated into the total social welfare losses, because, from the view of the entire society, tolls payment will only lead to the transfer and redistribution of wealth, not the reduction of total social wealth.

III. Numerical Analysis

In this part, we will construct a theoretical model when L_j is under uniform distribution and truncated normal distribution. Further, we will, based upon the field research data made on the Nanjing Yangtze River Tunnel and the Nanjing Yangtze River Bridge by a project team, conduct numerical analysis on the model constructed using mathematical software Matlab and Mathematica, and discuss the optimal level of congestion charges.

(I) Data description

s_1 indicates the road capacity of the Nanjing Yangtze River Tunnel (hereinafter referred to as the "Tunnel"), and s_2 indicates the road capacity of the Nanjing Yangtze River Bridge (hereinafter referred to as the "Bridge"). According to historical records, daily traffic flow designed at the Tunnel is of 40,000, and that of the Bridge is of 20,000, then we assume that

$$s_1 = 2s_2 \tag{4.1}$$

In order to obtain actual and specific s_1 and s_2, the Team members have carried out field research on Tunnel road capacity on August 13, 2012, and obtained the statistical data in Table 1. It indicates that the Tunnel traffic flow during peak hours is 3,816 per hour, and researchers also find that vehicles will, even in peak hours from 17:30 to 18:30, be able to pass through the Tunnel in 3 minutes on average, therefore, we may assume that $s_1 = 4000$ and $s_2 = 2000$ according to data in Table 1 and the actual situation.

Table 1 Traffic Flow Statistics of the Tunnel made on August 13, 2012

Time	Vehicles out (unit)	Vehicles in (unit)	Traffic Flow per hour in corresponding time period
12:35 to 12:40	67		1716
12:45 to 12:50		76	
16:00 to 16:05	98		2520
16:06 to 16:11		112	

17:46 to 17:51*	131		3816
17:52 to 17:57*		187	

Note: The symbol * indicates that the time period is in peak hours for daily traffic.

(II) Optimal level of congestion charges under uniform distribution

This part examines the way to adjust road capacity and improve social welfare through the congestion charge system when unit cost of vehicles satisfies the uniform distribution. We will firstly construct a theoretical model under uniform distribution based upon part II of this paper, and secondly, by numerical analysis, seek after the optimal level of charges and reveal and discuss the way for the price mechanism based upon the congestion charge system to affect social welfare, travel road selection of vehicles and transportation efficiency.

(1)Theoretical model

If L_j satisfies uniform distribution on $[0,\bar{L}]$, the following conditions can be derived from (2.2), (2.3) and (2.4):

$$\begin{cases} \frac{N_1^m}{s_1}L_0^m + P = \frac{N_2^m}{s_2}L_0^m \\ N_1^m = N\frac{\bar{L}-L_0^m}{\bar{L}} \\ N_2^m = N\frac{L_0^m}{\bar{L}} \end{cases} \tag{4.2}$$

Then we have:

$$L_0^m = \frac{s_2\bar{L} + \sqrt{(s_2\bar{L})^2 + \frac{4(s_1+s_2)s_1s_2\bar{L}P}{N^m}}}{2(s_1+s_2)} \tag{4.3}$$

Total social welfare losses are:

$$W = \sum_m \frac{(N^m)^2}{2\bar{L}^2}\left[\frac{(\bar{L}-L_0^m)^2(\bar{L}+L_0^m)}{s_1}+\frac{(L_0^m)^3}{s_2}\right] \quad (4.4)$$

(2)Calculations and analysis

We will firstly calculate the optimal level of congestion charges at the Tunnel minimizing the total welfare losses when actual traffic flow at peak hours N^H are respectively of 10,000, 15,000 and 20,000, actual traffic flow at off peak hours N^L are respectively of 6,000 and 10,000, and maximum unit time costs of vehicles are respectively of RMB 10, 20, 30, 50 and 100. Results are shown in Table 2.

Table 2 Optimal Level of Congestion Charges and Social Welfare Improvement under Uniform Distribution

N^H (ten thousand units)	N^L (ten thousand units)	$\bar{L}$ (RMB)	P^* (RMB)	W ($\times 10^5$ RMB)	W' ($\times 10^5$ RMB)	Social Welfare Improvement ($1-W/W'$)
1.0	0.6	10	3.67	1.06	1.13	6.66%
1.5	0.6	10	5.06	2.04	2.18	6.24%
2.0	1.0	10	7.01	3.90	4.17	6.47%
1.0	0.6	20	7.34	2.12	2.27	6.66%
1.5	0.6	20	10.11	4.08	4.35	6.24%
2.0	1.0	20	14.02	7.79	8.33	6.47%
1.0	0.6	30	11.01	3.17	3.40	6.66%
1.5	0.6	30	15.17	6.12	6.53	6.24%
2.0	1.0	30	21.03	11.69	12.50	6.47%
1.0	0.6	50	18.34	5.29	5.67	6.66%
1.5	0.6	50	25.28	10.20	10.88	6.24%
2.0	1.0	50	35.06	19.48	20.83	6.47%
1.0	0.6	100	36.69	10.58	11.33	6.66%

1.5	0.6	100	50.56	20.39	21.75	6.24%
2.0	1.0	100	70.12	38.97	41.67	6.47%

Note: W indicates the total social welfare losses of the Tunnel under the optimal level of congestion charges P^*, W' indicates the total social welfare losses if the congestion charge system is unavailable. $1-W/W'$ reflects the reduction of total social welfare losses after tolls charged at the Tunnel.

It can be seen from Table 2 that, when unit cost of vehicles satisfies uniform distribution, tolls charged at the Tunnel based upon minimized total social time costs will effectively reduce the total social time costs by 6% to 7%. The congestion charge system enables crowd with higher unit cost (that is, higher requirements on road traffic) to pass through in a shorter time after completion of payment less than their time costs. From the perspective of society as a whole, we may give priority to crowd with higher requirements on road traffic to pass through by use of the price mechanism and meanwhile control losses thus caused to crowd with lower requirements on road traffic within certain limits, thereby effectively improving the transportation efficiency aimed at the overall social welfare.

We can also come to conclusions as below by further observations towards Table 2. Firstly, the higher N^H (that is, the higher actual traffic flow during peak hours together with the higher maximum unit time costs of vehicles), the higher optimal tolls charged by the Tunnel, it fully reflects the economic laws of "high demand - high price". Secondly, we also find that, under the uniform distribution, improvement of the total social welfare losses is mainly affected by traffic flow at peak hours and off peak hours, while welfare improvement will not be affected by the changes of $\bar{L}$.

Based upon the theoretical model constructed in this part, we can analyze variations of travel road selection and speed of vehicles in Tunnels by numerical analysis (See Table 3 and Table 4).

Table 3 shows effects of variations of actual traffic flow on travel road selection of vehicles. The left figure of Table 3 shows the increase of actual traffic flow during peak hours (keep traffic flow at off peak hours unchanged), the right figure of Table 3 shows the increase of actual traffic flow during off peak hours (keep traffic flow at peak hours unchanged). The horizontal axis represents the actual traffic flow, and the vertical axis represents the proportion of vehicles selecting to travel in Tunnel by paying tolls, that is $\frac{N_1^m}{N^m}$ ($m \in \{H, L\}$). On the whole, proportion of vehicles selecting the Tunnel at peak hours is relatively stable, less affected by variations of actual traffic flow; on the contrary, proportion of vehicles selecting the Tunnel at off peak hours will encounter a great change when actual traffic flow varies. Specifically, the left figure of Table 3 indicates that amount of vehicles selecting the Tunnel will increase slowly as traffic flow at peak hours increases, while vehicles selecting the Tunnel will reduce rapidly at off peak hours. This is because that increase of traffic flow enables the improvement of P^*, thereby greatly reducing the tendency of road users to select the Tunnel at off peak hours. Similarly, we can observe from the right figure of Table 3 that P^* increases due to increase of traffic flow at off peak hours and that amount of vehicles selecting the Tunnel during peak hours is reduced.

Table 3 Variations of Travel Road Selection of Vehicles

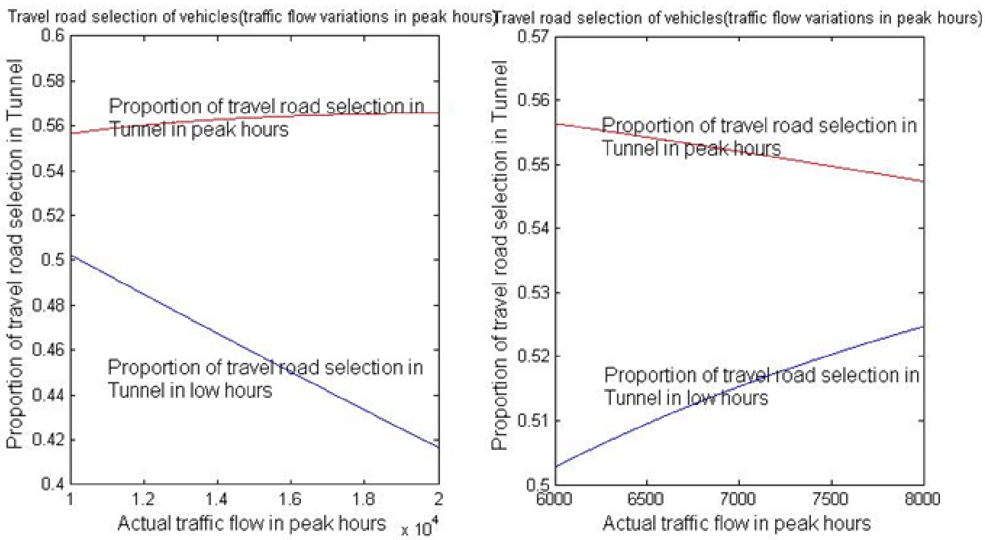
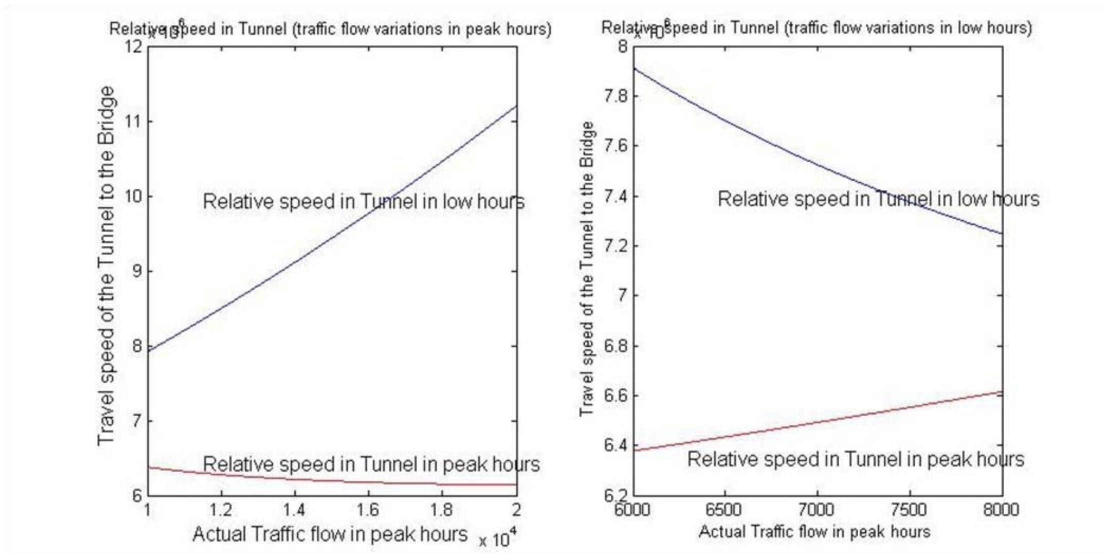


Table 4 reflects the way for travel speed in Tunnel to change as traffic flow varies. The travel speed in Tunnel stated in Table 4 is a relative speed to that on the Bridge, that is, $\frac{t_2^m}{t_1^m}$ ($m \in \{H, L\}$). Similar to Table 3, whether in peak hours or off peak hours, the changes to the actual traffic flow will not significantly impact the travel speed in Tunnel during peak hours, but will have an obvious impact on that during off peak hours. In view of the left figure of Table 4, travel speed in Tunnel in peak hours declines gradually as actual traffic flow increases at the same time, while the travel speed in Tunnel in off peak hours increases rapidly. This phenomenon can be explained by use of the price mechanism according to Table 3. As shown in Table 3, as traffic flow in peak hours increases, P^* will be adjusted accordingly and enable the proportion of vehicles traveling in Tunnel to be relatively stable, and thus keep the relative speed in Tunnel stable. However, for off peak hours, relative time for vehicles passing through the Tunnel will be rapidly shortened as vehicles traveling in Tunnel drastically reduce. Similarly, we can explain the phenomenon shown in the right figure of Table 4.

In view of the above analysis, it can be seen that the price mechanism is vital in improving social welfare, regulating road capacity and enhancing traffic efficiency.

Table 4 Variations for Travel Speed in Tunnel



(III) Optimal level of congestion charges under truncated normal distribution

(1) Theoretical model

The $\overline{L}$ is the maximum unit time cost for vehicles, and we assume that the standard deviation of original normal distribution σ satisfies:

$$2\sigma = \overline{L} \tag{4.5}$$

According to the property of normal distribution, the assumption enables $\overline{L}$ to be effectively in line with the meaning of "maximum". To simplify the analysis, we further assume that the mean value of normal distribution is 0 (zero). According to the property of normal distribution,

$$\frac{2L_j}{\bar{L}} \sim N(0,1) \tag{4.6}$$

The L_j is truncated at 0 and $\bar{L}$, according to the property of truncated normal distribution,

$$\begin{aligned} \Pr\left(L_j > L_0^m \middle| 0 \le L_j \le \bar{L}\right) &= \Pr(\frac{2L_j}{\bar{L}} > \frac{2L_0^m}{\bar{L}} \middle| 0 \le \frac{2L_j}{\bar{L}} \le 2) \\ &= \frac{\Phi(2) - \Phi(\frac{2L_0^m}{\bar{L}})}{\Phi(2) - \Phi(0)} \\ &\approx 2 - 2\Phi(\frac{2L_0^m}{\bar{L}}) \end{aligned} \tag{4.7}$$

$$\begin{aligned} \Pr\left(L_j \le L_0^m \middle| 0 \le L_j \le \bar{L}\right) &= \frac{\Phi(\frac{2L_0^m}{\bar{L}}) - \Phi(0)}{\Phi(2) - \Phi(0)} \\ &\approx 2\Phi(\frac{2L_0^m}{\bar{L}}) - 1 \end{aligned} \tag{4.8}$$

In which, $\Phi(\cdot)$ is the cumulative distribution function of normal distribution. For any value of m, we can solve for the following by substitution of (4.7) and (4.8) into (3.2):

$$P = N^m L_0^m \left[\frac{2\Phi(\frac{2L_0^m}{\bar{L}}) - 1}{s_2} - \frac{2 - 2\Phi(\frac{2L_0^m}{\bar{L}})}{s_1} \right] \tag{4.9}$$

According to the property of truncated normal distribution, for $\varepsilon \sim N(0,1)$,

$$E(\varepsilon | \varepsilon > c) = \frac{\phi(c)}{1 - \Phi(c)} \tag{4.10}$$

$$E(\varepsilon | 0 \le \varepsilon \le c) = \frac{\phi(0) - \phi(c)}{\Phi(c) - \Phi(0)} \tag{4.11}$$

In which, $\phi(\cdot)$ is the probability density function of standard normal distribution.

By use of (4.10) and (4.11), we can conclude

$$
\begin{aligned}
\int_{L_0^m}^{\bar{L}} L_j dF(L_j \,|\, 0 \le L_j \le \bar{L}) &= \int_{L_0^m}^{\bar{L}} L_j \frac{f(L_j, 0 \le L_j \le \bar{L})}{\Phi(2)-\Phi(0)} dL_j \\
&= \frac{1}{\Phi(2)-\Phi(0)} \int_{-\infty}^{+\infty} L_j f(L_j, L_0^m \le L_j \le \bar{L}) dL_j \\
&= \frac{1}{\Phi(2)-\Phi(0)} \int_{-\infty}^{+\infty} L_j f(L_j \,|\, L_0^m \le L_j \le \bar{L}) \left[\Phi(2) - \Phi(\frac{2L_0^m}{\bar{L}})\right] dL_j \\
&= \left[\frac{\Phi(2)-\Phi(\frac{2L_0^m}{\bar{L}})}{\Phi(2)-\Phi(0)}\right] E(L_j \,|\, L_0^m < L_j \le \bar{L}) \\
&= \left[\frac{\Phi(2)-\Phi(\frac{2L_0^m}{\bar{L}})}{\Phi(2)-\Phi(0)}\right] \frac{\bar{L}}{2} E(\frac{2L_j}{\bar{L}} \,\Big|\, \frac{2L_0^m}{\bar{L}} < \frac{2L_j}{\bar{L}} \le 2) \\
&\approx \bar{L}\phi(\frac{2L_0^m}{\bar{L}})
\end{aligned}
$$

(4.12)

Similarly

$$
\int_0^{L_0^m} L_j dF(L_j) \approx \bar{L}\left[\phi(0) - \phi(\frac{2L_0^m}{\bar{L}})\right] \quad (4.13)
$$

By substitution of (4.12) and (4.13) into the total social welfare losses (3.6), we can solve for L_0^m, that is, the solution of the constrained optimization problem as below:

$$
\min_{\{L_0^H, L_0^L\}} W = \sum_{m\in\{L,H\}} (N^m)^2 \bar{L} \left[\frac{\left(2-2\Phi(\frac{2L_0^m}{\bar{L}})\right)^2 \phi(\frac{2L_0^m}{\bar{L}})}{s_1} + \frac{\left(2\Phi(\frac{2L_0^m}{\bar{L}})-1\right)^2 \left(\phi(0)-\phi(\frac{2L_0^m}{\bar{L}})\right)}{s_2} \right]
$$

$$
s.t. \quad N^H L_0^H \left[\frac{2\Phi(\frac{2L_0^H}{\bar{L}})-1}{s_2} - \frac{2-2\Phi(\frac{2L_0^H}{\bar{L}})}{s_1}\right] = N^L L_0^L \left[\frac{2\Phi(\frac{2L_0^L}{\bar{L}})-1}{s_2} - \frac{2-2\Phi(\frac{2L_0^L}{\bar{L}})}{s_1}\right] \ge 0
$$

(3.14)

By solving for the solution of the optimization problem L_0^m above and substituting it into (4.9), we will conclude the optimal level of congestion charge P^* when the unit time cost for vehicles is under truncated normal distribution.

(2) Calculations and analysis

We also calculate the optimal level of congestion charges and corresponding social welfare improvement when actual traffic flow at peak hours N^H are respectively of 10,000, 15,000 and 20,000, actual traffic flow at off peak hours N^L are respectively of 6,000 and 10,000, and maximum unit time costs of vehicles are respectively of RMB 10, 20, 30, 50 and 100. Calculations are shown in Table 5.

Table 5 Optimal Level of Congestion charges and Corresponding Social Welfare Improvement under Truncated Normal Distribution

N^H (ten thousand units)	N^L (ten thousand units)	$\bar{L}$ (RMB)	P^* (RMB)	W ($\times10^5$ RMB)	W' ($\times10^5$ RMB)	Welfare Improvement ($1-W/W'$)
1.0	0.6	10	4.11	0.41	0.904	54.65%
1.5	0.6	10	5.78	0.81	1.744	53.56%
2.0	1.0	10	7.97	1.53	3.336	54.14%
1.0	0.6	20	8.28	0.82	1.816	54.85%
1.5	0.6	20	11.69	1.61	3.48	53.74%
2.0	1.0	20	15.93	3.05	6.664	54.23%
1.0	0.6	30	12.10	1.23	2.72	54.78%
1.5	0.6	30	17.53	2.42	5.224	53.68%
2.0	1.0	30	23.80	4.58	10	54.20%
1.0	0.6	50	20.67	2.06	4.536	54.59%
1.5	0.6	50	29.02	4.03	8.704	53.70%
2.0	1.0	50	39.83	7.63	16.664	54.21%
1.0	0.6	100	41.16	4.11	9.064	54.66%

1.5	0.6	100	58.11	8.05	17.4	53.74%
2.0	1.0	100	80.55	15.25	33.336	54.25%

Note: The W indicates the total social welfare losses of the Tunnel under the optimal level of congestion charges P^*, the W' indicates the total social welfare losses if the congestion charge system is unavailable. The $1-W/W'$ reflects the reduction of total social welfare losses after tolls charged at the Tunnel.

The most significant difference between calculations in Table 5 and Table 2 is that the optimal congestion charges charged in Tunnel will greatly reduce the total social cost by at least 60% when unit time cost for vehicles satisfies truncated normal distribution, while the congestion charge system under uniform distribution can only reduce the total social costs by 6% to 7%. The calculations demonstrate that the price mechanism based on the congestion charge system has different effects for different distributions of unit time costs, and that public regulators should, during the establishment of charging policies, take into account the traffic flow, road capacity and other actual factors, and comprehensively examine related information to road users. In addition to welfare improvement, other natures of optimal congestion charges under truncated normal distribution are basically in line with that under uniform distribution.

IV. Conclusion

As an important tool to regulate traffic demand by public regulators, a reasonable congestion charge system is effective to alleviate traffic congestion and can enhance the level of social welfare. This paper has constructed the optimal pricing model of congestion charges based upon total social welfare losses minimization. And by using numerical analysis, we conclude that overall social welfare losses will be effectively reduced by allowing road users to bear a higher time cost in order to use road resources with priority after certain payment, or to bear a lower time cost to use other alternative roads. This paper also concludes that if there are multiple roads for selection and road capacities are different, relative travel speed and proportion for selection of roads with high road capacity during peak hours may be relatively stable, but may encounter large changes during off peak hours as traffic flow varies.

This paper concludes that public regulators, when managing the road demand by use of the price mechanism, need pay attention to the price mechanism that may have complicated influence to behaviors of road users. For example, in order to ensure the transport efficiency during peak hours, congestion charges on roads with high road capacity would be higher, therefore, road users would not select the road with high road capacity, thereby resulting in a waste of road resources.

On the basis of this paper, we can further examine the price mechanism of optimal congestion charges under conditions of other time cost distributions and availability of multiple alternative roads. At the same time, we can also discuss how to charge peak and off peak hours with different fees, and the way to use different charging methods in the most efficient manner (for example, charged by frequency, day, month and year, and charged by bundling with other transport services), etc.

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Appendix

1. Matlab program code calculated in Table 2

```
function y=findminWelfareUniform(NH,NL,s1,s2,Lup)
        y=fminbnd(@WelfareUniform,0,Lup);
            function y=WelfareUniform(P)
            y=NH^2/(2*Lup^2)*((Lup-((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup
        *P/NH))/(2*(s1+s2))))^2*(Lup+((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup
        *P/NH))/(2*(s1+s2))))/s1+((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/N
        H))/(2*(s1+s2)))^3/s2)+NL^2/(2*Lup^2)*((Lup-((s2*Lup+sqrt((s2*Lup)^2+4*(s1
        +s2)*s1*s2*Lup*P/NL))/(2*(s1+s2))))^2*(Lup+((s2*Lup+sqrt((s2*Lup)^2+4*(s1
        +s2)*s1*s2*Lup*P/NL))/(2*(s1+s2))))/s1+((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s
        1*s2*Lup*P/NL))/(2*(s1+s2)))^3/s2);
            end
end

function y=averageWelfare(x)
NH=x(1);
NL=x(2);
s1=x(3);
s2=x(4);
Lup=x(5);
y=(NH^2+NL^2)*Lup/(2*(s1+s2));
end

function y = WelfareUniform(x)
P=x(1);
NH=x(2);
NL=x(3);
s1=x(4);
s2=x(5);
Lup=x(6);
y=NH^2/(2*Lup^2)*((Lup-((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NH))/(2*(s1
+s2))))^2*(Lup+((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NH))/(2*(s1+s2))))/s1
+((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NH))/(2*(s1+s2)))^3/s2)+NL^2/(2*Lu
p^2)*((Lup-((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NL))/(2*(s1+s2))))^2*(Lup
+((s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NL))/(2*(s1+s2))))/s1+((s2*Lup+sqrt(
(s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NL))/(2*(s1+s2)))^3/s2);
end

clear;
s1=4000;
s2=s1/2;
Lup=50;
```

```
NH=10000;
NL=6000;
P=findminWelfareUniform(NH,NL,s1,s2,Lup);
x1=[P NH NL s1 s2 Lup];
x2=[NH NL s1 s2 Lup];
Wuniform=WelfareUniform(x1);
Waverage=averageWelfare(x2);
Wratio=Wuniform/Waverage;
L0H=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NH))/(2*(s1+s2));
L0L=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P/NL))/(2*(s1+s2));
N1H=1-L0H/Lup;
N1L=1-L0L/Lup;
N1Haverage=s1/(s1+s2);
t1H=NH*N1H/s1;
tHaverage=NH/(s1+s2);
tHratio=((1-N1H)*s1)/N1H*s2;
tLratio=((1-N1L)*s1)/N1L*s2;
result=[NH NL Lup P Wuniform Waverage Wratio N1H N1L t1H tHaverage tHratio tLratio];
```

2. Matlab program code calculated in Table 3

```
clear;
s1=4000;
s2=s1/2;
Lup=50;
NH=10000:2:20000;
NL=6000;
P=zeros(numel(NH),1);
L0H=zeros(numel(NH),1);
L0L=zeros(numel(NH),1);
N1H=zeros(numel(NH),1);
N2H=zeros(numel(NH),1);
for i=1:numel(NH)
    P(i)=findminWelfareUniform(NH(i),NL,s1,s2,Lup);
L0H(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NH(i)))/(2*(s1+s2));
    L0L(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NL))/(2*(s1+s2));
    N1H(i)=1-L0H(i)/Lup;
    N1L(i)=1-L0L(i)/Lup;
end
plot(NH,N1H)
hold on
plot(NH,N1L)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;
s1=4000;
```

```
s2=s1/2;
Lup=50;
NH=10000;
NL=6000:2:8000;
P=zeros(numel(NL),1);
L0H=zeros(numel(NL),1);
L0L=zeros(numel(NL),1);
N1H=zeros(numel(NL),1);
N2H=zeros(numel(NL),1);
for i=1:numel(NL)
    P(i)=findminWelfareUniform(NH,NL(i),s1,s2,Lup);
    L0H(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NH))/(2*(s1+s2));
    L0L(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NL(i)))/(2*(s1+s2));
    N1H(i)=1-L0H(i)/Lup;
    N1L(i)=1-L0L(i)/Lup;
end
plot(NL,N1H)
hold on
plot(NL,N1L)
```

3. Matlab program code calculated in Table 4

```
clear;
s1=4000;
s2=s1/2;
Lup=50;
NH=10000:2:20000;
NL=6000;
P=zeros(numel(NH),1);
L0H=zeros(numel(NH),1);
L0L=zeros(numel(NH),1);
N1H=zeros(numel(NH),1);
N2H=zeros(numel(NH),1);
tHratiozeros(numel(NH),1);
tLratio=zeros(numel(NH),1);
for i=1:numel(NH)
    P(i)=findminWelfareUniform(NH(i),NL,s1,s2,Lup);
   L0H(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NH(i)))/(2*(s1+s2));
    L0L(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NL))/(2*(s1+s2));
    N1H(i)=1-L0H(i)/Lup;
    N1L(i)=1-L0L(i)/Lup;
    tHratio(i)=((1-N1H(i))*s1)/N1H(i)*s2;
    tLratio(i)=((1-N1L(i))*s1)/N1L(i)*s2;
end
plot(NH,tHratio)
```

```
hold on
plot(NH,tLratio)
hold off
%%%%%%%%%%%%%%%%%
clear;
s1=4000;
s2=s1/2;
Lup=50;
NH=10000;
NL=6000:2:8000;
P=zeros(numel(NL),1);
L0H=zeros(numel(NL),1);
L0L=zeros(numel(NL),1);
N1H=zeros(numel(NL),1);
N2H=zeros(numel(NL),1);
tHratio=zeros(numel(NL),1);
tLratio=zeros(numel(NL),1);
for i=1:numel(NL)
    P(i)=findminWelfareUniform(NH,NL(i),s1,s2,Lup);
    L0H(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NH))/(2*(s1+s2));
    L0L(i)=(s2*Lup+sqrt((s2*Lup)^2+4*(s1+s2)*s1*s2*Lup*P(i)/NL(i)))/(2*(s1+s2));
    N1H(i)=1-L0H(i)/Lup;
    N1L(i)=1-L0L(i)/Lup;
    tHratio(i)=((1-N1H(i))*s1)/N1H(i)*s2;
    tLratio(i)=((1-N1L(i))*s1)/N1L(i)*s2;
end
plot(NL,tHratio)
hold on
plot(NL,tLratio)
hold off
```

4. Mathematica program code calculated in Table 5

```
(* 1. Calculate expression of the objective function and the constraints according to parameters in the first line of the Table*)
    (0)=PDF[NormalDistribution[0,1],0]
    ((2 Lh)/Lup)=PDF[NormalDistribution[0,1],(2 Lh)/ Lup]
    ((2 Ll)/ Lup)=PDF[NormalDistribution[0,1],(2 Ll)/ Lup]
    ((2 Lh)/ Lup)=CDF[NormalDistribution[0,1],(2 Lh)/ Lup]
    ((2 Ll)/ Lup)=CDF[NormalDistribution[0,1],(2 Ll)/ Lup](*define CDF and PDF of standard normal distribution *)(*Define CDFand PDF of standard normal distribution*)
Nh=10000
Nl=6000
Lup =10
s1=4000
```

s2=2000 (*Use parameters in the first line of the Table*)
W=Nh^2 Lup ((((0)- ((2 Lh)/ Lup)) (2 ((2 Lh)/ Lup)-1)^2)/s2+(((2 Lh)/ Lup) (2-2 ((2 Lh)/ Lup))^2)/s1)+Nl^2 Lup ((((0)- ((2 Ll)/ Lup)) (2 ((2 Ll)/ Lup)-1)^2)/s2+(((2 Ll)/ Lup) (2-2 ((2 Ll)/ Lup))^2)/s1)
(*W refers to the objective function required to be optimized*)
B=Lh *Nh ((2 ((2 Lh)/ Lup)-1)/s2-(2-2 ((2 Lh)/ Lup))/s1)-Ll *Nl ((2 ((2 Ll)/ Lup)-1)/s2-(2-2 ((2 Ll)/ Lup))/s1)
(*B refers to the constraints*)
(*2. Find the minimum of the objective function satisfying constraints and the corresponding Lh and Ll by drawings*)
Manipulate[ContourPlot[10000 Lh ((erf(Lh/(5 Sqrt[2]))-1)/4000+erf(Lh/(5 Sqrt[2]))/2000)-6000 Ll((erf(Ll/(5Sqrt[2]))-1)/4000+erf(Ll/(5Sqrt[2]))/2000) 0,{Lh,0,10},{Ll,0,10},RegionFunction ({Lh,Ll,z} 1000000000 (((Lh^2/50) (1-erf(Lh/(5 Sqrt[2])))^{2})/(4000 Sqrt[2])+((1/Sqrt[2]- (Lh^2/50)/Sqrt[2])erf(Lh/(5Sqrt[2]))^{2})/2000)+360000000(((Lh^2/50)(1-erf(Ll/(5Sqrt[2])))^{2})/(4000 Sqrt[2])+((1/Sqrt[2]- (Lh^2/50)/ Sqrt[2]) erf(Ll/(5Sqrt[2]))^{2})/2000) M)],{M,40000,90000}]
(*After concluding the expression of the objective function W and the constraint B, draw the constraint line in a two-dimensional plane with a horizontal axis of Lh and a vertical axis of Ll. Assume that the objective function W is less than or equal to M, then find the points set on the constraint line satisfying the condition assumed. Then we will continuously reduce the maximum value M of the objective function, until the points set on the constraint line is finally compressed to only one point, that is, a projection of the minimum of the objective function satisfying the constraints on the two-dimensional plane, then, the value of M is the minimum of the objective function satisfying the constraints, and thus we can find the minimum of the objective function and the corresponding Lh and Ll. The M in the following figure is a controllable switch, the leftmost takes the value of 40000, and the rightmost takes the value of 90000, when we adjust the M button from the right to the left, we can see that corresponding points set on the constraint line will be getting smaller and smaller, until finally to be a point, as gradually depressing the M button.*)
(*3. Calculate the value of P*)
(0)=PDF[NormalDistribution[0,1],0]
((2 Lh)/Lup)=PDF[NormalDistribution[0,1],(2 Lh)/ Lup]
((2 Ll)/ Lup)=PDF[NormalDistribution[0,1],(2 Ll)/ Lup]
((2 Lh)/ Lup)=CDF[NormalDistribution[0,1],(2 Lh)/ Lup
((2 Ll)/ Lup)=CDF[NormalDistribution[0,1],(2 Ll)/ Lup]
Nh=10000
Nl=6000
Lup)=10
s1=4000
s2=2000
Lh=3.35
Ll=3.9
Ph=Lh *Nh ((2 ((2 Lh)/ Lup))-1)/s2-(2-2 ((2 Lh)/ Lup)))/s1)
Pl=Ll* Nl ((2 ((2 Ll)/ Lup))-1)/s2-(2-2 ((2 Ll)/ Lup)))/s1)
(*After we conclude that Lh=3.35 and Ll=3.9, we will respectively calculate values of Ph and Pl

under high and low unit time costs*)