3D Surface Fabrication Using Conformal Geometry

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Abstract

The developing 3D printing technology has revolutionized the manufacturing industry. Although the existing 3D printing method is used in a variety of fields to produce complete models in a single process, it requires expensive hardware and successive layers of materials.

This paper proposes a novel approach for fabricating 3D shapes based on surface foliation in conformal geometry. All oriented metric surfaces are Riemann surfaces. The holomorphic differentials on a Riemann surface induce horizontal and vertical trajectories. The horizontal (vertical) trajectories give a foliation of the surface singular at zeros. The surface is decomposed to two families of orthogonal leaves, and can be reconstructed by weaving the two families of leaves.

Comparing to conventional 3D printing technologies, this method has advantages: 1. General: the method handles surfaces with different topologies; 2. Rigorous: the method has concrete foundation; 3. Automatic: the algorithm is automatic; 4. Economical: the method only requires paper and scissors. The method has potential for fabricating 3D shapes in real life.

Key words: 3D printing; conformal geometry; foliation; trajectory; holomorphic differential; fabrication

1 Introduction

The three-dimensional(3D) printing is a developing process that creates 3D models of any shape from digital models by laying successive layers of material in different shapes, which eventually form the cross sections of the object. Distinct from traditional machining methods, 3D printing, also known as rapid prototyping, is related to automated fabrication (computer numerical controlled machining). Originated in the late 1980s, the 3D printing industry is relatively young. In the past few years, the evolution in production, information technology, and communication stimulated the development of 3D printing. The application of 3D printing has expanded to diverse fields: education, jewelery, footwear, biomedical industries, aerospace, architecture and so on. It holds a promising future for science, education and industry. However, it is inevitable that several upcoming applications have some drawbacks. First of all, 3D printers only manufacture products out of limited materials, for instance, plastic, resin, certain metals, and ceramics. Some mixed materials, like circuit boards, cannot be used in 3D printing yet. In addition, 3D printers are limited with the size of the products that they can create. Moreover, another challenge is the high expense of the printing services. Printer parts can fail; calibration can be delicate; software glitches occur frequently. Printers worth hundreds of thousands of dollars are prone to break.

To tackle the above obstacles, we developed a novel method for 3D surface foliation based on conformal geometry. In comparison with the conventional 3D printing technology, this method has several advantages. It is more economic since it can produce 3D solid objects with simple tools like paper-scissors instead of expensive machines. High end 3D products can be produced when using a laser paper cutter. Moreover, this technology is not restricted by either the size or the material of the products. Any easy-to-bend material regardless its size will work well. In this paper, we introduce the new approach for fabricating 3D shapes based on surface foliation in conformal geometry.

All surfaces in real life are Riemann surfaces. The Abel differentials on a Riemann surface form a group. An Abel differential induces two foliations of the surface, the leaves consist of the horizontal and the vertical trajectories of the differential, which are orthogonal everywhere. The surfaces can be reconstructed by weaving the two families of leaves together. The Section 2 briefly introduces the fundamental theories necessary for the current work; Section

3 explains the computational algorithms; Section 4 reports our preliminary experimental results; the paper is concluded in Section 5.

2 Theoretic Foundation

This section briefly introduces the concepts and theorems necessary for the current work. For thorough explanations, we refer the readers to the textbook [3].

Riemann Surface Suppose $f : \mathbb{C} \to \mathbb{C}$ is a complex function, $w = f(z)$. Define complex differential operators

$$\partial_z = \frac{1}{2}(\partial_x - i\partial_y), \partial_{\bar{z}} = \frac{1}{2}(\partial_x + i\partial_y).$$

We say f is a *holomorphic function* if

$$\partial_{\bar{z}} f = 0.$$

If f is invertible, and f^{-1} is also holomorphic, then f is called *biholomorphic*.

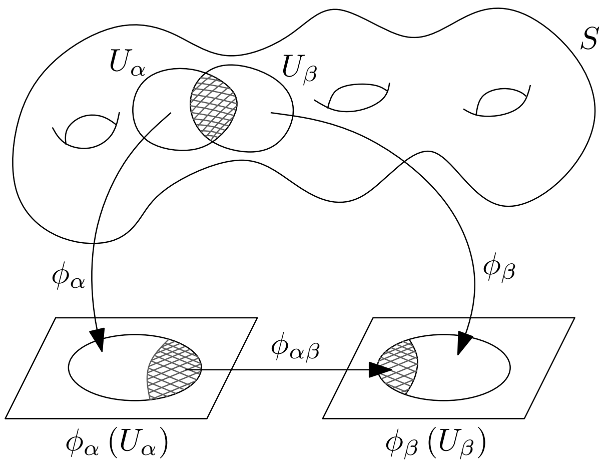


Figure 1: Conformal structure.

As shown in figure 1, let S be a topological surface, covered by an atlas $\{(U_\alpha, \phi_\alpha)\}$, where (U_α, ϕ_α) is a local chart, U_α is an open set on S, $\phi_\alpha : U_\alpha \to$

$\mathbb{C}$ is a homeomorphism mapping U_α to the complex plane $\mathbb{C}$, and the local complex coordinates is denoted as z_α. Suppose the intersection $U_\alpha \cap U_\beta$ is covered by two local charts (U_α, ϕ_α) and (U_β, ϕ_β), the coordinates transition function is

$$\phi_{\alpha\beta} = \phi_\beta \circ \phi_\alpha^{-1}.$$

If all coordinates transition functions are biholomorphic, then the atlas is a *conformal atlas*. The maximal conformal atlas is called the *conformal structure* of the surface. A topological surface with a conformal structure is called a *Riemann surface.*

Isothermal coordinates Suppose $(S, \mathbf{g})$ is a surface embedded in the three dimensional Euclidean space $\mathbb{R}^3$, $\mathbf{g}$ is the induced Euclidean metric. Let (u, v) be a local coordinate system, such that the metric can be represented as

$$\mathbf{g} = e^{2\lambda(u,v)}(du^2 + dv^2),$$

where $\lambda : S \to \mathbb{R}$ is a function defined on the surface. Then (u, v) is called an *isothermal coordinate system*, λ the *conformal factor*. For any point $p \in S$, there exists a neighborhood $U(p)$, such that it can be covered by the isothermal coordinates. All the local isothermal coordinate charts form the conformal structure induced by the Riemannian metric $\mathbf{g}$, namely

Theorem 2.1 (Chern[5]) *For each point p an oriented metric surface, there exists a neighborhood, which admits an isothermal coordinate system. Namely, all oriented metric surfaces are Riemann surfaces.*

de Rham Cohomology Suppose S is covered by isothermal coordinates (u, v), let ω be a *differential 1-form* with local representation,

$$\omega = f(u, v)du + g(u, v)dv.$$

Let d be the *exterior differential operator*

$$d\omega = (\partial_u g - \partial_v f)du \wedge dv.$$

ω is called a *closed 1-form* if $d\omega = 0$. Suppose $f : S \to \mathbb{R}$ is a function,

$$df = \partial_x f dx + \partial_y f dy,$$

then df is called an *exact 1-form.* Exact forms must be closed. Two closed 1-forms ω_1 and ω_2 are called *cohomological*, if they differ by an exact 1-form df

$$\omega_1 - \omega_2 = df.$$

All the cohomological classes form the de Rham cohomology group.

Definition 2.2 (de Rham Cohomology Group) *Suppose S is a smooth surface, the first de Rham cohomology group is defined by*

$$H^1(S, \mathbb{R}) := \frac{Ker\ d}{Img\ d}$$

Hodge Theory Let * be the *Hodge star* operator, then the *conjugate* of ω is $^*\omega$

$$^*\omega = f(u,v)dv - g(u,v)du.$$

The co-differential operator is defined as

$$\delta = {}^*d^*.$$

If both $d\omega$ and $\delta\omega$ equal to zeros, then ω is called a *harmonic 1-form.* The Hodge theorem claims that each cohomological class has a unique harmonic 1-form.

Theorem 2.3 (Hodge[2]) *Each cohomological class in $H^1(S, \mathbb{R})$ has a unique harmonic 1-form.*

All the harmonic 1-forms on the surfaces form a group, which is isomorphic to the first de Rham cohomology group $H^1(S, \mathbb{R})$. Furthermore, if ω is harmonic, then its conjugate $^*\omega$ is also harmonic.

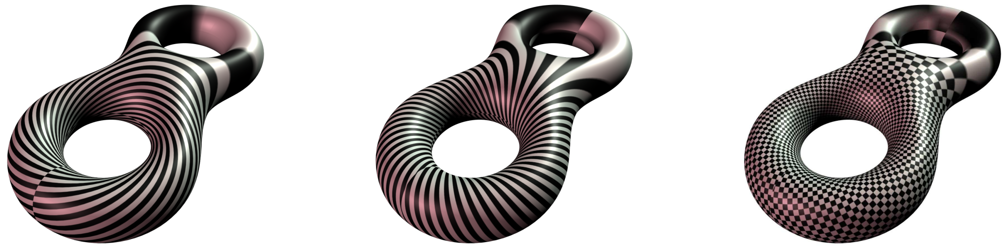
Holomorphic Differentials As shown in figure 2, suppose S is a Riemann surface, with conformal atlas $\{(U_\alpha, z_\alpha)\}$. Let Ω be a complex differential 1-form, which has local representation

$$\Omega = f_\alpha(z_\alpha)dz_\alpha,$$

where f_α is a holomorphic function. Then Ω is called a *holomorphic 1-form.* Each holomorphic 1-form can be decomposed to two real harmonic differential 1-forms,

$$\Omega = \omega + i^*\omega,$$

where both ω and $^*\omega$ are harmonic 1-forms. All the holormphic 1-forms form a linear space which is isomorphic to the first de Rham cohomology group.



Harmonic 1-form ω conjugate harmonic form ${}^*\omega$ holomorphic 1-form $\Omega = \omega + i^*\omega$

Figure 2: A holomorphic 1-form on a genus 2 Riemann surface.

Horizontal and Vertical Trajectories As shown in Figure 3, a holomorphic 1-form has zero points, where the indices are negative, therefore the number of zero points is less than the absolute value of the Euler characteristic number of the surface.

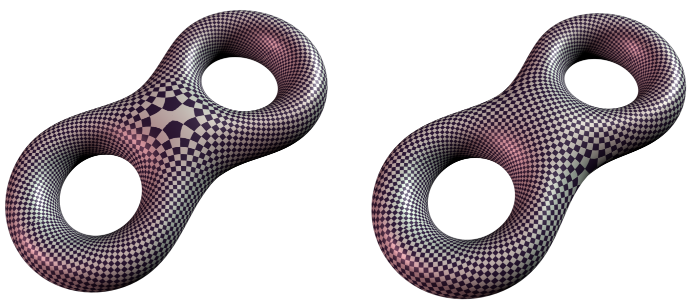


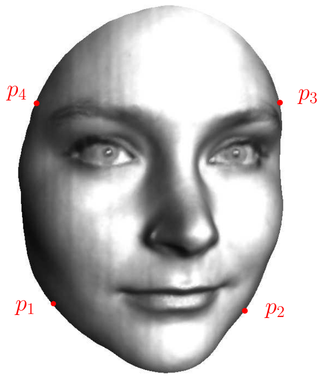
Figure 3: Zero points of a holomorphic 1-form on a genus 2 Riemann surface.

Fix a holomorphic 1-form Ω on a Riemann surface S, $p \in S$ is a point on the surface, $\mathbf{v} \in T_pS$ is a tangent vector at p, if $\langle \Omega, \mathbf{v} \rangle^2 > 0$, then $\mathbf{v}$ is called a *horizontal direction*; if $\langle \Omega, \mathbf{v} \rangle^2 < 0$, then $\mathbf{v}$ is a *vertical direction*.

Definition 2.4 (Horizontal Trajectories) *Suppose* Ω *is a holomorphic 1-form on a Riemann surface* S, $\gamma \subset S$ *is a curve on the surface, if all its tangential directions are horizontal directions, then* γ *is called a horizontal trajectory.*

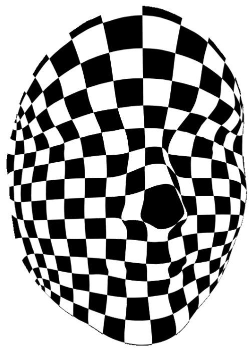
Vertical trajectories can be defined in the similar way. The horizontal trajectory may terminate at the zero points. If all horizontal trajectories are closed loops with finite lengths, then the holomorphic 1-form is a Strebel differential [6].

Roughly speaking, the horizontal trajectories of a Strebel differential form a foliation of the surface, each fiber is a trajectory. So do the vertical trajectories. Hence, the surface can be reconstructed by weaving the horizontal and vertical trajectories.



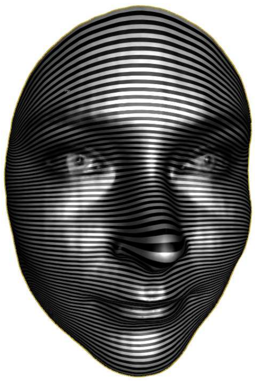
a. Original surface

b. Conformal image

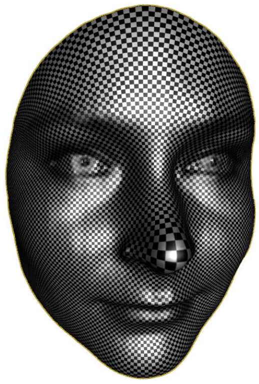


c. Weaving model

d. harmonic 1-form ω

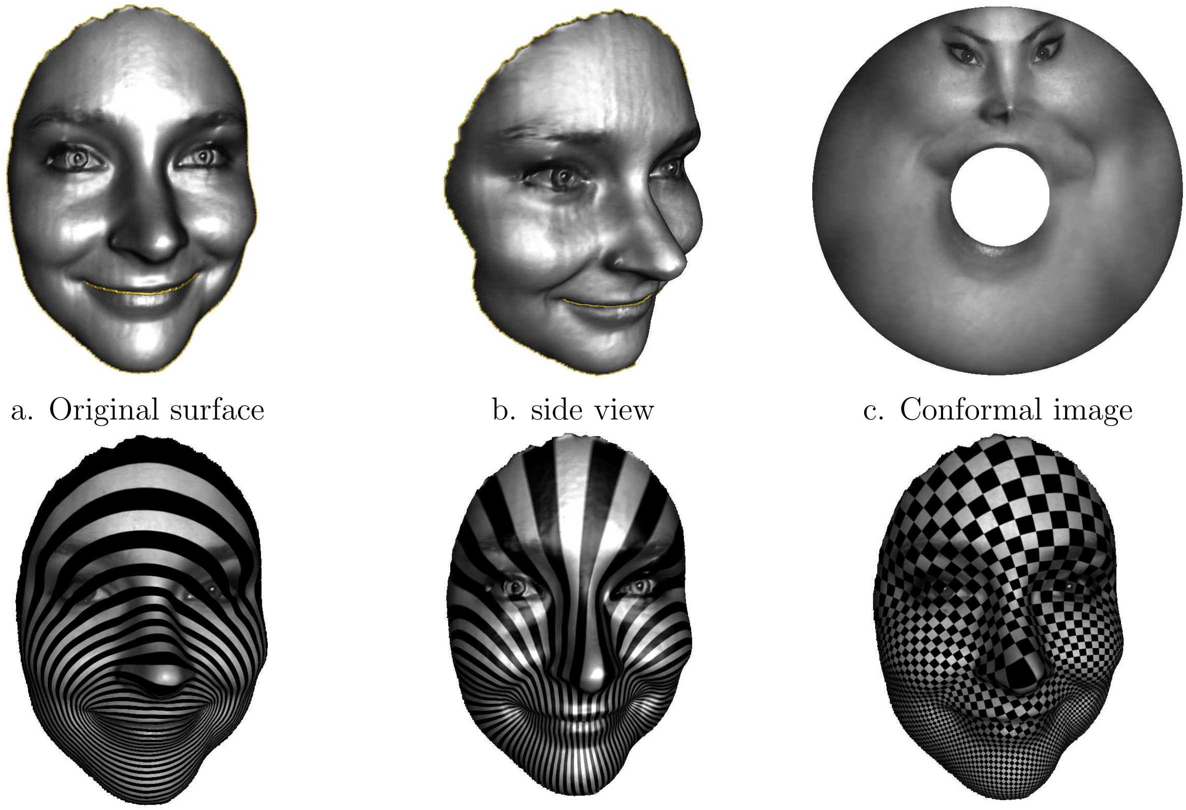


e. conjugate harmonic form $^*\omega$



f. holomorphic 1-form

Figure 4: Converting the 3D surface of a female face to a weaving model.



a. Original surface b. side view c. Conformal image

d. harmonic 1-form ω e. conjugate harmonic form $^*\omega$ f. holomorphic 1-form

Figure 5: Holomorphic 1-form on a topological annulus.

3 Computational Algorithms

The computational algorithms are mainly based on the computational conformal geometry by Gu and Yau's method [1].

Data Acquisition The geometric surfaces are scanned using phase shifting method [7] from real objects from different view angles. The acquired point clouds are filtered, denoised, merged together, then triangulated. The surface is conformally parameterized then remeshed to improve the meshing quality using Ruppert's Delaunay Refinement method [4]. The surface is represented as a triangular polyhedral surface, namely a triangle mesh.

Figure 6: 3D model of the female face woven by paper strips.

Suppose M is a genus g closed triangle mesh, with vertex, edge and face sets V, E and F respectively. Let v_i denote a vertex of M, $[v_i, v_j]$ an oriented edge, $[v_i, v_j, v_k]$ an oriented face. The boundary operator is defined as

$$\partial[v_i, v_j] = v_j - v_i, \partial[v_i, v_j, v_k] = [v_i, v_j] + [v_j, v_k] + [v_k, v_i].$$

The exterior differential operator is dual to the boundary operator. Let $f : V \to \mathbb{R}$ be a function (0-form),

$$df([v_i, v_j]) = f \circ \partial([v_i, v_j]) = f(v_j) - f(v_i).$$

Let $\omega : E \to \mathbb{R}$ be a 1-form,

$$d\omega([v_i, v_j, v_k]) = \omega\partial([v_i, v_j, v_k]) = \omega([v_i, v_j]) + \omega([v_j, v_k]) + \omega([v_k, v_i]).$$

The co-differential operator δ is defined as

$$d\omega(v_i) = \sum_{[v_i, v_j] \in M} w_{ij}\omega([v_i, v_j]),$$

where w_{ij} is the cotangent edge weight. Suppose two faces $[v_i, v_j, v_k]$ and $[v_j, v_i, v_l]$ share the edge $[v_i, v_j]$, then

$$w_{ij} = \cot\theta_k^{ij} + \cot\theta_l^{ji},$$

where θ_k^{ij} is the angle at vertex v_k in the face $[v_i, v_j, v_k]$, θ_l^{ji} the angle at vertex v_l in the face $[v_j, v_i, v_l]$.

Given an triangular face $[v_i, v_j, v_k]$, which is isometrically embedded on the plane, given two 1-forms $\omega_k = a_k dx + b_k dy$, where $k = 1, 2$, then the integration of $\omega_1 \wedge \omega_2$ on the triangle is

$$\int_{[v_i,v_j,v_k]} \omega_1 \wedge \omega_2 = (a_1 b_2 - a_2 b_1) Area([v_i, v_j, v_k]).$$

Therefore

$$\int_M \omega_1 \wedge \omega_2 = \sum_{[v_i,v_j,v_k]\in F} \int_{[v_i,v_j,v_k]} \omega_1 \wedge \omega_2.$$

The conjugate of ω_2 is given by $\omega_2^* = a_2 dy - b_2 dx$. Therefore

$$\int_{[v_i,v_j,v_k]} \omega_1 \wedge^* \omega_2 = (a_1 a_2 + b_1 b_2) Area([v_i, v_j, v_k]).$$

Fundamental Group Generators The Poincaré dual of M is $\bar{M}$. The dual of a vertex $v_i \in V$, an edge $e_j \in E$ and a face $f_k \in F$ are a face $\bar{v}_i$, an edge $\bar{e}_j$ and a vertex $\bar{f}_k$ in $\bar{M}$ respectively. Let $\bar{T}$ be a spanning tree of $\bar{M}$, which connects all the vertices of $\bar{M}$. The *cut graph* of M is defined as

$$C(M) := \{e \in M | \bar{e} \notin \bar{T}\}.$$

Let T be a spanning tree of the cut graph $C(M)$, then $C(M) - T = \{e_1, e_2, \cdots, e_{2g}\}$, then the union of the tree T with each edge e_i has a unique loop γ_i. $\{\gamma_1, \gamma_2, \cdots, \gamma_{2g}\}$ forms a basis of the fundamental group of M, which is also the homology group generators $H_1(S, \mathbb{R})$.

Harmonic Cohomology Basis The mesh M is sliced along a loop γ_k to get a mesh with two boundary components $\tilde{M}_k$, $\partial \tilde{M}_k = \gamma_k^+ - \gamma_k^-$, define a function $f_k : \tilde{M}_k \to \mathbb{R}$, such that

$$f_k(v_i) = \begin{cases} 1 & v_i \in \gamma_k^+ \\ 0 & v_i \in \gamma_k^- \\ random & otherwise \end{cases}$$

The gradient of the function on $\tilde{M}_k$

$$df_k([v_i, v_j]) = f_k(v_j) - f_k(v_i)$$

is well-defined on the original mesh M, denoted as τ_k, then $\{\tau_1, \tau_2, \cdots, \tau_{2g}\}$ forms a basis of $H^1(M, \mathbb{R})$.

According to Hodge theory, each cohomology class has a unique harmonic 1-form. Therefore, for each τ_k, there is a function $h_k : V \to \mathbb{R}$ unique up to a constant, such that $\omega_k = \tau_k + dh_k$ is harmonic. This leads to an elliptic partial differential equation, $\delta dh_k = -\delta\tau_k$. In the discrete setting, this elliptic PDE is reduced to a linear system, whose coefficient matrix is positive definite. So, $\{\omega_1, \omega_2, \cdots, \omega_{2g}\}$ is the basis of the harmonic 1-form group on M.

Holomorphic 1-form The conjugate of a harmonic 1-form is also harmonic.

$$^*\omega_k = \sum_{j=1}^{2g} \lambda_{kj}\omega_j.$$

We obtain

$$\int_M \omega_i \wedge {}^*\omega_k = \sum_{j=1}^{2g} \lambda_{kj} \int_M \omega_i \wedge \omega_j.$$

This leads to a linear system to solve λ_{kj}'s.

Foliation The mesh is sliced along the cut graph $C(M)$ to get a topological disk $\tilde{M}$. The mesh is conformally mapped onto the plane by integrating a holomorphic 1-form Ω, the mapping is denoted as $\phi : \tilde{M} \to \mathbb{C}$. A base vertex $v_0 \in \tilde{M}$ is fixed, for any vertex $v_i \in \tilde{M}$,

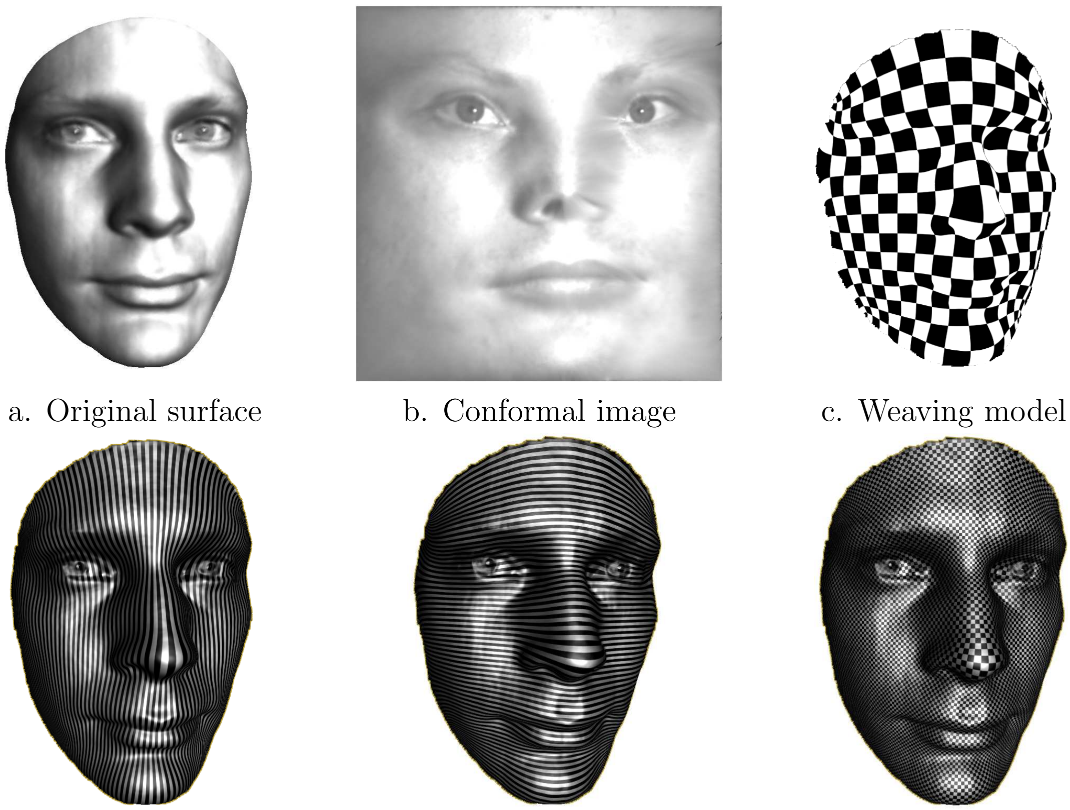
$$\phi(v_i) = \int_{v_0}^{v_i} \Omega,$$

where the integration path is arbitrarily chosen in $\tilde{M}$. The image of the mesh $\phi(\tilde{M})$ is decomposed into horizontal strips and vertical strips. Each strip is mapped back onto the original surface. The curved strips on the surface are approximated by developable strips, which can be isometrically flattened onto the plane.

Surfaces with boundaries Suppose M is a triangular mesh with boundaries, then we use M^- to denote the same mesh with opposite orientation. For each vertex $v \in \partial M$, there is a corresponding one $v^- \in \partial M^-$, this gives an equivalence relation $v \sim v^{-}$. Then we can construct a symmetric closed mesh

$$\widehat{M} := M \cup_{\sim} M^{-},$$

which is called the double covering of M. The holomorphic 1-forms on M can be computed by those on their double coverings.



a. Original surface b. Conformal image c. Weaving model

d. harmonic 1-form ω e. conjugate harmonic form $^{*}\omega$ f. holomorphic 1-form

Figure 7: Converting the 3D surface of a male face to a weaving model.

4 Experimental Results

Table 1 shows the time cost for each model. The whole process is as follows:

Figure 8: 3D model of the male face woven by paper strips.

Table 1: Time cost for each model

Model	# vertices	# faces	Holomorphic form	Segmentation Printing	Cutting	Assembly
Sophie	21043	41587	35 s	15 m	90 m	180m
Alex	80598	160058	130 s	90 m	240m	420m
Venus	704	1375	20 s	60 m	180m	360m

1. Compute the holomorphic form basis, by integrating the holomorphic 1-form. The surface is conformally mapped to planar domains.

2. Slice the conformal image into strips, which are mapped back onto the surface, and decompose the surface to strips. Each strip is approximated by a sequence of planar quadrilaterials. All the strips are printed out onto paper.

3. The paper strips are cut out using scissors.

4. The two families of paper strips are woven together to assembly the 3D model.

Figure 4 demonstrates one of our experimental results. The female face is scanned using phase shifting method, and four vertices on the boundary are selected p_1, p_2, p_3, p_4, as shown in frame (a). Therefore the input surface is a topological quadrilateral. By double covering along the boundary segments p_1p_2 and p_3p_4, we obtain a topological cylinder; the cylinder is double covered again to get a topological torus. Then the harmonic 1-form ω on the original quadrilateral can be computed as shown in frame (d), its conjugate harmonic 1-form $^*\omega$ is shown in frame (e), and the holomorphic 1-form Ω is shown in frame (f). By integrating the holomorphic 1-form, the surface is conformally mapped onto planar rectangle (b). The horizontal and vertical strips on the rectangle can be mapped back onto the original surface to get a two foliations, the two families of leaves form a weaving pattern. A coarse weaving model is illustrated in frame (c).

Figure 5 illustrates another one of our experimental results. The female face surface is sliced along the mouth, so it becomes a topological annulus. It can be conformally mapped to an planar annulus (c), the holomorphic 1-form is shown on (f), horizontal and vertical trajectories are demonstrated on (d) and (e).

The horizontal leaves are approximated by developable surface strips, which are isometrically embedded on the 2D planes and printed out on paper. The two family paper strips are woven together: the horizontal ones are in white color while the vertical ones in black color. The 3D surface model is constructed easily without using any advanced device, only paper and scissors as shown in Fig. 4 and Fig. 7. The venus surface model in Fig. 9 is a topological annulus, and its double covering is a topological torus. The Kitten model in Fig.11 is of genus one.

The most time-consuming step is cutting the paper strips using scissors, which can be greatly improved by using laser paper cutter. The manual weaving step is difficult to perform automatically. In order to save time, the resolutions of the models fibrilated in the current project are relatively low. The resolutions can be increased by using a laser paper cutter, so the quality of the models can be further improved as well. Furthermore, the fabricated models have distortions caused by the free boundary and the mechanical property of the paper material. The distortion can be reduced by choosing more rigid paper and fixing the boundary condition in the future experiments.

Figure 9: The Venus surface model.

Figure 10: 3D model of the Venus surface woven by paper strips.

5 Conclusions

This work proposes a novel approach for 3D shape fabrication based on conformal geometry. In principle, the method is rigorous and has concrete the-

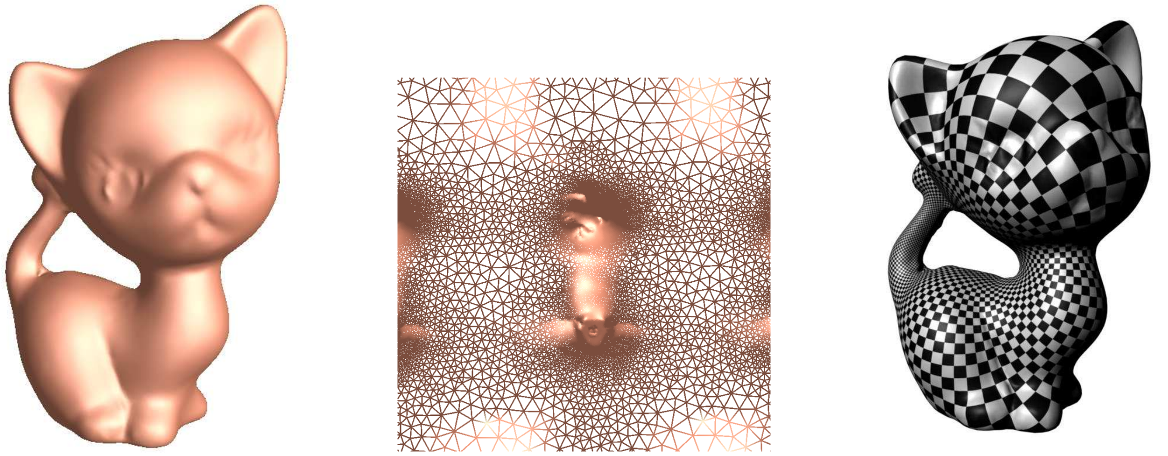


Figure 11: The kitten surface model.

oretic foundation. The method is general, it is capable of handling surfaces with different topologies. The method is automatic, only requires minimal manual input. This method is economical, it does not use special equipments or materials, requiring only paper and scissors. Furthermore, the resolutions of the fabricated models can be adjusted. The computed weaving pattern can be applied for building 3D models using different materials, such as leather and plastics.

On the other hand, current method requires manually cutting, which is time consuming and error-prone. This can be improved by using laser cutting technology. In addition, conformal mapping can induce large area distortions, which induces big approximation error and non-uniformity of the weaving pattern. This can be mitigated by selecting the optimal holomorphic differential or using quasi-conformal mapping. The method depends on manually assembly, which is time consuming. Because the assembly is performed in $\mathbb{R}^3$, we will explore further to find an automatic method for assembly.

In summary, the method is rigorous and practical, it has great potential to be applied to conventional manufacturing industry for fabricating shapes, which cannot be handled through the traditional methods, such as applications in fashion industry, toy industry and medicine fields.

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************************************ panel report ************************************

The project proposes a novel approach for 3D printer by fabricating 3D surfaces based on surface foliation in conformal geometry. The key observation is that holomorphic differentials on a Riemann surface induce horizontal and vertical trajectories. The surface is thus decomposed into two families of orthogonal leaves, and it can be reconstructed by weaving the two families of leaves. The novelty is the application of beautiful differential geometry theories and tools to a real and trendy problem, i.e., 3D printing. The idea and method may have real impact for practice. The effort in the project includes a good explanation of the theoretical background, implementation of the finite element method for computing harmonic maps, numerical experiments and even hand crafted product. The panel regards it as a well-rounded applied mathematics project that includes novel mathematics ideas and promising applications.

12--YHMA Evaluation Form -- Regional Competition

Instruction: Please fill in all sections. This form is to help the organizers to communicate your assessments and rationales to others in the evaluation process.

Project Title	3D Surface Fabrication using Conformal Geometry				
Evaluation level Choose one:	X Referee Report	Regional Committee		Regional Presentation	
Selection Criteria (check one in each area below)	Very strong	Strong	Modest	Weak	Not Applicable
Mathematical Contents (1, 4, 5)		X			
Creativity, Originality (2, 3)	X				
Scholarship, Presentation (7)		X			
Demonstrated Teamwork (8)					X
Impact outside Math (9)	X				
COMMENTS					
Comments related to Criteria 1,4,5	The project is aimed to develop a surface representation that provides a new and hopefully easy way for 3D printer. The representation is based on conformal structure for a Riemann surface which renders a foliation for the surface. The mathematics theory behind the interesting idea of application is rigorous but already available in differential geometry. So there is not much mathematical derivation. The computational algorithm is based on the mentor's previous work.				
Comments related to Criteria 2,3	I think the originality and the application very highly. It gives an interesting and potential application of differential geometry in real life. It may have a real impact for industrial practice.				
Overall Recommendation For Presentation	Highly Competitive X	Perhaps Competitive	Not Competitive		

12--YHMA Evaluation Form -- Regional Competition

Instruction: Please fill in all sections. This form is to help the organizers to communicate your assessments and rationales to others in the evaluation process.

Project Title	3D surface fabrication using conformal geometry				
Evaluation level Choose one:	Referee Report	Regional Committee		Regional Presentation	
Selection Criteria (check one in each area below)	Very strong	Strong	Modest	Weak	Not Applicable
Mathematical Contents (1, 4, 5)	X				
Creativity, Originality (2, 3)			X		
Scholarship, Presentation (7)		X			
Demonstrated Teamwork (8)			X		
Impact outside Math (9)			X		
COMMENTS					
Comments related to Criteria 1,4,5	This work uses the recent results from computational conformal geometry, an emerging interdisciplinary field involving mathematics (theory of Riemann surface) and computer science (algorithm and numerics).				
Comments related to Criteria 2,3	This paper applies the results from computational conformal geometry to 3D shape fabrication, which is new and interesting.				
Overall Recommendation For Presentation	Highly Competitive	Perhaps Competitive	Not Competitive	It is perhaps competitive	

12--YHMA Evaluation Form -- Regional Competition

Instruction: Please fill in all sections. This form is to help the organizers to communicate your assessments and rationales to others in the evaluation process.

Project Title	3D Surface Fabrication using Conformal Geometry by Y. Zhang				
Evaluation level Choose one:	Referee Report (*)	Regional Committee		Regional Presentation	
Selection Criteria (check one in each area below)	Very strong (X)	Strong	Modest	Weak	Not Applicable
Mathematical Contents (1, 4, 5)	(X)				
Creativity, Originality (2, 3)	(X)				
Scholarship, Presentation (7)	(X)				
Demonstrated Teamwork (8)	(X)				
Impact outside Math (9)	(X)				
COMMENTS					
Comments related to Criteria 1,4,5	PLEASE USE SEPARATE PARAGRAPH TO ELABORATE ON YOUR RATING FULLY See I below				

I. The project is on a computational geometry problem of surface reconstruction from point clouds motivated from 3D printing. The methodology is based on rigorous mathematics of Riemann surfaces, holomorphic differential forms, harmonic and conjugate harmonic forms, conformal mapping of surfaces to planes. The work may lead to further computational research of fast surface reconstruction from point clouds.

Comments related to Criteria 2,3	PLEASE USE SEPARATE PARAGRAPH TO ELABORATE ON YOUR RATING FULLY See II below				
Overall Recommendation For Presentation	Highly Competitive (X)	Perhaps Competitive	Not Competitive		

II. The project is high original in finding an elementary solution of surface reconstruction using paper-scissors, which contributes to low cost solution of 3D printing and public awareness of mathematics. The methodology of using two families of paper strips woven together to assembly a 3D model is genuinely creative.