# The Construction and Application of Heating-and-Cooling-Effect of Information Diffusion 

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#### Abstract

This article targets on understanding the influence of a specific message during a message-transmit process.

Drawing an analogy between the law of thermodynamics and heat transport theory and the growth of the influence and between Newton's law of cooling and the regression, two basic models named the Heating-Effect-Model and the Cooling-Effect-Model of information are induced. By defining the variables as information quantity-G, Total of transmitting points during one process-x, the order of a transmitting point during the process-n, transmitting duration-t and the influence of the message- H , models show how t and n work to produce the result of H . Based on the test results of real data, a mathematical model to describe the process through an expression is created

And the model is applied to public opinion transmission and message transmission of relief supplies in disaster areas. And it is also applied in financial crisis, which introduces a new method in solving financial problems.


Key words: information diffusion, heat conduct, Newton's law of cooling, public opinion, relief supplies, financial crisis

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## 1. Introduction

With the advancement of information technology, increasingly rapid dissemination of information, transmission continues to expand.

Facebook, Twitter, and their Chinese versions micro-blog, Wechat, QQ are becoming the primary tools of communication and transmission of information. In 1991 the adaptation of the film Six Degrees of Separation was staged on Broadway. The leading actor brought up the concept of "six degrees of separation" in a joke. Not until the United States Professor of social psychology at Harvard University Miguel• Rahm summarized its meanings did "six degrees of separation" become noticed in social networks research. In 2006, the United States Microsoft researchers' network experiments clearly proved this rule. While Complex network founder Balabaxi proposed "19 degrees of separation" on the basis of six degrees of separation.

The fast growth of Internet information transmission productivity leads us to explore information diffusion.

Communication was formed initially in 1940 in the United States as a new social subject. Mainland China introduced communication science in the early 80s. Earlier researches on the particular propagation established models qualitatively oriented. In recent years, researchers have developed some more mathematical models, attempting quantitatively to analyze the information diffusion mechanism, leading to the development of Transmission dynamics.

Mainly two mathematical methods were applied in past models. One is to use certain differential equation to describe Dynamics State at any time between changes and to forecast spread results of information as shown in SIS and SIR models. Another is to establish stochastic model based on probability analysis during random process. Through transfer probability of the system's state, various probabilities of the end state can be calculated, as shown by BA model. Recently, some scholars used social physics research to establish analogies between the propagation and dissemination of the information.

However, based on the understanding of the target type of information diffusion process, draw an analogy between the dissemination of information and heat transmission is more appropriate considering its crucial characters during diffusion. The conclusion came from identifying target transmission body and comparison between two models originated in separate academic realms. Therefore, this paper will focus on introducing the construction and application of a new model of information diffusion named heating-and-cooling-effect Model

With the knowledge of heat transfer and dissipation laws, essential elements of the physics theory were extracted and used to compare with the principal
characters in information diffusion process. As a result, heating-and-cooling-effect Model, comprised in the form of two basic models, was deduced. After setting mathematical foundation, further researches on applications of the model are produced.

Amazing to discover, applications can be widely distributed in different realms of subjects. In this treatise, three forms are exhibited to show the value of the established Model.

They are: 1. Evaluation of impacts of diffused public opinion.
2. Spread of information concerning disaster relief goods within the limit of disaster aftermath.
3. Rough predictability of the recovery of the financial crisis.

Every explanation of Application contains aspects: 1. Mathematical adoption of Model function to calculate the variables value. 2. How is the model useful? 3. Statistical analysis

Dissemination of information is a complex process. Simple mathematical models are naturally incapable of giving immaculate accurate summary or prediction. However, studies heretofore on this subject are not yet mature, and it is foreseeable how illimitable potential the Internet information is to bring. However, this state of being premature is what results in this paper and many other papers. It has been a great honor to be aware of it and be determined to go on the journey of exploring it.

## 2. Modeling

### 2.1 Denotation

1. ' $m$ ' denotes the order of a transmitting point during the process, the propagagtion distance, meaning that the message spread from the propagation source (denoted by 0 ) has traveled through m points.
2. ' $t$ ' denotes the transmitting duration.
3. 'H' denotes the strength of the influence of the message, which interpreted as $\mathrm{H}(\mathrm{m}, \mathrm{t})$.
4. ' $G$ ' denotes the information quantity.
5. ' $v$ ' denotes the propagation rate.
6. ' $\mathrm{H}_{0}$ ' denotes the greatest strength of influence.
7. ' $a$ ' is a constant, denoting the transmission coefficient, which determines the rate at which the information travels one point to another.
8. ' $b$ ' is a constant, influenced by the attractiveness and specificity of information.
9. ' $x$ ' denotes the total of transmitting points during one process, signifying the range of information spreading.

### 2.2 Mathematical Modeling Process

First of all, it should be emphasized that the model is based on the system of Oral Communication, and is established on a limited range of spreading.

Core laws of thermodynamics shows that, if two sets of system with different temperatures contact, heat Exchange occurs until they reach the same temperature. Let's apply this model to information Diffusion. Understandings are: an article starts to spread and gain attention during its diffusion process. Information absorbs heat to become a piece of hot information. While spreading, getting downloaded, receiving comments, its range of influence continuous to expand. However, with the passage of time, the strength of influence ceases to decline, and People's enthusiasm decreases - information cools down.

Move on to modeling:

### 2.2.1 Heating-Effect of Information Diffusion

Assume that information has the greatest strength of influence at its starting point. Moreover, its strength of influence decreases with the progression of time at any other point while decreases along the degree diffusion every moment. In
addition, the speed of diffusion is directly proportional to the strength of influence.

Though the variable $m$ is on discrete, it could be seen as continuous when the range of spreading $x$ is defined as a huge number.

So the information quantity at Point $m$ get from Point ( $m-1$ ) in an infinitesimal period of time can interpreted as:

$$
\begin{equation*}
\mathrm{G}=-\mathrm{a} \frac{\partial \mathrm{H}}{\partial \mathrm{~m}} d \mathrm{t} \tag{1}
\end{equation*}
$$

And during the same time, Point $m$ pass on information to the next Point ( $\mathrm{m}+1$ ), the quantity of it can interpreted as:

$$
\begin{equation*}
\mathrm{G}_{1}=-\mathrm{a}\left(\frac{\partial \mathrm{H}}{\partial \mathrm{~m}}+\frac{\partial^{2} \mathrm{H}}{\partial \mathrm{~m}^{2}} d \mathrm{~m}\right) d \mathrm{t} \tag{2}
\end{equation*}
$$

Then the accumulation is:

$$
\begin{equation*}
\Delta \mathrm{G}=\mathrm{G}-\mathrm{G}_{1}=\mathrm{a} \frac{\partial^{2} \mathrm{H}}{\partial \mathrm{~m}^{2}} d \mathrm{~m} d \mathrm{t} \tag{3}
\end{equation*}
$$

(In reality, the information a person passes on is also the accumulation, which is different from the model. This process is to remove the calculation of information passed twice. Considering that the model is aimed to analyze the range of the spreading, the process has no influence.)

Assuming that the increment of H interpreted as $\frac{\partial \mathrm{H}}{\partial \mathrm{t}} \mathrm{dt}$ is connected with the information quantity, the accumulation of information at Point $m$ also interpreted as:

$$
\begin{equation*}
\Delta \mathrm{G}=\mathrm{b} \frac{\partial \mathrm{H}}{\partial \mathrm{t}} d \mathrm{~m} d \mathrm{t} \tag{4}
\end{equation*}
$$

Combine (3) and (4), and the equation is get:

$$
\begin{equation*}
\frac{\partial \mathrm{H}}{\partial \mathrm{t}}=\mathrm{k}^{2} \frac{\partial^{2} \mathrm{H}}{\partial \mathrm{~m}^{2}} \quad, \quad \mathrm{k}=\sqrt{\frac{\mathrm{a}}{\mathrm{~b}}} \tag{5}
\end{equation*}
$$

(Compared with the thermal conduction differential equation, it is almost the same.)

Next, put it into a limit space.

Assume that information only passed x rounds. In order to make the calculation easier, another function n is defined, interpreted as:

$$
\begin{equation*}
\mathrm{n}=\mathrm{x}-\mathrm{m} . \tag{6}
\end{equation*}
$$

So the source becomes x while the last point is 0 . And formula (5) turns to:

$$
\begin{equation*}
\frac{\partial \mathrm{H}}{\partial \mathrm{t}}=\mathrm{k}^{2} \frac{\partial^{2} \mathrm{H}}{\partial \mathrm{n}}{ }^{2} \quad, \quad \mathrm{k}=\sqrt{\frac{\mathrm{a}}{\mathrm{~b}}} \tag{7}
\end{equation*}
$$

Now assume that the strength of influence in the place $n=x$ is at its greatest all the time. And the place $n=0$ (the last point) can't pass on any information, and the boundary conditions are:

$$
\begin{gather*}
H(x, t)=H_{0} \quad(t \geq 0)  \tag{8}\\
\frac{\partial H(0, t)}{\partial \mathrm{n}}=0 \quad(\mathrm{t} \geq 0) \tag{9}
\end{gather*}
$$

Use function $\mathrm{f}(\mathrm{n})(0<n \leq x \quad)$ to describe the relationship between H and n when $\mathrm{t}=0$, and initial conditions are:

$$
\begin{equation*}
\mathrm{H}(\mathrm{n}, 0)=\mathrm{f}(\mathrm{n})(0<n \leq x) \tag{10}
\end{equation*}
$$

A new function is introduced for easier calculation:

$$
\begin{equation*}
\mathrm{H}=\mathrm{H}_{0}+\mathrm{I} \tag{11}
\end{equation*}
$$

Substitute (11) into (7), and get:

$$
\begin{equation*}
\frac{\partial \mathrm{I}}{\partial \mathrm{t}}=\mathrm{k}^{2} \frac{\partial^{2} \mathrm{I}}{\partial \mathrm{n}}{ }^{2} \quad, \quad \mathrm{k}=\sqrt{\frac{\mathrm{a}}{\mathrm{~b}}} \tag{12}
\end{equation*}
$$

Then the boundary conditions turn to:

$$
\begin{equation*}
I(x, t)=0 \quad(t \geq 0) \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
\frac{\partial \mathrm{I}(0, \mathrm{t})}{\partial \mathrm{n}}=0 \quad(\mathrm{t} \geq 0) \tag{14}
\end{equation*}
$$

And the initial condition becomes :

$$
\begin{equation*}
\mathrm{I}(\mathrm{n}, 0)=\mathrm{f}(\mathrm{n})-\mathrm{H}_{0}(0<n \leq x) \tag{15}
\end{equation*}
$$

Now, set:

$$
\begin{equation*}
\mathrm{I}(\mathrm{n}, \mathrm{t})=\mathrm{N}(\mathrm{n}) \mathrm{T}(\mathrm{t}) \tag{16}
\end{equation*}
$$

The equation (12) can be transformed into:

$$
\begin{equation*}
\mathrm{NT}^{\prime}=\mathrm{k}^{2} \mathrm{~N}^{\prime \prime} \mathrm{T} \tag{17}
\end{equation*}
$$

Then set:

$$
\begin{equation*}
\frac{T^{\prime}}{T}=\frac{k^{2} N^{\prime \prime}}{N}=-k^{2} \lambda^{2} \quad(\lambda>0) \tag{18}
\end{equation*}
$$

It is easy to get a equation group:

$$
\left\{\begin{array}{l}
\mathrm{T}^{\prime}=-\mathrm{k}^{2} \lambda^{2} \mathrm{~T}  \tag{19}\\
\mathrm{~N}^{\prime \prime}+\lambda^{2} \mathrm{~N}=0
\end{array}\right.
$$

First, solve equation (19). The solution is as follows:

$$
\begin{align*}
& \frac{d T}{d t}=-k^{2} \lambda^{2} T  \tag{21}\\
& \frac{d T}{T}=-k^{2} \lambda^{2} d t \tag{22}
\end{align*}
$$

Integrate both sides and simplify it, and then get:

$$
\begin{equation*}
T=C e^{-k^{2} \lambda^{2} t} \tag{23}
\end{equation*}
$$

Second, solve the equation (20), and get:

$$
\begin{equation*}
N=A \cos \lambda n+B \sin \lambda n \tag{24}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d N}{d n}=-\lambda A \sin \lambda n+\lambda B \cos \lambda n \tag{25}
\end{equation*}
$$

Plug (23) and (24) into (15), and get the expression of I :

$$
\begin{equation*}
I=(A \cos \lambda n+B \sin \lambda n) C e^{-k^{2} \lambda^{2} t} \tag{26}
\end{equation*}
$$

To satisfy the boundary conditions, a function group is get :

$$
\left\{\begin{array}{l}
\mathrm{I}(\mathrm{x}, 0)=(\mathrm{A} \cos \lambda x+\mathrm{B} \sin \lambda x) \cdot \mathrm{Ce}^{-\mathrm{k}^{2} \lambda^{2} \mathrm{t}}=0  \tag{27}\\
\frac{\partial \mathrm{I}(0, \mathrm{t})}{\partial \mathrm{n}}=(-\lambda \mathrm{A} \sin \lambda x+\lambda \mathrm{B} \operatorname{scos} \lambda x) \cdot \mathrm{Ce}^{-\mathrm{k}^{2} \lambda^{2} \mathrm{t}}=0
\end{array}\right.
$$

Solve it, and get :

$$
\left\{\begin{array}{l}
\cos \lambda x=0  \tag{29}\\
B=0 \\
C=1
\end{array}\right.
$$

The values of $\lambda$ can get from (29) :

$$
\begin{equation*}
\lambda_{1}=\frac{\pi}{2 x} \quad, \lambda_{2}=\frac{3 \pi}{2 x} \quad, \lambda_{3}=\frac{5 \pi}{2 x} \ldots \ldots \lambda_{i}=\frac{(2 i-1) \pi}{2 x} \quad(i=1,2,3,4 \ldots \ldots) \tag{30}
\end{equation*}
$$

Define $a_{i}=A$, which is correspond $\lambda_{i}$, and get the particular solution:

$$
\begin{equation*}
I_{i}=a_{i} e^{-k^{2} \lambda^{2} t} \cos \lambda_{\mathrm{i}} \mathrm{n} \quad(\mathrm{i}=1,2,3,4 \ldots \ldots) \tag{31}
\end{equation*}
$$

Then get the general solution :

$$
\begin{equation*}
I(n, t)=\sum_{i=1}^{\infty}\left(a_{i} e^{-k^{2} \lambda_{i}^{2} t} \cos \lambda_{i} n\right)(i=1,2,3,4 \ldots \ldots) \tag{32}
\end{equation*}
$$

And the initial condition be transformed into :

$$
\begin{equation*}
\mathrm{I}(\mathrm{n}, 0)=\sum_{\mathrm{i}=1}^{\infty}\left(\mathrm{a}_{\mathrm{i}} \cos \lambda_{\mathrm{i}} \mathrm{n}\right) \quad(\mathrm{i}=1,2,3,4 \ldots \ldots) \tag{33}
\end{equation*}
$$

Substitute (30) into (33), and get:

$$
\begin{equation*}
\mathrm{I}(\mathrm{n}, 0)=\sum_{\mathrm{i}=1}^{\infty}\left(\mathrm{a}_{\mathrm{i}} \cos \frac{(2 \mathrm{i}-1) \pi}{2 \mathrm{x}} \mathrm{n}\right)=\mathrm{f}(\mathrm{n})-\mathrm{H}_{0} \quad,(\mathrm{i}=1,2,3,4 \ldots \ldots) \tag{34}
\end{equation*}
$$

And it is known that :

$$
\begin{equation*}
\mathrm{a}_{\mathrm{i}}=\frac{2}{\mathrm{x}} \int_{0}^{\mathrm{x}} \mathrm{f}(\mathrm{n}) \cos \frac{(2 \mathrm{i}-1) \pi \mathrm{n}}{2 \mathrm{x}} \mathrm{dn}-\frac{4}{\pi} \mathrm{H}_{0} \cdot \frac{1}{2 \mathrm{i}-1} \tag{35}
\end{equation*}
$$

Substitute (32) into (11), and get the expression of H :

$$
\begin{equation*}
H(n, t)=H_{0}+\sum_{i=1}^{\infty}\left(a_{i} e^{-\mathrm{k}^{2} \lambda_{\mathrm{i}}^{2} \mathrm{t}} \cos \lambda_{\mathrm{i}} \mathrm{n}\right) \quad(\mathrm{i}=1,2,3,4 \ldots \ldots) \tag{36}
\end{equation*}
$$

Because of $f(n)=H(n, 0)=0$, (35) transforms into:

$$
\begin{equation*}
\mathrm{a}_{\mathrm{i}}=-\frac{4}{\pi} \mathrm{H}_{0} \cdot \frac{1}{2 \mathrm{i}-1} \tag{37}
\end{equation*}
$$

Plug (37) and (29) into (36), and get the expression of H :

$$
\begin{array}{r}
H(n, t)=H_{0}-\frac{4 H_{0}}{\pi} \sum_{i=1}^{\infty}\left(\frac{1}{2 i-1} e^{-k^{2}\left[\frac{(2 i-1) \pi}{2 x}\right]^{2} t} \cos \frac{(2 i-1) \pi}{2 x} n\right) \\
(i=1,2,3,4 \ldots \ldots) \tag{38}
\end{array}
$$

Here consider a simple condition where audiences at every points are also communicators. Assume that $H_{0}=100$, transmission coefficient ' $a$ ' is 0.8 , and $\mathrm{b}=2000$, so $\mathrm{k}^{2}=0.02$.

Plug constants above in (38), and draw the graph of the function.
1, $\mathrm{x}, \mathrm{y}, \mathrm{z}$ stand for $\mathrm{t}(\times 10), \mathrm{m}$ (rounds) and H (\%) , when $0 \leq \mathrm{m} \leq 10,1 \leq \mathrm{t} \leq 10$

(Fig 1)
2, $\mathrm{x}, \mathrm{y}, \mathrm{z}$ stand for $\mathrm{t}(\times 100), \mathrm{m}$ (rounds) and H (\%), when $0 \leq \mathrm{m} \leq 10,1 \leq \mathrm{t} \leq 10$

(Fig 2)
3, $\mathrm{x}, \mathrm{y}, \mathrm{z}$ stand for t ( $\times 1000$ ) , m (rounds) and H (\%), when $0 \leq \mathrm{n} \leq 10,1 \leq \mathrm{t} \leq 10$

(Fig 3)
Conclusion: it is easily read from the graph that the strength of influence increases as time goes on at any point and it decreases with the rounds increasing at every moment.

Giving t different values to draw another graph, it is easy to draw the same conclusion.

When $\mathrm{t}=1000,2000,3000,4000,5000,6000,7000,8000,9000,10000$, $0<m \leq 10$

(Fig 4)
Giving $m$ different values to draw another graph, it is also easy to draw the same conclusion.

When $\mathrm{m}=1,2,3,4,5,6,7,8,9,10,500 \leq \mathrm{t} \leq 10000$

(Fig 5)

Draw the top view of Fig 2

(Fig 6)
Conclusion: it is read from the graph that the number of spreading round enlarges and the propagation rate raises as time goes on.

### 2.2.2 Cooling-Effect of Information Diffusion

Similarly, heat dissipation is compared to information diffusion to establish the cooling-effect model. We assume at there is no information diffusion between every two points. The influence of infected information is dropping to near 0 at the end of the diffusion duration. It's similar to the progress during which people lose their memories.

Assume that the propagation rate of information is directly proportional to the strength of influence, so:

$$
\begin{equation*}
v=-\beta \Delta H \tag{39}
\end{equation*}
$$

( $\beta$ is a constant, which is negative)
Then, get the equation:

$$
\begin{equation*}
\frac{\mathrm{dH}}{\mathrm{dt}}=-\beta(0-\mathrm{H}) \tag{40}
\end{equation*}
$$

Solve the equation:

$$
\begin{align*}
& \frac{\mathrm{dH}}{\mathrm{H}}=\beta \mathrm{dt}  \tag{41}\\
& \mathrm{H}=\mathrm{Ce}^{-\beta t} \tag{42}
\end{align*}
$$

Because of $H(0)=H_{0}$, we can get:

$$
\begin{equation*}
\mathrm{C}=\mathrm{H}_{0} \tag{43}
\end{equation*}
$$

Finally, get the expression:

$$
\begin{equation*}
\mathrm{H}=\mathrm{H}_{0} \mathrm{e}^{-\beta \mathrm{t}} \tag{44}
\end{equation*}
$$

Make $\beta=0.06$, which is the exponential of the function of the forgetting curve.
And substitute it and $\mathrm{H}_{0}=100$ into (44)
The graphs as follows:

(Fig 7)

(Fig 8)

(Fig 9)
A typical graph of the forgetting curve purports to show that

(Fig 7)
Conclusion: Comparison of the Ebbinghaus forgetting curve and cooling-effect model graphs shows this model portrays the memory loss happening in the human brain. For the extremely abstract and unpredictable information diffusion phenomenon, the process has an explanation based on the human cognitive rules of learning and forgetting.

## 3. The 1st Application of Heating-Effect and Cooling-Effect of Information Diffusion : The Transmission of Public Opinion

### 3.1 Introduction

Living in the information age, human has to deal with tons of it every day. Daily information consists of public opinion, among which some individual irresponsible rumor may spread as infectious disease to ruin people's life. In this case, authorities have to take the responsibility to monitor the spreading of information. The first application will explain the impact on communication by two separate phases.

And an essential prerequisite must be pointed out that all models are built on the simple network of communication, instead of some complex networks.

### 3.2 Application

Assume a starter transmitting a piece of opinion, and after x rounds of transmission, the diffusion function get from the Heating-effect Model function is:

$$
H(n, t)=H_{0}-\frac{4 H_{0}}{\pi} \sum_{i=1}^{\infty}\left(\frac{1}{2 i-1} e^{-k^{2}\left[\frac{(2 i-1) \pi}{2 x}\right]^{2} t} \cos \frac{(2 i-1) \pi}{2 x} n\right)
$$

$$
(\mathrm{i}=1,2,3,4 \ldots \ldots)
$$

Unlike in the previous example, here consider that $30 \%$ of the receivers whom are referred to as 'passively immune receivers' are immune to the info and do not pass it on to others and that $70 \%$ of receivers are transmitters. Therefore, $\mathrm{a}=$ $0.8 \times 0.7, \mathrm{k}^{2}=0.014$ and $\mathrm{H}_{0}=100$

Then draw the following graph showing the process of a piece of public opinion diffusing over time.

(Fig 8)
Suppose at $\mathrm{n}=10$, the government makes an intervention to stop the further spread of the opinion, all receivers of that info will be the only people ever to know. Then with modeling we obtain figure on relation between time and the strength of the opinion's influence at $\mathrm{n}=10$. (Same as in previous models, $\beta=0.06$ )

| $\mathrm{H}_{0}$ | t |
| :--- | :--- |
| 7.966423 | 1000 |
| 36.10843 | 2000 |
| 54.82654 | 3000 |
| 68.02369 | 4000 |
| 77.36371 | 5000 |
| 83.97552 | 6000 |
| 88.65608 | 7000 |
| 91.96951 | 8000 |
| 94.31512 | 9000 |
| 95.97561 | 10000 |

(Tab. 1)
Substitute values into the Cooling-effect Function:

$$
\mathrm{H}=\mathrm{H}_{0} \mathrm{e}^{-\beta \mathrm{t}}
$$

Draw the following graph describing the cooling process of the opinion:

(Fig 9)
And draw another graph showing the relationship between H and t .

(Fig 10)
Suppose at $\mathrm{t}=30$, the government makes an intervention to stop the further spread of the opinion, then we obtain the following figure:

| H | m |
| :--- | :--- |
| 92.93562 | 1 |
| 86.04486 | 2 |
| 79.49712 | 3 |
| 73.45352 | 4 |
| 68.06288 | 5 |
| 63.45812 | 6 |
| 59.75292 | 7 |
| 57.03886 | 8 |
| 55.38306 | 9 |
| 54.82654 | 10 |

(Tab. 2)
Substitute values into the Cooling-effect Function:

(Fig 11)
Putting two graphs together looks like:

(Fig 12)
This Figure shows the relation between the impacts of an opinion under different occasions of intervention.

### 3.3. Analysis

The Heating-Effect Model and Cooling-Effect Model as shown can be applied to the simplest form of opinion dissemination.

When a piece of public opinion goes through the process, it first continuously spreads and then gradually cools down as it is.

The application can allow the government, crisis management or epidemic scientists to make predictions about the social option on a particular issue. It, therefore, enables the government to make plan more scientifically and crisis PR to determine best time to release an announcement for their best interest.

## 4. The 2nd Application of Heating-Effect of Information Diffusion: Disaster Relief Goods Distribution

### 4.1 Introduction

Earthquakes, landslides, snowstorms, tsunamis, and hurricanes. We live on a planet where a variety of disasters occur and have a tremendous impact on people's lives. The 2008 Wenchuan earthquake displaced millions of people. At the time rescue was the priority. However, communications, which were crucial, were severely cut off. Moreover, transportation route to the outside world were disconnected and thereby, the quake-stricken areas were inaccessible to trucks carrying disaster relief supplies. Most of the materials had to reach their target by helicopters and be dispensed through manual handling at the scene. To help figure out the dissemination of the information of supplies under that circumstance and make the relevant decision, the Heating-Effect Model can be applied.

### 4.2 Application

Assumed that there are $P$ victims in a disaster-stricken area, helicopters have to air drop goods due to blocked transportations. As people can only have the simplest information diffusion there, it is the system of Oral Communication. Moreover, due to victims' inferior health status and living environment, each person can only speak to totally R people after they are told the message.

Heating-effect modeling shows:

$$
\begin{array}{r}
H(n, t)=H_{0}-\frac{4 H_{0}}{\pi} \sum_{i=1}^{\infty}\left(\frac{1}{2 i-1} e^{-k^{2}\left[\frac{(2 i-1) \pi}{2 x}\right]^{2} t} \cos \frac{(2 i-1) \pi}{2 x} n\right) \\
(i=1,2,3,4 \ldots \ldots)
\end{array}
$$

Suppose that $R=20, P=10000$, so the total number of receiver is $x=\frac{P}{R}=500$
Substitute $H_{0}=100, \mathrm{k}^{2}=0.02$, into the function, and get the graph following $(\mathrm{t}(\times 200000))$ :

(Fig 13)
Like the method used, draw top view of Fig 13:


If it is impossible to get the expression of the curve in Fig14 which is expressed as $f(t)$, the function $g(t)=R^{f(t)}$ can be used to calculate the total number of receivers in the disaster area, help the government to make plants.

### 4.3 Analysis

Heat-Effect modeling on dispersion of the information of relief goods is successful. It is more practical and reality-based than other practices of applications. Using this model, agencies can be better at monitoring. If a salvation expense is taken into consideration, what to do to achieve the maxim efficiency within minimum expense is predictable. The Wenchuan earthquake in 2008 could have received better care.

## 5. The 3rd Application of Cooling -Effect of Information Diffusion: The Recovery of Financial Crisis

### 5.1 Introduction

The Global Financial Crisis of the late 2000s, triggered by the US subprime mortgage fiasco, severely hit the world economy. It has cost countries and companies billions of dollars and has forced thousands of people out of job around the world. Five years since the burst of the US housing market, butterfly effect of the subprime crisis is still salient and profound as European countries struggle to resolve their sovereign debt crisis. This paper will examine how the US housing market had come to play a role as the epicenter of the global financial crisis. It will illustrate state of the US housing market before the subprime crisis followed by detailed analysis on cause of the crisis.

### 5.2 Application

Cooling-effect modeled can be applied to the recovery of the subprime crisis.
Considering the crisis is an abstract concept, we use the amount of bankrupted bank to make it intuitional. And the Cooling-Effect Model used to describe the strength of the influence of the subprime crisis.

Here is the function:

$$
\mathrm{H}=\mathrm{H}_{0} \mathrm{e}^{-\beta \mathrm{t}}
$$

### 5.3 Case

Statistics on US bank failures 2008-2013 are presented in the following graph.
All bank subjects were put into three categories: the large bank with total assets of one hundred million US dollars, medium-sized banks with one hundred million to one million US dollars, and small banks with 1 million. We add these numbers up, and draw the following graphs.

(Fig 15)

| Date (month/year) | Large | Middle-sized | Small | Total |
| :--- | :--- | :--- | :--- | :--- |
| $1 / 2008$ | 0 | 0 | 1 | 1 |
| $2 / 2008$ | 0 | 0 | 0 | 0 |
| $3 / 2008$ | 0 | 1 | 0 | 1 |
| $4 / 2008$ | 0 | 0 | 0 | 0 |
| $5 / 2008$ | 1 | 0 | 2 | 3 |
| $6 / 2008$ | 0 | 0 | 0 | 0 |
| $7 / 2008$ | 2 | 3 | 3 | 8 |
| $8 / 2008$ | 1 | 3 | 3 | 7 |
| $9 / 2008$ | 2 | 3 | 3 | 8 |
| $10 / 2008$ | 0 | 2 | 4 | 6 |
| $11 / 2008$ | 3 | 5 | 3 | 13 |
| $12 / 2008$ | 0 | 2 | 6 | 5 |
| $1 / 2009$ | 0 | 6 | 10 | 21 |
| $2 / 2009$ | 2 | 9 | 5 | 10 |
| $3 / 2009$ | 0 | 5 | 8 | 18 |
| $4 / 2009$ | 2 | 8 | 7 | 16 |
| $5 / 2009$ | 2 | 6 | 9 | 46 |
| $6 / 2009$ | 0 | 7 | 24 |  |
| $7 / 2009$ | 4 | 18 |  |  |
|  |  |  |  |  |


| 8/2009 | 4 | 12 | 15 | 31 |
| :---: | :---: | :---: | :---: | :---: |
| 9/2009 | 3 | 9 | 11 | 23 |
| 10/2009 | 4 | 15 | 20 | 39 |
| 11/2009 | 2 | 6 | 9 | 17 |
| 12/2009 | 5 | 14 | 16 | 35 |
| 1/2010 | 5 | 11 | 15 | 31 |
| 2/2010 | 1 | 4 | 7 | 12 |
| 3/2010 | 2 | 15 | 19 | 36 |
| 4/2010 | 9 | 19 | 23 | 51 |
| 5/2010 | 1 | 11 | 14 | 26 |
| 6/2010 | 1 | 5 | 8 | 14 |
| 7/2010 | 1 | 18 | 22 | 41 |
| 8/2010 | 0 | 8 | 10 | 18 |
| 9/2010 | 2 | 7 | 9 | 18 |
| 10/2010 | 0 | 9 | 12 | 21 |
| 11/2010 | 0 | 8 | 10 | 18 |
| 12/2010 | 2 | 6 | 8 | 16 |
| 1/2011 | 0 | 10 | 11 | 21 |
| 2/2011 | 0 | 10 | 12 | 22 |
| 3/2011 | 1 | 2 | 3 | 6 |
| 4/2011 | 0 | 11 | 13 | 24 |
| 5/2011 | 0 | 5 | 5 | 10 |
| 6/2011 | 2 | 3 | 4 | 9 |
| 7/2011 | 1 | 9 | 13 | 23 |
| 8/2011 | 0 | 6 | 7 | 13 |
| 9/2011 | 1 | 6 | 6 | 13 |
| 10/2011 | 0 | 10 | 11 | 21 |
| 11/2011 | 0 | 2 | 5 | 7 |
| 12/2011 | 1 | 2 | 2 | 5 |
| 1/2012 | 0 | 5 | 7 | 12 |
| 2/2012 | 0 | 3 | 4 | 7 |
| 3/2012 | 0 | 3 | 5 | 8 |
| 4/2012 | 0 | 5 | 6 | 11 |
| 5/2012 | 0 | 1 | 2 | 3 |
| 6/2012 | 0 | 4 | 7 | 11 |
| 7/2012 | 0 | 6 | 8 | 14 |
| 8/2012 | 0 | 0 | 1 | 1 |
| 9/2012 | 0 | 3 | 3 | 6 |
| 10/2012 | 0 | 3 | 4 | 7 |
| 11/2012 | 0 | 3 | 3 | 6 |
| 12/2012 | 0 | 0 | 1 | 1 |
| 1/2013 | 0 | 0 | 2 | 2 |
| 2/2013 | 0 | 0 | 1 | 1 |


| $3 / 2013$ | 0 | 1 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| $4 / 2013$ | 0 | 4 | 6 | 10 |
| $5 / 2013$ | 0 | 1 | 4 | 5 |
| $6 / 2013$ | 0 | 1 | 2 | 3 |
| $7 / 2013$ | 0 | 0 | 0 | 0 |
| $8 / 2013$ | 0 | 3 | 4 | 7 |
| $9 / 2013$ | 1 | 0 | 2 | 3 |
| $10 / 2013$ | 0 | 0 | 1 | 1 |

(Tab. 3)

(Fig 16)
According to USA Dow Jones stock index(below), We determine the period of Recovery of Subprime Crisis is the five years from October 2008 to October 2013.

(Fig 17)
Obtaining the starting and ending dates of 2008 crisis and using data from the bankrupted banks give the graph that describes the recovery of the subprime crisis.

(Fig 18)

Choose the climax point 64 as the value when $\mathrm{x}=0$ in the graph.

(Fig 19)
Compare to the cooling-effect model:

(Fig 20)

(Fig 21)
Conclusion: the Cooling-Effect model works well in describing the recovery of the subprime crisis.

### 5.4 Analysis

By comparison, the cooling-effect modeling in economics is roughly successful. The model can help institutions calculate their risk during the crisis according to their economic status (mentioned earlier) and make adjustments in a timely manner to reduce the danger of bankruptcy.

## 6. Summary and Expectations

Living in an information society stimulates our interest in information diffusion. Building a new model of it is just a bold idea, but two basic models eventually established through the effort, and have their worth in some applications.

This article aims to build a new model of information diffusion. Based on the basic laws of motion of matter, the new model provides a different perspective on studying information diffusion----making an analogy between information and heat. To some extent, the model is successful in describing dissemination and retrogression of information and being applied to public opinion transmission, message transmission of relief supplies in disaster areas and financial crisis.

But there are some deficiencies:

1. The modeling is a simplification, such as the assumption that the efficiency in receiving information for everyone is the same and the assumption that $70 \%$ of the people will pass on the message.
2. Plugging in some specific numbers is not fully reasonable.

Meanwhile, there are some deep researches to do:

1. Put the information diffusion in an unlimited space.
2. Make an analogy between heat conduct in a plane and information dissemination.
3. Study information diffusion in different original states.
4. Study information diffusion in the condition where there are multiple sources
5. Apply the model in more areas.

In the process of modeling, we have gained a lot. We have learned how to transform ideas into the reality, how to communicate and cooperate with each other, and also understand the relationship between math and our real life. And we believe the mathematical modes of thinking can be used not only in study but also in life.

Lastly, we would like to appreciate our teachers and such an opportunity enabling us to work together and think deeply.

## 7．Preparatory Knowledge

## 7．1．Thermodynamics

【1】 Newton＇s Cooling Effect：$\frac{\mathrm{dH}}{\mathrm{dt}}=-\mathrm{k}\left(\mathrm{H}-\mathrm{H}_{0}\right)$（ t is time， H is the correspondent Heat， $\mathrm{H}_{0}$ is Environmental Heat，here considered as a constant）

【2】The second law of thermodynamics：Heat can never pass from a colder to a warmer body without some other change，connected therewith，occurring at the same time

【3】Heat equation：$\frac{\partial \mathrm{u}}{\partial \mathrm{t}}=\alpha^{2} \frac{\partial^{2} \mathrm{u}}{\partial \mathrm{x}^{2}}$（ u is temperature， t is time， x is distance，and $\alpha$ is heat conductivity，a constant．）

7．2 Forgetting curve
【4】The forgetting curve：H．Ebbinghaus hypothesizes the decline of memory retention in time．This curve shows how information is lost over time when there is no attempt to retain it．A related concept is the strength of memory that refers to the durability those memory traces in the brain．

Suppose x hours after the first memory，the strength of memory rate fits the function： $\mathrm{y}=1-0.56 \mathrm{x}^{0.06}$

## 8．References

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## 9．Mathematica Program

```
In[1]:= Plot3D[100-4*(100/Pi)*Sum[(1/(2 i - 1)*E^(-0.02*((2 i - 1)*Pi/(2*10))^2*10 t)* Cos[(2 i -
1)*Pi*(10-n)/2/10]), {i, 1000}], {t, 1, 10}, {n, 0, 10}, AxesLabel -> {"t", "m", "H"}, PlotRange -> {0, 100}]
In[2]:= Show[%4, AxesStyle -> Directive[GrayLevel[0], AbsoluteThickness[1.13]]]
In[3]:= Plot3D[100-4*(100/Pi)* Sum[(1/(2 i - 1)*E^(-0.02*((2 i - 1)*Pi/(2*10))^2*100 t)* Cos[(2 i -
1)*Pi*(10-n)/2/10]), {i, 1000}], {t,1, 10}, {n, 0, 10}, AxesLabel -> {"t", "m", "H"}, PlotRange -> {0, 100}]
In[4]:= Show[%8, AxesStyle ->Directive[GrayLevel[0], AbsoluteThickness[1.52]]]
In[5]:= Plot3D[100-4*(100/Pi)*Sum[(1/(2 i - 1)*E^(-0.02*((2 i - 1)**Pi/(2*10))^2*1000 t)* Cos[(2 i -
1)*Pi*(10-n)/2/10]), {i, 1000}], {t, 1, 10}, {n, 0, 10}, AxesLabel -> {"t", "m", "H"}, PlotRange -> {0, 100}]
In[6]:= Show[%10, AxesStyle -> Directive[GrayLevel[0], AbsoluteThickness[1.68]]]
```

$\operatorname{In}[7]:=\operatorname{Show}\left[\operatorname{Pot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{* P i} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 1000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.\right.$
1)*Pi*(10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Red\},PlotLegends ->
$\{" \mathrm{t}=1000 \mathrm{n}\}], \operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 2000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$
1)*Pi* (10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Blue\}, PlotLegends ->
$\{" \mathrm{t}=2000 \mathrm{O}\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 3000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.$
1)*Pi* (10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Magenta\}, PlotLegends ->
$\{" \mathrm{t}=3000 \mathrm{O}\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 4000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.$
1)*Pi* (10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Pink\}, PlotLegends ->
$\{\mathrm{t}=4000$ " $\}], \operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{* 5000}\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$
1)*Pi*(10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Green\}, PlotLegends ->
$\{" \mathrm{t}=5000$ " $\}], \operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 6000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$
1)*Pi* $(10-n) / 2 / 10]),\{i, 1000\}],\{n, 0,10\}$, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Black\}, PlotLegends ->
$\{" \mathrm{t}=6000 \mathrm{O}\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E} \wedge\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 7000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.$
1)*Pi*(10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Cyan\}, PlotLegends ->
$\{" t=7000 "\}], \operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 8000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1) $\left.\left.\left.{ }^{2} \mathrm{Pi}^{*}(10-\mathrm{n}) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{n}, 0,10\}$, AxesLabel $->\{" \mathrm{~m} ", ~ " H "\}$, PlotStyle $->\{$ Orange $\}$, PlotLegends $->$ $\{" t=8000 "\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 9000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1) $\left.\left.\left.{ }^{*} \mathrm{Pi}^{*}(10-\mathrm{n}) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{n}, 0,10\}$, AxesLabel $->\{" \mathrm{~m} "$, "H"\}, PlotStyle $->\{$ Brown $\}$, PlotLegends $->$ $\{" t=9000 "\}]$, Plot[100-4* $(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 10000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.$ 1) $\left.\left.\left.{ }^{*} \operatorname{Pi}^{*}(10-n) / 2 / 10\right]\right),\{i, 1000\}\right],\{n, 0,10\}$, AxesLabel $->\{" m ", " H "\}$, PlotStyle $->$ \{Purple\}, PlotLegends $->$ $\{" t=10000 "\}]]$
$\operatorname{In}[8]:=\operatorname{Show}\left[\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.\right.$ 1)* $\left.\left.\left.\operatorname{Pi}^{*}(10-1) / 2 / 10\right]\right),\{i, 1000\}\right],\{t, 5,100\}$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->\{$ Red $\}$, PlotLegends $->$ $\{" m=1 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4*(100/Pi)*Sum[(1/(2 i $\left.\left.1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}((2 \mathrm{i}-1) * \mathrm{Pi} /(2 * 10))^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-2) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel -> \{"t", "H"\}, PlotStyle $->$ \{Blue $\}$, PlotLegends $->\{" m=2 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4* $(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*}\right.\right.$ $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-3) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Magenta\}, PlotLegends $->\{" \mathrm{~m}=3 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1) * \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-4) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5$, 100\}, AxesLabel $->\{" \mathrm{t} ", ~ " H "\}$, PlotStyle $->\{$ Pink $\}$, PlotLegends $->\{" \mathrm{~m}=4 "\}$, PlotRange $->\{\{10,100\},\{0$, $100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100\right.\right.\right.\right.$ t)* $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-5) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Green $\}$, PlotLegends $->\{" \mathrm{~m}=5 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100 $-4^{*}(100 / \mathrm{Pi})^{*}$ $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1) * \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-6) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5$, 100\}, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->$ \{Black $\}$, PlotLegends $->\{" m=6 "\}$, PlotRange $->\{\{10,100\},\{0$, $100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4* $(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100\right.\right.\right.$ t) $\left.\left.{ }^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-7) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->\{\mathrm{Cyan}\}$, PlotLegends $->\{" m=7 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1) * \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-8) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5$, 100\}, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->\{$ Orange $\}$, PlotLegends $->\{" m=8 "\}$, PlotRange $->\{\{10,100\},\{0$, $100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100\right.\right.\right.\right.$ t) $\left.\left.{ }^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-9) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Brown $\}$, PlotLegends $->\{" m=9 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-10) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5$, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle $->$ \{Purple\}, PlotLegends $->\{" m=10 "\}$, PlotRange $->\{\{10,100\},\{0$, 100\}\}, AxesOrigin $->\{10,0\}]$ ]
$\operatorname{In}[9]:=\operatorname{DensityPlot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 5000 \mathrm{t}\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1) $\left.\left.\left.\left.{ }^{2} \operatorname{Pi}^{*}(10-m) / 2 / 10\right]\right),\{i, 1000\}\right],\{t, 1,10\},\{m, 0,10\}\right]$
$\operatorname{In}[10]:=\operatorname{Plot}\left[E^{\wedge}(-0.06 t+\log [100]),\{t, 0,20\}\right.$, PlotRange $->\{0,100\}$, AxesLabel $\left.->\{" t ", " H "\}\right]$

In $\{11\}:=$ Show[\%61, AxesStyle $->$ Directive[GrayLevel[0], AbsoluteThickness[1.185]], Method $->$ \{"DefaultBoundaryStyle" -> Automatic, "ScalingFunctions" -> None\}]
$\operatorname{In}[12]:=\operatorname{Plot}\left[E^{\wedge}(-0.06 t+\log [100]),\{t, 0,100\}\right.$, PlotRange $->\{0,100\}$, AxesLabel $\left.->\{" t ", " H "\}\right]$

In [13]:= Show[\%65, AxesStyle -> Directive[GrayLevel[0], AbsoluteThickness[1.31]], Method -> \{"DefaultBoundaryStyle" -> Automatic, "ScalingFunctions" -> None\}]
$\operatorname{In}[14]:=\operatorname{Plot}\left[\mathrm{E}^{\wedge}(-0.06 \mathrm{t}+\log [100]),\{\mathrm{t}, 0,500\}\right.$, PlotRange $->\{0,100\}$, AxesLabel $->\{$ "t", "H"\}]

In[15]:= Show[\%64, AxesStyle -> Directive[GrayLevel[0], AbsoluteThickness[1.35]], Method -> \{"DefaultBoundaryStyle" -> Automatic, "ScalingFunctions" -> None\}]
$\operatorname{In}[16]:=\operatorname{Show}\left[\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}((2 \mathrm{i}-1))^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 1000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $\left.\left.\left.\left.{ }^{2} 10-n\right) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{n}, 0,10\}$, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Red\}, PlotLegends -> $\{" \mathrm{t}=1000$ " $\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 2000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $\left.\left.\left.\left.{ }^{*} 10-n\right) / 2 / 10\right]\right),\{i, 1000\}\right],\{n, 0,10\}$, AxesLabel $->\{" m ", " H "\}$, PlotStyle $->$ \{Blue\}, PlotLegends $->$ $\{" t=2000 "\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 3000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $(10-n) / 2 / 10]),\{i, 1000\}],\{n, 0,10\}$, AxesLabel $->\{" m ", " H "\}$, PlotStyle $->$ \{Magenta\}, PlotLegends -> $\{" \mathrm{t}=3000 \mathrm{C}\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 4000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $\left.\left.\left.\left.{ }^{*} 10-n\right) / 2 / 10\right]\right),\{i, 1000\}\right],\{n, 0,10\}$, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Pink\}, PlotLegends -> $\{" \mathrm{t}=4000$ " $\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 5000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $\left.\left.\left.\left.{ }^{*} 10-n\right) / 2 / 10\right]\right),\{i, 1000\}\right],\{n, 0,10\}$, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Green\}, PlotLegends -> $\{" \mathrm{t}=5000 \mathrm{n}\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 6000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $(10-n) / 2 / 10]),\{i, 1000\}],\{n, 0,10\}$, AxesLabel -> $\{" m ", " H "\}$, PlotStyle $->$ \{Black\}, PlotLegends $->$ $\{" \mathrm{t}=6000 \mathrm{C}\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 7000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi*(10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Cyan\}, PlotLegends -> $\{" \mathrm{t}=7000 \mathrm{C}\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 8000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $(10-n) / 2 / 10]),\{\mathrm{i}, 1000\}],\{n, 0,10\}$, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Orange\}, PlotLegends -> $\{" \mathrm{t}=8000 \mathrm{n}\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 9000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi* $\left.\left.\left.{ }^{*}(10-n) / 2 / 10\right]\right),\{i, 1000\}\right],\{n, 0,10\}$, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Brown\}, PlotLegends -> $\{" t=9000 "\}], \operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 10000\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)*Pi*(10-n)/2/10]), \{i, 1000\}], \{n, 0, 10\}, AxesLabel -> \{"m", "H"\}, PlotStyle -> \{Purple\}, PlotLegends -> $\{" t=10000 "\}]]$
$\operatorname{In}[17]:=\mathrm{m}=$ Table[100-4*(100/Pi)$)^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 1000 \mathrm{t}\right) * \operatorname{Cos}[(2 \mathrm{i}-\right.\right.$ 1)*Pi*n/2/10]), \{i, 1000\}], \{t, 0, 10\}, \{n, 0, 10\}]m//TableForm

In [18]:=Show[Plot[E^(-0.06 t + Log[7.966422604]), \{t, 0, 10\}, PlotStyle -> \{Red\}, AxesLabel -> \{"t", "H"\}, PlotLegends -> \{"t=1000"\}], Plot[E^(-0.06t + Log[36.10842639]), \{t, 0, 10\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Blue\}, PlotLegends $->\{" t=2000 "\}]$, Plot[E^( $-0.06 t+\log [54.82653631]),\{t, 0,10\}$, AxesLabel $->\{" t ", ~ " H "\}$, PlotStyle $->$ \{Magenta\}, PlotLegends $->\{" t=3000 "\}]$, Plot[E^(-0.06 t + Log[68.02368988]), \{t, $0,10\}$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Pink\}, PlotLegends -> \{"t=4000"\}], Plot[E^(-0.06 t + $\log [77.36371333]),\{\mathrm{t}, 0,10\}$, AxesLabel $->\{" \mathrm{t}$ ", "H"\}, PlotStyle $->$ \{Green\}, PlotLegends $->\{" \mathrm{t}=5000 \mathrm{n}\}]$, Plot[E^(-0.06t+Log[83.97551671]), \{t, 0, 10\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Black\}, PlotLegends -> $\{" t=6000 "\}], \operatorname{Plot}\left[\mathrm{E}^{\wedge}(-0.06 \mathrm{t}+\log [88.65608378]),\{\mathrm{t}, 0,10\}\right.$, AxesLabel $->\{" \mathrm{t} ", ~ " H "\}$, PlotStyle $->$ \{Cyan\}, PlotLegends -> $\{" t=7000 "\}], \operatorname{Plot}\left[E^{\wedge}(-0.06 t+\log [91.96951095]),\{t, 0,10\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle -> \{Orange\}, PlotLegends -> \{"t=8000"\}], $\operatorname{Plot}\left[E^{\wedge}(-0.06 t+\log [94.31512423]),\{t, 0,10\}\right.$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Brown\}, PlotLegends -> \{"t=9000"\}], Plot[E^(-0.06 t +

Log[95.97561091]), \{t, 0, 10\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Purple\}, PlotLegends -> \{"t=10000"\}], PlotRange -> $\{0,100\}]$
$\operatorname{In}[19]:=\operatorname{Show}\left[\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.\right.$ 1)*Pi* (10-1)/2/10]), \{i, 1000\}], \{t, 5, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Red\}, PlotLegends -> $\{" m=1 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4*(100/Pi)* Sum[(1/(2 i $\left.\left.1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-2) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Blue\}, PlotLegends -> \{"m=2"\}, PlotRange -> \{\{10, 100\}, \{0, 100\}\}, AxesOrigin -> \{10, 0\}], Plot[100-4*(100/Pi)*Sum[(1/(2i-1)*E^(-0.014*((2i-1)*Pi/(2*10))^2*100t)* $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-3) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel -> $\{" \mathrm{t} ", ~ " \mathrm{H} "\}$, PlotStyle -> \{Magenta\}, PlotLegends $->\{" m=3 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-4) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5$, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Pink\}, PlotLegends $->\{" m=4 "\}$, PlotRange $->\{\{10,100\},\{0$, $100\}\}$, AxesOrigin -> \{10, 0\}], Plot[100-4*(100/Pi)* Sum[(1/(2 i - 1)*E ${ }^{\wedge}\left(-0.014^{*}((2\right.$ i -
1)*Pi/(2*10) $\left.\left.\left.)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-5) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel -> \{"t", "H"\}, PlotStyle $->$ \{Green\}, PlotLegends $->\{" m=5 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-\right.\right.\right.\right.$ 6)/2/10]), \{i, 1000\}], \{t, 5, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Black\}, PlotLegends -> \{"m=6"\}, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}[(1 /(2 \mathrm{i}-\right.$ $\left.\left.1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-7) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Cyan\}, PlotLegends $->\{" m=7 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin -> \{10, 0\}], Plot[100-4*(100/Pi)* Sum[(1/(2i-1)*E^(-0.014* ((2i-1)*Pi/(2*10))^2*100t)* $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-8) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Orange $\}$, PlotLegends $->\{" m=8 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4*(100/Pi)* $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-9) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5$, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Brown\}, PlotLegends -> \{"m=9"\}, PlotRange -> \{\{10, 100\}, \{0, $100\}\}$, AxesOrigin -> $\{10,0\}]$, Plot[100-4* $(100 / P i)^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}((2 \mathrm{i}-\right.\right.\right.$
1)*Pi/(2*10) )^2*100 t)* $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-10) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 5,100\}$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Purple\}, PlotLegends $->\{" m=10 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]]$
$\operatorname{In}[20]:=\mathrm{m}=$ Table $\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 1000 \mathrm{t}\right)^{*} \operatorname{Cos}[(2 \mathrm{i}\right.\right.\right.$ $\left.\left.\left.\left.-1)^{*} \mathrm{Pi}^{*} \mathrm{n} / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 0,10\},\{\mathrm{n}, 0,10\}\right]$

In[21]:=m // TableForm
$\operatorname{In}[22]:=\operatorname{Show}\left[\operatorname{Plot}\left[\mathrm{E}^{\wedge}(-0.06(\mathrm{t}-30)+\log [92.93562]),\{\mathrm{t}, 30,100\}\right.\right.$, PlotStyle $->\{$ Red $\}$, AxesLabel $->\{" \mathrm{t} "$,
"H"\}, PlotLegends -> \{"m=1"\}], Plot[E^(-0.06 (t-30) + Log[86.04486]), \{t, 30, 100\}, AxesLabel -> \{"t", "H"\},
PlotStyle -> \{Blue\}, PlotLegends $->\{" m=2 "\}], \operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [79.49712]),\{t, 30,100\}\right.$,
AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Magenta\}, PlotLegends -> \{"m=3"\}], Plot[E^(-0.06 (t-30) +
$\log [73.45352]),\{\mathrm{t}, 30,100\}$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Pink\}, PlotLegends $->\{" m=4 "\}]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [68.06288]),\{t, 30,100\}\right.$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Green\}, PlotLegends -> \{"m=5"\}], Plot[E^(-0.06 (t-30) + Log[63.45812]), \{t, 30, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Black\}, PlotLegends -> \{"m=6"\}], Plot[E^(-0.06 (t-30) + Log[59.75292]), \{t, 30, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Cyan\}, PlotLegends -> \{"m=7"\}], Plot[E^(-0.06 (t-30) + Log[57.03886]), \{t, 30, 100\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Orange\}, PlotLegends -> \{"m=8"\}],
$\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [55.38306]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->\{$ Brown $\}$, PlotLegends $->\{" m=9 "\}], \operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [54.82654]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->$ \{Purple\}, PlotLegends $->\{" m=10 "\}]$, PlotRange $->\{0,100\}]$
$\operatorname{In}[23]:=\operatorname{Show}\left[\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.\right.$ 1)* $\left.\left.\left.\mathrm{Pi}^{*}(10-1) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 10,30\}$, AxesLabel $->\{" \mathrm{t} ", ~ " H "\}$, PlotStyle $->\{$ Red $\}$, PlotLegends $->$ $\left\{" m=1^{\prime \prime}\right\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $\left.->\{10,0\}\right], \operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}[(1 /(2 \mathrm{i}-\right.$ $\left.\left.1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \operatorname{Pi}^{*}(10-2) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 10,30\}$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->$ \{Blue $\}$, PlotLegends $->\{" m=2 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100-4* $(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \operatorname{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*}\right.\right.$ $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1) * \mathrm{Pi}^{*}(10-3) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 10,30\}$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Magenta\}, PlotLegends $->\{" m=3 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100 $-4^{*}(100 / \mathrm{Pi})^{*}$ $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}((2 \mathrm{i}-1) * \mathrm{Pi} /(2 * 10))^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-4) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}$, 10, 30\}, AxesLabel -> \{"t", "H"\}, PlotStyle $->$ \{Pink\}, PlotLegends $->\{" m=4 "\}$, PlotRange $->\{\{10,100\},\{0$, $100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}((2\right.\right.\right.\right.$ i 1)* $\left.\left.\left.\left.\mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-5) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 10,30\}$, AxesLabel $->\{" \mathrm{t} ", " \mathrm{H} "\}$, PlotStyle $->\{$ Green $\}$, PlotLegends $->\{" m=5 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-\right.\right.\right.\right.$ 6)/2/10]), \{i, 1000\}], \{t, 10, 30\}, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Black\}, PlotLegends $->$ \{"m=6"\}, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}[(1 /(2\right.$ i $\left.\left.1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1) * \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-7) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 10,30\}$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Cyan\}, PlotLegends $->\{" m=7 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*}\right.\right.\right.$ $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-8) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 10,30\}$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Orange $\}$, PlotLegends $->\{" m=8 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, Plot[100 $-4^{*}(100 / \mathrm{Pi})^{*}$ $\operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}\left((2 \mathrm{i}-1)^{*} \mathrm{Pi} /\left(2^{*} 10\right)\right)^{\wedge} 2^{*} 100 \mathrm{t}\right)^{*} \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(10-9) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}$, 10, 30\}, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->\{$ Brown $\}$, PlotLegends $->\{" m=9 "\}$, PlotRange $->\{\{10,100\},\{0$, $100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.014^{*}((2\right.\right.\right.\right.$ i -
1)*Pi/(2*10))^2*100 t)* $\left.\left.\operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \operatorname{Pi}^{*}(10-10) / 2 / 10\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 10,30\}$, AxesLabel $->\{" \mathrm{t} ", " H "\}$, PlotStyle $->$ \{Purple\}, PlotLegends $->\{" m=10 "\}$, PlotRange $->\{\{10,100\},\{0,100\}\}$, AxesOrigin $->\{10,0\}]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [92.93562]),\{t, 30,100\}\right.$, PlotStyle $->\{$ Red $\}$, AxesLabel $\left.->\{" t ", " H "\}\right]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [86.04486]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->\{$ Blue $\left.\}\right]$, $\operatorname{Plot}\left[\mathrm{E}^{\wedge}(-0.06(\mathrm{t}-30)+\log [79.49712]),\{\mathrm{t}, 30,100\}\right.$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Magenta $\left.\}\right]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [73.45352]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->\{$ Pink $\left.\}\right]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [68.06288]),\{t, 30,100\}\right.$, AxesLabel -> \{"t", "H"\}, PlotStyle -> \{Green\}], $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [63.45812]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->$ \{Black $\left.\}\right]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [59.75292]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->$ \{Cyan $\left.\}\right]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(\mathrm{t}-30)+\log [57.03886]),\{\mathrm{t}, 30,100\}\right.$, AxesLabel $->\{" \mathrm{t} "$, "H"\}, PlotStyle $->$ \{Orange\}], $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [55.38306]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->$ \{Brown $\left.\}\right]$, $\operatorname{Plot}\left[E^{\wedge}(-0.06(t-30)+\log [54.82654]),\{t, 30,100\}\right.$, AxesLabel $->\{" t ", " H "\}$, PlotStyle $->$ \{Purple $\left.\}\right]$, PlotRange $->\{0,100\}$ ]
$\operatorname{In}[24]:=\operatorname{Plot} 3 \mathrm{D}\left[100-4^{*}(100 / \mathrm{Pi})^{*} \operatorname{Sum}\left[\left(1 /(2 \mathrm{i}-1)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}((2 \mathrm{i}-1) * \mathrm{Pi} /(2 * 500))^{\wedge} 2^{*} 200000 \mathrm{t}\right) * \operatorname{Cos}[(2 \mathrm{i}-\right.\right.\right.$ 1)* $\left.\left.\left.\mathrm{Pi}^{*}(500-\mathrm{m}) / 2 / 500\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 1,10\},\{\mathrm{m}, 0,500\}$, AxesLabel $->\{" \mathrm{t} ", " \mathrm{~m} ", " H "\}$, PlotRange $->\{0$, 100\}]

In[25]:=Show[\%32, AxesStyle -> Directive[GrayLevel[0], AbsoluteThickness[1.27]]]
$\operatorname{In}[26]:=$ DensityPlot[100-4*(100/Pi)* Sum[(1/(2 i - 1) $)^{*} \mathrm{E}^{\wedge}\left(-0.02^{*}((2 \mathrm{i}-\right.$

1) $\left.\left.\left.\left.{ }^{2} \mathrm{Pi} /(2 * 500)\right)^{\wedge} 2^{*} 200000 \mathrm{t}\right) * \operatorname{Cos}\left[(2 \mathrm{i}-1)^{*} \mathrm{Pi}^{*}(500-\mathrm{m}) / 2 / 500\right]\right),\{\mathrm{i}, 1000\}\right],\{\mathrm{t}, 1,10\},\{\mathrm{m}, 0,500\}$, AxesLabel -> \{"t", "m", "H"\}, PlotRange -> \{0, 100\}]
$\operatorname{In}[27]:=$ DateListPlot $[\{\{1,0,1,0,3,0,8,7,8,6,13,5,12,21,10,18,15,16,46,31,23,39,17,35,31,12,36$, $51,26,14,41,18,21,18,16,21,22,6,24,10,9,23,13,21,7,5,12,7,8,11,3,11,14,1,6, ~, ~, 6,1,2,1,2,10$, $5,3,0,7,3,1\},\{0,0,0,0,1,0,2,1,2,0,3,0,0,2,0,2,2,0,4,4,3,4,2,5,5,1,2,9,1,1,1,0,2,0,0,2,0,0,1,0$, $0,2,1,0,1,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0\},\{1,0,0,0,2,0,3,3,3,4,5,3,6,0,5,8$, $7,9,4,15,1,20,9,6,15,7,9,23,4,8,2,10,9,2,10,8,1,12,3,3,5,4,3,7,6,1,5,2,7,4,5,6,2,7,8,1,3,4,3$, $1,2,1,1,6,4,2,0,4,2\},\{0,0,1,00,0,3,3,3,2,5,2,6,9,5,8,6,7,18,12,9,15,6,14,11,4,15,19,11,5,18$, $8,7,9,8,6,10,10, ~, 11,5,3,9,6,6,10,2,2,5,3,3,5,1,4,6,0,3,3,3,0,0,0,1,4,1,1,0,3,0,0\},\},\{2008,1\}$, Joined -> True, PlotLegends -> \{"Total", "Large", "Middle-sized", "Small"\}, PlotRange -> 55]
$\operatorname{In}[28]:=$ duibi $=\operatorname{ListPlot}[\{6,13,5,12,21,10,18,15,16,46,31,23,39,17,35,31,12,36,51,26,14,41,18$, $21,18,16,21,22,6,24,10,9,23,13,21,7,5,12,7,8,11,3,11,14,1,6,8,6,1,2,1,2,10,5,3,0,7,3,1\}$, Filling -> Axis, AxesLabel -> \{"m", "H"\} ]
$\operatorname{In}[29]:=$ duibi $=\operatorname{ListPlot}[\{46,31,23,39,17,35,31,12,36,51,26,14,41,18,21,18,16,21,22,6,24,10,9$, $23,13,21,7,5,12,7,8,11,3,11,14,1,6,8,6,1,2,1,2,10,5,3,0,7,3,1\}$, Filling $->$ Axis, AxesLabel $->\{" m "$, "H"\}]
$\operatorname{In}[30]:=\operatorname{leng}=\operatorname{Plot}\left[\mathrm{E}^{\wedge}(-0.06 t+\log [46]),\{\mathrm{t}, 0,60\}\right.$, AxesLabel $->\{" \mathrm{~m} "$, "H"\}]
$\operatorname{In}[31]:=$ Show[leng, duibi]
