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保险公司综合偿付能力的统计学评估

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Abstract

The main objective of this paper is to use altogether three statistical methods to figure out the integrated solvency of Chinese insurance companies in the year 2013 as compared to that of American insurance companies and provide suggestions for improvement.

For data analysis, we selected 4 companies among the top insurance corporations in China and 4 of their counterparts in America. By studying the 8 companies' annual reports, we derived the values of 7 important solvency indices (X_1 through X_7) for each company. In order to avoid partial evaluation that may result from studying only one index at a time, we applied two statistical methods—-the Principal Component Analysis (PCA) and the Factor Analysis (FA) to integrate the 7 inter-related solvency indices so as to determine each company's solvency in a comprehensive way. Our analytical results showed that, generally, Chinese insurance companies still fell behind American ones in terms of integrated solvency.

In order to help Chinese insurance companies to boost their integrated solvency, we found out the most significant index influencing our PCA and FA results and constructed a linear regression model comparing its relationship to other indices. We thus determined the two core approaches for integrated solvency enhancement: expanding investments and ensuring steady premium sources. In addition, since enhancing integrated solvency requires much more consideration, we included additional analysis on the current competitiveness of the Chinese insurance industry and arrived at a set of strategic suggestions for improvement.

Key words: Factor Analysis, insurance company, integrated solvency evaluation, linear regression model, Principal Component Analysis

Contents

1 Introduction

The issue of insurance and risk management has always been crucial for both the operation of insurance companies and the decision-making of regulatory agencies. Among all the elements that influence risk management, the issue of solvency has gained increasing significance in recent years, in that the failure of solvency tends to threaten the whole financial market in addition to individual insurance companies. Compared to the more sophisticated American insurance market, the insurance industry in China had a relatively slow start. Fortunately, in the past few decades, Chinese insurance companies have actively explored new management approaches and benefited from promotive policies made by supervisory agencies. Nevertheless, how well have Chinese insurance companies managed their financial operations, especially in terms of fulfilling solvency? Is there still much space for improvement?

Therefore, in order to assess the solvency performance and progress of Chinese insurance companies in the year 2013, we selected 4 prominent insurance companies in China (Ping An Insurance, People's Insurance, China Pacific Insurance, China Life Insurance) and compared them to 4 of their counterparts in the USA (American International Group, Berkshire Hathaway, Hartford Financial Services, MetLife). To make this comparison, we analyzed the 2013 annual reports of the 8 companies and gathered the useful data to compute the 7 indices that influence solvency: "loss ratio", "proportion of net income to total revenues", "investment income ratio", "reinsurance ratio", "rate of return on total assets", "asset-gross premium ratio", and "rate of change of premiums earned". However, rather than determine a company's solvency by analyzing one single index at a time, we need methods that could potentially generate several integrated solvency indices that reflect a company's overall solvency. As a result, we applied both the Principal Component Analysis (PCA) and Factor Analysis (FA) to the integration of the 7 indices so as to determine each company's solvency comprehensively.

After finding out the deficiency of Chinese insurance companies in terms of integrated solvency, we constructed a linear regression model comparing the most determining variable exhibited in PCA and FA to other indices. This statistical approach would give us insight on the most efficient ways for Chinese insurance companies to enhance solvency. Also in this paper, we analyzed other problems and opportunities in the Chinese insurance industry nowadays. Thus, we were able to conceive multiple ways for Chinese insurance companies to improve solvency and management. These suggestions will hopefully help Chinese insurance industry to prosper in the global market.

2 Solvency Indices as Variables

2.1 Analysis of the 7 solvency indices

There are multiple indices that influence an insurance company's solvency performance ([1]). Here, we choose 7 important indices that not only allow easy computations but also effectively represent a company's solvency.

1). Loss ratio (X_1)

Loss ratio is the ratio of total claims and expenses divided by total premiums earned. It is an important index that reflects a company's ability to fulfill its contract liabilities. A low loss ratio usually indicates a high reserve ratio, meaning the company is solvent enough to collect more premiums than the amount paid in claims. In short:

$$\text{loss Ratio}(X_1) = \frac{\text{claims and expenses}}{\text{premiums earned}} \times 100\%.$$

2). Proportion of net income to total revenues (X_2)

Proportion of net income to total revenues reflects the insurance company's ability to make a profit. A successful company tends to apply the rules of profit maximization to its operation, and a high value of this index usually indicates high profitability, and therefore, high solvency. Net income includes the income from both the insurance business and the investment business the company conducts. In short:

$$\text{proportion of net income to total revenues}(X_2) = \frac{\text{net income on business}}{\text{total revenues}} \times 100\%.$$

3). Investment income ratio (X_3)

Investment income ratio also reflects an insurance company's capacity for profit, but it focuses on the investment business the company carries out. In short:

$$\text{investment income ratio}(X_3) = \frac{\text{net investment income}}{\text{total investments}} \times 100\%.$$

4). Reinsurance ratio (X_4)

Since an insurance company conducts liability business, reinsurance ratio also affects its solvency. A company has to make deliberate decisions on how much premiums they are going to cede to other companies or financial organization based on its actual solvency. A low reinsurance ratio usually indicates that this company has high solvency and is able to handle a large portion of the insurance business it has been assigned. In short:

$$\text{reinsurance ratio}(X_4) = \frac{\text{premiums ceded}}{\text{premiums earned}} \times 100\%.$$

5). Rate of return on total assets (X_5)

Rate of return on total assets measures the potential solvency of an insurance company. A high value of this index indicates that the company has enough surplus reserves to carry out business and fulfill adequate solvency in the future. In short:

$$\text{rate of return on total assets}(X_5) = \frac{\text{surplus reserves}}{\text{total assets}} \times 100\%.$$

6). Asset-gross premium ratio (X_6)

Asset-gross premium ratio measures the short-term operating stability of an insurance company. By computing the change in total assets divided by total premiums earned, we can determine the stability of the company's asset pool and whether it can maintain solvency in the near future. A high ratio usually shows a large change in the total assets, and thus suggests instability and potential failure of solvency. In short:

$$\begin{aligned}&\text{asset-gross premium ratio}(X_6)\\&= \frac{|\text{total assets of this year-total assets of last year}|}{\text{premiums earned}} \times 100\%.\end{aligned}$$

7). Rate of change of premiums earned (X_7)

Like the asset-gross premium ratio, rate of change of premiums earned also reflects the operating stability. A company is usually successful if it remains steady in the premiums earned. In short:

$$\begin{aligned}&\text{rate of change of premiums earned}(X_7)\\&= \frac{|\text{premiums earned this year-premiums earned last year}|}{\text{premiums earned last year}} \times 100\%.\end{aligned}$$

2.2 Original data

We first downloaded the 2013 annual reports of the 8 targeted insurance companies from the Internet([2]), including four Chinese companies, Ping An Insurance (PAIC), People's Insurance (PIC), China Pacific Insurance (CPIC), China Life Insurance (CLIC) and four USA companies, American International Group (AIG), Berkshire Hathaway (BH), Hartford Financial Services (HFS) and MetLife. Then we drew useful data from the information-rich Consolidated Statement charts in each report and conducted computations when needed. Finally, we could determine the values of the 7 indices for each company for the year 2013. The original data is displayed in **Table 1.**

Companies	X_1	X_2	X_3	X_4	X_5	X_6	X_7
PAIC	60.4	12.8	5.1	13.7	1.1	7.8	13.7
PIC	67.9	5.8	5.2	11.0	1.6	21.7	15.5
CPIC	66.0	4.9	5.0	8.6	3.6	23.7	8.4
CLIC	42.7	6.8	4.9	0.2	5.6	22.8	0.8
AIG	71.9	13.2	4.4	24.2	10.0	18.6	0.5
BH	76.7	23.9	5.0	2.2	4.0	15.5	6.2
HFS	75.6	6.7	9.6	4.9	6.3	18.6	3.0
MetLife	70.1	5.0	4.5	6.0	4.7	12.8	0.5

Table 1: The original data matrix X

In **Table 1**, the variable X_1 represents Loss ratio, X_2–Proportion of net income to total revenues, X_3– Investment income ratio, X_4–Reinsurance ratio, X_5–Rate of return on total assets, X_6–Asset-gross premium ratio and X_7–Rate of change of premiums earned.

3 Principal Component Analysis

In this paper, we applied two dimension reduction statistical methods - Principal Component Analysis and Factor Analysis to evaluate the solvency of the above mentioned 8 insurance companies. The main purpose of using these two methods is that individual solvency indices cannot reflect the integrated solvency performance of each company. Since there are so many indices that influence solvency and evaluating each one of them will produce different results, we need to find ways to integrate these indices so as to evaluate each company's solvency comprehensively. In this section, we first introduce the basic ideas of PCA, and then we use this method to analyze the data.

3.1 Basic ideas of PCA

It is well known that PCA is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components ([3-6]). PCA can supply the user with a lower-dimensional picture, a projection or "shadow" of this object when viewed from its most informative viewpoint. The first principal component accounts for as much of the variability in the data as possible, and each succeeding component accounts for as much of the remaining variability as possible. The objectives of principal component analysis are to discover or to reduce the dimensionality of the data set and to identify new meaningful underlying variables. Mathematically speaking, the principal components are the linear combinations of the observed variables. That is,

$$
\begin{aligned}
&Z_1 = w_{11}X_1 + \cdots + w_{1p}X_p, \\
&Z_2 = w_{21}X_1 + \cdots + w_{2p}X_p, \\
&\cdots \\
&Z_p = w_{p1}X_1 + \cdots + w_{pp}X_p,
\end{aligned}
$$

where p is the number of the observed original variables; Z_1 is the first principal component, $\cdots$, Z_p is the p-th principal component; $W_{(k)} = (w_{k1}, \cdots, w_{kp})$ is the loading vector, which maps the observed variables to the k-th principal component; $W = (w_{kj})_{p\times p}$ is called the loading matrix.

The purpose of PCA is to find the loading matrix W so that one can get the principal components $Z_1, \cdots, Z_p$. PCA is sensitive to the relative scaling of the original variables, so the first step is to standardize the original data. That is, let

$$x^*_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j},$$

where x_{ij} is the original data, $i = 1, \cdots, n$ and $j = 1, \cdots, p$, n is the number of the observations,

$$\bar{x}_j = \sum_{i=1}^{n} x_{ij}, \quad s_j = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}.$$

Now consider the standardized data matrix, $X^* = (x^*_{ij})$. Mathematically, the PCA transformation is defined by a set of p-dimensional vectors of weights or loadings $W_{(k)}$, $k = 1, \cdots, p$, that map each row vector $X^*_{(i)}$ of X^* to a new vector of principal component scores $z_{(i)} = (z_{i1}, \cdots, z_{ip})$, given by

$$z_{ik} = X^*_{(i)} \cdot W_{(k)}, \ \ k = 1, \cdots, p,$$

in such a way that the individual variables of z_{ik} considered over the data set successively inherit the maximum possible variance from X^*, with each loading vector $W_{(k)}$ constrained to be a unit vector. The first loading vector $W_{(1)}$ thus has to satisfy

$$W_{(1)} = \text{argmax}\Big\{ \frac{w^T (X^*)^T X^* w}{w^T w} \Big\}.$$

A standard result for a symmetric matrix such as $(X^*)^T X^*$ is that the quotient's maximum possible value is the largest eigenvalue of the matrix, which occurs when w is the corresponding eigenvector. The full principal components decomposition $Z = (z_{ik})_{n \times p}$ of X^* can therefore be given as

$$Z = X^* W,$$

where W is a $p \times p$ matrix whose columns are the eigenvectors of $(X^*)^T X^*$. It is obvious that here W is the loading matrix.

Finally we need to choose the number of components. The eigenvalues of the matrix X^* represent the distribution of the source data's energy among each of the eigenvectors, where the eigenvectors form a basis for the data. Let D denote the diagonal matrix of eigenvalues. The cumulative energy content vector $g = (g_1, \cdots, g_p)$ for the jth eigenvector is the sum of the energy content across all of the eigenvalues from 1 through j:

$$g_j = \sum_{i=1}^{j} D_{ii}.$$

We can use the vector g as a guide in choosing an appropriate number of components. The way is to choose a value of L as small as possible while achieving a reasonably high value of g on a percentage basis. Usually, we choose a value of L so that the cumulative energy g is above 85 percent. In this case,

$$g_L / g_p \geq 0.85.$$

3.2 PCA results

Now we give the details of the PCA analysis for the original data in **Table 1**.

Step 1. Standardize the original data

In order to avoid the influence of the different scaling of the original data, we use the standardization calculation to get the data matrix X^* as shown in the following **Table 2**. In **Table 2**, each element x_{ij}^* of X^* is computed using

$$x_{ij}^* = \frac{x_{ij} - \bar{x}_j}{s_j}.$$

	X_1^*	X_2^*	X_3^*	X_4^*	X_5^*	X_6^*	X_7^*
PAIC	-0.55058	-0.21382	0.44418	0.63637	-1.24555	-1.82049	1.27125
PIC	0.13621	-0.62337	-0.15484	0.28210	-1.06824	0.73878	1.57134
CPIC	-0.03777	-0.76062	-0.27281	-0.03280	-0.35903	1.10702	0.38762
CLIC	-2.17141	-0.47086	-0.33179	-1.13496	0.35017	0.94131	-0.87945
AIG	0.50250	0.50517	-0.62672	2.01406	1.91042	0.16800	-0.92947
BH	0.94205	2.13699	-0.27281	-0.87254	-0.21719	-0.40276	0.02084
HFS	0.84132	-0.48611	2.44054	-0.51827	0.59839	0.16800	-0.51266
MetLife	0.33767	-0.74537	-0.56773	-0.37394	0.03102	-0.89988	-0.92947

Table 2: The standardized data matrix X^*

Step 2. Compute the correlation matrix

Based on **Table 2**, we get the correlation matrix $(X^*)^T X^*$ in **Table 3**. From **Table 3**, we see that the 7 variables indeed have positive or negative correlations with each other. That is why we use the principal component analysis and factor analysis methods to convert them into uncorrelated variables and reduce the dimension simultaneously.

	X_1^*	X_2^*	X_3^*	X_4^*	X_5^*	X_6^*	X_7^*
X_1^*	1.00000	0.33257	0.30243	0.2545	0.15182	-0.20576	0.03093
X_2^*	0.33257	1.00000	-0.18536	0.04995	0.05826	-0.39159	0.03162
X_3^*	0.30243	-0.18536	1.00000	-0.26728	0.10260	0.08372	-0.06796
X_4^*	0.25450	0.04995	-0.26728	1.00000	0.31941	-0.15935	0.13725
X_5^*	0.15182	0.05826	0.10260	0.31941	1.00000	0.27021	-0.83213
X_6^*	-0.20576	-0.39159	0.08372	-0.15935	0.27021	1.00000	-0.13805
X_7^*	0.03093	0.03162	-0.06796	0.13725	-0.83213	-0.13805	1.00000

Table 3: The correlation matrix $(X^*)^T X^*$

Step 3. Carry out PCA

Using the PRINCOMP command in R software, we carry out the principal component analysis for the data X^*. The summary of the results is shown in **Table 4** and **Figure 1**.

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7
Standard deviation	1.31113	1.23262	1.06084	0.94453	0.75047	0.55192	0.03448
Proportion of Variance	0.28066	0.24805	0.18373	0.14565	0.09195	0.04973	0.00019
Cumulative Proportion	0.28066	0.52872	0.71246	0.85811	0.95007	0.99980	1.00000

Table 4: Importance of PCA components

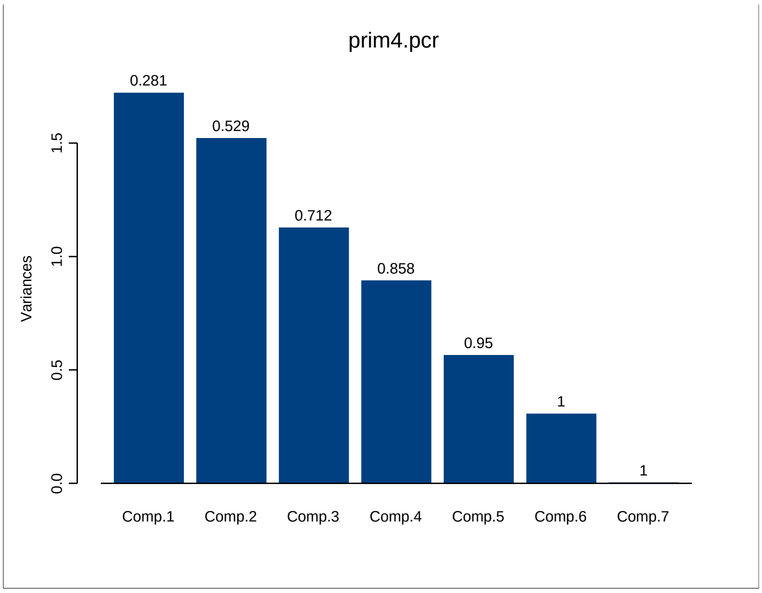

Figure 1: Screeplot of PCA components

Step 4. Calculate the principal component scores

The other important results of PCA are displayed in **Tables 5 and 6** and **Figures 2 and 3**. **Table 5** and **Figure 2** show the loadings of the PCA components. In **Table 6** we obtain the matrix of the scores of all of the components for 8 observations. **Figure 3** gives the biplot of the first two components showing their relationships with the original variables and the observations. In the biplot, the observations (the 8 companies) are plotted as points numbering 1-8 using the corresponding scores of the first two components. For example, the observation 1 (PAIC) has scores -2.33959 and 0.60902, then the coordinate of point 1 is (-2.33959, 0.60902). Moreover, the 7 variables are plotted as rays. For example, if the correlations (loadings) r between a particular variable and Comp. 1 and 2 are 0.660 and 0.253, respectively, then the ray starts at the origin (0, 0) and its headpoint is (.660, .253). Note that r ranges from -1 to 1. The more a correlation approaches -1 or 1, the stronger it is. The more it approaches 0, the weaker it is. The biplot can quickly tell us the variables' correlations with the components, and the relations and characteristics of the observations.

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7
X_1^*		0.523	0.465	0.272	0.253	0.609	
X_2^*	-0.143	0.530		-0.499	0.487	-0.433	-0.155
X_3^*	0.172		0.779	0.169	-0.210	-0.511	-0.148
X_4^*		0.435	-0.395	0.647	-0.157	-0.280	-0.365
X_5^*	0.660	0.253	-0.122			-0.188	0.670
X_6^*	0.339	-0.425		0.293	0.757		-0.203
X_7^*	-0.631			0.378	0.238	-0.252	0.575

Table 5: Loadings of PCA components

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7
X_1^*	-2.33959	0.60902	-0.35485	-0.08078	-1.04730	-0.59868	-0.01976
X_2^*	-1.37358	-0.82077	0.00683	1.28676	0.65724	0.12360	0.05755
X_3^*	-0.04565	-1.00359	-0.21539	0.76317	0.61392	0.37383	-0.07325
X_4^*	1.04076	-2.10000	-0.96459	-1.19321	-0.03107	-0.51821	0.01385
X_5^*	1.78325	1.93639	-1.33774	0.82949	0.08759	-0.28963	0.00615
X_6^*	-0.64500	1.38271	0.64661	-1.53799	1.17515	0.08576	0.00082
X_7^*	1.27104	-0.12762	2.38045	0.42065	-0.45298	-0.37089	0.00009
X_8^*	0.30877	0.12386	-0.16132	-0.48808	-1.00255	1.19422	0.01454

Table 6: Scores of PCA components

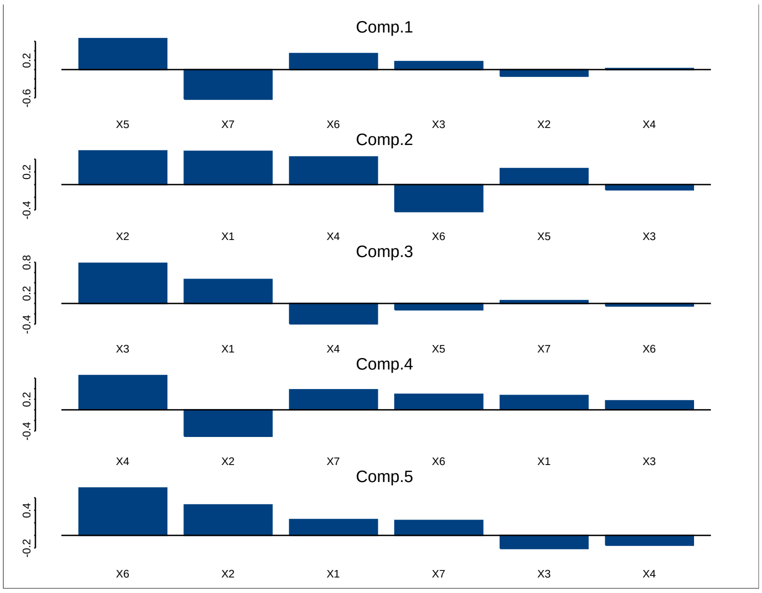

Figure 2: Plot of the loadings

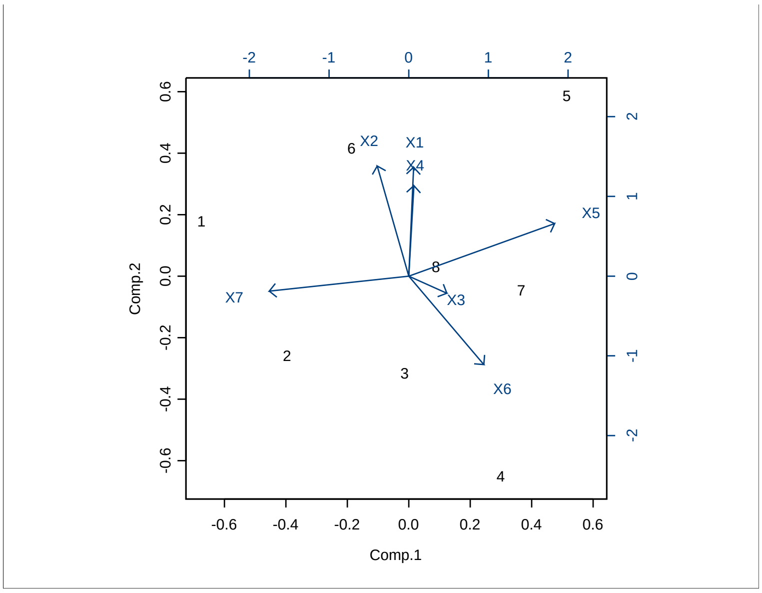

Figure 3: Biplot of Comp.1 and Comp.2

Step 5. Compute the solvency performance scores

According to **Table 4**, the cumulative proportion of variance for the first 4 components already exceeds 85%. Based on the principle in **Section 3.1**, we choose the number of components $L = 4$. Now using the first 4 components Z_1, Z_2, Z_3, Z_4 together with the contribution rate in **Table 4**, we finally obtain the **linear combination evaluation model** for assessing the solvency performance of the insurance companies.

$$\mathbf{Y} = \mathbf{0.28066Z_1} + \mathbf{0.24805Z_2} + \mathbf{0.18373Z_3} + \mathbf{0.14565Z_4}.$$

Based on this linear combination model and the matrix in **Table 6**, we can calculate the solvency performance scores for the 8 companies and get the ranking in **Table 7.** The scores for the 8 companies are computed using the formula: scores=$A \times B$, where

$$A = \begin{pmatrix} -2.33959 & 0.60902 & -0.35485 & -0.08078 \\ -1.37358 & -0.82077 & 0.00683 & 1.28676 \\ -0.04565 & -1.00359 & -0.21539 & 0.76317 \\ 1.04076 & -2.10000 & -0.96459 & -1.19321 \\ 1.78325 & 1.93639 & -1.33774 & 0.82949 \\ -0.64500 & 1.38271 & 0.64661 & -1.53799 \\ 1.27104 & -0.12762 & 2.38045 & 0.42065 \\ 0.30877 & 0.12386 & -0.16132 & -0.48808 \end{pmatrix},$$

and

$$B = \begin{pmatrix} 0.28066 \\ 0.24805 \\ 0.18373 \\ 0.14565 \end{pmatrix}.$$

	PAIC	PIC	CPIC	CLIC	AIG	BH	HFS	MetLife
Scores	-0.7002	0.7041	-1.8599	-0.1007	0.8235	0.3701	0.3385	0.4246
Rank	7	2	8	6	1	4	5	3

Table 7: **Ranks of the solvency performance using PCA**

3.3 Conclusions based on PCA

According to the loadings of PCA components in **Table 5**, X_1 and X_2 have the largest magnitudes in Component 2, and can be grouped together to represent the operating capability index of each company, since loss ratio and proportion of net income to total revenues are both indicators of a company's ability to sustain favorable operation. Similarly, X_3 belongs to the development capability index, determining Component 3. X_4 shows the operating scale index, determining Component 4. Component 1 is mainly determined by X_5, X_6 and X_7, which all represent the financial stability index.

Moreover, from the ranking we obtained in **Table 7** using PCA, we can see that the 4 American companies received ranks of 1, 4, 5, and 3, respectively, while the 4 Chinese companies received ranks of 7, 2, 8, and 6, respectively. Although PIC received a remarkable second place, PAIC, CPIC, and CLIC still fell behind. In other words, the 4 American insurance companies generally surpassed their Chinese counterparts in terms of integrated solvency for the year 2013. Therefore, there's still much space for improvement for the Chinese insurance industry.

4 Factor Analysis

In this section, we try to determine, through the Factor Analysis method, the solvency performance of the insurance companies. Like what we did in the PCA section, we first introduce the basic ideas of Factor Analysis and then give corresponding analysis.

4.1 Basic ideas of FA

Factor analysis is a statistical method used to describe variability among observed, correlated variables in terms of a potentially lower number of unobserved variables called factors ([3-6]). Computationally, this technique is equivalent to low rank approximation of the matrix of observed variables. FA is widely used in actuarial science, social sciences, marketing, and other applied sciences that deal with large quantities of data.The key concept of factor analysis is that multiple observed variables have similar patterns of responses because of their association with an underlying latent variable, the factor, which cannot be easily measured. For example, the 7 variables we considered above, Loss ratio, Proportion of net income to total revenues, Investment income ratio, Reinsurance ratio, Rate of return on total assets, Asset-gross premium ratio and Rate of change of premiums earned, are all associated with the latent variable the solvency performance.

In every factor analysis, there are usually less number of factors as there are variables. Each factor captures a certain amount of the overall variance in the observed variables, and the factors are always listed in order of how much variation they explain. The mathematical model is as follows:

$$\begin{aligned} X_1 &= a_{11}F_1 + \cdots + a_{1m}F_m + a_1\varepsilon_1, \\ X_2 &= a_{21}F_1 + \cdots + a_{2m}F_m + a_2\varepsilon_2, \\ &\cdots \\ X_p &= a_{p1}F_1 + \cdots + a_{pm}F_m + a_p\varepsilon_p, \end{aligned}$$

where p is the number of the observed original variables; m is the number of the factors ($m \leq p$); $F_1, \cdots, F_m$ are the factors; a_{ij} is the factor loading, which expresses the relationship of each variable to the underlying factor; $A = (a_{ij})_{p\times m}$ is called the factor loading matrix; $\varepsilon_1, \cdots, \varepsilon_p$ are the special factors which represent the remaining components of the original variables that can not be explained by factors.

The steps for implementing the factor analysis are quite similar to the PCA. We first standardize the original data to get a normalized data matrix X^*, where

$$x^*_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j},$$

$$\bar{x}_j = \sum_{i=1}^{n} x_{ij}, \quad s_j = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2}.$$

Then we compute the first m eigenvalues and the corresponding eigenvectors of $(X^*)^T X^*$. That is, we obtain the m eigenvalues

$$\lambda_1 \geq \cdots \geq \lambda_m \geq 0,$$

and the corresponding eigenvectors $u_1, \cdots, u_m$. This yields the loading matrix

$$A = \left(u_{ij}\sqrt{\lambda_j}\right)_{p\times m}.$$

The eigenvalue is a measure of how much of the variance of the observed variables a factor explains. Any factor with an eigenvalue ≥ 1 explains more variance than a single observed variable. The factors that explain the least amount of variance are generally discarded. The general method for determining the number of components in PCA can also be used here to decide how many factors (m) should be chosen. Since a set of factors and factor loadings is identical only up to orthogonal transformation, the orthogonal rotation of the loading matrix is usually useful for the factor analysis.

Finally, to derive the factor score matrix, we use the least squares method. That is, we estimate $F = (F_1, \cdots, F_m)^T$ by minimizing

$$\sum_{i=1}^{p} \varepsilon_i^2 = (X^* - AF)^T (X^* - AF) := \phi(F).$$

Let

$$\frac{\partial \phi(F)}{\partial F} = 2A^T (X^* - AF) = 0,$$

we arrive at

$$\hat{F} = (A^T A)^{-1} A^T X^*.$$

Therefore, for every observation $X^*_{(i)}$, $i = 1, \cdots, n$,

$$\hat{F}_{(i)} = (A^T A)^{-1} A^T X^*_{(i)}.$$

4.2 FA results

In this subsection, we implement the factor analysis based on the standardized data matrix X^* in **Table 2** to assess the performance of the 8 insurance companies. FA computation is also based on the correlation matrix $(X^*)^T X^*$ in **Table 3** and its eigenvalues. Here we choose $m = 4$ factors as in PCA method. Next we give the detailed results of the FA analysis.

Step 1. Calculate the factor scores

Tables 8-9 and **Figure 4** display the results of the FA using the FACTANAL command of R software. **Table 8** shows the loadings of the 4 factors and **Table 9** gives the matrix of the factor scores. We can see that the loadings and scores are different from **Tables 5-6** using PCA method, because we use the principal factor estimate and the Varimax orthogonal rotation method to obtain the loading matrix and then use least squares method to get the score matrix.

	Factor 1	Factor 2	Factor 3	Factor 4
X_1^*		0.415	0.295	0.583
X_2^*		0.706		
X_3^*		-0.171	-0.215	0.712
X_4^*			0.784	
X_5^*	0.960	-0.111	0.363	0.130
X_6^*	0.176	-0.564		
X_7^*	-0.933		0.196	

Table 8: Loadings of FA factors

	Factor 1	Factor 2	Factor 3	Factor 4
X_1^*	-2.0589162	1.41716152	-1.4785162	-0.99088863
X_2^*	0.4880135	-2.69516201	5.5481032	1.59768036
X_3^*	-3.0475948	2.02869211	-6.4450443	-2.02466786
X_4^*	1.2615525	-1.54024930	0.1116016	-0.78382997
X_5^*	1.7159503	-0.02836202	2.3790322	-0.05617151
X_6^*	0.06924067	1.5733653	-0.5306144	0.2150647
X_7^*	0.51657133	-0.4547970	-0.4014267	1.9050079
X_8^*	1.05518271	-0.3006486	0.8168646	0.1378050

Table 9: Scores of FA factors

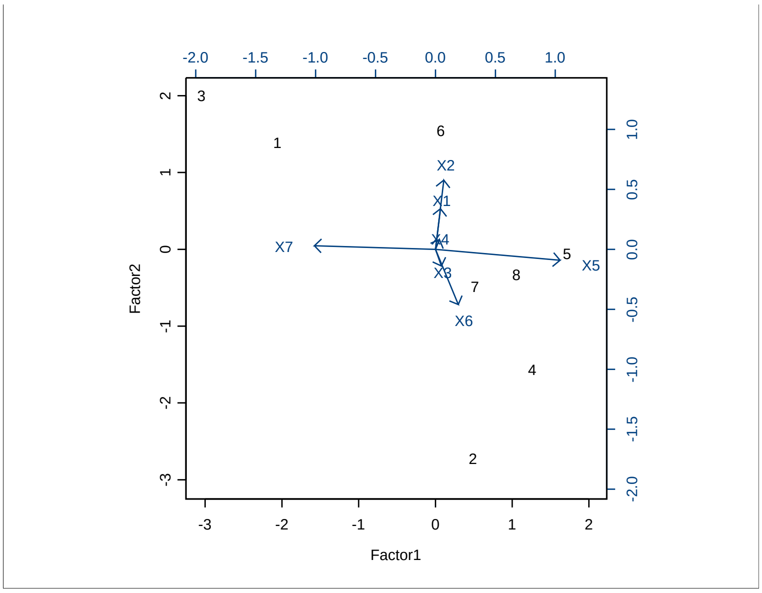

Figure 4: Biplot of Factor 1 and Factor 2

Step 2. Compute the solvency performance scores

Now using the matrix of factor scores, we can derive the solvency performance scores of the 8 insurance companies by the following **linear combination evaluation model**:

$$\mathbf{Y = 0.28066F_1 + 0.24805F_2 + 0.18373F_3 + 0.14565F_4},$$

where F_i denotes the factor i, $i = 1, 2, 3, 4$. Thus, based on the score matrix in **Table 9**, we calculate the solvency performance scores for the 8 companies and get the ranking in **Table 10**. The scores for the 8 companies=$A_1 \times B$, where

$$A_1 = \begin{pmatrix} -2.0589162 & 1.41716152 & -1.4785162 & -0.99088863 \\ 0.4880135 & -2.69516201 & 5.5481032 & 1.59768036 \\ -3.0475948 & 2.02869211 & -6.4450443 & -2.02466786 \\ 1.2615525 & -1.54024930 & 0.1116016 & -0.78382997 \\ 1.7159503 & -0.02836202 & 2.3790322 & -0.05617151 \\ 0.06924067 & 1.5733653 & -0.5306144 & 0.2150647 \\ 0.51657133 & -0.4547970 & -0.4014267 & 1.9050079 \\ 1.05518271 & -0.3006486 & 0.8168646 & 0.1378050 \end{pmatrix},$$

and

$$B = \begin{pmatrix} 0.28066 \\ 0.24805 \\ 0.18373 \\ 0.14565 \end{pmatrix}.$$

	PAIC	PIC	CPIC	CLIC	AIG	BH	HFS	MetLife
Scores	-0.5825	-0.4004	-0.1902	-0.5798	0.8559	0.0567	0.8237	0.0166
Rank	8	6	5	7	1	3	2	4

Table 10: **Ranks of the solvency performance using FA**

4.3 Conclusions based on FA

Compared to the loadings of PCA components in **Table 5**, the loadings of FA components in **Table 8** show similar physical natures for each solvency index. X_1 and X_2 have the large magnitudes in Factor 2, and can be grouped together to represent the operating capability index of each company. Moreover, X_3 determines Factor 4 and independently represents the development capability index. X_4 dominates in Factor 3 and shows the operating scale index. X_5,X_6, and X_7 together represent the financial stability index in Factor 1.

From the ranking we obtained in **Table 10** using FA, we can see that the 4 American companies received ranks of 1, 3, 2, and 4, respectively, while the 4 Chinese companies received ranks of 8, 6, 5, and 7, respectively. Similar to the results we obtained from PCA, the 4 American insurance companies surpassed their Chinese counterparts in terms of integrated solvency for the year 2013. Therefore, the Factor Analysis further confirmed the need for Chinese insurance companies to devise strategies to increase solvency.

5 Comparisons Between PCA and FA

5.1 Comparisons of the two analytical methods

Researchers have argued that the distinctions between the two techniques PCA and FA may mean that there are objective benefits for preferring one over the other based on the analytic goal ([6]). Fabrigar et al. ([7]) addressed a number of reasons used to suggest that principal component analysis is equivalent to factor analysis. For example, although it is sometimes suggested that principal component analysis is computationally quicker and requires fewer resources than factor analysis. Fabrigar et al. believed that the ready availability of computer resources has rendered this practical concern irrelevant. And, the point that PCA and FA can produce similar results is also addressed by Fabrigar et al.

However, Suhr ([8]) illustrated some differences between principal component analysis and factor analysis. For instance, PCA results in principal components account for a maximal amount of variance for observed variables, while FA accounts for common variance in the data. The component scores in PCA represent a linear combination of the observed variables weighted by eigenvectors, while the observed variables in FA are linear combinations of the underlying and unique factors.

5.2 Comparisons of the two rankings

Just as Fabrigar et al. had addressed, from the two rankings we obtained using PCA and FA, we can discover that the two methods produced similar results. The specific ranks for each company may vary due to the differences illustrated by Suhr. Despite a great disparity between rank "2" and rank "6" for the company PIC, both rankings show that American insurance corporations were generally more solvent than Chinese ones in 2013. Therefore, we can safely conclude that the two methods are equally helpful and precise when used to determine the integrated solvency of insurance companies.

6 Linear Regression Analysis

6.1 Constructing linear regression model

Both of the loading matrices from the PCA and FA in **Tables 5 and 8** show that the index X_5 (rate of return on total assets) is the most significant variable in Component 1 and Factor 1. This means that X_5 plays an important role in assessing an insurance's solvency performance. Since X_5 equals "surplus reserves" divided by "total assets", in order for Chinese insurance companies to catch up with American ones, much effort should be made to increase "surplus reserves". However, as we all know, "surplus reserves" isn't something that can be directly enhanced. Companies must devise various specific measures to build up its pool of reserves. Therefore, in this section, we will analyze the relationship of X_5 to the other indices X_1, X_2, X_3, X_4, X_6, and X_7 using linear regression models in order to find efficient measures to increase X_5. We assume that

$$X_5 = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_6 + \beta_6 X_7 + \varepsilon,$$

where β_i, $i = 0, \cdots, 6$, are the unknown coefficients and ε is the random error having normal distribution $N(0, \sigma^2)$ with the variance $\sigma^2 > 0$.

Based on the data matrix in **Table 1**, we use the least squares method to estimate the unknown parameters $\beta_0, \cdots, \beta_6$. Let $Y = (X_{15}, \cdots, X_{85})^T$, and $\tilde{X} = (X_{ij})_{8\times 6}$, where $i = 1, \cdots, 8$, $j = 1, 2, 3, 4, 6, 7$. Then, $Y = (1.1, 1.6, 3.6, 5.6, 10.0, 4.0, 6.3, 4.7)^T$, and

$$\tilde{X} = \begin{pmatrix} 60.4 & 12.8 & 5.1 & 13.7 & 7.8 & 13.7 \\ 67.9 & 5.8 & 5.2 & 11.0 & 21.7 & 15.5 \\ 66.0 & 4.9 & 5.0 & 8.6 & 23.7 & 8.4 \\ 42.7 & 6.8 & 4.9 & 0.2 & 22.8 & 0.8 \\ 71.9 & 13.2 & 4.4 & 24.2 & 18.6 & 0.5 \\ 76.7 & 23.9 & 5.0 & 2.2 & 15.5 & 6.2 \\ 75.6 & 6.7 & 9.6 & 4.9 & 18.6 & 3.0 \\ 70.1 & 5.0 & 4.5 & 6.0 & 12.8 & 0.5 \end{pmatrix}.$$

Now the least squares estimator $\hat{\beta} = (\hat{\beta}_0, \cdots, \hat{\beta}_6)^T$ can be calculated from the formula

$$\hat{\beta} = (\tilde{X}^T \tilde{X})^{-1} \tilde{X}^T Y.$$

(1). Model 1

Using LSFIT command in R software, we obtain the following results:

	coef	std.err	t.stat	p.value
Intercept	0.2306	1.2441	0.1853	0.8833
X_1	-0.0105	0.0179	-0.5859	0.6626
X_2	0.0992	0.0286	3.4741	0.1784
X_3	0.3645	0.1108	3.2897	0.1879
X_4	0.2007	0.0233	8.6002	0.0737
X_6	0.1569	0.0317	4.9560	0.1268
X_7	-0.4025	0.0263	-15.3044	0.0415

Table 11: Linear regression analysis: **Model 1**

Residual Standard Error = 0.4101, Multiple R-Square = 0.997, N = 8, F-statistic = 54.9958 on 6 and 1 df, p-value = 0.1029.

(2). Model 2

Examining the p-value in **Table 11**, we decide to delete the intercept in the linear model due to its large p-value 0.8833. We thus get **Table 12** after removing the intercept:

	coef	std.err	t.stat	p.value
X_1	-0.0086	0.0105	-0.8160	0.5002
X_2	0.1002	0.0201	4.9768	0.0381
X_3	0.3680	0.0786	4.6843	0.0427
X_4	0.2010	0.0167	12.0084	0.0069
X_6	0.1604	0.0181	8.8477	0.0125
X_7	-0.4016	0.0186	-21.6183	0.0021

Table 12: Linear regression analysis: **Model 2**

Residual Standard Error = 0.2949, Multiple R-Square = 0.9992, N = 8, F-statistic = 432.4351 on 6 and 2 df, p-value = 0.0023.

(3). Model 3

The p-value 0.5002 for X_1 implies that we should delete X_1 from **Model 2**. Thereafter, we arrive at **Table 13**. The Multiple R-Square value and the p-values all show the fitness of **Model 3**.

Residual Standard Error = 0.278, Multiple R-Square = 0.999, N = 8, F-statistic = 583.8 on 5 and 3 df, p-value = 0.0001.

	coef	std.err	t.stat	p.value
X_2	0.0896	0.0145	6.1884	0.0085
X_3	0.3211	0.0505	6.3597	0.0079
X_4	0.1938	0.0134	14.4948	0.0007
X_6	0.1534	0.0151	10.1904	0.0020
X_7	-0.4035	0.0174	-23.2458	0.0002

Table 13: Linear regression analysis: **Model 3**

(4). Final Model

Hence, we can construct a linear model describing the relationship between the index X_5 and the other indexes:

$$\mathbf{X_5 \approx 0.0896X_2 + 0.3211X_3 + 0.1938X_4 + 0.1534X_6 - 0.4035X_7}.$$

6.2 Analysis on the model

From the model obtained above, we can discover that X_3 and X_7 have the largest magnitudes of coefficients, namely 0.3211 and -0.4035. Therefore, X_5 is most significantly influenced by X_3 and X_7. Other things equal, when X_3 increases by 1%, X_5 will increase by 0.3211%. Other things equal, when X_7 increases by 1%, X_5 will decrease by 0.4035%. **Figure 5** also shows that X_5 has an apparent positive linear relationship with X_3 and negative linear relationship with X_7, respectively.

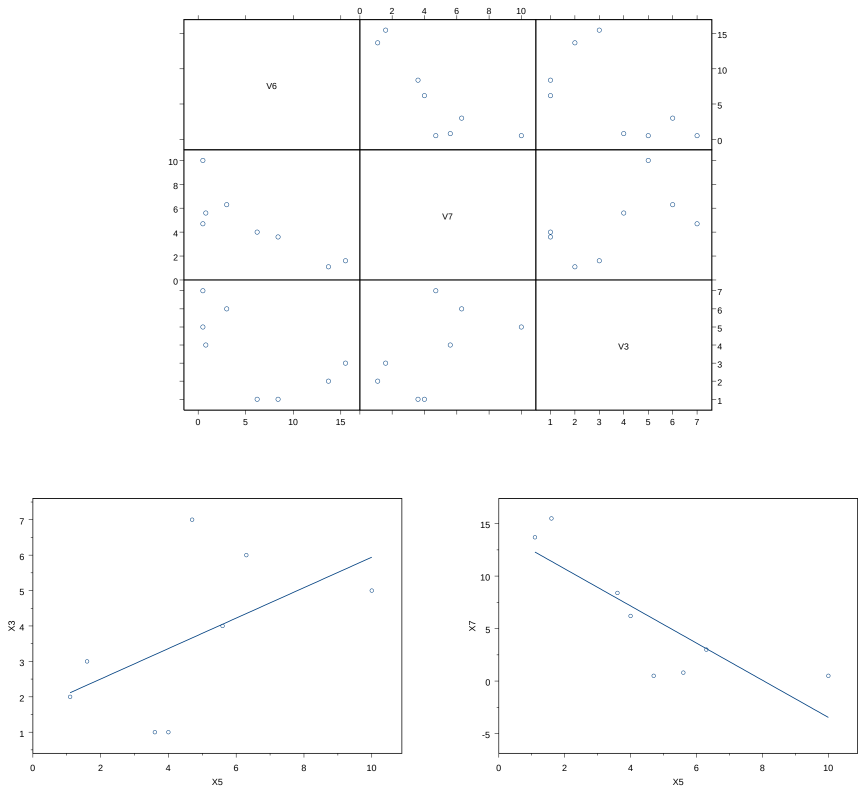

Figure 5: Linear regression

Since X_5 (rate of return on total assets) has been proven to be the most deciding index in solvency evaluation, an insurance company must try to increase its X_3 (investment income ratio) and decrease its X_7 (rate of change of premiums earned) in order to enhance its X_5 (rate of return on total assets). Once rate of return on total assets is improved, the company's integrated solvency will surely exhibit a significant rise.

As a result, the two core approaches for Chinese insurance companies to improve integrated solvency are **1) to expand investments in other companies and industries**, and **2) to ensure steady sources of premiums**.

For the first approach, since the investment income ratio is the development capability index of a company, deliberate decisions should be made regarding what is best for the company's long-time development. A Chinese insurance company should increase its interactions with other companies from both domestic and foreign financial industries. Through expanded investments, the company can not only create more profit channels but also familiarize itself with the advanced businesses other companies conduct. This form of globalization will generate immense revenues and incomes that build up the company's capital and will also provide renewed insights on how to improve its development capability as a whole.

For the second approach, since the rate of change of premiums earned is a financial stability index of a company, much effort should be made to bring down the rate of change in order to ensure stability. The Chinese insurance industry nowadays is marked by an unsteady flow of premiums due to the fact that it's still in the developing stage. Therefore, a Chinese insurance company should strengthen its management of financial resource as well as emphasize employee teamwork to ensure a nice corporate image. We'll talk more about financial stability in **Section 7.1 B**.

Ideally, for the year 2014 and beyond, paying special attention to the two approaches mentioned above will help Chinese insurance companies catch up with American ones in terms of integrated solvency.

7 More Analysis on the Competitiveness of Chinese Insurance Companies

From the Principal Component Analysis and Factor Analysis, we found out that American insurance companies nowadays still surpass Chinese ones. From the linear regression model, we learned that expanding investments and ensuring steady premium sources could potentially increase Chinese insurance companies' integrated solvency. However, for an industry as big as the Chinese insurance market, we still need to take into account more factors and devise even more suggestions. Therefore, in this section, we point out additional problems facing Chinese insurance companies and give extra suggestions on how to improve solvency. Moreover, while American insurance industry has already reached sophistication, Chinese insurance industry is still in its developing stage. Therefore, we analyze the opportunities waiting for Chinese insurance companies and the prospects of their future development.

7.1 Problems facing Chinese insurance companies

In this part, we examine the reasons why Chinese insurance companies nowadays still fall behind their American counterparts and discover the problems they have to overcome. Also, by finding out the three major problems, we can formulate a set of suggestions for enhancement for Chinese companies.

A. Lack of total assets

The amount of total assets is an important factor that determines a company's competitiveness. It also proves to be especially essential in the insurance industry, mainly because carrying out liability business requires a large pool of assets, which guarantees the capacity to pay claims and expenses as well as the opportunity for more profit. Since Chinese insurance industry is still in the developing phase, even the most competent Chinese insurance companies experience a lack of assets to carry out insurance business to increase solvency. On the other hand, American insurance companies possess enough capital and usually don't have to worry about paying their claims and expenses.

Below is a table of the total asset values of several Chinese and American insurance companies till the end of 2013. (in millions of Chinese RMB; 1$=6.2 RMB, 1 HKD=1.26RMB at the end of 2013) The data are extracted from corporations' annual reports or from the China Finance Information website ([9]).

MetLife	5188042
Prudential Financial	4537042
TIAA-CREF	3664200
American International Group	3401524
Ping An Insurance	3360312
Berkshire Hathaway	3006572
China Life Insurance	1972941
Hartford Financial Services	1722880
People's Insurance	755319
China Pacific Insurance	723533
Xinhua Insurance	565849
Taiping Life Insurance	442623

Table 14: Total assets of several insurance companies

From **Table 14**, we can see that, except the two most prominent Chinese insurance companies—Ping An and China Life Insurance, the Chinese insurance industry is still far behind USA's insurance market in terms of asset pool. Therefore, Chinese insurance companies need to devise effective strategies to increase total assets.

Suggestions for improvement:

1) Diversify financial services and products

Part of the reason many American insurance companies have strong asset pools is that they not only conduct insurance business, but also carry out many other forms of business. For example, Berkshire Hathaway (BH) contributed a large portion of their capital to railroad and banking. The diversification of business can bring potential sources of revenues for insurance companies so as to increase their asset pools.

2) Explore personnel's creativity

In order to stand out from a large number of competitors, creativity can be a sharp spear that guarantees success. However, the problem nowadays facing Chinese insurance companies is that they tend to imitate the insurance products designed by foreign companies. Imitation should be avoided since customers tend to favor characteristic products instead of familiar ones. Chinese insurance companies should encourage their personnel to design products that are particularly Chinese rather than imitate those already devised by others. In this way, customers can be attracted to these companies and bring more business.

B. Insurance industry is not stable enough

Financial stability is an important indicator of solvency for a company or financial organization. Since the Chinese insurance industry had a relatively slow start, it is often characterized by considerable instability, in that total assets and premium revenues experienced sharp increase/decrease over the past few decades. In **Section 6.2**, we have already noticed the unsteadiness of Chinese insurance industry's premium sources. Despite the fact that the capital pool of the insurance industry has been increasing prominently in the recent five years, severe changes may still foreshadow considerable risk and can be a sign of inadequate risk management.

1) The "financial stability index" of Chinese insurance companies indicate the instability of their operations, which might be a sign of solvency risk in the future. **Table 15** below is extracted from the original data table in **Section 2.2**. The three solvency indices shown, namely rate of return on total assets (X_5), asset-gross premium ratio(X_6) and rate of change of premiums earned(X_7), are designated as "financial stability index" that reflect each company's degree of

financial stability. From the descriptions of solvency indices in **Section 2.1**, we know that high values of X_5 and low values of X_6 and X_7 suggest high stability.

	Rate of return on total assets	Asset-gross premium ratio	Rate of change of premiums earned
PAIC	1.1	7.8	13.7
PIC	1.6	21.7	15.5
CPIC	3.6	23.7	8.4
CLIC	5.6	22.8	0.8
AIG	10.0	18.6	0.5
BH	4.0	15.5	6.2
HFS	6.3	18.6	3.0
MetLife	4.7	12.8	0.5

Table 15: Subset of original data

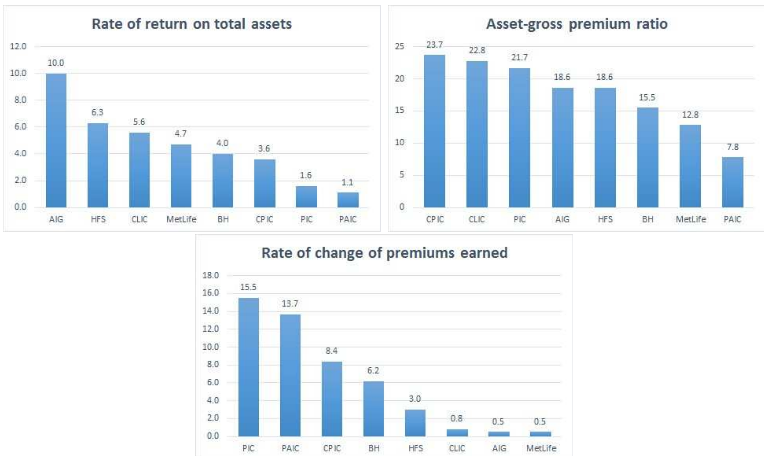

Figure 6: Histograms

From the three histograms in **Figure 6**, we can see that Chinese insurance companies generally fall behind American ones in terms of financial stability. This may be explained by the fact that Chinese insurance industry is still in its developing phase. Thus, Chinese insurance companies still have a long way to go to stabilize their financial operations.

2). The solvency issue is especially serious for Life Insurance in China. Although this paper determined each insurance company's solvency using Principal Component Analysis and Factor Analysis, Chinese insurance companies in particular have a distinct index—the solvency margin ratio, which they calculated by themselves. By collecting the Life-Insurance solvency margin ratios from year 2009 to 2013 of the four Chinese insurance companies we researched (PAIC, PIC, CPIC, CLIC), we arrive at the following **Table 16** and four line charts in **Figure 7**.

	2013	2012	2011	2010	2009
PAIC	172	191	156	180	227
PIC	202	130	159	124	288
CPIC	191	211	187	241	208
CLIC	226	236	170	212	304

Table 16: Solvency margin ratio

Figure 7: Line charts

From the charts in **Figure 7**, we can see that the top four insurance companies in China have shown severe instability in their respective Life Insurance business over the past five years, and each company's solvency margin ratio is in decreasing trend. For instance, PIC's solvency margin ratios of life insurance decreased from 288 to only 124 in 2010 and jumped sharply from 130 to 202 in 2013. Therefore, Chinese insurance companies really need to devise strategic plans to stabilize their solvency.

Suggestions for improvement:

1) Strengthen management of financial resources

Excellent management requires high-quality management personnel. Chinese insurance companies should actively absorb technical experts with risk assessment, actuarial science, and foreign language expertise. In this way, the companies will be able to make accurate decisions on how they are going to allocate financial resources and how much profit they could potentially gain. As a result, financial stability can be secured.

2) Strengthen teamwork

A corporation cannot achieve financial stability if the personnel itself is not stable. To succeed, a company has to have a cohesive team. Too much conflict between employees can delay decision-making and may sometimes cause the company to miss the best opportunity. If active cooperation rather than bitter argument is ensured, it will be much easier for the company to grasp favorable chances and ensure financial stability.

C. Lack of adequate corporate image and work ethics

Occupational credibility and professional ethics are two core values that define a company's corporate image. With a positive corporate image, insurance companies can attract more and more customers and businesses to build up their asset pools so that adequate solvency can be guaranteed. Thus, Chinese insurance companies generally place a high value on the quality of customer service and proper management of personnel and financial resources. However, due to the lack of experience in such management, their corporate image still can't reach the level accomplished by American insurance companies. Moreover, in recent years, some Chinese customers have complained about bad attitudes and illegal marketing devises of some insurance companies. This can be explained by the lack of management and low working morale in the Chinese insurance industry. Therefore, in order to increase solvency, Chinese insurance companies must conceive ways to improve corporate image.

Suggestions for improvement:

1) Raise the wages of the service personnel

One of the main causes of low working morale is the low wages received by the service personnel. Without the motivation of high wages, the personnel tend to fail to exhibit friendly attitudes toward customers. Therefore, raising wages is a great strategy to improve the quality of customer service.

2) Strengthen management of marketing and service

In order to improve the quality of customer service, insurance companies should also provide professional training for the personnel. Also, to prevent illegal activities, companies should establish related departments to check and approve marketing materials before they come into application. Good management is essential to the improvement of corporate image and can set up the very prerequisite of a qualified insurance company—credibility.

7.2 Opportunities waiting for Chinese insurance companies

In this subsection, we analyze the basic trends and prospects in today's Chinese insurance industry that can serve to increase integrated solvency and enhance risk management skills. The first two trends reflect the opportunities created by outside factors, while the next two trends discuss the opportunities Chinese insurance companies have created or could potentially create by themselves. We anticipate a more prosperous future for Chinese insurance companies if they can seize these opportunities.

A. Chinese insurance industry is increasingly connected with the global market

As China continues along the road of globalization, Chinese insurance companies have also increased their interactions with one another as well as with foreign insurance companies. Major insurance companies in China are now actively investing in the businesses of both domestic and overseas companies. Also, the four Chinese companies we researched earlier (PAIC, PIC, CPIC, CLIC) are in fact four of the five insurance corporations in China that have been listed in the global market. Moreover, all of the four American insurance companies we studied (AIG, BH, HFS, MetLife) have branches in China. All these provide Chinese companies with great opportunities to learn how to resolve the solvency issue. Therefore, considering the increasing connections between China and the rest of the world, Chinese insurance companies should capitalize on this favorable trend and seize the opportunity of more interactions with foreign insurance industry. Through countless investment channels and personnel exchange programs, Chinese companies can not only increase the value of their capital, but also absorb advanced knowledge on risk management and solvency improvement.

B. Chinese regulatory agencies are implementing more and more favorable policies for insurance companies

In recent years, the China Insurance Regulatory Commission has formulated a series of policies that would help domestic insurance companies overcome multiple problems such as lack of assets and instability. For instance, in 2013, the Commission implemented three large-scale market reforms, namely the reform on premium rate, the reform on the access of capital, and the deepened reform on exit mechanism. These reforms have shown positive effects to various extents. The Commission has also established standards on the proper values of statistical indices and drawn up the appropriate norms on management and operation for insurance companies. Therefore, with the support of the regulatory agencies, we are convinced that Chinese insurance companies will make the most of the opportunities created for them and exhibit better solvency and more advanced risk management skills.

C. Demand for insurance is increasing in China

Despite the image issue of some Chinese insurance companies, it is very clear that more and more Chinese people are in need of insurance products. Along with the rapid economic development in China, a growing number of people are able to afford their insurance and are therefore contributing more premiums to insurance companies. Moreover, with the advent of the information age, Chinese citizens are also constantly informed of news and advertisement about insurance companies. Nevertheless, due to the large income gap between the rich and the poor, many people from more distant areas in China are still unaware of insurance companies and how to purchase insurance. Therefore, Chinese insurance companies should actively take part in the economic development in China so as to create opportunities by themselves, since people will begin purchasing insurance when their living standards improve.

D. The growing popularity of telephone and Internet sale

Telephone and Internet sale is a defining component of the auto-insurance industry. Although it is not uncommon in American and European markets, it is still relatively new in China. Most of the telemarking and net-marketing business of the Chinese insurance industry is concentrated in the few biggest insurance companies such as PAIC and PIC. Nevertheless, the good news is that telephone and Internet sale is growing rapidly. Along with the technological improvement in recent years, many Chinese insurance companies have begun telemarketing business, and some people are now able to purchase insurance products on telephone. Even so, the net-marketing needs more dissemination, since most companies are still not fully equipped with Internet sale services. The benefits of telephone and Internet sale are beyond doubt. It not only increases the popularity of individual insurance companies by allowing marketing on social media, but also gives customers a more convenient platform to purchase insurance products. Therefore, Chinese insurance companies should create opportunity for themselves by adding or improving telephone and Internet sale. Through enhancing the quality of their service, these companies could improve their corporate image and potentially gain more premiums and capitals to increase solvency.

8 Conclusions

This paper used three statistical methods to analyze original data. By applying both the Principal Component Analysis (PCA) and the Factor Analysis (FA) to the integrated solvency evaluation of 4 outstanding Chinese and 4 American insurance companies, we overcame the disadvantage of evaluating only one index at a time and provided two practical rankings of the 8 companies in terms of integrated solvency. Despite differences in the specific results, the two rankings both showed that Chinese insurance companies were generally less solvent than American ones for the year 2013. This conclusion signals the need for Chinese insurance companies to devise ways to increase solvency.

By constructing a linear regression model, we confirmed the need for Chinese insurance companies to expand investment and stabilize premium sources. We also examined the three major problems facing them and the four major opportunities waiting for them nowadays and gave valuable suggestions for improvement. Our analysis and suggestions will hopefully prove beneficial to Chinese insurance companies and help them come up with effective strategies to cope with potential danger and seize future opportunities while they are still in the developing phase. In prospect, the Chinese insurance industry will be able to achieve further prosperity and reach the sophisticated level accomplished by American and European insurance markets in the near future.

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10 Appendix: R codes

```
# input the original data X
x<-c(60.4, 12.8,   5.1,    13.7,   1.1,    7.8,    13.7,
67.9,    5.8,   5.2,    11.0,   1.6,    21.7,   15.5,
66.0,   4.9,   5.0,   8.6,   3.6,   23.7,   8.4,
  42.7,   6.8,   4.9,    0.2,    5.6,    22.8,   0.8,
71.9,   13.2,   4.4,    24.2,   10.0,   18.6,   0.5,
76.7,    23.9,      5.0,    2.2,    4.0,    15.5,   6.2,
75.6,       6.7,    9.6,    4.9,    6.3,    18.6,   3.0,
70.1,   5.0,    4.5,    6.0,    4.7,    12.8,   0.5)

# standardize the original data to get matrix X^*
y<-matrix(x, 7, 8)
  y<-t(y)
  for (i in 1:7)  {y[, i]<-(y[,i]-sum(y[,i]/8))/sqrt(sum((y[,i]-sum(y[,i])/8)^2)/7)}

# PCA results for X^*
prim1<- princomp(y)
screeplot(prim1)
plot(loadings(prim1))
print(loadings(prim1), cutoff=.5)
summary(prim1)
prim1$scores
prim1$correlation
loadings(prim1)
biplot(prim1)

# FA results for X^*
prim2<-factanal(y, factors=4, method="principal", data=NULL,
```

```
covlist=NULL, scores=T, type="regression", rotation="varimax" )
prim2$correlation
biplot(prim2)
prim2
summary(prim2)
prim2$scores

# linear regression analysis
lx1<-c(60.4,    12.8,    5.1,      13.7,          7.8, 13.7)
lx2<-c( 67.9,     5.8,    5.2,      11.0,    21.7,    15.5)
lx3<-c(   66.0, 4.9, 5.0, 8.6,    23.7, 8.4)
lx4<-c(42.7,      6.8,    4.9,      0.2,            22.8,    0.8)
lx5<-c(71.9,    13.2,    4.4,      24.2,      18.6,  0.5)
lx6<-c( 76.7,     23.9,        5.0,      2.2,            15.5,    6.2)
lx7<-c(75.6,          6.7,      9.6,      4.9,            18.6,    3.0)
lx8<-c(70.1,    5.0,      4.5,      6.0,      12.8,    0.5)
lxx<-matrix(0, 8, 6)
lxx[1, ]<-lx1
lxx[2, ]<-lx2
lxx[3, ]<-lx3
lxx[4, ]<-lx4
lxx[5, ]<-lx5
lxx[6, ]<-lx6
lxx[7, ]<-lx7
lxx[8, ]<-lx8
ly<-c(1.1, 1.6, 3.6, 5.6, 10.0, 4.0, 6.3, 4.7)
rexy<-lsfit(lxx,ly)
ls.print(rexy)
```