Research on the Mathematical Model of Online Shopping

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Abstract

The rapid development of internet and the emergence of modern logistics have promoted the development of consumers' online shopping. This paper mainly studies the issue of optimal pricing in online shopping.

Based on the research, the major factors influencing dealers' turnover include product performance, price, consumers' economic status, sales volume, pageview, credibility and product discount rate. These factors restrain each other and influence the profits of products together with cost. By taking the abstract psychological factors influencing customers' shopping into full account and exploring the potential rule between factors influencing consumers' shopping probability, this paper establishes a differential equation model to obtain the expression formula. Following that, normal distribution and other methods are adopted to simplify the profit expression formula, which is solved to gain the optimal pricing. Then, this paper adopts the color gradual change of the three-dimensional color graph to analyze the influence of various parameters on ways of promotion and put forward the strategies to adjust to the environmental changes: 1) Increase the price and reduce the discount rate for the rich buyers and the opposite for buyers who are not rich; 2) When the cost rises, the price and discount rate should be increased; 3) When the product performance and initial sales decrease, there should be no significant changes for the pricing, but promotional measures are necessary, etc. Quantitative value can be calculated with MATLAB Program.

The paper mainly consists of two parts, which analyzes two conditions, namely with promotional activities and without promotional activities, and puts forwards pricing strategies.

Keywords: online shopping pricing; purchase probability; differential equation; model; three-dimensional color graph; Quantitative; strategy adjustment.

Content

I. Introduction

With the ascent of the 21st century, the global informationalization process is changing people's lifestyle and the whole world is gradually entering in a network economy era. The increasing popularity of internet has made online shopping an emerging shopping style. More and more people know and love online shopping. To many office staffs, online shopping can help them save time and buy products they like at a price lower than that in the physical stores. Besides, online dealers have seized the business opportunities brought by online shopping. They launch promotional activities by discounting their products or reducing the price of their products. A case in point is the "Shopping Carnival" on Singles' Day (November 11) to attract netizens.

The pricing of different products can influence the purchase will of different customers and the sales volume of dealers, and the influence of different discount and promotional activities is different. Currently, researches into the pricing and promotional methods adopted by the dealers for online shopping are insufficient. Few academic literatures discuss about the issue. Therefore, to study the netizens' consumption behaviors and psychology and establish a mathematical model of online shopping based on the economics and mathematics knowledge so as to analyze the relationship more scientifically and put forward some feasible suggestions about pricing of online shopping and strategies adopted by online dealers to respond to the environmental changes can contribute to the promotion and improvement of the profits of online shopping.

II. Explanation of the mathematical model

2.1 Fundamental hypotheses

Concerning the research issue, this paper puts forward the following hypotheses:

Hypothesis 1: The postage of the online dealer's products is the same to the average level and remains unchanged, and is borne by the consumer;

Hypothesis 2: Consumer's consumption behavior is based on thorough and rational consideration, and the all consumers' psychology is similar;

Hypothesis 3: The products for sale are not food, and these products are life necessities that all level of affluence of consumers will buy.

2.2 Explanation of symbols commonly used

The symbols frequently appearing in the paper are shown in Table 1 below:

Table 1 Explanation of symbols

Symbols	Definition	Unit
ε	Product performance	1
P	Price (including postage)	Yuan
P_0	Average price (including postage)	Yuan
p	Selling price (excluding postage)	Yuan
$P_{\max}$	Price ceiling	Yuan
P_m	Postage (parameter)	Yuan
P'	Optimal price (including postage)	Yuan
p'	Optimal selling price (excluding postage)	Yuan
C	Product cost (excluding postage cost)	Yuan
C_m	Postage cost (parameter)	Yuan
α	Purchase probability	100%

α_P	Price component of purchase probability	100%
α_N	Selling component of purchase probability	100%
N	Sales volume	Digit
w	Probability weight of price factor in	100%
t	Credibility	100%
v	Praise rate (parameter)	100%
B	Pageview	Digit
P_f	Dealers' initial price (parameter)	Yuan
N_f	Dealers' initial sales volume (parameter)	Digit
B_f	Dealers' initial pageview (parameter)	Digit
β	Refund rate (parameter)	100%
I	Total profits	Yuan
E'	Engel coefficient of non-food products	100%
μ	Engel coefficient of average non-food	100%
d	Discount rate	1
d'	The optimal discount rate (parameter)	1

2.3 Problem Analysis

Generally speaking, online shoppers will usually consider the following factors before their online shopping, namely product performance, price, their own economic status, product credibility, sales volume and product discount rate. All the above factors will influence on the purchase behavior of an online shopper. To online dealers, the total pageviews of their products decide the number of their consumers. To sum up, major factors influencing dealers' turnover include product performance, price, consumers' economic status, pageview, sales volume, credibility and product discount rate.

Based on the above analysis, the following part will discuss the relationship between the factors stated above:

First, product performance, price, consumers' economic status, sales volume, credibility and product discount rate can directly decide consumers' purchase probability. Second, since pageviews are fluctuating, the analysis shows that sales volume, product discount rate and initial pageviews can influence the product pageviews. On the contrary, the pageviews might influence the purchase amount, which is a dynamic process.

The logic relationship is shown below:

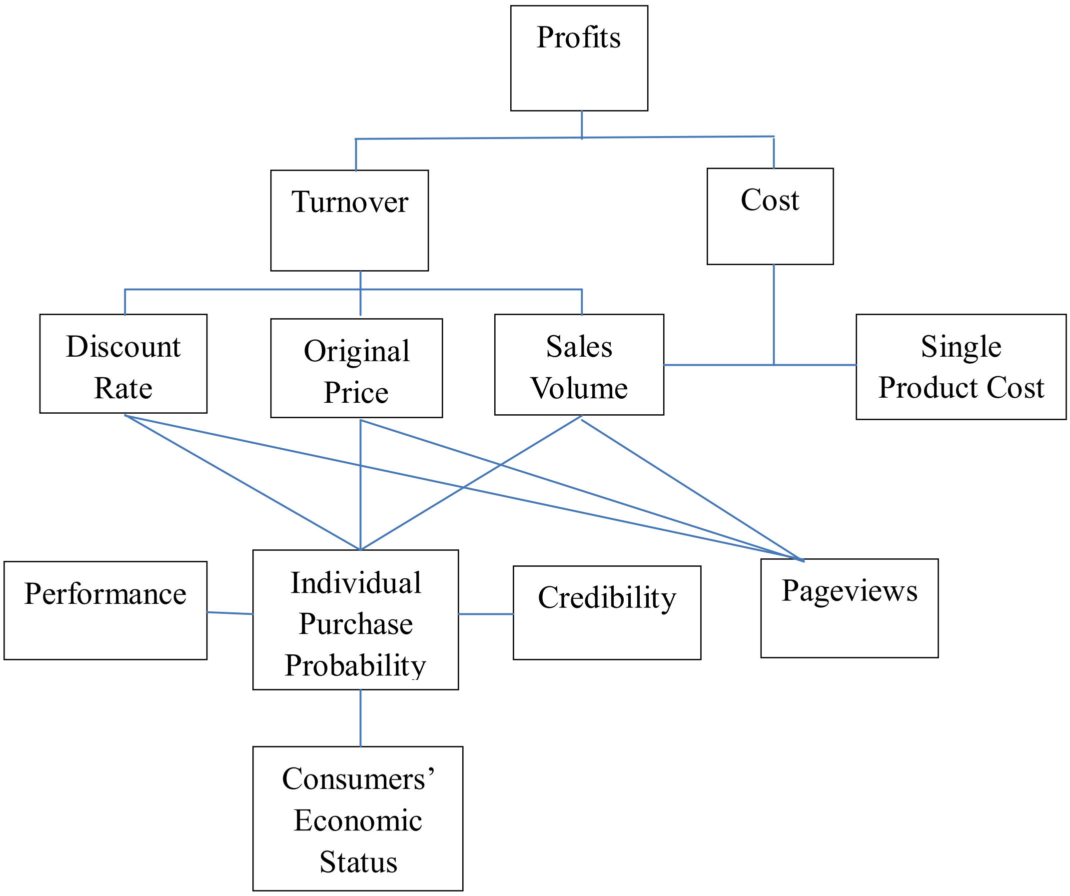

Chart 2-1 Logic relationship of the mathematical model

III. Purchase probability model

In the perspective of Statistics and Probability Theory, every consumer has certain probability to purchase a product. Here, the purchase probability is assumed to be α . The probability is the function relevant to price P and sales volume N , and is influenced by some parameters. Besides, it is decided by the subjective psychological factors of consumers.

According to the problem analysis and logical relationship, factors influencing consumers' purchase probability mainly include: product quality, price, sales volume, credibility rate and product discount rate. As to one's own economic status, its influence is reflected in every factor. Considering the characteristics of promotional activities and discount activities, they will be respectively discussed in the part of promotion models. Now, the influence of various parameters on consumers' purchase rate will be analyzed.

3.1 Parameter model

3.1.1 Product performance: ε

ε stands for product performance, which describes consumers' satisfaction degree of consumers' demands.

This paper assumes a consumer to be satisfied with a product if the product meets the consumer's demands and that α is fully influenced by the rest of factors. Under the circumstance, define $\varepsilon = 1$. Otherwise, the consumer will not buy it. Then $\varepsilon = 0$. The relationship can be shown in Formula (1).

$$\varepsilon = \begin{cases} 0 \\ 1 \end{cases}, \tag{1}$$

Formula (1) shows two conditions where product performance meets consumers' demands or not respectively.

3.1.2 Price: P

P_0 stands for the general price of a product. This paper defines $P \in (0,+\infty)$ and both P_0 and P include postage.

When P appears alone, it has no meaning. $\frac{P}{P_0}$ is used to measure the deviation degree of P from P_0. The function of the influence of price factor on consumers' purchase probability is defined as below:

$$\alpha_P\left(\frac{P}{P_0}\right)\ (\alpha_P \in [0,1))\ .$$

To work out the relationship between α_P and $\frac{P}{P_0}$:

First, α_P is a monotone decreasing function, namely $|\alpha'_P| < 0$.

It should be noticed that when $\frac{P}{P_0} \to 0$, namely when the price is extremely low, the consumers will be extremely willing to purchase the product and they are not sensitive to the insignificant price changes. The relationship can be expressed through the following mathematical formula:

$$\lim_{\frac{P}{P_0}\to 0} \alpha_P = 1,\ \lim_{\frac{P}{P_0}\to 0} |\alpha'_P| = 0\ .$$

When $\frac{P}{P_0}$ is close to 1, namely when the price is close to the consumers' psychological price, consumers will be willing to purchase the product and are sensitive to the price changes. In other words, $\alpha_P(1)$ is larger, as well as $|\alpha'_P(1)|$.

In fact, to every consumer, when the price of a product reaches $P_{\max}$, consumers will not buy the product, namely

$$\alpha_P\left(\frac{P_{\max}}{P_0}\right)=0\ .$$

However when $\frac{P_{\max}}{P_0} \to +\infty$ and when $\frac{P}{P_0}$ is very large, namely the price is very high, consumers will be extremely unwilling to purchase he product and are not sensitive to product changes. The relationship can be expressed by the following mathematical formula:

$$\lim_{\frac{P}{P_0} \to +\infty} \alpha_P = 0 \,, \lim_{\frac{P}{P_0} \to +\infty} |\alpha'_P| = 0 \quad .$$

To sum up,

$$\text{When} \quad \alpha'_P < 0 \,, \lim_{\frac{P}{P_0} \to 0} |\alpha'_P| = 0 \quad \text{and} \quad \alpha'(1) \quad \text{is larger,} \quad \lim_{\frac{P}{P_0} \to +\infty} |\alpha'_P| = 0 \quad ,$$

The following conditions can be obtained:

$$\forall \frac{P}{P_0} \in (0,1), \quad \alpha''_P < 0 \,, \forall \frac{P}{P_0} \in [1,+\infty) \quad , \quad \alpha''_P \geq 0 \quad .$$

α_P is formed through consumers' comparison of prices of various dealers.

When $\frac{P}{P_0}$ is of a certain value and the value of $\mathrm{d}\left(\frac{P}{P_0}\right)$ is changed, what the value of $\mathrm{d}\alpha_P$ is.

Consumers will adjust based on α_P before the price change, namely

$$\mathrm{d}\alpha_P = \mathrm{d} f_P \cdot \alpha_P \quad ,$$

Where, f_P is the function of $\frac{P}{P_0}$ and α_P is made to meet the analysis above. Then

$$\mathrm{d}\alpha_P = \mathrm{d} f_P \cdot \alpha_P \Leftrightarrow \frac{\mathrm{d}\alpha_P}{\mathrm{d}\frac{P}{P_0}} = \frac{\mathrm{d} f_P}{\mathrm{d}\frac{P}{P_0}} \cdot \alpha_P \quad .$$

In order to meet the analysis conditions, $\frac{\mathrm{d} f_P}{\mathrm{d}\frac{P}{P_0}}$ should meet the condition of $\lim_{\frac{P}{P_0} \to 0} \frac{\mathrm{d} f_P}{\mathrm{d}\frac{P}{P_0}} = 0$ and $\frac{\mathrm{d} f_P}{\mathrm{d}\frac{P}{P_0}}$ should be negative when $P \in (0,+\infty)$.

Besides, $\frac{\mathrm{d} f_P}{\mathrm{d} \frac{P}{P_0}}$ has the minimum value.

Since $\lim\limits_{\frac{P}{P_0}\to+\infty} \alpha_P = 0$ and $\lim\limits_{\frac{P}{P_0}\to+\infty} \left|\alpha'_P\right| = 0$, $\lim\limits_{\frac{P}{P_0}\to+\infty} \frac{\mathrm{d} f_P}{\mathrm{d} \frac{P}{P_0}} = 0$ is unnecessary.

Make

$$\frac{\mathrm{d} f_P}{\mathrm{d} \frac{P}{P_0}} = -\frac{k'_P \frac{P}{P_0}}{e^{\frac{P}{P_0}}} \left(k'_P > 0\right)$$

$$\Rightarrow f_P = \int -\frac{k'_P \frac{P}{P_0}}{e^{\frac{P}{P_0}}} \mathrm{d} \frac{P}{P_0}$$

$$\Rightarrow f_P = k'_P \left(\frac{P}{P_0} + 1\right) \cdot e^{-\frac{P}{P_0}} \quad ,$$

Obtain

$$\lim_{\frac{P}{P_0}\to 0} f'_P = 0 \, , f''_P < 0 \left(\frac{P}{P_0} \in (0,1)\right), f''_P \geq 0 \left(\frac{P}{P_0} \in [1,+\infty)\right) \quad .$$

Then

$$\mathrm{d}\alpha_P = \mathrm{d}\left(k'_P \left(\frac{P}{P_0} + 1\right) \cdot e^{-\frac{P}{P_0}}\right) \cdot \alpha_P$$

$$\Rightarrow \int \frac{\mathrm{d}\alpha_\mathrm{P}}{\alpha_\mathrm{P}} = \int \mathrm{d}\left(k'_P \left(\frac{P}{P_0} + 1\right) \cdot e^{-\frac{P}{P_0}}\right)$$

$$\Rightarrow \ln c'_P \cdot \alpha_P = k'_P \left(\frac{P}{P_0} + 1\right) \cdot e^{-\frac{P}{P_0}}$$

$$\Rightarrow \alpha_P = \frac{e^{k'_P \left(\frac{P}{P_0}+1\right) \cdot e^{-\frac{P}{P_0}}}}{c'_P} \quad .$$

Make

$$c'_P = e^{k'_P} - c_P \quad ,$$

Obtain

$$\alpha_P = \frac{e^{k'_P \left(\frac{P}{P_0}+1\right) \cdot e^{-\frac{P}{P_0}}}}{e^{k'_P} - c_P} \quad .$$

Since the function does not meet the required nature of $\lim_{\frac{P}{P_0}\to+\infty} \alpha_P = 0$ and $\lim_{\frac{P}{P_0}\to 0} \alpha_P = 1$, the function should be changed to make α_P meet conditions and be endowed with the characteristic similar to $\frac{e^{k'_P\left(\frac{P}{P_0}+1\right)\cdot e^{-\frac{P}{P_0}}}}{e^{k'_P}-c_P}$.

The function undergoes the following changes:

Adopt the parameter k_P.

Change the current α_P. Then,

$$\alpha_P = \frac{e^{k_P\left(\frac{P}{P_0}+1\right)\cdot e^{-\frac{P}{P_0}}}}{e^{k_P}-c_P} - \frac{c_P}{e^{k_P}-c_P} \quad .$$

So,

$$\lim_{\frac{P}{P_0}\to+\infty} \alpha_P = 0 \, , \lim_{\frac{P}{P_0}\to 0} \alpha_P = 1 \, ,$$

In other worlds, the conditions have been met.

In fact, to every consumer, when the price of a product reaches $P_{\max}$, the consumer will not buy it, namely

$$\alpha_P\left(\frac{P_{\max}}{P_0}\right) = 0 \, ,$$

Then, make

$$c_P = e^{k_p\left(\frac{P_{\max}}{P_0}+1\right)\cdot e^{-\frac{P_{\max}}{P_0}}} \qquad (2)$$

to meet the conditions

Verify the nature of the function:

$$\lim_{\frac{P}{P_0}\to 0} \alpha_P = 0 \, , \lim_{\frac{P}{P_0}\to 0} \left|\alpha'_P\right| = 0 \, , \alpha_P\left(\frac{P_{\max}}{P_0}\right) = 0, \ \left|\alpha'_P\right| < 0 \, ,$$

$$\alpha''_P < 0 \left(\frac{P}{P_0} \in (0,1)\right), \alpha''_P \ge 0 \left(\frac{P}{P_0} \in \left[1, \frac{P_{\max}}{P_0}\right]\right) \quad ,$$

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Conditions are met.

To sum up:

$$\alpha_P=\begin{cases}\dfrac{e^{k_P\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}}-c_P}{e^{k_P}-c_P}, & 0<\dfrac{P}{P_0}<\dfrac{P_{\max}}{P_0}\\ 0, & \dfrac{P}{P_0}\geq\dfrac{P_{\max}}{P_0}\end{cases}. \tag{3}$$

Discuss the value of $\frac{P_{\max}}{P_0}$:

To ordinary households, $\frac{P_{\max}}{P_0}$ stays at around 2. However, to poor households, it might be exist that $\frac{P_{\max}}{P_0}\leq 1.5$. Therefore, $\frac{P_{\max}}{P_0}$ varies with the family conditions.

Then discuss k_P should meet the condition that it is decided by individual consumers.

According to the nature of (3), it can be seen that, as long as $k_P\neq 0$, the conditions will be met.

k_P should meet the following conditions:

a) $\alpha_P(1)$ stays at a large value, around 80% or above.

Meaning: When the practical price is the same to the most acceptable price, consumers will be willing to purchase considering the price.

b) $\alpha_P(0.5)$ stays at a large, around 95%.

Meaning: When the practical price is half of the most acceptable price, consumers will be willing to purchase considering the price.

c) $\alpha_P(1.5)$ stays at a small value, around 50% or below or even 0;

Meaning: When the practical price is about one and a half times more than the most acceptable price, consumers will be unwilling to purchase considering the price.

3.1.3 Sales volume: *N*

It should be noticed that online shops are different from physical shops. Other consumers' purchase condition of a product will be listed out, which will have a great influence on the consumer's purchase behavior and decide the value of α. It is assumed that N_0 stands for the average sales volume of the product. Similar to P, N alone stands for no specific meaning. Therefore, $\frac{N}{N_0}$ is adopted to measure the difference between certain sales volume and the average sales volume.

Define the function of the factor of sales volume influencing consumers' purchase probability as:

$$\alpha_N\left(\frac{N}{N_0}\right)(\alpha_N \in [0,1)) \quad .$$

To work out the relations α_N and $\frac{N}{N_0}$:

First, α_N is the monotone increasing function, namely $\alpha'_N > 0$.

It should be noticed that when $\frac{N}{N_0} = 0$, namely when the sales volume is low or even 0, consumers will be unwilling to purchase, but are sensitive to the changes of the sales volume. The reason is that the sales volume has a great influence on consumers' purchase behavior. In other words, $\alpha_N(0) = 0$ and $|\alpha'_N(0)|$ are larger.

With the increase of $\frac{N}{N_0}$, consumers' sensitivity to the changes of sales volume will decrease gradually. In other words, $|\alpha'_N|$ will become smaller and smaller.

When $\frac{N}{N_0} \to +\infty$, namely when the sales volume is high, consumers will be very willing to purchase and remain insensitive to the changes of sales volume. The relationship can be expressed through the following mathematical formula.

$$\lim_{\frac{N}{N_0}\to+\infty} \alpha_N = 1,\ \lim_{\frac{N}{N_0}\to+\infty} |\alpha'_N| = 0 \ .$$

Summing up $|\alpha'_N(0)|$ is larger, $|\alpha'_N|$ decrease gradually, and $\lim_{\frac{N}{N_0}\to+\infty} |\alpha'_N| = 0$, the condition of $\alpha''_N < 0$ can be obtained.

α_N is generated through consumers' comparison of the sales volume of various dealers.

When consumers are comparing the sales volume and when $\frac{N}{N_0}$ stays at a certain value and the value of $\mathrm{d}\left(\frac{N}{N_0}\right)$ and $\mathrm{d}\alpha_N$ is changed, consumers will adjust based on α_N before the change.

Then,

$$\mathrm{d}\alpha_N = \mathrm{d}g_N \cdot \alpha_N \ ,$$

Where, g_N stands for the function of $\frac{N}{N_0}$. Make α_N meet the above analysis conditions.

Then,

$$\mathrm{d}\alpha_N = \mathrm{d}g_N \cdot \alpha_N \Leftrightarrow \frac{\mathrm{d}\alpha_N}{\mathrm{d}\frac{N}{N_0}} = \frac{\mathrm{d}g_N}{\mathrm{d}\frac{N}{N_0}} \cdot \alpha_N \ .$$

$\frac{\mathrm{d}g_N}{\mathrm{d}\frac{N}{N_0}}$ can meet the condition of $\lim_{\frac{N}{N_0}\to+\infty} \frac{\mathrm{d}g_N}{\mathrm{d}\frac{N}{N_0}} = 0$,and $\frac{\mathrm{d}g_N}{\mathrm{d}\frac{N}{N_0}}$ remains positive and monotone decreasing when $N \in [0,+\infty)$.

Make,

$$\frac{\mathrm{d}g_N}{\mathrm{d}\frac{N}{N_0}} = k'_N e^{-\frac{N}{N_0}} \ (k_N > 0)$$

$$\Rightarrow g_N = \int k'_N e^{-\frac{N}{N_0}} \mathrm{d}\frac{N}{N_0}$$

$$\Rightarrow g_N = -k'_N e^{-\frac{N}{N_0}} \quad ,$$

It is found $g'_N > 0$ and that, to $\forall N \in R^+$, $g''_N < 0, g'''_N > 0$ and $\lim\limits_{\frac{N}{N_0} \to +\infty} g''_N = 0$.

Then,

$$\mathrm{d}\alpha_N = \mathrm{d}\left(-k'_N \cdot e^{-\frac{N}{N_0}} \right) \alpha_N$$

$$\Rightarrow \int \frac{\mathrm{d}\alpha_{\mathrm{N}}}{\alpha_{\mathrm{N}}} = \int \mathrm{d}\left(-k'_{\mathrm{N}} \cdot e^{-\frac{N}{N_0}} \right)$$

$$\Rightarrow \ln c'_P \cdot \alpha_P = -k'_N \cdot e^{-\frac{N}{N_0}} \; (c'_N > 0)$$

$$\Rightarrow \alpha_N = \frac{e^{-k'_N \cdot e^{-\frac{N}{N_0}}}}{c'_N} \quad .$$

Make

$$c'_N = 1 - e^{-k'_N} \quad ,$$

Then, obtain

$$\alpha_N = \frac{e^{-k'_N \cdot e^{-\frac{N}{N_0}}}}{1 - e^{-k'_N}} \quad .$$

Since the function does not meet the required nature of $\alpha(0) = 0$ and $\lim\limits_{\frac{N}{N_0} \to +\infty} \alpha_N = 1$, it is necessary to transform the function so as to make α_N meet the condition and endowed with the characteristic similar to $\frac{e^{-k'_N \cdot e^{-\frac{N}{N_0}}}}{1 - e^{-k'_N}}$.

Transform the function through as below:

Adopt k_N as the parameter

Transform the current α_N,

Then

$$\alpha_N = \frac{e^{-k_N \cdot e^{-\frac{N}{N_0}}}}{1 - e^{-k_N}} - \frac{e^{-k_N}}{1 - e^{-k_N}} \quad . \tag{4}$$

Where, $\alpha_N(0)=0$ and $\lim\limits_{\frac{N}{N_0}\to+\infty} \alpha_N = 1$. So the conditions are met.

Verify other characteristics of the function

$$\lim_{\frac{N}{N_0}\to+\infty} \left|\alpha'_N\right| = 0\,, \alpha'_N < 0 \quad \alpha''_N < 0 \quad ,$$

All conditions are met.

Then discuss that k_N should meet the condition that it is decided by individual consumers.

Similarly, based on the nature of formula (4), it can be known that, as long as $k_N \neq 0$, the conditions will be met.

k_N should meet the following conditions:

a) $\alpha_N(1)$ stays at a large value, around 80% or above.

Meaning: when the sales volume of a dealer is equal to the average level, consumers will be willing to purchase considering the sales volume.

b) $\alpha_N(0.5)$ stays at a small value, around 60% or below.

Meaning: when the sales volume of a dealer is equal to half of the average level, consumers will be not so willing to purchase considering the sales volume.

c) $\alpha_N(2)$ stays at a high value, around 95%.

Meaning: when the sales volume of a dealer is twice as much as the average level, consumers will be very willing to purchase considering the sales volume.

3.1.4 Credibility: t

The credibility t is in direct correlation with the praise rate v, the latter of which can directly reflect product quality. Thus, v can directly serve as the expression formula of t:

$$t = v \ . \tag{5}$$

3.1.5 Weight of price factor: w

Life experiences show that when the price changes consumers will pay more attention to the price factor, but ignore the factor of sales volume. Therefore, a fluctuant parameter is required to describe consumers' attention shown to the price factor.

Here, $w(w \in (0,1])$ is adopted as a measurement. w changes with $\frac{P}{P_0}$.

Define that when $\frac{P}{P_0} = 1$, the weight is w_e. w_e is a parameter, which is decided by consumers. $w_e \in (0,1]$.

To work out the relationship between w and $\frac{P}{P_0}$:

First, w is monotone increasing, namely $w' > 0$.

It should be noticed that when $\frac{P}{P_0} \to 0$, namely when the price is extremely low, consumers will not pay attention to the price and be insensitive to price changes. The relationship can be expressed through the following mathematical formula:

$$\lim_{\frac{P}{P_0} \to 0} w = 0 \ , \lim_{\frac{P}{P_0} \to 0} w' = 0 \ .$$

When $\frac{P}{P_0}$ is close to 1, namely when the price is close to consumers' psychological price, consumers will pay more attention to the price, In other words, when $w'(1)$ is larger and $\frac{P}{P_0}=1, w = w_e$, namely $w(1) = w_e$.

When $\frac{P}{P_0} \to +\infty$, namely when the price is very high, consumers will show great attention to the price and remain insensitive to price changes. The relationship can be expressed through the following mathematical formula:

$$\lim_{\frac{P}{P_0}\to+\infty} w = 1 \quad , \lim_{\frac{P}{P_0}\to+\infty} w' = 0 \quad .$$

Summing up, $w' > 0$, $\lim_{\frac{P}{P_0}\to 0} w' = 0$; $w'(1)$ is larger, $\lim_{\frac{P}{P_0}\to+\infty} w' = 0$,

The following conditions can be obtained:

$$\forall \frac{P}{P_0} \in (0,1), w'' > 0, \forall \frac{P}{P_0} \in [1,+\infty), w'' < 0 \quad .$$

The formation of w can be regarded as a result of the price comparison.

When $\frac{P}{P_0}$ stays at a certain value and the change of $\mathrm{d}w$ along with the change of $\mathrm{d}\left(\frac{P}{P_0}\right)$, consumers will properly increase based on w before the price change, namely

$$\mathrm{d}w = \mathrm{d}f_w \cdot w \quad ,$$

Where, f_w is the function of $\frac{P}{P_0}$. w is made to meet the above analysis conditions.

Then,

$$\mathrm{d}\,w = \mathrm{d}\,f_w \cdot w \Leftrightarrow \frac{\mathrm{d}\,w}{\mathrm{d}\frac{P}{P_0}} = \frac{\mathrm{d}\,f_w}{\mathrm{d}\frac{P}{P_0}} \cdot w \quad .$$

$\frac{\mathrm{d}\,f_w}{\mathrm{d}\frac{P}{P_0}}$ can meet the condition of $\lim\limits_{\frac{P}{P_0}\to+\infty} \frac{\mathrm{d}\,f_w}{\mathrm{d}\frac{P}{P_0}} = 0$, and $\frac{\mathrm{d}\,f_w}{\mathrm{d}\frac{P}{P_0}}$ remains positive and has the maximum value when $P \in (0,+\infty)$.

Since $w(0) = 0$ and $w'(0) = 0$, $\lim\limits_{\frac{P}{P_0}\to 0} \frac{\mathrm{d}\,f_w}{\mathrm{d}\frac{P}{P_0}} = 0$ is unnecessary.

Make

$$\frac{\mathrm{d}\,f_w}{\mathrm{d}\frac{P}{P_0}} = \frac{k'_w \frac{P}{P_0}}{e^{\frac{P}{P_0}}} (k'_w > 0)$$

$$\Rightarrow f_w = \int \frac{k'_w \frac{P}{P_0}}{e^{\frac{P}{P_0}}} \mathrm{d}\frac{P}{P_0}$$

$$\Rightarrow f_w = -k'_w \left(\frac{P}{P_0} + 1\right) e^{-\frac{P}{P_0}} \quad ,$$

It is found that

$$f'_w > 0\,,\ \lim_{\frac{P}{P_0}\to+\infty} f'_w = 0\,, f''_w > 0 \left(\frac{P}{P_0} \in (0,1)\right), f''_P \le 0 \left(\frac{P}{P_0} \in [1,+\infty)\right) \quad .$$

Then,

$$\mathrm{d}\,w = \mathrm{d}\left(-k'_w \left(\frac{P}{P_0} + 1\right) e^{-\frac{P}{P_0}}\right) \cdot w$$

$$\Rightarrow \int \frac{\mathrm{d}\,w}{w} = \int \mathrm{d}\left(-k'_w \left(\frac{P}{P_0} + 1\right) e^{-\frac{P}{P_0}}\right)$$

$$\Rightarrow \ln c'_w \cdot w = -k'_w \left(\frac{P}{P_0} + 1\right) e^{-\frac{P}{P_0}}$$

$$\Rightarrow w = \frac{e^{-k'_w\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}}}{c'_w} \quad .$$

Make

$$c_w = 1 \, , c'_w = c_w - e^{-k_w} \quad ,$$

To obtain

$$w = \frac{e^{-k'_w\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}}}{c_w - e^{-k_w}} \quad .$$

However, the function does not meet the required condition of $w(0) = 0$ and $\lim\limits_{\frac{P}{P_0}\to+\infty} w = 1$.Therefore, it is necessary to transform the function, making w meet the condition and endowed with the characteristic similar to $\frac{e^{-k'_w\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}}}{c_w - e^{-k_w}}$.

Make the following changes to the function:

Adopt k_w as the parameter

Change the current w',

Then,

$$w = \frac{e^{-k_w\left(\frac{P_{\max}}{P_0}+1\right)e^{-\frac{P_{\max}}{P_0}}}}{c_w - e^{-k_w}} - \frac{e^{-k_w}}{c_w - e^{-k_w}} \quad ,$$

Then $\lim\limits_{\frac{P}{P_0}\to 0} w = 0$, $\lim\limits_{\frac{P}{P_0}\to+\infty} w = 1$. The condition has been met.

Since $P_{\max}$ is considered to exist in the analysis of α_P, here w should meet the condition of $w\left(\frac{P_{\max}}{P_0}\right)=1$.

Then, make to obtain Formula (6) as below:

$$c_w = e^{-k_w\left(\frac{P_{\max}}{P_0}+1\right)e^{-\frac{P_{\max}}{P_0}}}, \qquad (6)$$

To verify the characteristic of the nature of the function:

$$\lim_{\frac{P}{P_0}\to 0} w = 0,\ w\left(\frac{P_{\max}}{P_0}\right)=1,\ \lim_{\frac{P}{P_0}\to 0} w' = 0,\ w'>0,$$

$$w''>0\left(\frac{P}{P_0}\in(0,1)\right),\ w''<0\left(\frac{P}{P_0}\in(1,P_{\max})\right),$$

The conditions have been met.

Finally, formula (7) is obtained as below:

$$w=\begin{cases}\dfrac{e^{-k_w\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}}-e^{-k_w}}{c_w-e^{-k_w}}, & 0<\dfrac{P}{P_0}<\dfrac{P_{\max}}{P_0}\\[2ex] 1, & \dfrac{P}{P_0}\geq\dfrac{P_{\max}}{P_0}\end{cases}. \qquad (7)$$

Similarly, based on the nature of formula (7), it can be seen that, as long as $k_w \neq 0$, conditions will be met, but there should be a specific k_w to make $w(1)=w_e$

The following function can be obtained:

$$c_w = e^{-k_w\left(\frac{P_{\max}}{P_0}+1\right)e^{-\frac{P_{\max}}{P_0}}},\quad \frac{e^{-2k_we^{-1}}-e^{-k_w}}{c_w-e^{-k_w}}=w_e. \qquad (8)$$

It is hard to work out the analytical solution based on the function. In practical operation, numerical values can be adopted to work out k_w.

3.2 Model establishment of purchase probability model

The following part will analyze the influence of every factor on α and obtain the expression of α.

1) Product performance: ε

Since product performance can directly influence consumers' purchase behavior, ε should be a multiplier.

2) Price and consumers' economic status should be combined to exert their influence. Any one of them cannot play a decisive role alone.

Therefore, the factors of the group should be added and connected, and w and $(1-w)$ are adopted to describe their weight among the whole item respectively.

3) Credibility: t

Credibility can directly influence consumers' purchase behavior. If consumers' credibility is low for a certain product, it suggests the product quality is poor, which might directly restrain consumers' purchase behavior.

Based on the above analysis, and considering the expression formula of $(1)(3)(4)(5)(7)$, the function between $\frac{P}{P_0}$ and $\frac{N}{N_0}$ can be obtained:

$$\alpha\left(\frac{P}{P_0},\frac{N}{N_0}\right) = \varepsilon \cdot t \cdot \left[w\alpha_P + (1-w)\alpha_N\right] \quad . \tag{9}$$

Where,

$$\varepsilon = \begin{cases} 0 \\ 1 \end{cases},$$

It stands for the condition that the product performance meets consumers' demands or not.

$$t = v \ ,$$

$$\alpha_P = \begin{cases} \dfrac{e^{k_P\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}} - c_P}{e^{k_P} - c_P}, 0 < \dfrac{P}{P_0} < \dfrac{P_{\max}}{P_0} \\ 0 \quad , \quad \dfrac{P}{P_0} \geq \dfrac{P_{\max}}{P_0} \end{cases},$$

$$c_P = e^{k_p\left(\frac{P_{\max}}{P_0}+1\right)\cdot e^{-\frac{P_{\max}}{P_0}}} \ ,$$

$$\alpha_N = \frac{e^{-k_N \cdot e^{-\frac{N}{N_0}}} - e^{-k_N}}{1 - e^{-k_N}} \ ,$$

$$w = \begin{cases} \dfrac{e^{-k_w\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}} - e^{-k_w}}{c_w - e^{-k_w}}, \qquad 0 < \dfrac{P}{P_0} < \dfrac{P_{\max}}{P_0} \\ 1 \quad , \quad \dfrac{P}{P_0} \geq \dfrac{P_{\max}}{P_0} \end{cases},$$

c_w and k_w meet the following condition:

$$\frac{e^{-2k_w e^{-1}} - e^{-k_w}}{c_w - e^{-k_w}} = w_e \, , c_w = e^{-k_w\left(\frac{P_{\max}}{P_0}+1\right)e^{-\frac{P_{\max}}{P_0}}} \ .$$

3.3 Influence rules of purchase probability α

Analyze the graph of α :

In order to draw the graph, the value of ε, v, $\frac{N}{N_0}$, w_e, k_P and k_N should be confirmed.

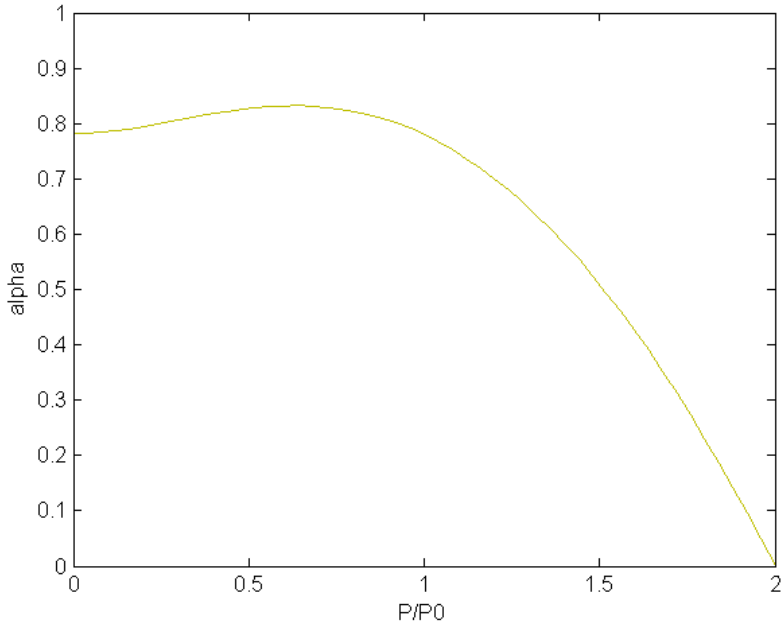
It can be assumed that the product meet the requirement, namely $\varepsilon = 1$, the research shows $v = 97\%$.

Assume

$$k_P = -3.9, k_N = -1.7,$$

Both k_P and k_N meet the required conditions and assume $w_e = 0.7$.

When $\frac{N}{N_0} = 1$, the graph of $\alpha - \frac{P}{P_0}$ is shown below (See MOS_alpha_figureN in the Attachment)



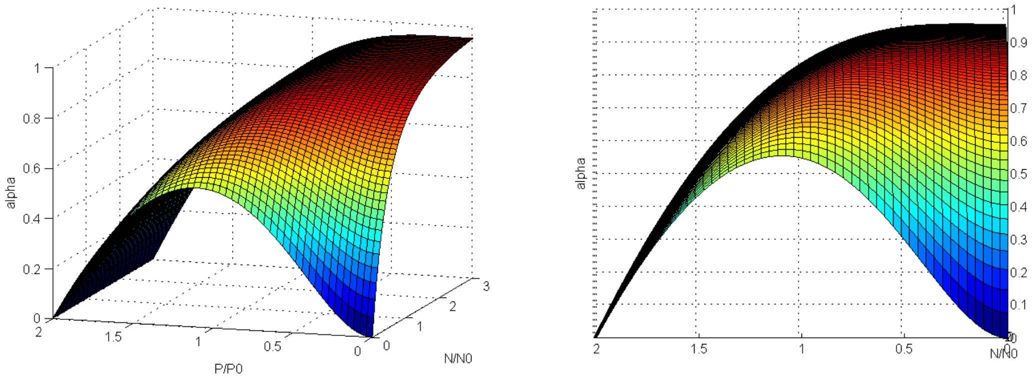
Graph 3-1 Standard $\alpha - \frac{P}{P_0}$ graph

Analyze the influence of the major variables in expression formula of α on it and draw the graph for further analysis.

3.3.1 The influence of sales volume $\frac{N}{N_0}$ on purchase probability α

$\frac{N}{N_0}$ and w_e are regarded as the important influencing factor apart from $\frac{P}{P_0}$ (independent variable). Use MATLAB to draw the graph of $\alpha - \frac{P}{P_0}$ when N and w_e change.

Maintain the other parameters unchanged and the graph of $\alpha - \frac{N}{N_0} - \frac{P}{P_0}$ is obtained as below (See MOS_alpha_figureN in the Attachment)



Graph 3-2 The graph of the influence of sales volume $\frac{N}{N_0}$ on α

From the graph, it can be seen that the larger $\frac{N}{N_0}$ is, the larger α is generally, which means consumers are more willing to purchase.

When $\frac{N}{N_0} < 1$, the graph $\alpha - \frac{P}{P_0}$ undergoes great changes.

When $\frac{N}{N_0} \geq 1$, the graph $\alpha - \frac{P}{P_0}$ undergoes relatively smaller changes, and α has the peak value when $\frac{P}{P_0} \in (0,2]$. In other words, the peak value stands for consumers' greatest purchase will. The horizontal axis of the peak value decreases with the increase of N .

It is found that, when $\frac{N}{N_0}$ is smaller and $\frac{P}{P_0}$ is smaller or larger, the changes of α will be larger:

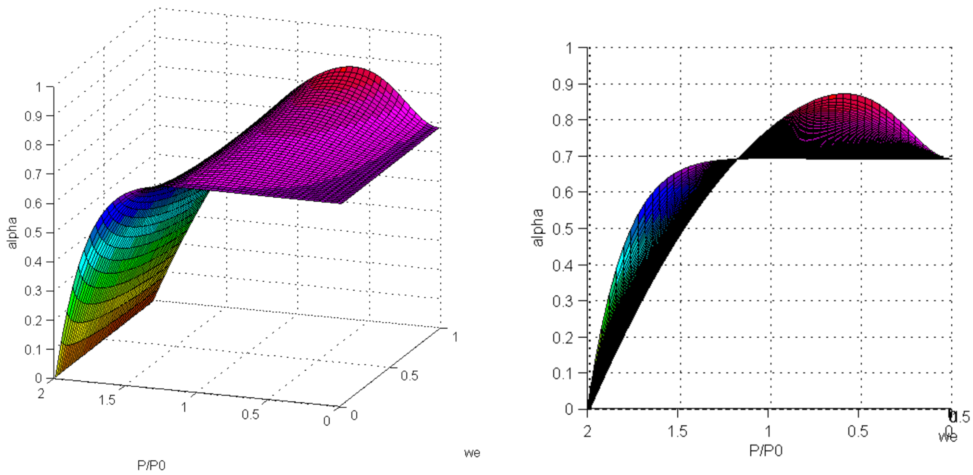
When $\frac{N}{N_0}$ is larger, and only when $\frac{P}{P_0}$ is larger, the changes of α will be larger.

This means when the sales volume is smaller, consumers will pay more attention to the price. However, when the price is lower, due to consumers' lack of faith for the dealers, they would not purchase either. Due to different pricing level, consumers will pay attention to different aspects. As a result, the function first ascends and then descends.

When the sales volume is high, consumers will pay less attention to the price. When the price is lower, , α will only undergo small changes. Only when the price is comparatively high will consumers refuse to purchase.

3.3.2 The influence of the weight of price factor w_e on purchase probability α under the condition of average price

Under the circumstance, set $\frac{N}{N_0}=1$ and obtain the graph of $\alpha - w_e - \frac{P}{P_0}$. (See MOS_alpha_figurewe in the attachment)



Graph 3-3 The graph showing the influence of the weight of price factor w_e on purchase probability α under the condition of average price

From the graph, it can be seen that the larger w_e, the value of α will be higher in the low price section, which suggests the consumers are more willing to purchase. However, the value of α is low, which suggests consumers are reluctant to purchase.

Besides, α has its peak value when $\frac{P}{P_0}$ is within the section. In other words, consumers are most willing to purchase. The peak value increases with the increase of w_e.

Besides, it can be found that, in the low price section, the changes of α are smaller. However, due to different prices, consumers pay attention to different aspects, which result in the function to first ascend and then descend. In the high price section, consumers are affected by the price, which result in the rapid decrease of α.

Besides, it is found that there is a price ratio which can make α stand at the same value despite the value of w_e. However, the meaning is not clear. It is supposed that it is related to w_e. It can be studied in the future.

Here, w_e can be regarded as another expression of E. Generally speaking, the smaller E is, the smaller w_e is. Therefore, it can reflect the influence of E on α.

To sum up the above graph analysis, the analysis result is generally in line with the practical life experiences. The mathematical model can generally describes the changes of consumers' purchase probability.

IV. Model of total profits

4.1 Sales volume model

When the pageview is B, the quantity that a consumer decides to purchase is the sales volume N.

Number the number of B consumers as j and $j \in [1, B]$. Then, the purchase probability of the " j " consumer is:

$$\alpha_j = \varepsilon_j \cdot t \cdot \left[w_j \alpha_{P_j} + (1 + w_j) \alpha_{N_j} \right].$$

Where,

$$\varepsilon_j = \begin{cases} 0 \\ 1 \end{cases},$$

It stands for two conditions, where product performance meets the consumer's demands or not.

$$t = v,$$

$$\alpha_{Pj} = \begin{cases} \dfrac{e^{k_P\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}} - c_P}{e^{k_P} - c_P}, 0 < \dfrac{P}{P_0} < \dfrac{P_{\max}}{P_0} \\ 0 \quad , \quad \dfrac{P}{P_0} \geq \dfrac{P_{\max}}{P_0} \end{cases},$$

$$c_{Pj} = e^{k_{Pj}\left(\frac{P_{\max j}}{P_0}+1\right) \cdot e^{-\frac{P_{\max j}}{P_0}}},$$

$$\alpha_{Nj} = \frac{e^{-k_{Nj} \cdot e^{-\frac{N}{N_0}}} - e^{-k_{Nj}}}{1 - e^{-k_{Nj}}},$$

$$w_j == \begin{cases} \dfrac{e^{-k_w\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}} - e^{-k_w}}{c_w - e^{-k_w}}, & 0 < \dfrac{P}{P_0} < \dfrac{P_{\max}}{P_0} \\ 1 \quad , & \dfrac{P}{P_0} \geq \dfrac{P_{\max}}{P_0} \end{cases},$$

c_{wj} and k_{wj} meet the following conditions:

$$\frac{e^{-2k_{w_j}e^{-1}} - e^{-k_{w_j}}}{c_{w_j} - e^{-k_{w_j}}} = w_{e_j} \text{ , } c_{wj} = e^{-k_{w_j}\left(\frac{P_{\max j}}{P_0}+1\right)e^{-\frac{P_{\max j}}{P_0}}}.$$

The mathematical expression of the sales volume N can be expressed as below:

$$N = \sum_{j=1}^{B} \alpha_j . \quad (10)$$

4.2 Pageview model

Analyze the how the changes of price and sales volume result in the changes of the pageview B.

As to online shopping, low price holds great attraction to consumers, leading to the increase of B. The high sales volume will make the online shop rank in the top list, which will attract more consumers and result in the increase of B. Take the price reduction of an online shop for example. (The price reduction and increase follows the same rule.)

The first process: Due to price reduction, more consumers will visit the page. However, within a short period of time, the sales volume will not undergo great changes. At the moment, B is unstable and will be increased to B_1.

The second process: After a period of time, due to the increase of sales volume, the shop ranking will be improved. The sales volume will attract more pageview. Later, the shop will not undergo great changes and B will tend to be stable until it stays at B_2.

As to the first process, B_1 after changes is related to $\frac{P}{P_f}$. P_f stands for the initial price of the online dealers.

To work out the relationship between B_1 and $\frac{P}{P_f}$:

When $\frac{P}{P_f} \to 0$, namely when the price is extremely low, the pageviews will be extremely high, which can be regarded as tending to be positive infinity, namely

$$B_1 \to +\infty \ .$$

When $\frac{P}{P_f} = 1$, namely no changes are undergone, the pageviews will stay unchanged, namely

$$B_1(1) = B_f \ .$$

B_f stands for the initial pageviews of the online dealer.

When $\frac{P}{P_f} \to +\infty$, namely when the price is extremely high, the pageviews will be very extremely low, but some rich consumers will still purchase. However, the purchase probability can be regarded as tending to be 0, namely

$$\lim_{\frac{P}{P_0} \to 0} B_1 = 0 \,.$$

Constitute

$$B_1 = f_B \cdot B_f \,,$$

Where, f_B stands for the function of $\frac{P}{P_f}$ and B_1 is made to meet the analysis conditions.

f_B should meet the condition that when $P \in (0,+\infty)$, $f_B \in (0,+\infty)$ and f_B features monotone increasing.

Since pageview is involved with many factors, a qualitative analysis is conducted of B.

First, employ the simplest inverse proportional function to make

$$f_B = \frac{k}{\frac{P}{P_0}} \quad ,$$

It is found that when and only when $k = 1$, conditions are met.

The following formula is obtained:

$$B_1 = \frac{P}{P_f} \cdot B_f \quad .$$

However, it is found that when $P = 2P_f$, $B = \frac{1}{2} \cdot B_f$. While the practical value should be extremely small in fact. Therefore, the function does not meet the required conditions.

Make

$$f_B = \frac{e^{-\frac{P}{P_f}}}{\frac{P}{P_f} \cdot e^{-1}} \quad ,$$

Obtain

$$B_1 = \frac{e^{-\frac{P}{P_f}}}{\frac{P}{P_f} \cdot e^{-1}} \cdot B_f \quad . \tag{11}$$

Put $\frac{P}{P_f} = \frac{1}{2}$ and $\frac{P}{P_f} = 2$ into the formula, and it is found that the formula is generally in line with the practical situation.

In terms of the second process, B_2 after changes is related to $\frac{N}{N_f}$ and the changes are based on B_1. N stands for the sales volume after the changes and N_f for the initial sales volume of the dealer.

To work out the relationship between B_2 and $\frac{N}{N_f}$:

When $\frac{N}{N_f} \to 0$, namely when there are no purchasers, due to factors including price, the pageviews will be extremely low. However, consumers are sensitive to the changes of the sales volume. Therefore, the change rate is comparatively large, which can be expressed through the following mathematical formula:

$$\lim_{\frac{N}{N_f} \to 0} B_2 = 0 \quad , \quad \left.\frac{\partial B_2}{\partial\left(\frac{N}{N_f}\right)}\right|_{\frac{N}{N_f}=0} \text{ is larger.}$$

When $\frac{N}{N_f}$=1, namely when there are no changes, pageviews will remain unchanged, namely

$$B_2(1) = B_1 .$$

When $\frac{N}{N_f} \to +\infty$, namely when the sales volume is very high, the pageviews will be extremely high, which can be regarded as positive infinity. However, consumers are not sensitive to the changes of the sales volume. Therefore, the change rate is relatively small, which can be expressed through the following mathematical formula:

$$\lim_{\frac{N}{N_f} \to +\infty} \frac{\partial B_2}{\partial\left(\frac{N}{N_f}\right)} = 0 \quad .$$

It can be concluded

$$B_2 = g_B \cdot B_1 \quad ,$$

Where g_B stands for the function of $\frac{N}{N_f}$ and B is made to meet the analysis conditions.

Make

$$g_B = k\sqrt{\frac{N}{N_f}} \quad ,$$

It can be found that if and only if $k = 1$, it meets the conditions, namely

$$B_2 = \sqrt{\frac{N}{N_f}} B_1 \quad . \tag{12}$$

Put $\frac{N}{N_f} = 0.5$ and $\frac{N}{N_f} = 2$ in the function, and it is found that the function is in line with the practical situation. Therefore, the function meets the conditions.

To combine the above two processes, and change B into function of $\frac{P}{P_f}, \frac{N}{N_f}$. Based on the comprehensive consideration of formula $(11)(12)$, the following formula (13) can be obtained:

$$B\left(\frac{P}{P_f}, \frac{N}{N_f}\right) = \frac{e^{-\frac{P}{P_f}}}{\frac{P}{P_f} \cdot e^{-1}} \cdot \sqrt{\frac{N}{N_f}} \cdot B_f \quad . \tag{13}$$

4.3 Establishment of profit model (without sales promotion)

Based on the three models of purchase probability, sales volume and pageview, profit $I\left(\frac{P}{P_0}\right)$ can be obtained.

In order to obtain the profit formula, sales volume is firstly calculated. Sales volume is the major factor.

It should be noticed after adjustment B might not be an integer. In order to make formula (9) meaningful, the expression is adapted to the one below:

$$N = \sum_{j=1}^{[B]} \alpha_j .$$

The analysis shows that when P changes, pageview will first change. At the same time, the individual purchase probability will be changed. At the moment, the historical sales volume will not undergo significant changes. All these factors work together to contribute to the first significant change of the sales volume, which is called the first process. The change of the sales volume result in the increase of the dealer's ranking, which might further significantly influence the pageview. Due to the changes of the sales volume and the pageview, α will change again, which result in the second change of the total sales volume (the second process). At the moment, the sales ranking of the dealer has undergone significant change. Further change has a little influence on consumers. Compared with the previous two changes, it can be ignored in the whole change process. To sum up, the price change results in two changes of various values [3]. The change time having happened are demonstrated as i . $i = 0,1,2$, i=0 means the initial value.

To sum up the function in 3.1,4.1 and 4.2 , it can be concluded that

$$\alpha_{ij} = \varepsilon \cdot t \cdot [w \cdot \alpha_{P_{ij}} + (1-w) \cdot \alpha_{N_{ij}}] ,$$

$$N_i = \sum_{j=1}^{[B_i]} \alpha_{ij} \quad \text{and define} \quad N_{i-1}\big|_{i=1} = N_f ,$$

$$B_i = \frac{e^{-\frac{P}{P_0}}}{\frac{P}{P_f}e^{-1}}\sqrt{\frac{N_{i-1}}{N_f}}B_f \quad .$$

Work out various variable sin turn:

In the first process, $i = 1$

$$B_1 = \frac{e^{-\frac{P}{P_0}}}{\frac{P}{P_f}e^{-1}}B_f \quad , \quad \left(\frac{N_{i-1}}{N_f} = \frac{N_f}{N_f} = 1\right) \quad .$$

$$\alpha_{1j} = \varepsilon \cdot t \cdot \lfloor w \cdot \alpha_{P_{1j}} + (1-w) \cdot \alpha_{N_{1j}} \rfloor \quad .$$

Where

$$\alpha_{N_{1j}} = \frac{e^{-k_{N_j}e^{-1}} - e^{k_{N_j}}}{1 - e^{-k_{N_j}}} \quad ,$$

$$\Rightarrow N_1 = \sum_{j=1}^{[B_1]} \alpha_{1j} \quad .$$

In the second process, $i = 2$

$$B_2 = \frac{e^{-\frac{P}{P_0}}}{\frac{P}{P_f}e^{-1}}\sqrt{\frac{N_1}{N_f}}B_f \quad ,$$

$$\alpha_{2j} = \varepsilon \cdot t \cdot \lfloor w \cdot \alpha_{P_{2j}} + (1-w) \cdot \alpha_{N_{2j}} \rfloor \quad .$$

Where

$$\alpha_{N_{2j}} = \frac{e^{-k_{N_j}e^{-\frac{N_1}{N_0}}} - e^{k_{N_j}}}{1 - e^{-k_{N_j}}} \quad ,$$

$$\Rightarrow N_2 = \sum_{j=1}^{[B_2]} \alpha_{2j} \quad .$$

Since various variables in the former calculations are relatively complex, the latter calculation will not expand them.

According to the former analysis, N_2 stands for the final sales volume, namely

$$N = N_2$$

Due to the characteristics of online shopping, products are sent out through post. The total price of the products is the result by adding the selling price with the postage. Dealers can also obtain profits from postage. The product cost can be assumed to be

C, the postage to be P_m and the actual postage cost of the dealer to be C_m,. All these items are of a specific value. There is certain probability that online shoppers might return products to the dealers. However, the dealers are only allowed to return the selling price to the shoppers. There is no permission for them to return postage. This means dealers could still earn profits. Assume the products return rate to be β [4]. Considering all the above factors, the expression form of the total profits I can be expressed as below:

$$\begin{aligned} I &= (1-\beta)(P-C-C_m)N + \beta(P_m - C_m)N \\ &= [(1-\beta)(P-C-C_m) + \beta(P_m - C_m)]N \quad . \end{aligned} \tag{14}$$

4.4 Solution of profit model (without promotion sales)

I obtained in 4.1 cannot be studied or estimated. In order to obtain a relatively accurate value, set $k_{P_j}, k_{N_j}, \varepsilon_j$ to be a fixed value k_P, k_N and ε. However, w_{e_j} and $\frac{P_{\max}}{P_0}$ cannot be directly estimated, because to different consumption group with different amount of wealth the gap of consumers' w_{e_j} and $\frac{P_{\max}}{P_0}$ is huge.

First, estimate the value between k_P and k_N:

Estimate k_P,

Make $k_P = -3.9$,

Then,

$$\alpha_P(1) \approx 0.80, \alpha_P(0.5) \approx 0.95, \alpha_P(1.5) \approx 0.49 \quad .$$

To estimate k_N,

Make $k_N = -1.7$, namely

$$\alpha_N(1) \approx 0.81,\ \alpha_N(0.5) \approx 0.59, \alpha_N(2) \approx 0.94 \quad .$$

To estimate ε_j,

The average value $\bar{\varepsilon}$ of ε_j can be used to express ε_j:

$$\bar{\varepsilon}_i = \frac{\sum_{j=1}^{[B_i]} \varepsilon_j}{[B_i]} \quad .$$

Assume $\bar{\varepsilon}_i$ to be a fixed value $\bar{\varepsilon}$, which can be solved based on P_f and N_f already known.

Then analyze w_{e_j},

People of different family economic statuses pay different attention to price, namely w. However, w_{e_j} of every household cannot be obtained. Therefore, it can be regarded that w_{e_j} is related to the wealthy degree of a family.

E_j' is adopted to measure the economic status of a family. By employing Engel coefficient E (the percentage of food expense among the total expenses). However, it has been assumed that online products are confined to non-food categories. Therefore, it is defined

$$E_j' = 1 - E_j \quad , \tag{15}$$

Formula (15) shows the percentage of non-food expense of the total. E_j' meets the condition that, the larger E_j' is, the wealthier the family is. Assume E_0 to be the average Engel coefficient of the consumption group, then

$$E_0' = 1 - E_0 \quad .$$

Use the function to generally describe how w_{ej} changes with E'_j.

The change of w_{e_j} along with the change of E'_j should meet the following conditions:

When $E'_j \to 0\%$, namely when the family is extremely poor, consumers will only pay attention to the price. Besides, with the change of E'_j, the change of w_{ej} is quite small. The relationship can be expressed as the mathematical formula below:

$$\lim_{E'_j \to 0} w_{e_j} = 1, \lim_{E'_j \to 0} w'_{e_j} = 0 \ .$$

When E'_j is between 40%~80%, the family's wealthy degree changes rapidly. Therefore, when E'_j is larger, w_{e_j} changes rapidly. Besides, when $E'_j = E'_0$, $w_{e_j} = w_0$. w_0 stands for the weight of the family's expected price of E'_0. The relationship can be expressed through the mathematical formula as $w_{e_j}\left(E'_0\right) = w_0$.

When $E'_j \to 100\%$, namely when the family is extremely rich, consumers will almost ignore the price. Besides, when there are some slight changes happening to E', w_e remains unchanged generally. The mathematical formula is shown below:

$$\lim_{E' \to 1} w_{e_j} = 0, \lim_{E' \to 1} w'_{e_j} = 0 \ .$$

Adopt

$$w_{ej} = \frac{1 - e^{-k_{w_e} E_j'^2}}{1 - e^{-k_{w_e}}} \tag{16}$$

to express the relationship between w_{ej} and E'_j, in which k_{w_e} stands for a parameter. Then assume k_{w_e} of every individual to be the same value.

The function meets all the conditions analyzed above. However, k_{w_e} should make

$w_{ej}\left(E_0'\right)=w_0$.

The function is obtained below:

$$\frac{1-e^{-k_{w_e}E_0'^2}}{1-e^{-k_{w_e}}}=w_0 \quad . \tag{17}$$

It is hard for the function to get its solution. Therefore, specific values can be input in the function to express k_{w_e} with specific value.

Finally, $\frac{P_{\max}}{P_0}$ is analyzed.

Since $\frac{P_{\max}}{P_0}$ is decided by E_j', function simulation is conducted of $\frac{P_{\max}}{P_0}$ as related to E'.

The following function is adopted.

$$\frac{P_{\max}}{P_0}=3E'^3+1 \quad . \tag{18}$$

Input $E'=0.5$, $E'=0.65$ and $E'=0.8$ into the function, which stands for the poverty-stricken family, the comparatively well-off family and the rich family respectively. $P_{\max}$ obtained is $1.38P_0$, $1.82P_0$ and $2.54P_0$,respectively, which are generally in line with the practical condition.

Analyze the characteristics of E_j'.

The reason why estimation cannot be conducted in4.1is that α_{ij} is different from each other. N_2 can be obtained by working out the average value of α_{ij}.

According to the hypothesis, consumers of all level of affluence of life have purchase behaviors. Therefore, the ratio of consumers of various classes browse the product is generally the same to the ratio of the population of various classes in the total population.

Assume the percentage of population whose new Engel coefficient E' among the total to be φ . φ is the probability density function related to E'.

E' mainly follows the normal distribution[5].

Then, the following formula is obtained:

$$\varphi = \frac{e^{-\frac{(E'-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \quad . \tag{19}$$

According to the statistics offered by National Bureau of Statistics, $\mu \approx 63\%, \sigma \approx 5$ [6][7], in which σ is the sample standard deviation.

Verify the gap between φ and the practical situation:

Since the National Bureau of Statistics provided the percentage value, φ undergoes the following treatment:

$$\overline{E'_k} = \frac{\int_{s_{k-1}}^{s_k} E' \cdot \varphi \, \mathrm{d}\, E'}{\int_{s_{k-1}}^{s_k} \varphi \, \mathrm{d}\, E'} \quad .$$

Based on the formula, the value of E' under the normal distribution can be obtained. Where, k stands for the number of every range of value divided by the National Bureau of Statistics. s_k stands for the percentage upper limit and define $s_0 = 0$.

Table 2-1 Average value of E' according to percentage division

s_k	Actual percentage	$\overline{E'_k}$
5%	53.2%	52.1%
10%	55.8%	55.8%
20%	56.8%	57.7%
40%	59.1%	60.3%
60%	61.4%	63%
80%	64.2%	65.6%
90%	66.8%	68.2%
100%	72.6%	71.8%

The error is not large, which is generally in line with the requirement. (See MOS_E_NormalityRegression in the Attachment)

The average value of α_i is obtained:

$$\overline{\alpha}_i = \int_0^1 \alpha_i \varphi \, \mathrm{d}E' \quad . \tag{20}$$

To sum up the above analysis, the estimable expression formula of I.

Since there exists no summation symbol, there is no need to adopt Gaussian function for B.

Then

$$I = [(1-\beta)(P - C - C_m) + \beta(P_m - C_m)]N \quad .$$

Where

$$N = B_2 \cdot \overline{\alpha}_2 \quad ,$$

$$B_2 = \frac{e^{-\frac{P}{P_f}}}{\frac{P}{P_f} e^{-1}} \sqrt{\frac{N_1}{N_f}} B_f \quad ,$$

$$\overline{\alpha}_2 = \int_0^1 \alpha_2 \varphi \, \mathrm{d}E' ,$$

$$\alpha_2 = \overline{\varepsilon} \cdot t \cdot \lfloor w \cdot \alpha_P + (1-w) \cdot \alpha_{N_2} \rfloor \quad ,$$

$$t = v \quad ,$$

$$w = \begin{cases} \dfrac{e^{-k_w\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}} - e^{-k_w}}{c_w - e^{-k_w}}, & 0 < \dfrac{P}{P_0} < \dfrac{P_{\max}}{P_0} \\ 1 \quad , & \dfrac{P}{P_0} \ge \dfrac{P_{\max}}{P_0} \end{cases} ,$$

c_w and k_w meet the following conditions:

$$c_w = e^{-k_w\left(\frac{P_{\max}}{P_0}+1\right)e^{-\frac{P_{\max}}{P_0}}} , \quad \frac{e^{-2k_w e^{-1}} - e^{-k_w}}{c_w - e^{-k_w}} = w_e \quad ,$$

$$\frac{P_{\max}}{P_0} = 3E'^3 + 1 ,$$

$$w_e = \frac{1 - e^{-k_{w_e}(E'-1)^2}}{1 - e^{-k_{w_e}}} ,$$

k_{w_e} meets the following conditions:

$$\frac{1-e^{-k_{w_e}(E_0'-1)^2}}{1-e^{-k_{w_e}}}=w_0 \quad ,$$

$$\alpha_P = = \begin{cases} \dfrac{e^{k_P\left(\frac{P}{P_0}+1\right)e^{-\frac{P}{P_0}}}-c_P}{e^{k_P}-c_P}, & 0<\dfrac{P}{P_0}<\dfrac{P_{\max}}{P_0} \\ 0 \quad , & \dfrac{P}{P_0}\geq\dfrac{P_{\max}}{P_0} \end{cases} ,$$

$$c_P = e^{k_p\left(\frac{P_{\max}}{P_0}+1\right)\cdot e^{-\frac{P_{\max}}{P_0}}} ,$$

$$\alpha_{N_2}=\frac{e^{-k_N e^{-\frac{N_1}{N_0}}}-e^{k}}{1-e^{-k}},$$

$$N_1 = B_1 \cdot \overline{\alpha}_1 \quad ,$$

$$B_1=\frac{e^{-\frac{P}{P_f}}}{\frac{P}{P_f}e^{-1}}B_f \, ,$$

$$\overline{\alpha}_i=\int_0^1 \alpha_i \varphi \, \mathrm{d}\, E' \quad ,$$

$$\alpha_1 = \overline{\varepsilon}\cdot t\cdot\left[w\cdot\alpha_P+(1-w)\cdot\alpha_{N_1}\right]$$

$$\alpha_{N_1}=\frac{e^{-k_N e^{-\frac{N_f}{N_0}}}-e^{k}}{1-e^{-k}},$$

$\overline{\varepsilon}$ meets the condition of $N_f = B_f \cdot \overline{\alpha}_1\left(P_f\right)$, namely

$$\overline{\varepsilon} = \frac{N_f}{B_f \cdot t \cdot \left[w \cdot \alpha_P\left(P_f\right)+\left(1-w\right)\cdot\alpha_{N_1}\right]} \; . \qquad (21)$$

Solve the maximum value of I and its corresponding value of P , P' .

Generally speaking, the method of P derivation can be adopted. However, since α_P and w are piecewise functions, P derivation is impossible.

The expression formula of $\overline{\alpha}_1$ and $\overline{\alpha}_2$ can be adapted into:

$$\overline{\alpha}_1 = \sum_{j=0}^{100} \alpha_1 \varphi\left(E'_j\right)\Delta E', \overline{\alpha}_2 = \sum_{j=0}^{100} \alpha_2 \varphi\left(E'_j\right)\Delta E' \quad ,$$

Where, $E'_j = 5\%$ and $\Delta E' = 1\%$.

Though α_1 meets the following condition,

$$\sum_{j=0}^{100} \alpha_1 \varphi\left(E'_j\right)\Delta E' - \int_0^1 \alpha_1 \varphi \, \mathrm{d}\, E' = \delta_1 \neq 0 \quad ,$$

$|\delta_1| << \int_0^1 \alpha_1 \varphi \, \mathrm{d}\, E'$ can be ignored.

Similarly, to $\overline{\alpha}_2$, δ_2 can also be ignored.

Based on that, the approximate integration calculation can be conducted.

However, I derivation is still hard. Considering the price is mainly integer only one integer is required to make I reach the maximum value of the positive integer domain. Though such treatment features large amounts of calculation, the programming will be easier and MATLAB software can be adopted for the solution of the function.

Since P and P_0 both include the postage P_m, the price posted by the dealer (namely selling price) p and the general price p_0 meets the following condition:

$$p = p - P_m , p_0 = p_0 - P_m \quad . \tag{22}$$

According to the formula, the corresponding value of $\frac{p}{p_0}$, $\frac{p'}{p_0}$ when I reaches the maximum value, and p' stands for the optimal price.

Solution result

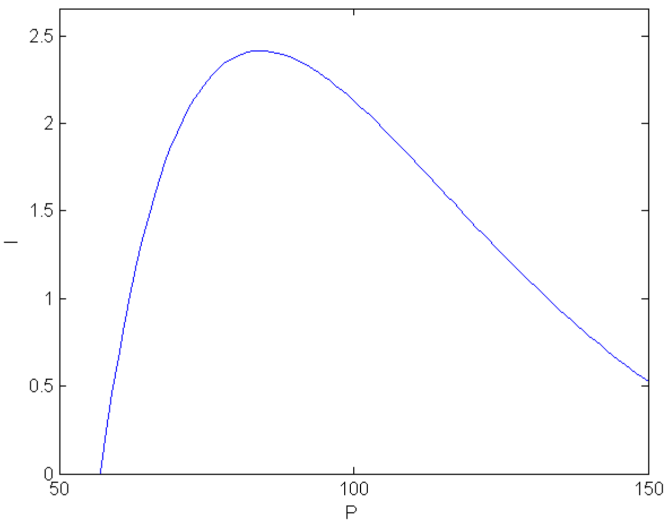
$$\boldsymbol{p' = 94\% p_0}$$

Where, $N_f = N_0, E_0' = 63\% \quad \mu = 63\% \quad C = 70\% p_0$ **.** (See MOS_I in the Attachment)

Dealers can assign value to the parameters according to their own situation. Here only once case is listed.

4.5 Result analysis

Draw the graph of $I - \frac{p}{p_0}$ when $N_f = N_0, E_0' = 63\% \quad \mu = 63\%$ and $C = 60\%\ p_0$, in which the unit of $\frac{p}{p_0}$ is 1%. (See MOS_I in the Attachment.)

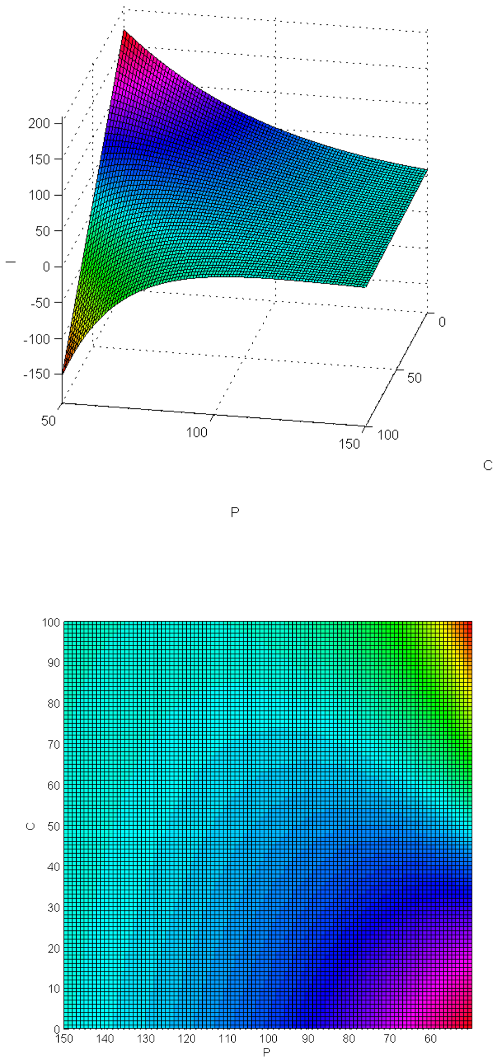


Graph 4-1 Standard graph of $I - \frac{p}{p_0}$

Then analyze the influence of various major parameters on I:

4.5.1 The influence of cost C on the total profits I

Set the other parameters to remain unchanged and obtain the graph of $I - C - \frac{p}{p_0}$. (See MOS_I_figureC in the Attachment.)



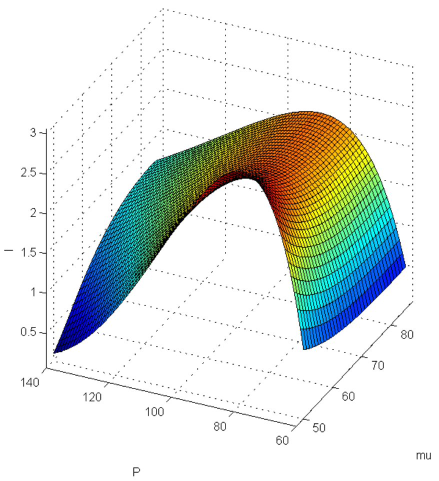
Graph 4-2 The graph showing The influence of cost C on the total profits I

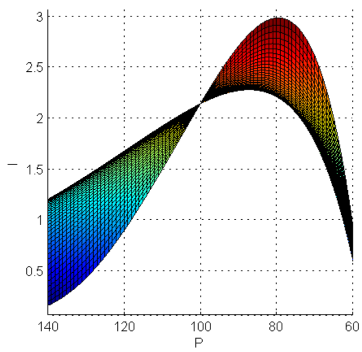
MATLAB bases on the method of contour line to fill the graph with color. By observing the color gradual change of the upper section projected on $C-\frac{p}{p_0}$, it can be seen that, **with the increase of C, the value of $\frac{p'}{p_0}$ increases, whose change rate is close to that of C.** The conclusion is obvious. **Therefore, C is the significant factor deciding the value of $\frac{p'}{p_0}$.**

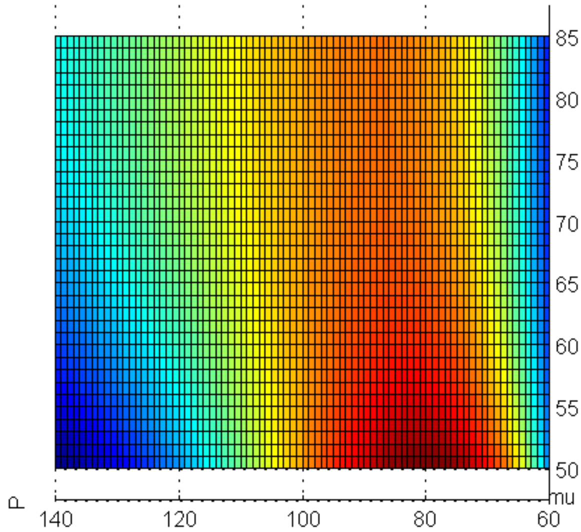
Here, the strategy for dealers to adopt in response to the environmental changes is given: **when the cost rises, the pricing should also be promoted.**

4.5.2 The influence of the average wealth degree of the consumption group, μ, on the profits, I

Set the other parameters to remain unchanged and obtain the graph of $I-\mu-\frac{p}{p_0}$. (See MOS_I_figuremu in the Attachment.)







Graph 4-3 The influence of the average wealth degree of the consumption group μ on the profits I

Based on the graph projected on $I-\frac{p}{p_0}$, it can be seen that **the smaller μ is, the larger change rate of I along with the change of $\frac{p}{p_0}$.** The reason is that: the less wealthier the family is, the more sensitive the family is to the price. Therefore, **even if the price change is insignificant, it might have a great influence on profits.**

By observing the change of color projected on $\mu-\frac{p}{p_0}$, it can be seen that **with the increase of μ, the value of $\frac{p'}{p_0}$ is increased gradually.** The reason behind this is that: **the wealthier the family is, the less sensitive the family is to the price. Therefore, the dealers can slightly increase the price so as to make more profits.**

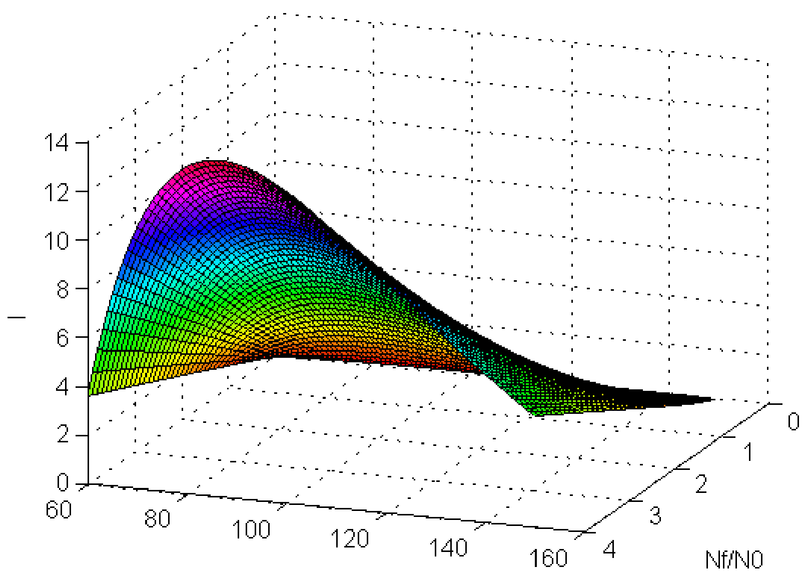
Here, the strategy given for the dealers to respond to the environmental change is that: **increase the price when the consumers are rich, and decrease the price when the consumers are not.**

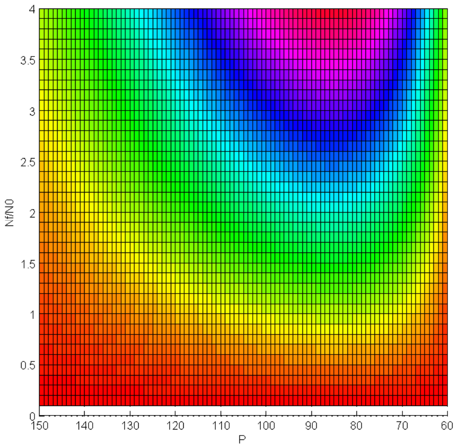
4.5.3 The influence of product performance ε and initial sales volume $\frac{N_f}{N_0}$ on the total profits I

$\bar{\varepsilon}$ is obtained by solving the function. It cannot be directly studied. Therefore, P_f and N_f can be employed to indirectly describe $\bar{\varepsilon}$.

For the convenience of research, it is assumed that $\frac{P_f}{P_0}=1$. Then the changes of N_f reflect the changes of $\bar{\varepsilon}$.

Set the other parameters to remain unchanged, and obtain the graph of $I-\frac{N_f}{N_0}-\frac{p}{p_0}$.(See MOS_I_figureNf in the Attachment.)





Graph 4-4 The influence of product performance ε and initial sales volume $\frac{N_f}{N_0}$ on the total profits I

Based on the projection on the graph of $I-\frac{p}{p_0}$, it can be seen that, to **the value of every $\frac{p}{p_0}$, the larger $\frac{N_f}{N_0}$ is, the higher the profits I are. The reason behind this is that the pageview and purchase probability are both high.**

By observing the gradual change of the color projected on the graph of $\frac{N_f}{N_0}-\frac{p}{p_0}$, it can be concluded that: **though the value of $\frac{p}{p_0}$ undergoes minor changes along with the value change of $\frac{N_f}{N_0}$, the value change of I is significant.**

Here, the strategy given to the dealers in response to the environmental change is that: **When product performance and initial sales volume decrease, the pricing can remain stable without significant changes.**

Based on the above analysis, some valuable conclusions have been reached in terms of the model. This paper thinks that the model is generally in line with the practical situation of life.

V. Promotion models

5.1 The establishment of promotion models

In terms of online shopping, on some specific days, dealers will launch promotion sales and create online shopping festivals. As to the dealers' promotion sales, this paper thinks that promotion sales are related to purchase probability α, credibility t and pageview B.

$$d(d \in [0.5,1]) \ .$$

5.1.1 The influence of discount rate d on purchase probability α and credibility t

First, analyze the relationship between $t(d)$ and d:

In the following analysis, it is temporarily assumed that $v = 1$for the convenience of analysis.

Though the definition domain of d is $[0.5,1]$, it can be expanded to $(0,1]$ so as to find the function relationship.

First, t features monotone increasing, namely $t' > 0$.

It should be noticed that when $d \to 0$, consumers have almost lost credibility for the dealer. Besides, the insignificant changes of d will not exert a great influence on consumers, namely

$$\lim_{d \to 0} t = 0 \, , \lim_{d \to 0} t' = 0 \ .$$

However, when $d = 1$, then

$$t(1) = v \ ,$$

In other words, when without discounts, credibility is decided by v.

t is still regarded to be formed by every consumer's comparison of promotion degree.

Assume that when d is a specific value, and $-\mathrm{d}d$ changes, $\mathrm{d}t$ will changes, consumers will change d based on t before the change, namely

$$-\mathrm{d}t = \mathrm{d}h_t \cdot t \ ,$$

Where, h_t stands for the function of d .

Then,

$$-\mathrm{d}t = \mathrm{d}h_t \cdot t \Leftrightarrow \frac{\mathrm{d}t}{\mathrm{d}d} = -\frac{\mathrm{d}h_t}{\mathrm{d}d} \cdot t.$$

$\frac{\mathrm{d}h_t}{\mathrm{d}d}$ should meet the condition of $\frac{\mathrm{d}h_t}{\mathrm{d}d} > 0$,and $\frac{\mathrm{d}h_t}{\mathrm{d}d}$ has the maximum value when $d \in (0,1)$.

Because $t(0) = 0$, $t'(0) = 0$, $\lim_{d \to 0} \frac{\mathrm{d}h_t}{\mathrm{d}d} = 0$ is unnecessary.

Make

$$\frac{\mathrm{d}h_t}{\mathrm{d}d} = -\frac{k'_t\ d}{e^d} \left(k'_t \in (0,1)\right)$$

$$\Rightarrow \int \mathrm{d}\, h_t = \int -\frac{k'_t\ d}{e^d} \mathrm{d}d$$

$$\Rightarrow h_t = k'_t (d+1) e^{-d} \quad ,$$

It can be found that

$$h'' < 0 \quad .$$

Then,

$$\mathrm{d}t = \frac{k'_t d}{e^d} \mathrm{d}d \cdot t$$

$$\Rightarrow \int \frac{\mathrm{d}t}{t} = \int \frac{k'_t d}{e^d} \mathrm{d}d$$

$$\Rightarrow \ln \frac{t}{c'_t} = e^{-k'_t (d+1) e^{-d}}$$

$$\Rightarrow t = c'_t \cdot e^{-k'_t (d+1) e^{-d}} \quad .$$

Make

$$c'_t = \frac{v}{e^{-2k'_t \cdot e^{-1}} - e^{-k'_t}},$$

The following formula is obtained:

$$t = \frac{e^{-k'_t (d+1) e^{-d}}}{e^{-2k'_t \cdot e^{-1}} - e^{-k'_t}} v.$$

Since the function does not meet the required nature of $\lim_{d \to 0} t = 0$ and $t(1) = v$, it is necessary to change the function to make t meet the required condition and endowed the characteristic similar to $\frac{e^{-k_t'(d+1)e^{-d}}}{e^{-2k_t' \cdot e^{-1}} - e^{-k_t'}} v$.

Make the following changes to t:

Adopt k_t as the parameter

Change t

Then,

$$t = \left(\frac{e^{-k_t(d+1)e^{-d}}}{e^{-2k_t \cdot e^{-1}} - e^{-k_t}} - \frac{e^{-k_t}}{e^{-2k_t \cdot e^{-1}} - e^{-k_t}} \right) v, \qquad (23)$$

So,

$$\lim_{d \to 0} t = 0 \, , t(1) = v \ ,$$

Conditions are met.

Verify other characteristics of the function:

$$\lim_{d \to 0} t' = 0 \quad , \quad t' > 0 \quad ,$$

All meet the conditions

After the change of the function, based on the nature of formula (23), it can be seen that, k_t only has to meet the condition of $k_t \neq 0$.

Considering the online shopping festivals, k_t should meet the following conditions:

1) $t(0.5)$ is not large, around50%.

Meaning: when 50% off, only half of the consumers believe it.

2) $t(0.8)$ is large, about80%~90%.

Meaning: when 20% off, most consumers believe it.

Then analyze the influence of α . Assume the function relationship to be $\alpha'(d)$, then

$$\alpha' \in [0,1) \ .$$

However, in the elementary simple function, here are not many functions which can make α' conform to the practical situation.

Considering that,

$$\alpha' = \left(\frac{\alpha}{v}\right)^{d^3} \tag{24}$$

can be adopted to estimate α'.

Since $t(d)$ is also the function of d, the initial α should be divided by v before the change while the relationship between α' and d is studied.

α' meets the condition that, the larger d is, the smaller α' is. Besides, $\alpha'(1) = \frac{\alpha}{v}$ and compared with $\frac{\alpha}{v}$, α' registers a significant improvement, which is in line with the practical situation.

To sum up the relationship between α' and t, this paper thinks that t is directly decided by the value of α_d,

Then,

$$\alpha_d = \alpha' \cdot t \quad . \tag{25}$$

5.1.2 The influence of discount rate d on pageview B

It is regarded that the pageview is decided by every consumer's comparison of various promotion sales. When $d = 0.5$, B_f' usually has an upper limit, which does not tend to be positive infinity. Assume the upper limit to be k_B times of B_f $(k_B > 1)$. Though B_f' stands for discount, the pageview of the actual price is not changed.

Now, to find out the relationship between B_f' and d:

First, B_f' is a monotone decreasing function, which can be expressed through the following mathematical formula.

$$\frac{\mathrm{d}B_f}{\mathrm{d}d} \le 0 \quad .$$

Besides,

$$B'_f(0.5) = k_B \cdot B_f \,, B'_f(1) = B_f \quad .$$

However, when $d \to 0.5$, B'_f changes slightly, because consumers are not sensitive to changes of promotion sales. In other words, $\left|\frac{\mathrm{d}B'_f}{\mathrm{d}d}\right\|_{d=0.5}$ is comparatively smaller.

When $d \to 1$, B'_f changes significantly. At the moment, consumers are very sensitive to the changes of promotion sales. In other words, $\left|\frac{\mathrm{d}B'_f}{\mathrm{d}d}\right\|_{d=1}$ is comparatively larger. .

To sum up, when $\frac{\mathrm{d}B_f}{\mathrm{d}d} \le 0$, $\left|\frac{\mathrm{d}B'_f}{\mathrm{d}d}\right\|_{d=0.5}$ is comparatively smaller while $\left|\frac{\mathrm{d}B'_f}{\mathrm{d}d}\right\|_{d=1}$ is comparatively larger.

Thus, it can be concluded that

$$\frac{\mathrm{d}^2 B'_f}{\mathrm{d}d^2} < 0 \quad .$$

B'_f is still qualitatively described:

Adopt the simplest power function,

Assume

$$B'_f = \left(k_d d^2 + c_d\right) B_f \quad .$$

In order to make the condition of $\frac{\mathrm{d}^2 B'_f}{\mathrm{d}d^2} < 0$ more significant,

Adapt the function into

$$B'_f = \left(k_d (d-0.5)^2 + c_d\right) B_f \,(k_d < 0, c_d > 0).$$

Then

$$B'_f(0.5) = k_B \cdot B_f \,, B'_f(1) = B_f \quad .$$

After solution,

$$k_d = -4(k_B - 1), c_d = k_B \quad .$$

Put the value of d into the function and find it is in line with the practical situation. Then

$$B_f' = \left[-4(k_B - 1)(d - 0.5)^2 + k_B\right]B_f \quad . \tag{26}$$

5.2 The solution of promotion sales models

Adopt the method mentioned in 4.4 to work out I, and replace $\overline{\alpha}_i$ in the original expression formula with $\overline{\alpha}_{di}$.

It should be noticed that, when α_{N_2} is calculated, the value of N_0 is changed. Since other dealers also launch promotion sales, the value of N_0 is increased. The investigation shows that most dealers sell their products at 20% off. Considering that, adopt $\frac{p}{p_0} = 1$ and the value of N_1 with 20% off as N_0 and put it into α_{N_2} for the calculation.

Assume k_t=-5.3to be a specific value, and put it into $k_B = 4$.

Since the pageviews each time are the $\frac{B_f'}{B_f}$ times of the original one. Adopt B_f' as the initial pageviews value and the result is the same.

Use MATLAB to work out the value of P' and the value of d'. d' stands for the optimal discount rate.

Solution result:

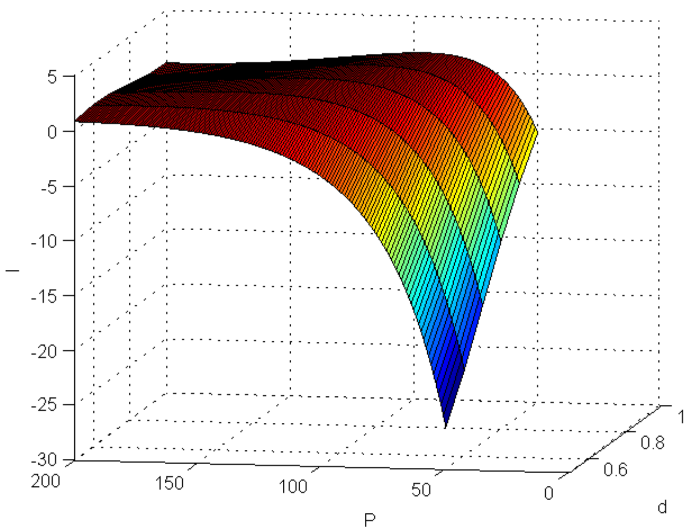
$$\boldsymbol{p' = 109\% p_0,\ d' = 0.9}$$

Where $N_f = N_0$, $E_0' = 63\%$, $\mu = 63\%$ and $C = 70\% p_0$ **. (See** MOS_d in the Attachment.)

Dealers can assign a specific value for the parameter according to their own situation. Here is only one case.

5.3 Result analysis

Draw the graph of $I-d-\frac{p}{p_0}$, when $N_f=N_0$, $E_0'=63\%$, $\mu=63\%$ and $C=70\% p_0$, in which the unit of $\frac{p}{p_0}$ is 1%. (See MOS_d in the Attachment.)

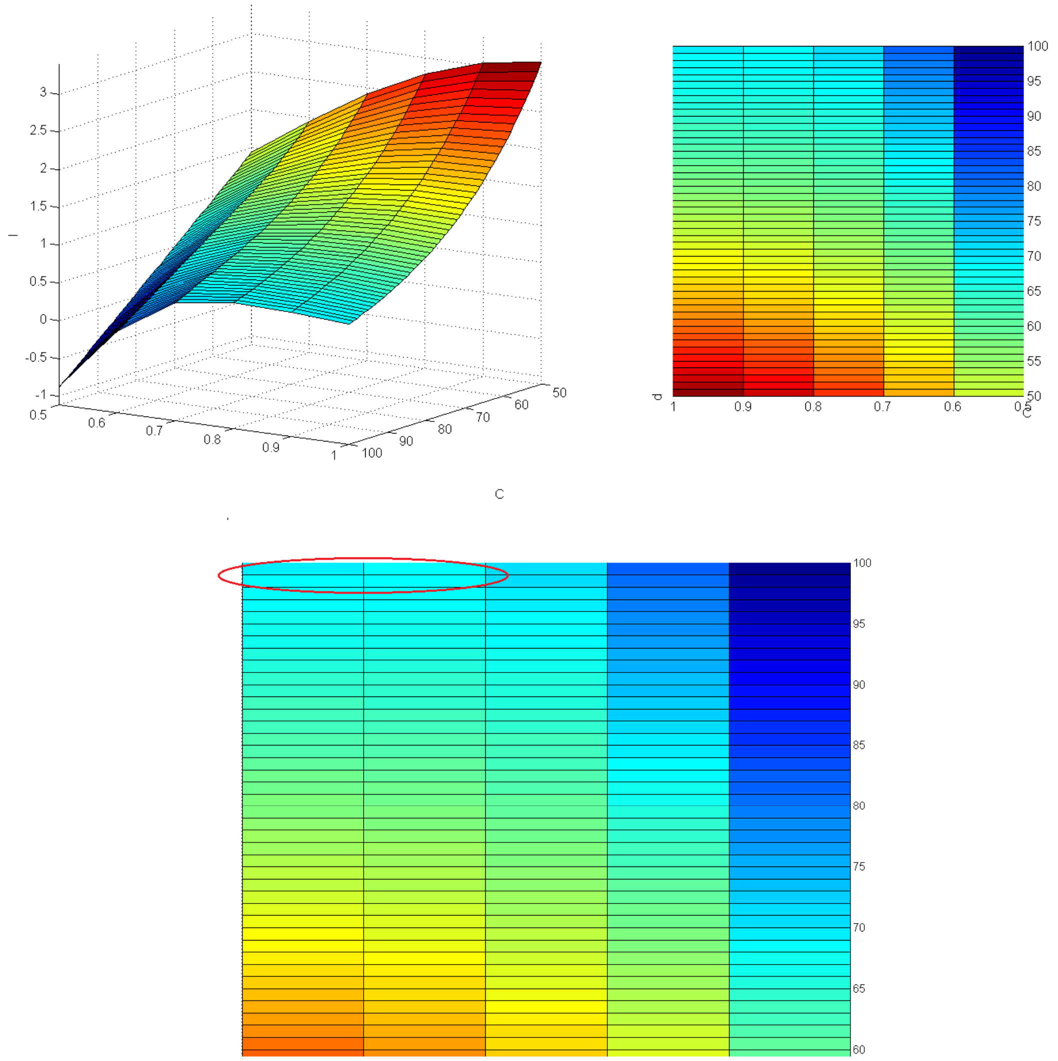


Graph 5-1 Standard $I-d-\frac{p}{p_0}$ graph

Then analyze the influence of various parameters on I and d'.

5.3.1 The influence of cost C on the total profits I and the optimal discount rate d'

Draw a three-dimensional color graph of $I-d-C$. (See MOS_d_figuredC in the Attachment.)

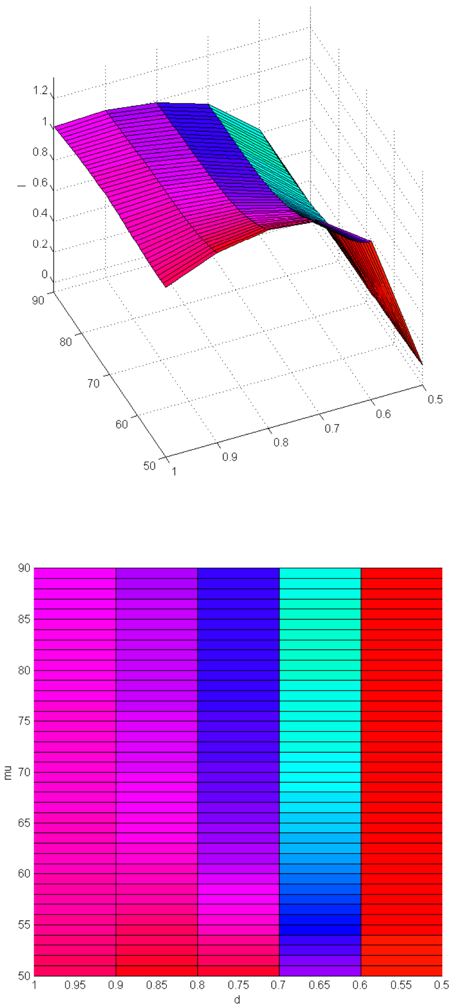


Graph 5-2 The graph showing the influence of cost C on the total profits I and the optimal discount rate d'

Based on the observation of the gradual color change trend projected on $C-d$, it can be seen that, **with the decrease of** C**, the value of** d' **gradually increases.** The reason behind this is that **the lower cost is, the less necessary promotion sales are required for the increase of profits, which might even be counterproductive.** Here the strategy given to the dealers in response to the environmental changes: **when cost rises, the discount rate can be increased accordingly.**

5.3.2 The influence of consumers' average wealth degree μ on the total profits I and the optimal discount rate d'

Draw the three-dimensional color graph of $I-d-\mu$ (See MOS_d_figuredmu in the Attachment)



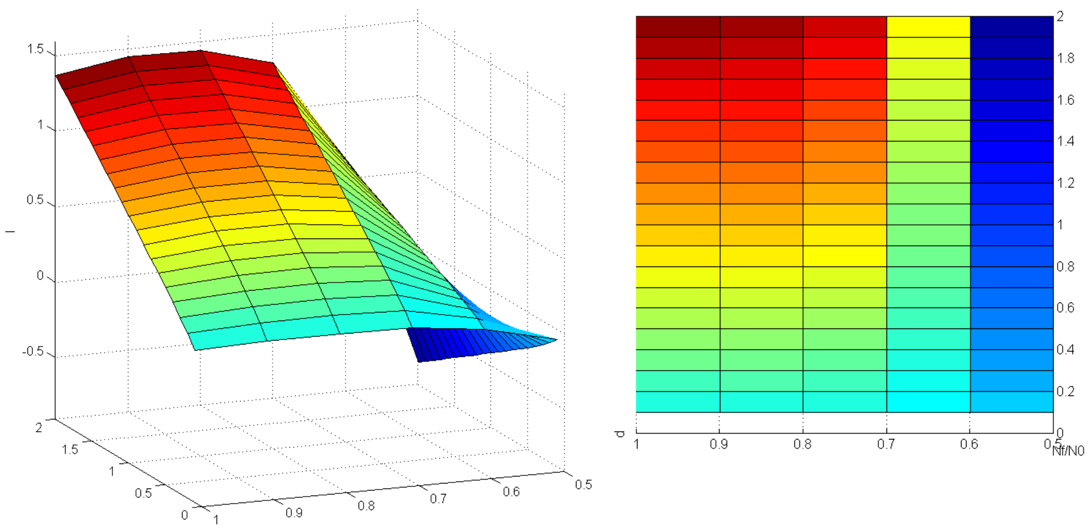
Graph 5-3 The influence of consumers' average wealth degree μ on the total profits I and the optimal discount rate d'

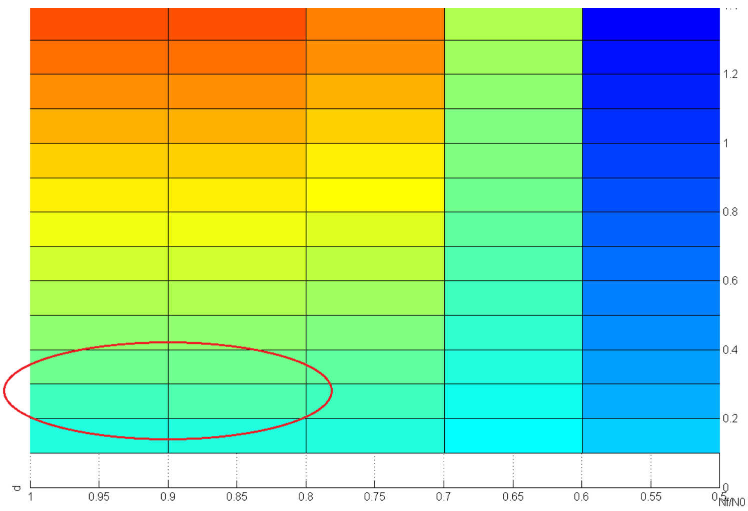
From the color gradual change trend of $\mu - d$ projected on the graph, it can be seen that, **with the decrease of μ, the value of d' also decreases accordingly.** The reason behind this is that **the poorer the family is, the more sensitive it is to the promotion sales. Dealers can make use of the rule to stimulate consumers' purchase behaviors through promotion sales.**

Here, the strategy given for the dealers in response to the environmental changes: **low discount strength is applicable to the rich consumers but not to those who are not rich.**

5.3.3 The influence of product performance ε and initial sales volume $\frac{N_f}{N_0}$ on the total profits I and optimal discount rate d'

$\frac{N_f}{N_0}$ is still adopted to reflect ε ,and the three-dimensional color graph of $I-d-\frac{N_f}{N_0}$ is drawn (See MOS_d_figuredNf in the Attachment.)





Graph 5-4 The influence of product performance ε and initial sales volume $\frac{N_f}{N_0}$ on the total profits I and optimal discount rate d'

From the color gradual change trend of the graph projected on $\frac{N_f}{N_0} - d$, it can be seen that, **with the increase of** $\frac{N_f}{N_0}$**, the value of** d' **will also be gradually increased..**

The reason behind this is: **the higher** ε **or** $\frac{N_f}{N_0}$ **is, the less necessary for dealers to use promotion sales to increase their profits. If** d **is comparatively small, the result might be counterproductive.**

Here the strategy given to dealers in response to the environmental changes: **when product performance and original sales volume decrease, promotion sales are required to stimulate consumption.**

VI. Conclusions

According to problem analysis, we built mathematical model of online shopping as follows:

Our model is B2C, C2C platform supplier provides theoretical basis for product pricing, it is expected to bring more commercial benefits for businessmen.

According to mathematical model, result analysis and set parameters, we conducted the pricing strategy as follows:

Pricing suggestions when without promotion sales:

$$\boldsymbol{p' = 94\% p_0}$$

Pricing suggestions when with promotion sales:

$$\boldsymbol{p' = 109\% p_0 \ \ d' = 0.9}$$

Parameters involved include:

1) The individual situation of dealers

$$P_m = 12 \ C = 70\% \ p_0 \ C_m = 10 \ \ N_f = N_0.$$

2) The average situation of the consumers

$$\frac{P_{\max}}{P_0} = 2 \ w_{e_0} = 0.7 \ k_P = -3.9 \ k_N = -1.7 \ k_t = -5.3 \ k_B = 3$$

3) The average economic status of the consumption group

$$E_0' = 63\% \quad \mu = 63\% \quad \sigma=5$$

Retailers can adjust the specific adjustment value according to their own practical economic status and platform, so that obtain optional pricing strategy quantitatively.

Adjustment suggestions:

1) **Adopt high price and low discount strength in terms of rich consumers, while the opposite in terms of those who are not rich;**
2) **When the cost increases, the pricing and discount rate can also be increased accordingly;**
3) **When product performance and original sales volume decrease, the pricing can remain stable without significant changes. However, if necessary,**

promotion sales should be promoted to stimulate consumption.

Retailers can adjust the specific adjustment value in the above three circumstance through procedural quantification according to the practical situations.

VII. Evaluation of the mathematical model and innovation point

This paper innovationally expounds and analyzes online shoppers' psychology, and express the abstract issues through mathematical expressions based on the differential equation established through mathematical knowledge. This seldom appears in the existing literatures. During the solution process, the normal distribution method and other methods are employed to simplify the expressions to make them readable. The model takes a full consideration of the factors, so it can be applied to a wide scope. Dealers can change the parameters according to their practical situation. The model targets at consumers of different wealth degree. So it is very useful. Finally, this paper shows the influence trend of different parameters on the result through the color changes of the three-dimensional color graphs, which can provide guidance to dealers' marketing strategies in response to the environmental changes.

The limits of the paper are mainly reflected as below: 1) This paper fails to give a detailed expression of every variable. The model result is not satisfying when the parameters are under the extreme conditions of the definition domain. Expression formulas more in line with the practical situation should be found in the future; 2) The deduction process is based on the ideal condition. For example, this paper thinks that the variable B only undergoes two change processes and that consumers' consumption is rational. However, the real life is more than that. In the future, study should be conducted as to the pervasive model for the unreasonable condition.

Innovation point

1. **This paper analyzes consumers' online shopping psychology, employs the mathematical method of variable control and simple function to establish the differential equation and work out the potential complex mathematical expression.**

2. **The model takes various factors into full consideration. It has a wide application scope. Dealers can adjust the parameters according to their practical situations to quantitatively gain the result. The model is applicable to different environment, it is very useful.**

3. **This paper shows the influence trend of different parameters on the result through the color changes of the three-dimensional color graphs, which can provide guidance to dealers' marketing strategies in response to the environmental changes.**

VIII. References

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IX. Attachment

MATLAB Program

MOS_alpha_figureN

```
e=1; v=0.97; we=0.7; kP=-3.9; kN=-1.7; Pmax_ratio=2;
minratio=0;maxratio=2;Nminratio=0.0;Nmaxratio=3;
syms kw cw
P_ratio=minratio:0.04:maxratio;
N_ratio=[Nminratio:0.04:Nmaxratio];
[p_ratio,n_ratio]=meshgrid(P_ratio,N_ratio);
cP=exp((kP.*(Pmax_ratio+1).*exp((-Pmax_ratio) ) ) );
alphaP=(exp((kP.*(p_ratio+1).*exp((-p_ratio) ) ))-cP)./(exp((
kP) )-cP);
alphaN=(exp((-kN.*exp((-n_ratio) ) ) )-exp((-kN) ))./(1-exp((-
kN) ) );
[kw,cw]=solve((exp((-2.*kw.*exp((-1) ) ) )-exp((-kw) ))./(cw-e
xp((-kw) ) )==we,cw==exp((-kw.*(Pmax_ratio+1).*exp((-Pm
ax_ratio) ) ) ),kw,cw);
kw=double(kw);
cw=double(cw);
w=(exp((-kw.*(p_ratio+1).*exp((-p_ratio) ) ) )-exp((-kw) ))./(
cw-exp((-kw) ) );
alpha=e.*v*(w.*alphaP+(1-w).*alphaN);
surf(n_ratio,p_ratio,alpha)
zlim([0,1])
xlabel('N/N0')
ylabel('P/P0')
zlabel('alpha')
figure(2)
syms kw cw
N_ratio=1;
cP=exp((kP.*(Pmax_ratio+1).*exp((-Pmax_ratio) ) ) );
alphaP=(exp((kP.*(p_ratio+1).*exp((-p_ratio) )) )-cP)./(exp((
kP) )-cP);
alphaN=(exp((-kN.*exp((-N_ratio) ) ) )-exp((-kN) ))./(1-exp((
-kN) ) );
[kw,cw]=solve((exp((-2.*kw.*exp((-1) ) ) )-exp((-kw) ))./(cw-e
xp((-kw) ) )==we,cw==exp((-kw.*(Pmax_ratio+1).*exp((-Pm
ax_ratio) ) ) ),kw,cw);
kw=double(kw);
cw=double(cw);
w=(exp((-kw.*(p_ratio+1).*exp((-p_ratio) ) ) )-exp((-kw) ))./(
cw-exp((-kw) ) );
alpha=e.*v*(w.*alphaP+(1-w).*alphaN);
plot(P_ratio,alpha)
ylim([0 1])
xlabel('P/P0')
ylabel('alpha')
```

MOS_alpha_figurewe

```
e=1; v=0.97; N_ratio=1; kP=-3.9; kN=-.7; Pmax_ratio=2;
minratio=0;maxratio=2;weminratio=0.0;wemaxratio=1;
syms kw cw
P_ratio=minratio:0.04:maxratio;
we=weminratio+0.02:0.02:wemaxratio-0.02;
[p_ratio,w_e]=meshgrid(P_ratio,we);
cP=exp((kP.*(Pmax_ratio+1).*exp((-Pmax_ratio) ) ) );
alphaP=(exp((kP.*(p_ratio+1).*exp((-p_ratio) ) ) )-cP)./(exp(
(kP))-cP);
alphaN=(exp((-kN.*exp((-N_ratio) ) ) )-exp((-kN) ))./(1-exp((
-kN) ) );
kw=ones(size(we));
cw=ones(size(we));
for i=1:length(we)
    syms kwi cwi
kwi=solve((exp((-2.*kwi.*exp((-1) ) ) )-exp((-kwi) ))./(exp((-k
wi.*(Pmax_ratio+1).*exp((-Pmax_ratio) ) ) )-exp((-kwi) ) )==
we(1,i),kwi);
    kw(1,i)=double(kwi);
cwi=exp((-kwi.*(Pmax_ratio+1).*exp((-Pmax_ratio) ) ) );
    cw(1,i)=double(cwi);
end
[p_ratio,k_w]=meshgrid(P_ratio,kw);
[p_ratio,c_w]=meshgrid(P_ratio,cw);
w=(exp((-k_w.*(p_ratio+1).*exp((-p_ratio) ) ) )-exp((-k_w) ))
./(c_w-exp((-k_w) ) );
alpha=e.*v*(w.*alphaP+(1-w).*alphaN);
surf(w_e,p_ratio,alpha)
zlim([0,1])
xlabel('we')
ylabel('P/P0')
zlabel('alpha')
```

MOS_d

```
syms E P kwe kw cw B0
v=0.97;  we0=0.7;  kP=-3.9;  kN=-1.7;  P0=100;  E0=63;
sigma=5; mu=63; beta=0.01; C=70; B0=1; Pm=12; Cm=10;
N0=0.05; Nf=.05; kt=-5.3; Bmax_ratio=3;
Pminratio=.5;Pmaxratio=2;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
```

```
P=Pfirstterm:Plastterm;d=0.5:.1:1;
P=P+Pm;P0=P0+Pm;
[p,D]=meshgrid(P,d);
p=p.*D;B0=(-4*(Bmax_ratio-1)*(D-.5).^2+Bmax_ratio)*B0;
E=0:100;
alpha1_e=0;W=cell(size(E));
alphaN1=(exp((-kN.*exp(-Nf/N0))  )-exp((-kN)  ))./(1-exp((-k
N) ) );
WE0=@(kwe)(1-exp((-kwe.*(1-E0/100)^2  )))./(1-exp((-kwe)
 ) )-we0;
kwe=double(fsolve(WE0,9));
for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe)));
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) ))./(
Cw(1)-exp((-Cw(2))  )  )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(p./P0+1).*exp((-p./P0))))-exp((-kw)))./(cw-ex
p((-kw)) );
    for j=1:length(d)
    for k=1:length(P)
        if w(j,k)>1
            w(j,k)=1;
            alphaP(j,k)=0;
        end
    end
    end
    W{1,i}=w;
alpha1_E=v.*(w.*alphaP+(1-w).*alphaN1).^(D.^3);
    alpha1_e=alpha1_e+alpha1_E.*phi*1;
end
for i=1:(Plastterm-Pfirstterm+1)
    if p(length(d),i)==P0
        break
    end
end
epsilon=Nf./(B0.*alpha1_e(length(d),i));
alpha1=alpha1_e.*epsilon.*(exp(-kt*(D+1).*exp(-D))-exp(-kt
))/(exp(-2*kt*exp(-1))-exp(-kt));
B1=exp(-p/P0)./((p/P0)*exp(-1)).*B0;
N1=B1.*alpha1;
N0B=N1(4,i);
alphaN2=(exp((-kN.*exp((-N1./N0B) ) ) )-exp((-kN) ))./(1-ex
p((-kN) ) )
B2=sqrt(N1./Nf).*B1;
alpha2=0;
for i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0)     )     )     );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
    for j=1:length(d)
    for k=1:length(P)
        if alphaP(j,k)<0
            alphaP(j,k)=0;
        end
    end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
.^(D.^3);
    alpha2=alpha2+alpha2_E.*phi*1;
end
N2=B2.*alpha2.*(exp(-kt*(D+1).*exp(-D))-exp(-kt))/(exp(-2*
kt*exp(-1))-exp(-kt));
clc
I=N2.*((p-C-Cm)*(1-beta)+(Pm-Cm)*beta);
surf(D,p./D-Pm,I)
xlabel('d')
ylabel('P')
zlabel('I')
for j=1:length(d)
for i=1:(Plastterm-Pfirstterm+1)
    if I(j,i)==max(max(I))
        break
    end
end
if I(j,i)==max(max(I))
    break
end
end
P=P(1,i)-Pm
d=d(1,j)
```

MOS_d_figure_dC

```
syms E P kwe kw cw B0
v=0.97;  we0=0.7;  kP=-3.9;  kN=-1.7;  P0=100;  E0=63;
sigma=5;  mu=63;  beta=0.01;  B0=1;  Pm=12;  Cm=10;
N0=0.05; Nf=.05; kt=-5.3; kb=4.1; Bmax_ratio=4;
Pminratio=.5;Pmaxratio=1.7;Cmin=50;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
P=Pfirstterm:Plastterm;d=0.5:.1:1;C0=Cmin:P0;
P=P+Pm;P0=P0+Pm;
[p,D]=meshgrid(P,d);
p=p.*D;B0=(-4*(Bmax_ratio-1)*(D-.5).^2+Bmax_ratio)*B0;
E=0:100;
alpha1_e=0;W=cell(size(E));
alphaN1=(exp((-kN.*exp(-Nf/N0))  )-exp((-kN)  ))./(1-exp((-k
N) ) );
WE0=@(kwe)(1-exp((-kwe.*(1-E0/100)^2)))./(1-exp((-kwe))
)-we0;
kwe=double(fsolve(WE0,9));
for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2      ))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe)));
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) ))./(
Cw(1)-exp((-Cw(2))  )  )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(p./P0+1).*exp((-p./P0) ) ) )-exp((-kw) ))./(cw-
```

```
exp((-kw) ) );
    for j=1:length(d)
    for k=1:length(P)
        if w(j,k)>1
            w(j,k)=1;
            alphaP(j,k)=0;
        end
    end
    end
    W{1,i}=w;
alpha1_E=v.*(w.*alphaP+(1-w).*alphaN1).^(D.^3);
    alpha1_e=alpha1_e+alpha1_E.*phi*1;
end
for i=1:(Plastterm-Pfirstterm+1)
    if p(length(d),i)==P0
        break
    end
end
epsilon=Nf./(B0.*alpha1_e(length(d),i));
alpha1=alpha1_e.*epsilon.*(exp(-kt*(D+1).*exp(-D))-exp(-kt
))/(exp(-2*kt*exp(-1))-exp(-kt));
B1=exp(-p/P0)./((p/P0)*exp(-1)).*B0;
N1=B1.*alpha1;
N0B=N1(4,i);
alphaN2=(exp((-kN.*exp((-N1./N0B) ) ) )-exp((-kN)))./(1-exp
((-kN) ) );
B2=sqrt(N1./N0).*B1;
alpha2=0;
for i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0)     )     )     );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0))))-cP)./(exp((kP))-
cP);
    for j=1:length(d)
    for k=1:length(P)
        if alphaP(j,k)<0
            alphaP(j,k)=0;
        end
    end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
.^(D.^3);
    alpha2=alpha2+alpha2_E.*phi*1;
end
N2=B2.*alpha2.*(exp(-kt*(D+1).*exp(-D))-exp(-kt))/(exp(-2*
kt*exp(-1))-exp(-kt));
clc
[c0,D]=meshgrid(C0,d);
I=double(size(D));
for i=1:length(C0)
    C=C0(1,i);
    I0=N2.*((p-C-Cm)*(1-beta)+(Pm-Cm)*beta);
    maxI0=max(I0');
    for j=1:length(d)
    I(j,i)=maxI0(1,j);
    end
end
surf(c0,D,I)
xlabel('C')
ylabel('d')
zlabel('I')
```

MOS_d_figuredmu

```
syms E P kwe kw cw B0
v=0.97;  we0=0.7;  kP=-3.9;  kN=-1.7;  P0=100;  E0=63;
sigma=5;  beta=0.01;  B0=1;  Pm=12;  Cm=10;  N0=0.05;
Nf=.05; kt=-5.3; kb=4.1; Bmax_ratio=4; C=80;
Pminratio=.5;Pmaxratio=1.7;mumin=50;mumax=90;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
P=Pfirstterm:Plastterm;d=0.5:.1:1;mu=mumin:1:mumax;
P=P+Pm;P0=P0+Pm;
[p,D]=meshgrid(P,d);[MU,Dmu]=meshgrid(mu,d);
I=double(size(Dmu));
p=p.*D;B0=(-4*(Bmax_ratio-1)*(D-.5).^2+Bmax_ratio)*B0;
E=0:100;
for l=1:length(mu)
mul=mu(1,l);
alpha1_e=0;W=cell(size(E));
alphaN1=(exp((-kN.*exp(-Nf/N0))  )-exp((-kN)  ))./(1-exp((-k
N) ) );
WE0=@(kwe)(1-exp((-kwe.*(1-E0/100)^2  ))).(1-exp((-kwe)
  ) )-we0;
kwe=double(fsolve(WE0,9));
for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mul).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0)     )     )     );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe)));
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) )).(
Cw(1)-exp((-Cw(2))  ) )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(p./P0+1).*exp((-p./P0))))-exp((-kw)))./(cw-ex
p((-kw)) );
    for j=1:length(d)
    for k=1:length(P)
        if w(j,k)>1
            w(j,k)=1;
            alphaP(j,k)=0;
        end
    end
    end
    W{1,i}=w;
alpha1_E=v.*(w.*alphaP+(1-w).*alphaN1).^(D.^3);
    alpha1_e=alpha1_e+alpha1_E.*phi*1;
end
for i=1:(Plastterm-Pfirstterm+1)
    if p(length(d),i)==P0
        break
    end
end
epsilon=Nf./(B0.*alpha1_e(length(d),i));
alpha1=alpha1_e.*epsilon.*(exp(-kt*(D+1).*exp(-D))-exp(-kt
))/(exp(-2*kt*exp(-1))-exp(-kt));
B1=exp(-p/P0)./((p/P0)*exp(-1)).*B0;
N1=B1.*alpha1;
N0B=N1(4,i);
alphaN2=(exp((-kN.*exp((-N1./N0B) ) ) )-exp((-kN) ))./(1-ex
p((-kN)) );
```

```
B2=sqrt(N1./N0).*B1;
alpha2=0;
for i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mul).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
    for j=1:length(d)
    for k=1:length(P)
        if alphaP(j,k)<0
            alphaP(j,k)=0;
        end
    end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
.^(D.^3);
    alpha2=alpha2+alpha2_E.*phi*1;
end
N2=B2.*alpha2.*(exp(-kt*(D+1).*exp(-D))-exp(-kt))/(exp(-2*
kt*exp(-1))-exp(-kt));
I0=N2.*((p-C-Cm)*(1-beta)+(Pm-Cm)*beta);
maxI0=max(I0');
for i=1:length(d)
    I(i,l)=maxI0(1,i);
end
end
clc
surf(MU,Dmu,I)
xlabel('mu')
ylabel('d')
zlabel('I')
```

MOS_d_figuredNf

```
syms E P kwe kw cw B0
v=0.97;  we0=0.7;  kP=-3.9;  kN=-1.7;  P0=100;  E0=63;
sigma=5;  beta=0.01;  B0=1;  Pm=12;  Cm=10;  N0=0.05;
kt=-5.3; kb=4.1; Bmax_ratio=4; C=90; mu=63;
Pminratio=.5;Pmaxratio=1.7;Nfmin_ratio=0;Nfmax_ratio=2;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
P=Pfirstterm:Plastterm;d=0.5:.1:1;Nf_ratio=Nfmin_ratio:.1:
Nfmax_ratio;Nf=Nf_ratio*N0;
P=P+Pm;P0=P0+Pm;
[p,D]=meshgrid(P,d);[nf_ratio,DNf]=meshgrid(Nf_ratio,d);
I=double(size(DNf));
p=p.*D;B0=(-4*(Bmax_ratio-1)*(D-.5).^2+Bmax_ratio)*B0;
E=0:100;
for l=1:length(Nf)
Nfl=Nf(1,l);
alpha1_e=0;W=cell(size(E));
alphaN1=(exp((-kN.*exp(-Nfl/N0))  )-exp((-kN)  ))./(1-exp((-k
N) ) );
WE0=@(kwe)(1-exp((-kwe.*(1-E0/100)^2  )))./(1-exp((-kwe)
 ) )-we0;
kwe=double(fsolve(WE0,9));
for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe)));
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) )).(
Cw(1)-exp((-Cw(2)) ) )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(p./P0+1).*exp((-p./P0))))-exp((-kw)))./(cw-ex
p((-kw)) );
    for j=1:length(d)
    for k=1:length(P)
        if w(j,k)>1
            w(j,k)=1;
            alphaP(j,k)=0;
        end
    end
    end
    W{1,i}=w;
alpha1_E=v.*(w.*alphaP+(1-w).*alphaN1).^(D.^3);
    alpha1_e=alpha1_e+alpha1_E.*phi*1;
end
for i=1:(Plastterm-Pfirstterm+1)
    if p(length(d),i)==P0
        break
    end
end
epsilon=Nfl./(B0.*alpha1_e(length(d),i));
alpha1=alpha1_e.*epsilon.*(exp(-kt*(D+1).*exp(-D))-exp(-kt
))/(exp(-2*kt*exp(-1))-exp(-kt));
B1=exp(-p/P0)./((p/P0)*exp(-1)).*B0;
N1=B1.*alpha1;
N0B=N1(4,i);
alphaN2=(exp((-kN.*exp((-N1./N0B) ) ) )-exp((-kN) )).(1-ex
p((-kN) ));
B2=sqrt(N1./Nfl).*B1;
alpha2=0;
for i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
    for j=1:length(d)
    for k=1:length(P)
        if alphaP(j,k)<0
            alphaP(j,k)=0;
        end
    end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
.^(D.^3);
    alpha2=alpha2+alpha2_E.*phi*1;
end
N2=B2.*alpha2.*(exp(-kt*(D+1).*exp(-D))-exp(-kt))/(exp(-2*
kt*exp(-1))-exp(-kt));
I0=N2.*((p-C-Cm)*(1-beta)+(Pm-Cm)*beta);
maxI0=max(I0');
for i=1:length(d)
    I(i,l)=maxI0(1,i);
end
end
clc
surf(nf_ratio,DNf,I)
```

```
xlabel('Nf/N0')
ylabel('d')
zlabel('I')
```

MOS_E_NormalityRegression

```
syms E X mu sigma

mu=63;sigma=5;
x=0:100;
phi=exp(-(x-mu).^2/(2*sigma^2))./(sqrt(2*pi)*sigma);
plot(x,phi)
phi=@(X)exp(-(X-mu).^2/(2*sigma^2))./(sqrt(2*pi)*sigma);
ExpectedValueE=@(X)X.*exp(-(X-mu).^2/(2*sigma^2))./(sq
rt(2*pi)*sigma);
PHI1=@(E)(100*quad(phi,0,E)-5);
E1=fsolve(PHI1,60);
A1=quad(ExpectedValueE,0,E1)/quad(phi,0,E1);
PHI2=@(E)(100*quad(phi,0,E)-10);
E2=fsolve(PHI2,60);
A2=quad(ExpectedValueE,E1,E2)/quad(phi,E1,E2);
PHI3=@(E)(100*quad(phi,0,E)-20);
E3=fsolve(PHI3,60);
A3=quad(ExpectedValueE,E2,E3)/quad(phi,E2,E3);
PHI4=@(E)(100*quad(phi,0,E)-40);
E4=fsolve(PHI4,60);
A4=quad(ExpectedValueE,E3,E4)/quad(phi,E3,E4);
PHI5=@(E)(100*quad(phi,0,E)-60);
E5=fsolve(PHI5,60);
A5=quad(ExpectedValueE,E4,E5)/quad(phi,E4,E5);
PHI6=@(E)(100*quad(phi,0,E)-80);
E6=fsolve(PHI6,60);
A6=quad(ExpectedValueE,E5,E6)/quad(phi,E5,E6);
PHI7=@(E)(100*quad(phi,0,E)-90);
E7=fsolve(PHI7,60);
A7=quad(ExpectedValueE,E6,E7)/quad(phi,E6,E7);
A8=quad(ExpectedValueE,E7,100)/quad(phi,E7,100);
[A1 A2 A3 A4 A5 A6 A7 A8]
[100*quad(phi,0,55)          100*quad(phi,0,56.5)
100*quad(phi,0,58.5)          100*quad(phi,0,61)
100*quad(phi,0,63.5)          100*quad(phi,0,66)
100*quad(phi,0,70)]
```

MOS_I

```
syms E P kwe kw cw B0
v=0.97;  we0=0.7;  kP=-3.9;  kN=-1.7;  P0=100;  E0=63;
sigma=5; mu=63; beta=0.01; C=60; B0=1; Pm=10; Cm=7;
N0=0.05; Nf=.05; Pf=100; Bf=1;
Pminratio=.5;Pmaxratio=1.5;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
P=Pfirstterm:Plastterm;
P=P+Pm*ones(size(P));P0=P0+Pm;Pf=Pf+Pm;
E=0:100;
alpha1_e=0;W=cell(size(E));
alphaN1=(exp((-kN.*exp(-Nf/N0)))-exp((-kN)))./(1-exp((-kN)
 ) );
WE0=@(kwe)(1-exp((-kwe.*(1-E0/100)^2  ))./(1-exp((-kwe)
 ) )-we0;
kwe=double(fsolve(WE0,9));
for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0)      )      )      );
alphaP=(exp((kP.*(P./P0+1).*exp((-P./P0) ) ) )-cP)./(exp((kP
) )-cP);
we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe))              );
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) ))./(
Cw(1)-exp((-Cw(2)) ) )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(P./P0+1).*exp((-P./P0))) )-exp((-kw)))./(cw-ex
p((-kw)));
    for j=1:length(P)
        if w(1,j)<0
            w(1,j)=0;
        end
    end
    W{1,i}=w;
    alpha1_E=v.*(w.*alphaP+(1-w).*alphaN1);
    alpha1_e=alpha1_e+alpha1_E.*phi*1;
end
for i=1:length(P)
    if P(1,i)==Pf
        break
    end
end
if P(1,i)==Pf
else
    display('ERROR IN Plastterm')
end
epsilon=Nf/(Bf.*alpha1_e(1,i));
if epsilon>1
    display('ERROR IN EPSILON')
end
alpha1=alpha1_e*epsilon;
B1=exp(-P/Pf)./((P/Pf)*exp(-1))*B0;
N1=B1.*alpha1;
alphaN2=(exp((-kN.*exp((-N1./N0) ) ) )-exp((-kN) ))./(1-exp(
(-kN) ) );
B2=sqrt(N1./Nf).*B1;
alpha2=0;
for i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(P./P0+1).*exp((-P./P0) ) ) )-cP)./(exp((kP
) )-cP);
    for k=1:length(P)
        if alphaP(1,k)<0
            alphaP(1,k)=0;
        end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
;
    alpha2=alpha2+alpha2_E.*phi*1;
end
N2=B2.*alpha2;
I=N2.*((P-C-Cm)*(1-beta)+(Pm-Cm)*beta);
plot(P-Pm,I)
ylim([0,max(I)*1.1])
xlabel('P')
ylabel('I')
```

```
for i=1:length(P)
    if I(1,i)==max(I)
        break
    end
end
P=P(1,i)-Pm
```

MOS_I_figureC

```
syms E P kwe kw cw B0
epsilon=0.1; v=0.97; we0=0.7; kP=-3.9; kN=-1.7; P0=100;
E0=63; sigma=5; mu=E0; beta=0.01; B0=1; Pm=10; Cm=7;
N0=0.05; Nf=.05;
Pminratio=.5;Pmaxratio=1.5;Cmin=0;Cmax=100;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
P=Pfirstterm:Plastterm;C=Cmin:Cmax;
P=P+Pm*ones(size(P));P0=P0+Pm;
[p,c]=meshgrid(P,C);
E=0:100;
alpha1_e=0;W=cell(size(E));
alphaN1=(exp((-kN.*exp(-Nf/N0))  )-exp((-kN)  ))./(1-exp((-k
N) ) );
kwe=double(solve((1-exp((-kwe.*(1-E0/100)^2  )))./(1-exp((-
kwe) ) )==we0,kwe));
for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0)     )     )     );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe)));
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) ))./(
Cw(1)-exp((-Cw(2))  )  )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(p./P0+1).*exp((-p./P0)  )))-exp((-kw))).(cw-ex
p((-kw)));
    for j=1:length(C)
    for k=1:length(P)
        if w(j,k)>1
            w(j,k)=1;
            alphaP(j,k)=0;
        end
    end
    end
    W{1,i}=w;
alpha1_E=epsilon.*v.*(w.*alphaP+(1-w).*alphaN1);
    alpha1_e=alpha1_e+alpha1_E.*phi;
end
for i=1:(Plastterm-Pfirstterm+1)
    if p(1,i)==P0
        break
    end
end
epsilon=Nf/(B0.*alpha1_e(1,i));
alpha1=alpha1_e*epsilon;
B1=exp(-p/P0)./((p/P0)*exp(-1))*B0;
N1=B1.*alpha1;
alphaN2=(exp((-kN.*exp((-N1./N0) ) ) )-exp((-kN) ))./(1-exp(
(-kN) ) );
B2=sqrt(N1./N0).*B1;
alpha2=0;
for i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0)     )     )     );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
    for j=1:length(C)
    for k=1:length(P)
        if alphaP(j,k)<0
            alphaP(j,k)=0;
        end
    end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
;
    alpha2=alpha2+alpha2_E.*phi;
end
N2=B2.*alpha2;
I=N2.*((p-c-Cm)*(1-beta)+(Pm-Cm)*beta);
surf(c,p-Pm,I)
xlabel('C')
ylabel('P')
zlabel('I')
```

MOS_I_figuremu

```
syms E P kwe kw cw B0

v=0.97; we0=0.7; kP=-3.9; kN=-1.7; P0=100; E0=63;
sigma=5; beta=0.01; B0=1; Pm=10; Cm=7; C=60; Nf=.05;
N0=.05;

Pminratio=.6;Pmaxratio=1.4;mumin=50;mumax=85;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
P=Pfirstterm:Plastterm;mu=mumin:mumax;
P=P+Pm;P0=P0+Pm;
E=0:100;
[p,MU]=meshgrid(P,mu);
I=double(size(p));
alphaN1=(exp((-kN.*exp(-Nf/N0))  )-exp((-kN)  ))./(1-exp((-k
N) ) );
WE0=@(kwe)(1-exp((-kwe.*(1-E0/100)^2  )))./(1-exp((-kwe)
  ) )-we0;
kwe=double(fsolve(WE0,9));
for j=1:length(mu)
    muj=mu(1,j);
    alpha1_e=0;W=cell(size(E));
   for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-muj).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP); we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe)));
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) ))./(
Cw(1)-exp((-Cw(2))  )  )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(p./P0+1).*exp((-p./P0))  ))-exp((-kw))).(cw-ex
```

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```
p((-kw)));
    for l=1:length(mu)
    for k=1:length(P)
        if w(l,k)>1
            w(l,k)=1;
            alphaP(l,k)=0;
        end
    end
    end
    W{1,i}=w;
    alpha1_E=v.*(w.*alphaP+(1-w).*alphaN1);
    alpha1_e=alpha1_e+alpha1_E.*phi;
    end
    for i=1:(Plastterm-Pfirstterm+1)
    if p(1,i)==P0
        break
    end
    end
    epsilon=Nf/(B0.*alpha1_e(1,i));
    alpha1=alpha1_e*epsilon;
    B1=exp(-p/P0)./((p/P0)*exp(-1))*B0;
    N1=B1.*alpha1;
alphaN2=(exp((-kN.*exp((-N1./N0) ) ) )-exp((-kN) ))./(1-exp(
(-kN) ) );
    B2=sqrt(N1./N0).*B1;
    alpha2=0;
    for i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-muj).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
    alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0)  )))-cP)./(exp(
(kP))-cP);
    for l=1:length(mu)
    for k=1:length(P)
        if alphaP(l,k)<0
            alphaP(l,k)=0;
        end
    end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
;
    alpha2=alpha2+alpha2_E.*phi;
    end
    N2=B2.*alpha2;
    for i=1:length(p)
I(j,i)=N2(1,i).*((p(1,i)-C-Cm)*(1-beta)+(Pm-Cm)*beta);
    end
end
surf(MU,p-Pm,I)
xlabel('mu')
ylabel('P')
zlabel('I')
```

MOS_I_figureNf

```
syms E P kwe kw cw B0
v=0.97;  we0=0.7;  kP=-3.9;  kN=-1.7;  P0=100;  E0=63;
sigma=5; mu=63; beta=0.01; C=60; B0=1; Pm=10; Cm=7;
N0=0.05;
Pminratio=.6;Pmaxratio=1.5;Nfmin_ratio=0;Nfmax_ratio=4;
Pfirstterm=P0*Pminratio;Plastterm=P0*Pmaxratio;
P=Pfirstterm:Plastterm;Nf_ratio=Nfmin_ratio:.1:Nfmax_rati
o;Nf=Nf_ratio*N0;
P=P+Pm*ones(size(P));P0=P0+Pm;
[p,nf]=meshgrid(P,Nf);
E=0:100;
alpha1_e=0;W=cell(size(E));
alphaN1=(exp((-kN.*exp(-nf./N0))  )-exp((-kN)  ))./(1-exp((-k
N) ) );
kwe=double(solve((1-exp((-kwe.*(1-E0/100)^2  )))./(1-exp((-
kwe) ) )==we0,kwe));
for i=1:length(E)
    syms kw cw
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0)      )      )      );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
we=(1-exp((-kwe.*(1-Ei/100)^2)))./(1-exp((-kwe)));
eqn=@(Cw)[(exp((-2.*Cw(2).*exp((-1) ) ) )-exp((-Cw(2)) ))./(
Cw(1)-exp((-Cw(2)) ) )-we;Cw(1)-exp((-Cw(2).*(Pmax./P0+
1).*exp((-Pmax./P0) ) ) )];
    fsolve(eqn,[1;-4.3]);
    kw=double(ans(2,1));
    cw=double(ans(1,1));
w=(exp((-kw.*(p./P0+1).*exp((-p./P0))))-exp((-kw)))./(cw-ex
p((-kw)) );
    for j=1:length(Nf)
    for k=1:length(P)
        if w(j,k)>1
            w(j,k)=1;
            alphaP(j,k)=0;
        end
    end
    end
    W{1,i}=w;
    alpha1_E=v.*(w.*alphaP+(1-w).*alphaN1);
    alpha1_e=alpha1_e+alpha1_E.*phi*1;
end
for i=1:(Plastterm-Pfirstterm+1)
    if p(1,i)==P0
        break
    end
end
epsilon=nf/(B0.*alpha1_e(1,i));
B1=exp(-p/P0)./((p/P0)*exp(-1))*B0;
N1=B1.*alpha1_e.*epsilon;
alphaN2=(exp((-kN.*exp((-N1./N0) ) ) )-exp((-kN) ))./(1-exp(
(-kN) ) );
B2=sqrt(N1./nf).*B1;
alpha2=0;
for                                          i=1:length(E)
Ei=E(1,i);Pmax_ratio=Ei^3/3e5+1;Pmax=Pmax_ratio*P0;
phi=exp(-(Ei-mu).^2./(2.*sigma^2))./(sqrt(2.*pi).*sigma);
    cP=exp((kP.*(Pmax./P0+1).*exp((-Pmax./P0) ) ) );
alphaP=(exp((kP.*(p./P0+1).*exp((-p./P0) ) ) )-cP)./(exp((kP
) )-cP);
    for j=1:length(Nf)
    for k=1:length(P)
        if alphaP(j,k)<0
            alphaP(j,k)=0;
        end
    end
    end
alpha2_E=epsilon.*v.*(W{1,i}.*alphaP+(1-W{1,i}).*alphaN2)
;
```

```
    alpha2=alpha2+alpha2_E.*phi*1;
end
N2=B2.*alpha2;
I=N2.*((p-C-Cm)*(1-beta)+(Pm-Cm)*beta);
surf(nf/N0,p-Pm,I)
xlabel('Nf/N0')
ylabel('P')
zlabel('I')
```