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The title of project: Revegetation plan for the Loess Plateau

Abstract:

This paper aims to discuss the problem of vegetation restoration of the loess plateau in the premise of the ecological and economic benefits, terrain slope and the present situation of soil erosion on the loess plateau are. We will give a comprehensive planning of vegetation which is based on various analyses and processing data; analyze the pros and cons of species, and then get the best tree species. Next, we will establish the Fibonacci sequence; the function model which is based on ecological benefit, the slope and the vegetation quantity; and comprehensive nonlinear programming model of efficiency index. Finally, the optimal solution of vegetation planning is obtained.

For refined processing, the loess plateau soil and water loss mainly depends on two factors: the number of slope and the vegetation. So we focus on the relationship between water and the soil protection or between the comprehensive benefit and the ecological benefit. It is divided into five parts: imaging simulation, the selection of tree species, the regional division, the planting mode and comprehensive relationship among the variables of them.

In terms of the simulation map, we got the digital elevation model of the loess plateau according to The United States the CGIAR Consortium for Spatial Information (CGIAR - CSI) institutions tif geographic data and using of Global Mapper software. We regarded he results as the basis of planning background and slope classification.

In the aspect of selection of tree species, we considered four factors in spending, growth speed, environmental adaptability and the survival rate. We selected nine common afforestation tree species combining with relevant materials and arrived at the evaluations of the different factors in different wood species. By using the principal component analysis (pca), eventually we got that the acacia is the best tree.

For regional division, we used the digital elevation model and the Grobal Mapper to map the transverse and longitudinal cutting image in the region. Through using the function fitting, the loess plateau can be divided into 0 ° ~ 20 °, 20 ° to 40 °, 40 ° above and analyzed in three regions respectively.

For selecting the way of planting, we first proposed 3 planning which have large difference between planting scheme. By using the Fibonacci sequence, we simulated and calculated the different planting scheme of the sum of crown density in three years (the amount of time spent on the plants grow), as a judge of the standard of using the alternate distribution. Finally, we obtained the best scheme.

For comprehensive relationship between the variables, this part is the key of this paper. We derived the relevant data, and got the relevant information by using fitting for many times. And then we got the relationship between the crown density and annual soil erosion modulus; slope and the number of soil loss; crown density and the number; quantity and the rate of water and soil protection; quantity, gradient and

comprehensive, ecological benefits. Finally, we obtained the corresponding optimal planning scheme in different gradients.

For the evaluation of the model, we evaluated four models together. Tree species selection model uses the scientific evaluation method in order to select the proper tree species. Planting form model can dynamically describe the way of development, evaluate the alternative distribution mode can adapt to the requirements of sustainable development. The planting density model can get both ecological benefit and economic benefit. Selection of planting density model can fit the conclusion of the first model, second model and the actual situation. Terrain slope model combines the conclusion of the full text; the best scheme in combination with the actual terrain was chose.

About the improvement and promotion of model, First, in order to improve the accuracy of prediction models, the death of tree in actual cases is inevitable which cannot be ignored. Second, the loess plateau of governance has stage division. Different stages have different vegetations, therefore scheme will change. Third, we can get evaluation by the variation coefficient and Moran index. Fourth, the loess plateau monsoon has to be considered. In allusion to the promotion of the model, in this paper, the model can also be used in residential area greening, the improvement of vegetation coverage, and other practical problems.

Keywords:

Digital Elevation Model (DEM); Principal Component Analysis; The high degree fitting; Nonlinear Programming; Surfer 3D Composition

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1. The background

The Loess Plateau has become the main region of environmental protection and vegetation recovery by Chinese government due to its serious soil erosion. According to the "Outline of Comprehensive Management of Loess Plateau (2010-2030)" issued by National Development and Reform Commission, China, the Loess Plateau is still suffering series of problems including serious soil erosion, the expansion of desertification and grassland degradation due to influence of fragment terrain, strong rainfall and other natural factors.

Since vegetation recovery is the key way to solve the soil erosion problem in Loess Plateau, it is urgent to establish a complete vegetation recovery plan.

2. The causes of soil erosion

There are many causes of soil erosion such as climate, soil, terrain, tree species, their adaptability to the environment, the ecology benefit of vegetation, etc. Therefore, in this project, we establish a comprehensive evaluation system for the Loess Plateau in order to select the most suitable species, and to make a reasonable afforestation plan under the consideration of the above factors.

2.1 The main steps of the project

a. Based on Global Mapper v 10.0, we built a typical topography of the Loess Plateau with the background plot (0.383 km2), and the slope was extracted in order to divide zoning.

b. Evaluate relevant indices of different tree species to select the most suitable species for the plot.

c, Calculated the functional relationships of relevant factors based on the related documents.

e, Evaluated and improved the model.

3. Assumptions and Symbol Description

We made the following assumption:

1. The regional hydrology, topography remains stable during planting.

2. The tree species and individuals are growing well and have no significant difference between individuals.

3. The slope of (x,y) is represented using the average of slope of section of x, y.

The description of symbol is as follows:

Canopy cover	ρ	Index of comprehensive evaluation	F
Index of comprehensive benefit	l	Annual soil erosion modulus	y
Economic benefit	λ	Numbers	n
Ecological benefit	δ	Slope	θ

4. Model establishment and analysis

4.1 Construction of simulation environment (plot)

The format of the data in this study was tif provided by The CGIAR Consortium for Spatial Information (CGIAR-CSI). The DEM of the Loess Plateau was extracted using Global Mapper. We then selected a plot from the centre of the Loess Plateau (N 37°10′03-37°10′31， E109°57′58-109°58′30) as the implementation place.

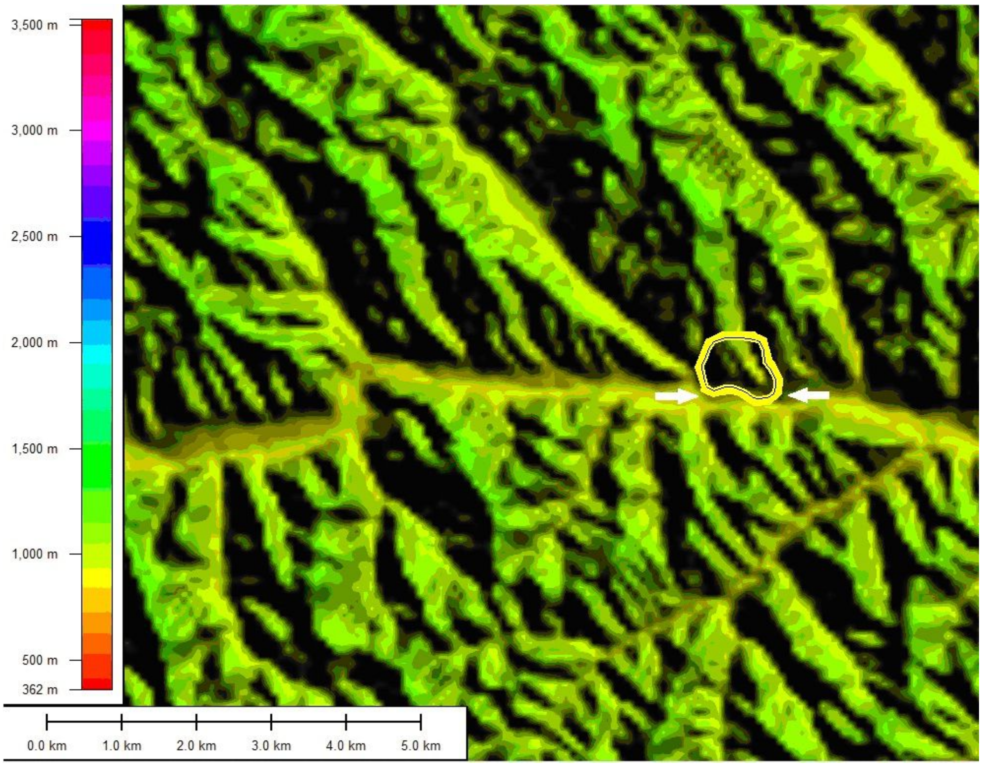

Figure 4.1.1 The environment of the study plot

Note: the color in the Figure represents elevation.

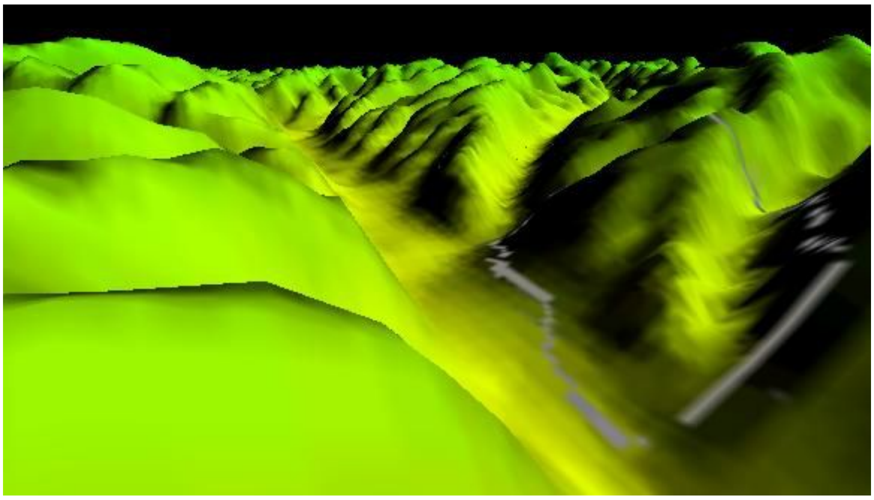

Figure 4.1.2 3D image of study plot

The detailed information of selected region

Left corner X=109.9663077363; Y=37.1754479201

Right corner X=109.9663077363; Y=37.1754479201

Extreme west longitude=109°57' 58.7079" E

Extreme north latitude=37°10' 31.6125" N

Extreme east longitude=109°58' 30.1708" E

Extreme south latitude=37°10' 3.1258" N

Projection Datum = WGS84

Projection Units = arc degrees

Coverage = 0.383 km^2

Through the analysis of the digital elevation model (DEM), we can also get the cross section, longitudinal section and slope of the study area. Section data will be described in the next chapter.

4.2 Tree species selection and indexes digitalization

According to "the main species and afforestation technology of the Loess Plateau", we selected the following nine species: *Ailanthus altissima M, Robinia pseucdoacacia, Pinus tabulaeformis Carr. Populus simonii Carr. Populus canadensis Moench. Ulmus pumila L. Hippophae rhamnoides L. Haloxylon ammodendron*. In this project, we referred to "the major skill of afforestation" and selected expense, growth rate, adaptability to environment and survival rate as the main indexes of the species. The detailed information of indexes and species is as table below.

Table 4.2.1 the species and indexes

	Cost	Growth rate	Adaptability to environment	Survival rate
Amorpha fruticosa	1.5	4.4	3.3	1.9
Ailanthus altissima M.	2.3	3.9	3.5	2.4
Pinus tabulaeformis Carr.	1.9	2.4	2.9	1.7
Populus simonii Carr.	1.3	6.0	4.5	2.0
Populus canadensis Moench.	2.1	5.2	4.4	2.1
Ulmus pumila L.	1.8	4.9	3.8	1.5
Hippophae rhamnoides L.	1.9	3.4	3.0	2.2
Robinia pseucdoacacia	3.1	5.8	4.0	2.4
Haloxylon ammodendron	2.2	4.2	3.7	3.0

Note: 1.Cost: λ=1%×(The price of sapling+ maintenance costs)

2. Growth rate $\xi=5\times\pi\times\left(\frac{x}{2}\right)^2/t$ (x is the crown of the tree)

3. Adaptability to environment $q=\sqrt{\lg\frac{1}{m\times c}}$ (m is the precipitation that a plant needs, c is temperature that a plant needs)

4. Survival rate $h=\left(2\times h'\right)^{\ln t}$ (h' is the real Survival rate)

In this project, the principle component analysis (PCA) was conducted to determine the most suitable species due to the complex relationship between indexes.

Principal component analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelatedvariables called principal components. The number of principal components is less than or equal to the number of original variables. PCA is the simplest of the true eigenvector-based multivariate analyses.

Often, its operation can be thought of as revealing the internal structure of the data in a way that best explains the variance in the data. The equation of PCA can be expressed as follows:

$$F_1 = a_{11}x_1 + a_{21}x_2 + \cdots + a_{n1}x_n \text{ ;}$$

$$F_2 = a_{12}x_1 + a_{22}x_2 + \cdots + a_{n2}x_n \text{ ;}$$

$$\cdots$$

$$F_p = a_{1p}x_1 + a_{2p}x_2 + \cdots + a_{np}x_n \text{ ,}$$

Where $a_{1i}, a_{2i}, \cdots, a_{ni}$ （$i = 1, 2, \cdots, p$） are the corresponding eigenvectors of the eigenvalues of covariance matrices of x ; $x_1, x_2, \cdots, x_n$ are the standardized values of original $x_1, x_2, \cdots, x_n$. The PCA was performed in SPSS. The results are as follows:

Table 4.2.2 the correlation analysis among indexes

Correlation Matrix

		Costs	GrowthRate	Adaptability	SurvivalRate
Correlation	Costs	1.000	.105	.048	.501
	GrowthRate	.105	1.000	.905	.076
	Adaptability	.048	.905	1.000	.114
	SurvivalRate	.501	.076	.114	1.000

Table 4.2.3The output of PCA analysis, total variance explained：

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	1.968	49.199	49.199	1.968	49.199	49.199
2	1.438	35.949	85.148	1.438	35.949	85.148
3	.505	12.614	97.763			
4	.089	2.237	100.000			

Extraction Method: Principal Component Analysis.

Table 4.2.4 The output of PCA analysis, component matrix:

Component Matrix[a]

	Component	
	1	2
Costs	.330	.802
GrowthRate	.933	-.286
Adaptability	.930	-.298
SurvivalRate	.353	.790

Extraction Method: Principal Component Ana

a. 2 components extracted.

Result analysis:

There are significant correlations between cost and survival rate, growth rate and Adaptability to environment. Since one factor may be strongly related to another, there may be some overlapping information between factors (Table 4.2.2).

There are three main criteria to determine the number of principle component (PC). In this project, we selected the principle components based on the Eigenvalue (greater than 1). Table 4.2.3 showed that two principle components should be included based on their eigenvalues. According to the percentage of variances, we named the first PC as Main_01 and the second PC as Main_02.

Besides, according to the Main_01, the growth rate and the Adaptability to environment contributed the most influence on Main_01. In contrast, cost and survival rate were two significant factors in Main_02.

The function of each PC

We calculated the coefficient of each factor in each PC using the following formula: C=value of component matrix/SQR(Eigenvalue of each PC)

In this study, the Eigenvalue of each Main_01 and Main_02 are 1.968and 1.438, respectively. The procedure of calculation in SPSS is as follows:

“Res_vec01=Main_01/SQR(1.968)”and“Res_vec02=Main_02/SQR(1.438)”

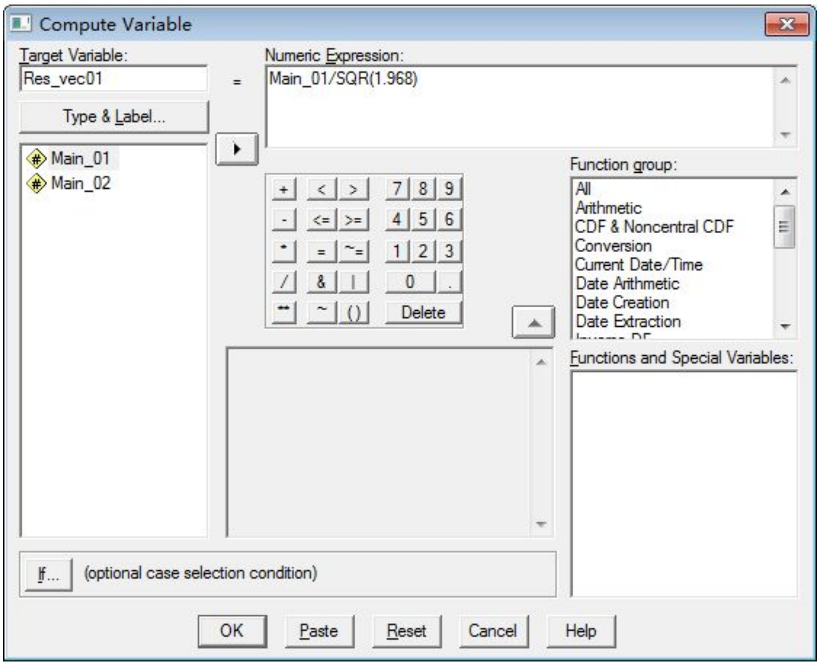

Figure 4.2.1 The procedure of Eigenvector in SPSS

Table 4.2.5 Principle components and eigenvectors

	Main_01	Main_02	Res_vec01	Res_vec02
Cost (X_1)	0.3300	0.8020	0.24	0.67
Growth rate (X_2)	0.9330	-0.2860	0.67	-0.24
Adaptability to environment (X_3)	0.9300	-0.2980	0.66	-0.25
Survival rate (X_4)	0.3530	0.7900	0.25	0.66

Table 4.2.5 showed the Principle components and eigenvectors. The two principle components were obtained based on the values in table 4.2.5.

$$F_1 = 0.240 \times X_1 + 0.670 \times X_2 + 0.660 \times X_3 + 0.250 \times X_4$$

$$F_2 = 0.670 \times X_1 - 0.240 \times X_2 - 0.250 \times X_3 + 0.660 \times X_4$$

We then determine the weight for the comprehensive model based on the eigenvalue of each PC (the eigenvalue of each PC divided by the sum of eigenvalues of all PCs). The function expression is as below:

$$F = \frac{k_1}{k_1 + k_2} F_1 + \frac{k_2}{k_1 + k_2} F_2$$

Namely we got the comprehensive index of evaluation F :

$$F = 0.422X_1 + 0.286X_2 + 0.276X_3 + 0.423X_4$$

We evaluated the tree species based on the F function. The evaluation was conducted in Matlab. The results revealed that the comprehensive benefit of *Robinia pseucdoacacia* was the greatest due to its highest F value. Therefore, we selected *Robinia pseucdoacacia* as the species of afforestation.

Species	*Populus canadensis Moench.*	*Amorpha frutic osa*	*Ailanthus altissima M.*	*Pinus tabulaeformis Carr.*	*Populus simonii Carr.*
F value	4.4761	3.6059	4.0672	3.0077	4.3526
Species	*Ulmus pumila L.*	*Hippophae rhamnoides L.*	*Robinia pseuc doacacia*	*Haloxylon ammodendro n*	
F value	3.8443	3.5328	5.0862	4.4198	

4.3 The planting plan.

The best plan of planting was based on the comprehensive benefits which included both ecological and economic benefits. We took a $8 \times 8m^2$ plot and four *Robinia pseucdoacacia* as study objects, and the evaluation process of the best planting plan is as Figure 4.3.1.

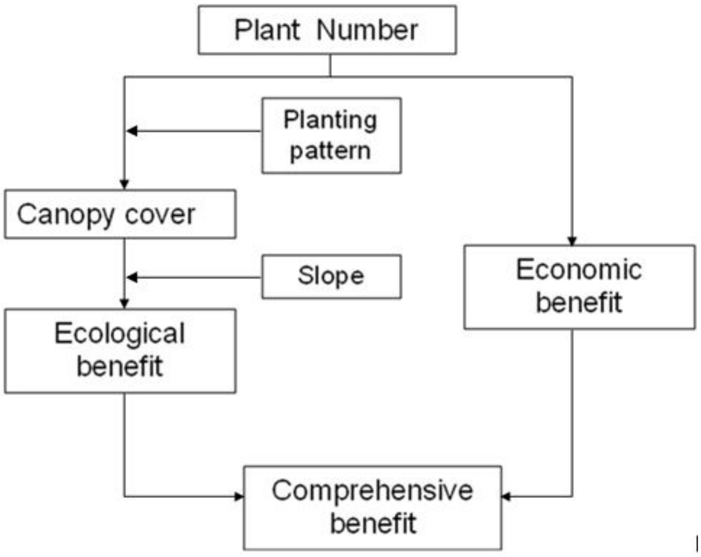

Figure 4.3.1 The evaluation process of the best planting plan

（1）The way of planting

We select three planting methods in this project according to “the main skill of afforestation”:

① Random planting: the tree was planted randomly in the plot, like sowing by plane.

② Matrix planting: the distribution of trees looks like a matrix, with rows and columns.

③ Alternating planting: insert the interspaces and lined tightly.

（2）The choice of terrain

The terrain, especially slope, has significant influence on the growth rate of tree, rate of soil erosion, the water infiltration and water flow speed. We also divided the study plot into three different terrain area according to the various slope (0°，20°)、(20°，40°) and (40°，50°).

（3）Regarding the comparison

① The costs λ. $\lambda = n \times a$ （a is price, *Robinia pseucdoacacia* is 10 RMB/ individual plant）

②The canopy cover ρ： the canopy cover is the ratio of the aboveground portion formed by plant crowns and the total ground area. The formula of canopy cover calculation is $\rho_0 = S_F$ (the area of valid crown)／S_0 (the area of ground surface).

We built the functional model of ρ and soil erosion modulus according to the data provided in "the Regularity of soil erosion and loss".

Table 4.3.1 The canopy cover and corresponding soil erosion modulus

Canopy cover ρ	Annual soil erosion modulus y （t/ km^2 ）
4.1	14984
13.2	7813
38.3	2413
80.0	223

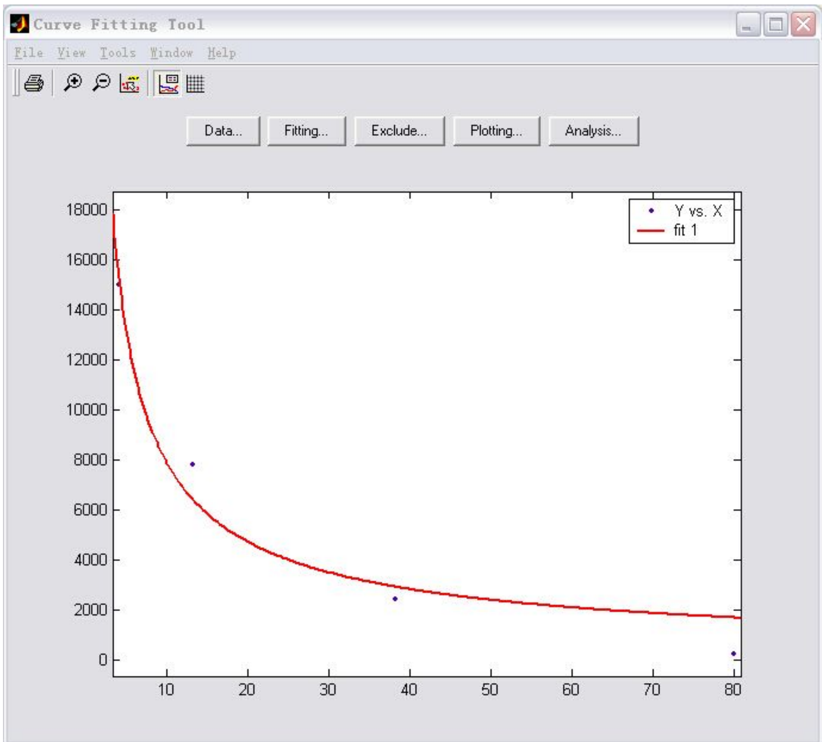

Figure 4.3.1 The fitting of canopy cover and soil erosion modulus

The result of model fitting is as follows:

General model Power1	Coefficients (with 95% confidence:		Goodness of fit			
f(x) = a*x^b	a	b	RMSE	SSE	R-square	Adjusted R-square
	4.377e+004 (-5174, 9.272e+004)	-0.7442 (-1.373, -0.1153)	1487	4.422e+006	0.9659	0.9489

The relationship between canopy cover and soil erosion modulus can be expressed as below:

$$y = 43770 \times \rho^{-0.7442}$$

③ The influence of terrain: The relationship between slope (θ) and soil loss was also tested using data in "the Regularity of soil erosion and loss". Table 4.3.2 is the detailed data and Figure 4.3.2 is the fitting curve.

Table 4.3.2 the data of slope and soil loss used for model fitting

	Slope θ (°)			
	3	13	25	50
Soil loss z (t/ha)	3.75	27.2	147.0	273.0

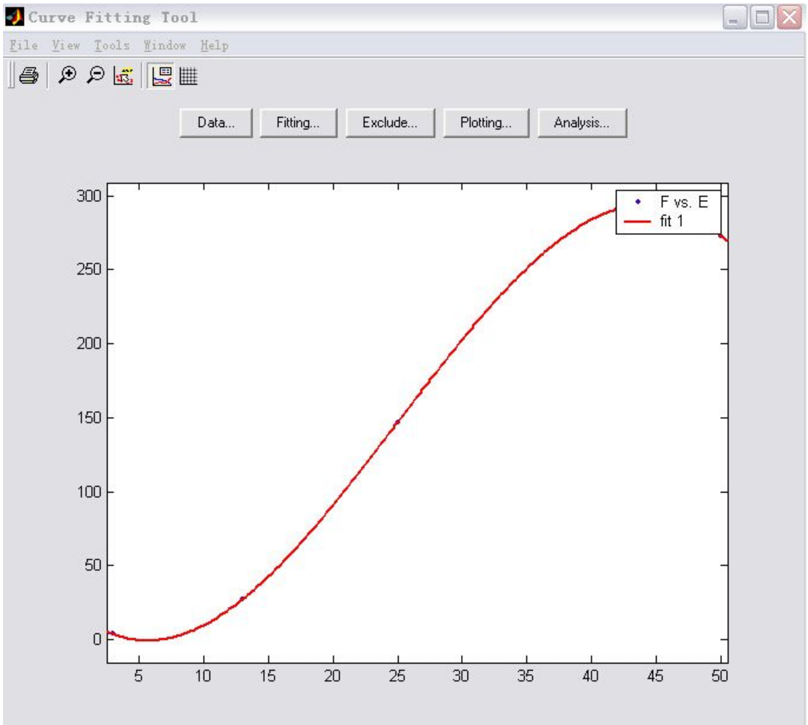

Figure 4.3.2 the fitting curve of slope and soil loss

Coefficients				Goodness of fit			
P1	P2	P3	P4	RMSE	SSE	R-square	Adjusted R-square
-0.01023	0.7666	-7.701	20.23	N/A	7.289e-025	1	N/A

Fitting model is exponential model as below:

f(x) = p1*x^3 + p2*x^2 + p3*x + p4

Combing the result of model fitting, the relationship between slope and soil loss can be expressed as $z = -0.01023 \times \theta^3 + 0.7666 \times \theta^2 - 7.701 \times \theta^1 + 20.03$

④ The ecological benefit: The soil loss may still occur during the growth of plant. In this study, we chose the first three years of planting as study temporal scale. We assumed that there is no plant cover in the plot in the first place, and then we can calculate the annual soil loss based on the formula we got. After one year planting, we got canopy cover ρ_1 and corresponding y_1. The soil conservation rate can be calculated as $\left(1 - \frac{y_1}{y_0}\right)$. We then can get the soil conservation rate of the second and the third year based on the same formula. In addition, the total soil conservation rate can be expressed as $\eta = \left(3 - \frac{y_{sum}}{y_0}\right)$, where η represents the ecological benefit.

The index of ecological benefit (δ) will be obtained according to the soil conservation rate to evaluate the reasonability of planting. The detailed model will be built in the following chapters.

4.3.1 The choice on the way of planting

In order to present the influence of plant growth on the land cover, we used Visual Basic to simulate the process of plant growth and calculate the changes of each index.

According to the relevant documents, the average radius of a three years old *Robinia pseucdoacacia* can reach up to 2m. A new branch can sprout after three month growth.

Fibonacci sequence was used to reflect the dynamic changes of the crown of *Robinia pseucdoacacia*. Fibonacci sequence are the numbers in the following integer sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ⋯

By definition, the first two numbers in the Fibonacci sequence are 1 and 1, or 0 and 1, depending on the chosen starting point of the sequence, and each subsequent number is the sum of the previous two.

In mathematical terms, the sequence Fn of Fibonacci numbers is defined by the recurrence relation

$$a_n = a_{n-1} + a_{n-2}$$

In our study, we need to calculate the Fn every three month and twelve times in total. The formula of Fn can be written as:

$$a_n = \frac{1}{\sqrt{5}} \times \left[\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right]$$

We conduct the formula in VB environment to simulate the dynamic growth of *Robinia pseucdoacacia*. The real length (m) vs length in VB (ppi) is 1:500. The VB code is attached in appendix. The calculation process of Fn in VB is as follows:

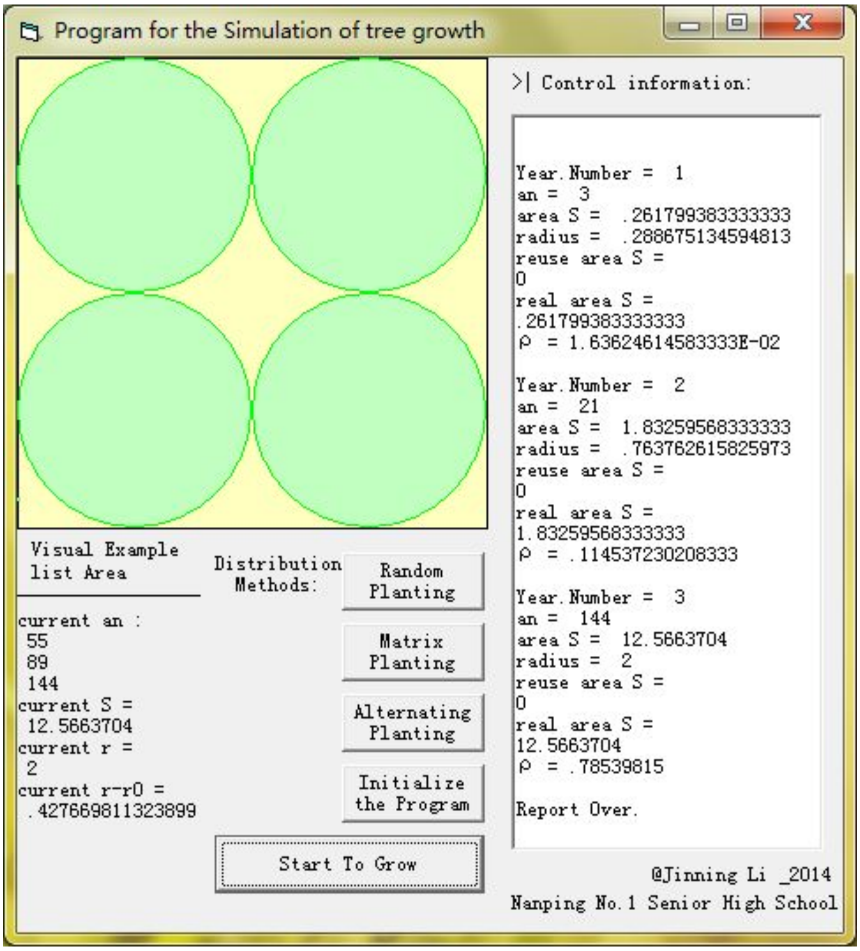

Figure 4.3.1 The interface of simulation program of tree growth

We calculated the increase-rate of crown (f_z) based on the following formula:

$$f_z = \frac{\pi \times 2^2}{a_{13}} = \frac{1}{36}\pi$$

The current crown area (S) can be calculated using the increase-rate of crown (f_z) multiply current number of branches ($current\ a_n$), and the formula is as follows:

$$S = f_z \times a_n$$

In VB environment, we can identify the overlap area of the tree crown (S_c), and the effective coverage area (S_f) can be reflected by S—S_c. In addition, the change of canopy cover is calculated by the following formula:

$$\rho = \frac{S_f}{S_{all}} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n - 36\pi\sqrt{5} \times \mathrm{S}_c}{2304\pi\sqrt{5}}$$

Note: Since some surplus parts (S_s) exists in the alternating planting, we can get a certain value for S_s in VB environment, $S_s = 16 \times \left(2-\sqrt{3}\right)$. The change of canopy cover can be correspondingly transformed to

$$\rho = \frac{S_f}{S_{all} - S_s} = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n - 36\pi\sqrt{5} \times \mathrm{S}_c}{2304\pi\sqrt{5} + 16\sqrt{3} - 32}$$

We then got the indexes of different planting ways by various time scales.

Table 4.3.1 the indexes of different planting ways by various time scales.

The end of the first year				
	S	S_c	r	ρ
Random planting	0.262	0.001	0.289	0.016
Matrix Planting	0.262	0	0.289	0.016
Alternating planting	0.262	0	0.289	0.017
The end of the second year				
	S	S_c	r	ρ
Random planting	1.832	0.057	0.764	0.111
Matrix Planting	1.832	0	0.764	0.115
Alternating planting	1.832	0	0.764	0.123
The end of the third year				
	S	S_c	r	ρ
Random planting	12.566	6.288	2.000	0.634
Matrix Planting	12.566	0	2.000	0.785
Alternating planting	12.566	0	2.000	0.842

Note: Because the bias caused by random planting may be large, we repeated the calculation of random planting many times and took the average value of the results.

Data analysis:

According to the table 4.3.1, there are not obvious differences among three planting ways in the beginning of planting. However, the variation is getting large along with time. There are three conclusions:

(1) For random planting, the overlap area of canopy is getting greater along with the time length of planting. It caused a low efficiency of plant using.

(2) The canopy cover of three planting ways is alternate planting> matrix planting > random planting. In addition, the overlap area of canopy of alternate and matrix

planting are zero. Thus, we concluded that these two planting ways would be more suitable in the study area.

(3) Compared with matrix planting, the alternate planting gains a greater canopy cover. Therefore, the alternate planting will have the lowest costs and highest ecological benefit when we have the same number of plants.

To sum up, the alternate planting would be our best choice for tree planting.

4.3.2 Functional Relationship between Canopy Density ρ and Number n

After determining a alternately distributed method, the canopy density has been calculated when $n=1$、2、3、4、9, resulted in the following table:

Table 4.3.2-1 The number of trees and protected area

Area / Year / Number	The first year	The second year	The third year
1	0.2616	1.8316	12.56
2	0.5232	3.6632	25.12
3	0.7848	5.4948	37.68
4	1.0464	7.3264	50.24
9	2.3544	16.4844	64.00

Note: When n=9, the overlap occurs in the third year, so the covered area equals to the total area.

So we got the relationship form of Canopy Density ρ and Number n: (Canopy Density ρ_0 = Effective canopy coverage area S_F ／Land area S_0)

Table 4.3.2-2 The number of trees and canopy cover

ρ / Year / Number	The first year	The second year	The third year
1	0.0040875	0.02861875	0.19625
2	0.008175	0.0572375	0.3925
3	0.0122625	0.08585625	0.58875
4	0.01635	0.114475	0.785
9	0.0367875	0.25756875	1.0000

And then, we used $y = 43770 \times \rho^{-0.7442}$, which we got before, to get y_1、y_2、y_3、y_{sum}, as follows:

Table 4.3.2-3 The number of trees and erosion modulus

Number \ Erosion modulus	y_1	y_2	y_3	$y_{和}$
1	92106.36334	21641.98455	5164.491316	113767.9729
2	54987.3908	12920.23937	3083.195253	67946.88017
3	40664.55251	9554.840569	2280.1001	50278.26808
4	32827.40776	7713.367739	1840.663947	40619.2755
9	17953.22029	4218.4199	1537.221693	22271.64019

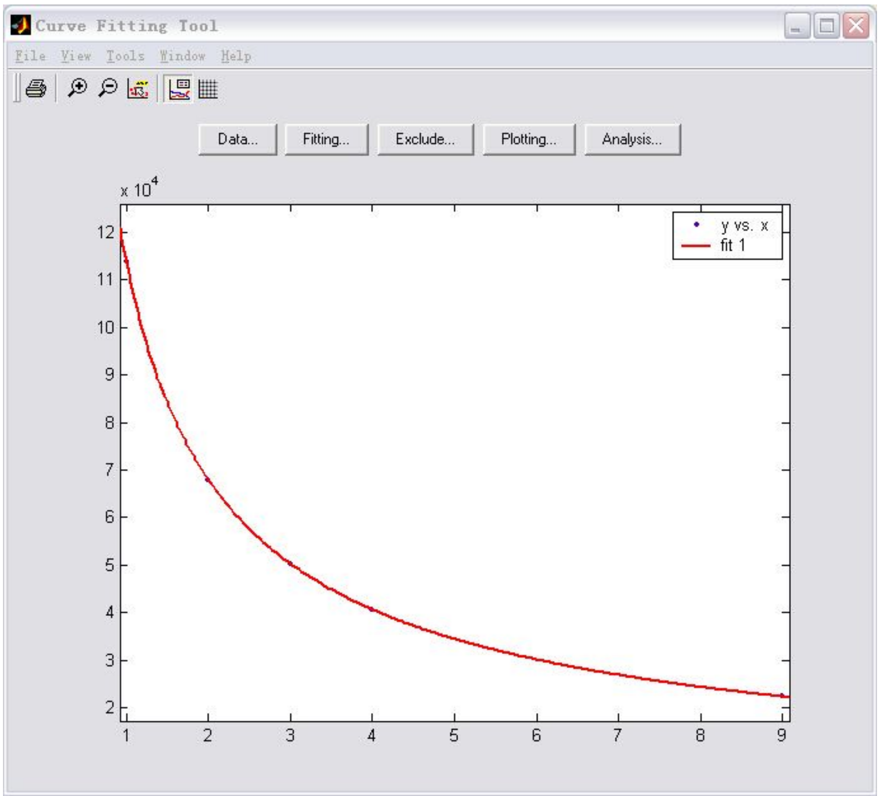

Figure 4.3.2-1 The fitting curve between erosion modulus and number of trees

By function-fitting, the functional relationship between y_{sum} and n is: $y_{sum} = 113600 \times n^{-0.745} + 168.2$. In practice, the soil and water conservation effect was determined by the terrain (means the slope) and the number of the plants. Specifically, the steeper slope, the faster soil erosion, and the less obvious effect of soil and water conservation; while the more plants, the better conservation effect. So this study defined that the vegetation ecological efficiency δ to calculate the effect of soil and water conservation.

Combined with the actual situation, δ (Ecological efficiency) should positively correlate with the soil and water conservation rate of bare land and plants number. In those two, the soil and water conservation rate of bare land is the base, and the vegetation protection effect is an addition reaction.

More specifically, δ = the soil and water conservation rate of bare land + the soil and water conservation rate of plants covered land（the soil and water conservation rate of bare land $=1-\frac{z_\theta}{z_{all}}$. z is soil erosion rate. And based on material resource, $z_{all}=325.9$, average annual soil erosion rate）

After simplifying, soil and water conservation rate of vegetation:

$$\eta=\left(3-\frac{113600\times n^{-0.745}+168.2}{260097.68}\right),\ \left(\eta=\left(3-\frac{y_{sum}}{y_0}\right),\ y_0\approx 260097.68\right)$$

After calculated，we got δ：

$$\delta=\left(4-\frac{z_\theta}{325.9}-\frac{113600\times n^{-0.745}+168.2}{260097.68}\right)$$

4.3.3 Planning for Economic Efficiency and Ecological Benefits

With reality, when the cost is higher, the ecological benefits is higher, while the focus of vegetation restoration is ecological benefits, and this issue needs to ensure the consistency and comparability of δ and λ. To make the results more visible, we defined that $l=\frac{\delta}{\lambda^{0.03}}$. When l is higher, the ecological benefits by per unit cost is higher, and this program is also more suitable.

① **0°~20° slope land (** using θ average value as 10° to calculate)

$$z_\theta = 10.03, \text{ so } \delta = \left(4 - \frac{10.03}{329.5} - \frac{113600 \times n^{-0.745} + 168.2}{260097.68}\right); \quad \lambda = 10 \times n$$

According to Lingo 12.0， we did a nonlinear programming to ecological benefits and economic efficiency. At the highest l value point, we got the related results as following：

At this point, N is 4.065007.

And when $n = 4.065007$, $l_{\max} = 0.0391972$, $\delta_{10^\circ} = 3.8153$. As a result, when the land slope degree is from 0° to 20°, planting 4.065007 trees on $64m^2$ land area is the best.

②. **20°~40° slope land** (using θ average value as 30° to calculate)

$$z_\theta = 196.58, \text{ so } \delta = \left(4 - \frac{196.58}{329.5} - \frac{113600 \times n^{-0.745} + 168.2}{260097.68}\right); \quad \lambda = 10 \times n; \quad l = \frac{\delta}{\lambda^{0.03}}$$

To do a nonlinear programming (lingo report, in appendix), we got:

When $n = 4.997707$, $l_{\max} = 0.0333982$, and $\delta_{10^\circ} = 3.2710$.

From the results, we got that the ecological benefits would be lower with a steeper slope, but the overall efficiency index was highest, and the economic efficiency had been taken into account. So this is a practical result.

As a result, when the slope degree is between 20° and 40°, planting 4.997707 trees on every $64\,m^2$ area of land is the best.

③ **Steeper than 40° slope land**

In the digital elevation model, the maximum slope is 50°, so in this study we used the average value 45° of 40° and 50° to conduct the research.

$$\text{When } z_\theta = 286.96, \quad \delta = \left(4 - \frac{286.96}{329.5} - \frac{113600 \times n^{-0.745} + 168.2}{260097.68}\right);$$

Using Lingo 12.0 to do the nonlinear programming, and got:

When $n = 5.594923$, $l_{\max} = 0.0306026$, and $\delta_{10^\circ} = 3.0074$。

So we concluded that, when the slope degree is between 40° to 60°, planting 5.594923 trees on every 64 m^2 area of land is the best.

Using the above date, the reasonable planting density in unit land area is showed as the following table:

Slope（°）	0-20	20-40	40-60
Reasonable density(individual plant/ m^2)	0.063516	0.078089	0.087421

4.4 The planting zoning based on terrain

According to the above studies, we know that both soil loss and the density of planting are closely related to slope. Due to the complex terrain of the study area, we established the fourth model to zone the suitable planting location a based on terrain.

The transverse section (Figure 4.4.1-1) and longitudinal section (Figure 4.4.1-2) were generated using Global Mapper and DEM.

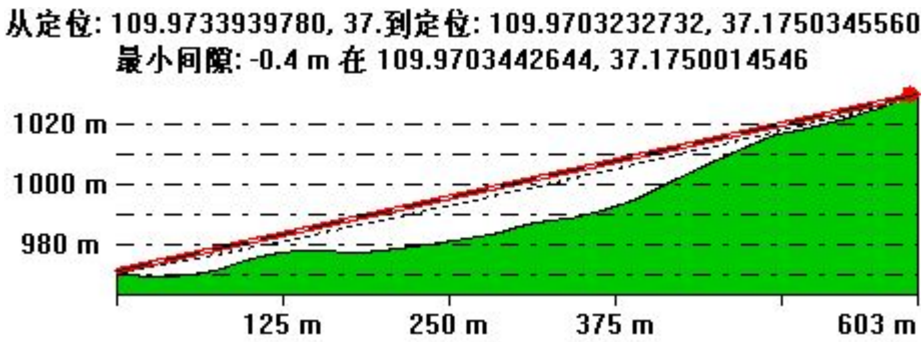

Figure 4.4.1-1 longitudinal section：z-y

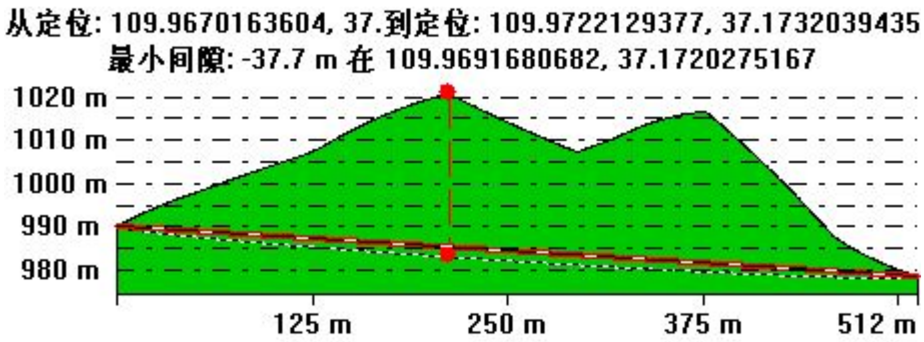

Figure 4.4.1-2 transverse section z-x

We selected some typical points and conducted interpolation for the plot in Matlab (The code is in the appendix). The functions of Z-Y and Z-X can also be calculated in Matlab. The process of model fitting is as follows:

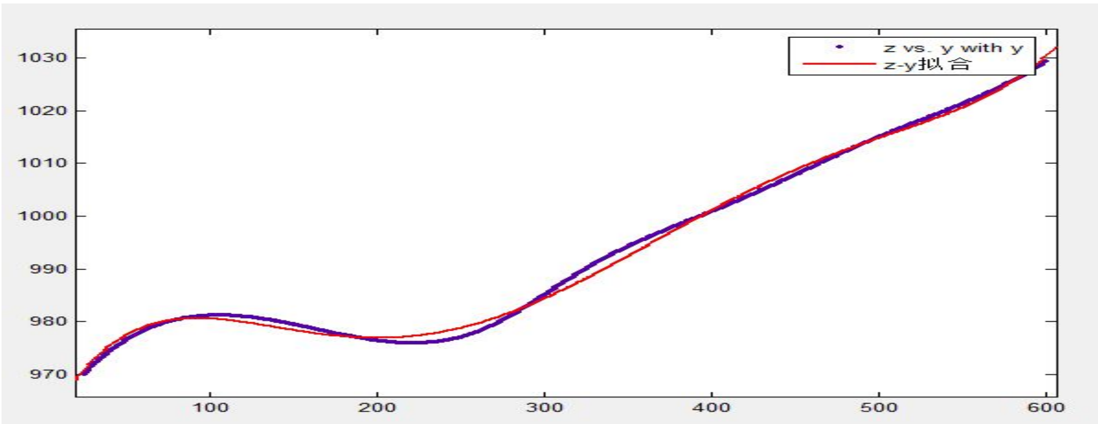

Figure 4.4.1-3 Fitting curve of Z-Y

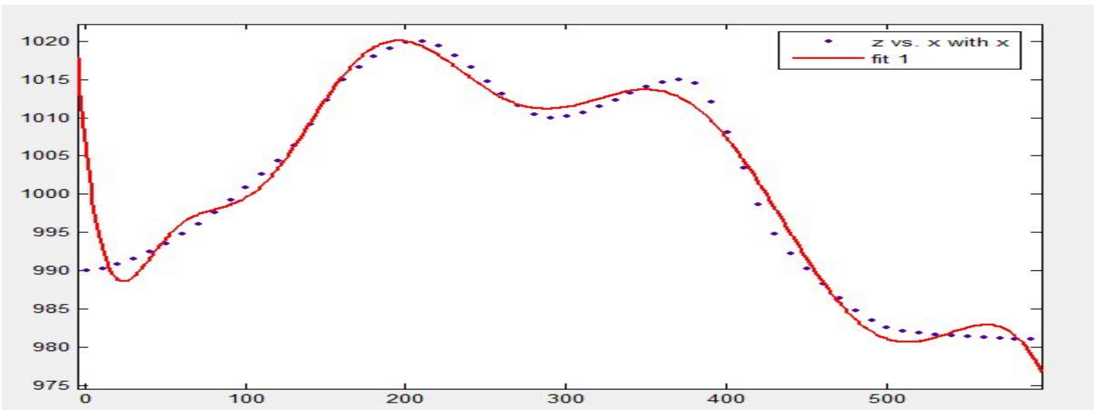

Figure 4.4.1-4 Fitting curve of Z-X

Note: the range of X is $25 \le x \le 600$.

After comparing the goodness of fit, the fifth power of polynomial had a good performance on data fitting. Linear model Poly5:

f(x) = p1*x^5 + p2*x^4 + p3*x^3 + p4*x^2 + p5*x + p6

Coefficients (with 95% confidence bounds)					
P1	P2	P3	P4	P5	P6
2.33e-011 (2.20e-011, 2.45e-011)	-3.929e-008 (-4.13e-008, -3.728e-008)	2.382e-005 (2.257e-005, 2.507e-005)	-0.006 (-0.006, -0.005)	0.615 (0.570, 0.660)	959 (957, 961)

Goodness of fit			
RMSE	SSE	R-square	Adjusted R-square
16.37	1.528e+005	0.997	0.997

The function of Z-Y is as follows:

$$F_1(y) = 2.33\times10^{-11}y^5 - 3.929\times10^{-8}y^4 + 2.382\times10^{-5}y^3 - 6\times10^{-3}y^2 + 0.6151y + 959$$

The same method was used to build the function of Z-X.

After comparing the goodness of fit, the fifth power of Gauss had a good performance on data fitting. General model Gauss5:

f(x) = a1*exp(-((x-b1)/c1)^2) + a2*exp(-((x-b2)/c2)^2) +
a3*exp(-((x-b3)/c3)^2) + a4*exp(-((x-b4)/c4)^2) +
a5*exp(-((x-b5)/c5)^2)

Coefficients		Upper 95% confidence	Lower 95% confidence
a1	770.5	-6441	7982
b1	123.9	-93.64	341.5
c1	166.2	-821.2	1154
a2	822.2	-2504	4148
b2	644.9	92.41	1197
c2	187.6	-341.4	716.5
a3	-6370	-6.382e+006	6.369e+006
b3	-67.14	-5641	5506
c3	94.08	-4009	4197
a4	830.7	-4824	6485
b4	368.6	24.42	712.7
c4	193.4	-1143	1529
a5	7828	-6.376e+006	6.391e+006
b5	-79.14	-5295	5137
c5	103.7	-4452	4660

Goodness of fit			
RMSE	SSE	R-square	Adjusted R-square
28.95	3.771e+004	0.9899	0.9868

The function of Z-X ($F_2(x)$) is as follows:

$$F_2(x)=770\times e^{-(\frac{x-124}{166})^2}+822\times e^{-(\frac{x-645}{188})^2}-6370\times e^{-(\frac{x-67.14}{94.08})^2}+830.7\times e^{-(\frac{x-369}{193})^2}+7828\times e^{-(\frac{x-79.4}{103.7})^2}$$

We determined the relationship between the absolute of slope (tan value of angle of terrain) and the change of y and x through calculating the first-order derivative of $F_1(y)$, $F_2(x)$ in matlab.

The first-order derivative of $F_1(y)$ is：

F1’=36055003644186705/309485009821345068724781056*y^4
-5937336931582347/377789318629571617 09568*y^3
+21091269304116555/295147905179352825856*y^2-3/250*y+6151/10000

Can be simplified as：

$$F_1'(y) = 1.16\times10^{-10}y^4 - 1.57\times10^{-7}y^3 + 0.715\times10^{-4}y^2 - 0.012y + 0.6151$$

The first-order derivative of $F_2(x)$ is：

F2’=770*(-1/13778*x+62/6889)*exp(-(1/166*x-62/83)^2)+822*(-1/17672*x+645/17672)*exp(-(1/188*x-645/188)^2)-6370*(-625/2765952*x-9325/614656)*exp(-(25/2352*x+1119/1568)^2)+8307/10*(-2/37249*x+738/37249)*exp(-(1/193*x-369/193)^2)+7828*(-200/1075369*x-15880/1075369)*exp(-(10/1037*x+794/1037)^2)

Can be simplified as：

$$F_2'(x) = 770\times\left(\frac{-1}{13778}x+\frac{62}{6889}\right)\times e^{(-(\frac{1}{166}x-\frac{62}{83})^2)} + 822\times(\frac{-1}{17672}x+\frac{645}{17672})\times\exp(-(\frac{1}{188}x-\frac{645}{188})^2)$$
$$-6370\times(-\frac{625}{2765952}x-\frac{9325}{614656})\times e^{(-(\frac{25}{2352}x+\frac{1119}{1568})^2} + 830.7\times(\frac{-2}{37249}x+\frac{738}{37249})\times e^{(-(\frac{1}{193}x-\frac{369}{193})^2)}$$
$$+7828\times(\frac{-200}{1075369}x-\frac{15880}{1075369})\times e^{(-(\frac{10}{1037}x+\frac{794}{1037})^2)}$$

The points that were used in the analysis of X-Z is between (1,1) and (600,600). Due to the inaccuracy of mode fitting of Z-X between 0 and 25, we removed the corresponding function value of $F_2'(x)$ upon X=0-25andThe adjusted function is $F_1'(y)$. The absolute of slope of the other points was calculated as $\left[\left|F_1'(y)\right|+\left|F_2'(x)\right|\right]/2$. Given the complex function expression, we used V.B to perform the calculation. The detailed code was in the appendix. We set up 10 as the minimum unit distance between X and Y .

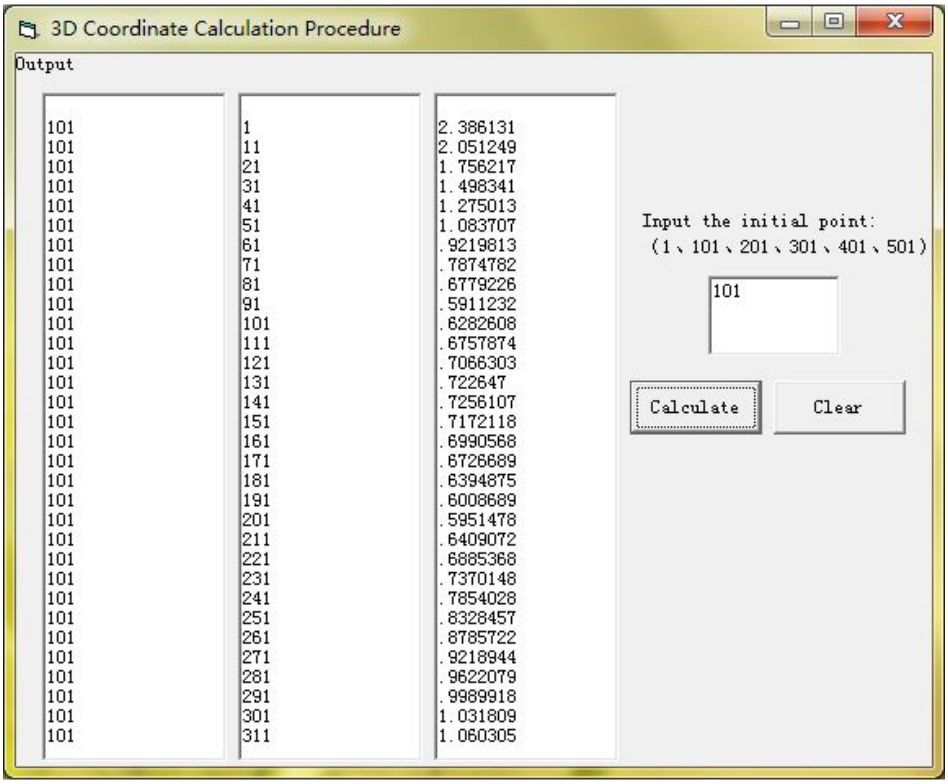

Figure 4.4.1-5 3D coordinate calculation procedure interface

A large amount of data of 3D space was obtained in this project. The data were inputted into EXCEL, three ranks (respectively x, y, z) and 3600 rows in total. A interpolation operation was conducted in Surfer 8.0. The file was converted to grid through linear interpolation triangulation method. The 3D surface of slope, 3D grid map and contour map (projection drawing) were got using the drawing function of Surfer8.0.

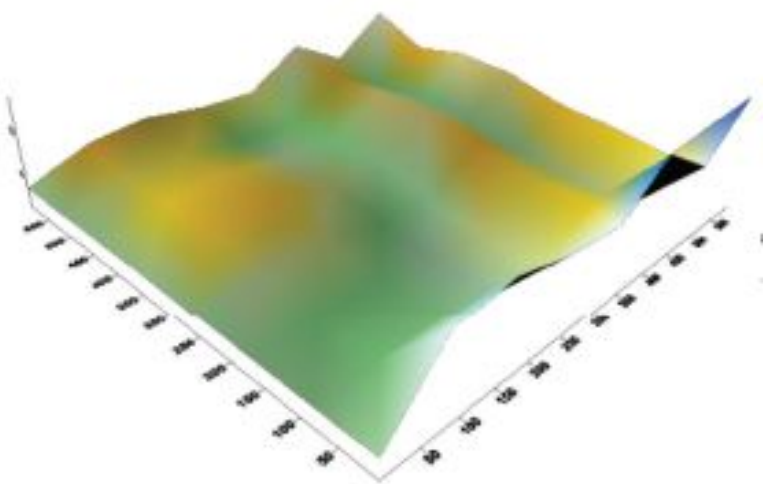

Figure4.4.1-6 3D surface

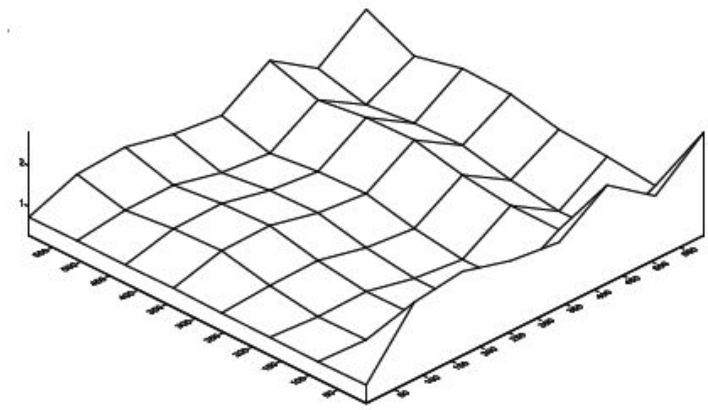

Figure 4.4.1-7 3D grid

After generating the contour map with surfer 8.0 (here it can be used as equal slope graph), the study area can be zoned based on the main consideration factor, slope. The zoned area is shown as follows:

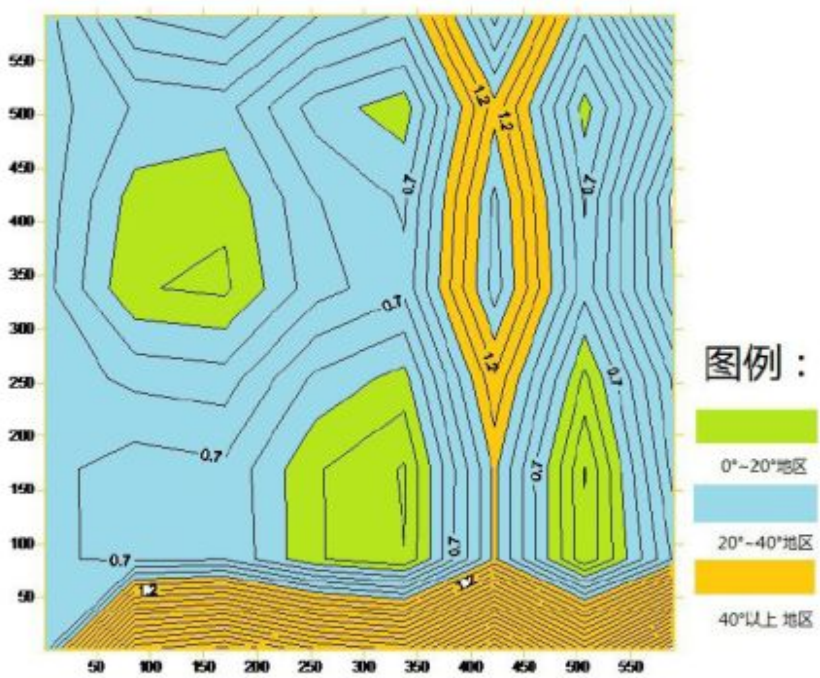

Figure 4.4.1-8 Schematic diagram of slope distribution

According to the output information of Surfer 8.0, the area of the regions can be calculated. Combing with the former study, we got a comprehensive table in which the information of reasonable density, total amount of trees and the costs of trees were listed.

Table 4.4.1.1 Reasonable density and the purchase of trees

Region(°)	0～20	20～40	Greater than 40
Area(m^2)	81538.145	193803.595	84658.259
Reasonable density(trees/m^2)	0.063516	0.078089	0.087421
The amount of trees	5179	15134	7401
The total amount	27714		

4.5 Model summary and conclusion

(1) Triennial acacia is selected as the best optimal tree by using principal component analysis.

(2) By simulating tree growth, alternative distribution is the most optimal planting pattern.

(3) We determined the planting densities with the maximum comprehensive benefits using nonlinear programming.

(4) The zoning and weight calculation of study area were conducted by model fitting and data interpolation.

Above all, the final conclusions are given as follows:

(1) Three years old acacia is suitable species for the ecological restoration of the Loess Plateau.

(2) The alternative planting is the suitable way to plant.

(3) The total number of 27714 trees are needed, in which 5179 trees should be planted on the region of 0°～20° slope at density of 0.064 trees/ m^2 ; 15134 trees should be planted on the region of 20°～40° slope at density of 0.078 trees/ m^2; 7401 trees should be planted on the region of 40°～60° slope at density of 0.087 trees/ m^2

(The regional distribution is shown in figure 4.4.1.8). It is expected that after three years, the local ecological environment will be well restored at the low cost.

5 Evaluation criteria and model evaluation

5.1 The establishment of optimal plan of ecological restoration

In order to establish the optimal plan of ecological restoration of the Loess Plateau, we used several methods to conduct species selection, canopy cover estimation and optimal planting density selection, respectively. We then proposed a comprehensive plan of ecological restoration of Loess Plateau based on the result of above study.

5.2 Evaluation of the optimal model

Due to the close interaction among four models, we evaluated them individually. The scientific evaluation method was applied in tree species selection model, and it can select the suitable tree. Planting pattern model can describe dynamic development of different planting ways. Alternative planting was identified as the most suitable planting way. Planting density model can take into account the ecological benefits and economic benefits. The planting density we got can fit the results of model 1 and model 2 and can also match the reality situation. The terrain slope model integrated the above three models and can consider the real situation more specifically.

In conclusion, the plan we proposed is a reasonable and scientific scheme for the ecological restoration in the Loess Plateau.

6 The Improvement and Promotion of the model

6.1 The improvement of the model

In order to improve the applicability of the prediction model, few improvements can be done in the future:

(1) After planting, the canopy cover would not be the only factor associated with n since the tree morality will be inevitable. To address this issue, we can take into

account the mortality rate when conducting the Fibonacci sequence, which will be more practical.

(2) After planting, the trees should be cared. In the subsequent work, we will combine TSP method and caretakers to maintain the plant.

(3) In reality, rainfall is relatively concentrated, which is not fully consistent with our original assumption (the weather condition keeps constant). More rain happens in the summer and the autumn. The model would be more accurate if real weather condition is considered.

(4) The depth and breadth of plant root can greatly impact the effect of soil and water conservation. So, similar to crown density, the project should also determine the relationship between root growth and ecological benefits by Fibonacci sequence, and then the results would be improved accordingly.

(5) In addition, the variation coefficient or Moran index method can also be used to determine the tree distribution.

(6) The process of ecological recovery can be divided into four stages including wasteland capturing, soil consolidating and original species introduction. Future studies can be conducted based on the difference stages.

6.2 Promotion of model

The project can be used on residential area planning or dealing with the issue of raising coverage rate. Besides, it can also provide the support on tree planting activities for schools on annual tree planting day.

Because the project involves 3D distribution, in addition to land planning, it can also be used on the airdropped supplies or sea fishery feeding operation planning. Moreover, the slope fitting method in the project can also be applied to the land surveying and mapping, geographic information engineering etc.

References:

[1] The National Development and Reform Commission, "the outline of comprehensive planning of the Loess Plateau (2010-2030 year) ", 2011

[2] " The main tree species and techniques of forestation in the Loess Plateau area ", pndasz, 2012

[3] "The technical points and main tree species of forestation", anonymous, 2009

[4] Soil and Water Conservation Bureau of Shanxi province, "the rules of soil and water conservation", 1999

Regarding the research group:

The team members came from honors program of school. They won First Prize (1st round) and the Grand Prize (1st round) of the 7th China Network Challenge for Mathematical Modeling, respectively; they were also awarded First Prize in Math, Physics and biology Leagues.

Appendix

1. The program of matlab for comprehensive evaluation index F

```
X1=[1.50 2.30 1.90 1.30 2.10 1.80 1.90 3.10 2.20]
X2=[4.40 3.90 2.40 6.00 5.20 4.90 3.40 5.80 4.20]
X3=[3.30 3.50 2.90 4.50 4.40 3.80 3.00 4.00 3.70]
X4=[1.90 2.40 1.70 2.00 2.10 1.50 2.20 2.40 3.00]
F=0.422*X1+0.286*X2+0.276*X3+0.423*X4
```

2. The program of Visual Basic for the simulation of plant growth

```
Dim i As Integer: Dim a As Single: Dim b As Single: Dim c As Single
Dim s As Double: Dim ti As Integer: Dim r As Double
Dim x1 As Double: Dim y1 As Double: Dim x2 As Double: Dim y2 As Double
Dim x3 As Double: Dim y3 As Double: Dim x4 As Double: Dim y4 As Double
Dim sc As Double: Dim sc1 As Double: Dim sc2 As Double: Dim sc3 As Double
Dim sc4 As Double: Dim sc5 As Double: Dim sc6 As Double: Dim r0 As Double:
Dim pd As Integer
Private Sub Command1_Click()
Text1.Text = ""
pd = 0
Randomize
R1.Left = 4000 * Rnd:
R1.Top = 4000 * Rnd: x1 = R1.Left: y1 = R1.Top
R2.Left = 4000 * Rnd
R2.Top = 4000 * Rnd: x2 = R2.Left: y2 = R2.Top
R3.Left = 4000 * Rnd
R3.Top = 4000 * Rnd: x3 = R3.Left: y3 = R3.Top
R4.Left = 4000 * Rnd
R4.Top = 4000 * Rnd: x4 = R4.Left: y4 = R4.Top
Label8.Visible = True: Label7.Visible = True
Label7.Caption = "(" & x1 & "," & y1 & ")" & "  " & "(" & x2 & "," & y2
& ")" & vbCrLf & "" & vbCrLf & "(" & x3 & "," & y3 & ")" & "  " & "(" &
x4 & "," & y4 & ")"
End Sub
Private Sub Command2_Click()
Label7.Visible = False: Label8.Visible = False
Text1.Text = ""
pd = 0
R1.Left = 950
R1.Top = 950: x1 = R1.Left: y1 = R1.Top
R2.Left = 2950
R2.Top = 950: x2 = R2.Left: y2 = R2.Top
```

```
R3.Left = 950
R3.Top = 2950: x3 = R3.Left: y3 = R3.Top
R4.Left = 2950
R4.Top = 2950: x4 = R4.Left: y4 = R4.Top
End Sub
Private Sub Command3_Click()
Label7.Visible = False: Label8.Visible = False
pd = 1
R1.Left = 950
R1.Top = 950: x1 = R1.Left: y1 = R1.Top
R2.Left = 2950
R2.Top = 950: x2 = R2.Left: y2 = R2.Top
R3.Left = 1950
R3.Top = 2950 - 1000 * (2 - Sqr(3)): x3 = R3.Left: y3 = R3.Top
Print R3.Top
R4.Left = 3950
R4.Top = 2950 - 1000 * (2 - Sqr(3)): x4 = R4.Left: y4 = R4.Top
Text1.Text = "special dispose:" & vbCrLf & "List Area surplus"
End Sub
Private Sub Command4_Click()
Label7.Visible = False: Label8.Visible = False
Timer1.Enabled = True
ti = 1
a = 0: b = 1: n = 12
r0 = 0.1
End Sub
Private Sub Command5_Click()
Unload Me
   Load Me
   Me.Show
Load Me
End Sub

Private Sub Timer1_Timer()
Form1.Cls
Me.CurrentY = 4500
Print "": Print "current an :"
Print a: Print b
c = a + b
Print c
s = c * 3.1415926 / 36
r = Sqr(s / 3.1415926)
Print "current S =  ": Print s: Print "current r =  ": Print r: Print
"current r-r0 =  ": Print r - r0
```

```
R1.Height = 2 * r * 500: R1.Width = 2 * r * 500
R1.Top = R1.Top - (r - r0) * 500: R1.Left = R1.Left - (r - r0) * 500
R2.Height = 2 * r * 500: R2.Width = 2 * r * 500
R2.Top = R2.Top - (r - r0) * 500: R2.Left = R2.Left - (r - r0) * 500
R3.Height = 2 * r * 500: R3.Width = 2 * r * 500
R3.Top = R3.Top - (r - r0) * 500: R3.Left = R3.Left - (r - r0) * 500
R4.Height = 2 * r * 500: R4.Width = 2 * r * 500
R4.Top = R4.Top - (r - r0) * 500: R4.Left = R4.Left - (r - r0) * 500
a = b: b = c
ti = ti + 1
r0 = r
If ti = 4 Or ti = 8 Or ti = 12 Then
a1:
If (2 * r - Abs(y1 - y2) / 500) < 0 Or (2 * r - Abs(x1 - x2) / 500) < 0
Then GoTo a2
sc1 = (2 * r - Abs(y1 - y2) / 500) * (2 * r - Abs(x1 - x2) / 500)
sc = sc + sc1
a2:
If (2 * r - Abs(y1 - y3) / 500) < 0 Or (2 * r - Abs(x1 - x3) / 500) < 0
Then GoTo a3
sc2 = (2 * r - Abs(y1 - y3) / 500) * (2 * r - Abs(x1 - x3) / 500)
sc = sc + sc2
a3:
If (2 * r - Abs(y1 - y4) / 500) < 0 Or (2 * r - Abs(x1 - x4) / 500) < 0
Then GoTo a4
sc3 = (2 * r - Abs(y1 - y4) / 500) * (2 * r - Abs(x1 - x4) / 500)
sc = sc + sc3
a4:
If (2 * r - Abs(y2 - y3) / 500) < 0 Or (2 * r - Abs(x2 - x3) / 500) < 0
Then GoTo a5
sc4 = (2 * r - Abs(y2 - y3) / 500) * (2 * r - Abs(x2 - x3) / 500)
sc = sc + sc4
a5:
If (2 * r - Abs(y2 - y4) / 500) < 0 Or (2 * r - Abs(x2 - x4) / 500) < 0
Then GoTo a6
sc5 = (2 * r - Abs(y2 - y4) / 500) * (2 * r - Abs(x2 - x4) / 500)
sc = sc + sc5
a6:
If (2 * r - Abs(y3 - y4) / 500) < 0 Or (2 * r - Abs(x3 - x4) / 500) < 0
Then GoTo a7
sc6 = (2 * r - Abs(y3 - y4) / 500) * (2 * r - Abs(x3 - x4) / 500)
sc = sc + sc6
a7:
sc = sc * 0.8
```

```
If pd = 0 Then
Text1.Text = Text1.Text & vbCrLf & "" & vbCrLf & "Year.Number =  " & ti
/ 4 & vbCrLf & "an =  " & c & vbCrLf & "area S =  " & s & vbCrLf & "radius
=  " & r & vbCrLf & "reuse area S =  " & vbCrLf & sc & vbCrLf & "real area
S =  " & vbCrLf & s - sc & vbCrLf & "ρ = " & (s * 4 - sc) / 64
sc = 0
If ti = 12 Then Text1.Text = Text1.Text & vbCrLf & "" & vbCrLf & "Report
Over."
End If
If pd = 1 Then
sc = 0
Text1.Text = Text1.Text & vbCrLf & "" & vbCrLf & "Year.Number =  " & ti
/ 4 & vbCrLf & "an =  " & c & vbCrLf & "area S =  " & s & vbCrLf & "radius
=  " & r & vbCrLf & "reuse area S =  " & vbCrLf & sc & vbCrLf & "real area
S =  " & vbCrLf & s - sc & vbCrLf & "ρ = " & (s * 4 - sc) / (64 - 1000
* (2 - Sqr(3)) * 4000 / 500 / 500)
sc = 0
If ti = 12 Then Line1.x2 = 4000: Text1.Text = Text1.Text & vbCrLf & ""
& vbCrLf & "Report Over.": Shape1.Visible = True
End If
If ti = 12 Then
Timer1.Enabled = False
End If
End If
End Sub
```

3. Non-linear planning of lingo (30°and 45°):

```
  Local optimal solution found.
  Objective value:                              0.3116890E-01
  Infeasibilities:                               0.000000
  Extended solver steps:                                5
  Total solver iterations:                             84


                       Variable           Value        Reduced Cost
                              N        4.997707        -0.3465460E-08
                              B        104.9453          0.000000
                              A       0.3000000E-01       0.000000

                            Row    Slack or Surplus      Dual Price
                              1       0.3116890E-01       1.000000
                              2        0.000000        -0.2970013E-03
                              3        0.000000        -0.5015996E-01
                              4        4.997707          0.000000
```

```
                         5      15.00229          0.000000
 Local optimal solution found.
  Objective value:                          0.2855957E-01
  Infeasibilities:                           0.000000
  Extended solver steps:                            5
  Total solver iterations:                         84


                   Variable          Value        Reduced Cost
                          N       5.594923          0.000000
                          B       105.3013          0.000000
                          A      0.3000000E-01       0.000000

                        Row    Slack or Surplus      Dual Price
                          1      0.2855957E-01       1.000000
                          2       0.000000        -0.2712176E-03
                          3       0.000000        -0.4918591E-01
                          4       5.594923          0.000000
                          5       14.40508          0.000000
```

4. The fitting of Cross and Longitudinal section

Z-Y Section:

```
clear
z1=[970 980 978 990 1003 1015 1030 1020]
y1=[25 141 260 325 416.6 500 603 540]
y=25:1:600
z=interp1(y1,z1,y,'spline')
f_ = clf;
figure(f_);
legh_ = []; legt_ = {};
xlim_ = [Inf -Inf];
ax_ = subplot(1,1,1);
set(ax_,'Box','on');
axes(ax_); hold on;
y = y(:);
z = z(:);
h_ = line(y,z,'Parent',ax_,'Color',[0.333333 0 0.666667],...
    'LineStyle','none', 'LineWidth',1,...
    'Marker','.', 'MarkerSize',12);
xlim_(1) = min(xlim_(1),min(y));
xlim_(2) = max(xlim_(2),max(y));
legh_(end+1) = h_;
legt_{end+1} = 'z vs. y with y';
if all(isfinite(xlim_))
```

```
  xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
  set(ax_,'XLim',xlim_)
end
fo_ = fitoptions('method','LinearLeastSquares','Weights',y);
set(fo_,'Weight',y);
ft_ = fittype('poly5' );
cf_ = fit(y,z,ft_ ,fo_);
if 0
  cv_ = {2.32959807651e-011, -3.928863315784e-008, 2.382104410597e-005,
-0.006006926229024, 0.6150574450703, 958.9758556603};
  cf_ = cfit(ft_,cv_{:});
end
h_ = plot(cf_,'fit',0.95);
legend off;
set(h_(1),'Color',[1 0 0],...
    'LineStyle','-', 'LineWidth',2,...
    'Marker','none', 'MarkerSize',6);
legh_(end+1) = h_(1);
legt_{end+1} = 'z-y拟合';
hold off;
legend(ax_,legh_, legt_);
```

Z-X Section:

```
clear
x1=[0 125 208 166.67 291.67 375 437.5 512 600]
z1=[990 1005 1020 1016 1010 1015 993 982 981]
x=1:10:600
z=interp1(x1,z1,x,'cubic')
f_ = clf;
figure(f_);
legh_ = []; legt_ = {};
xlim_ = [Inf -Inf];
ax_ = subplot(1,1,1);
set(ax_,'Box','on');
axes(ax_); hold on;
x = x(:);
z = z(:);
h_ = line(x,z,'Parent',ax_,'Color',[0.333333 0 0.666667],...
    'LineStyle','none', 'LineWidth',1,...
    'Marker','.', 'MarkerSize',12);
xlim_(1) = min(xlim_(1),min(x));
xlim_(2) = max(xlim_(2),max(x));
legh_(end+1) = h_;
legt_{end+1} = 'z vs. x with x';
if all(isfinite(xlim_))
```

```
  xlim_ = xlim_ + [-1 1] * 0.01 * diff(xlim_);
  set(ax_,'XLim',xlim_)
end
fo_ =
fitoptions('method','NonlinearLeastSquares','Weights',x,'Lower',[-Inf
-Inf 0 -Inf -Inf 0 -Inf -Inf 0 -Inf -Inf 0 -Inf -Inf 0 ]);
st_ = [1019.962354158 211 120.942500819 980.9719111556 561 132.9644487533
939.986071433 1 39.28405613799 716.1717698896 381 35.1011022096
689.7540643534 71 38.88582818163 ];
set(fo_,'Startpoint',st_);
set(fo_,'Weight',x);
ft_ = fittype('gauss5' );
cf_ = fit(x,z,ft_ ,fo_);
if 0
  cv_ = {770.4866376311, 123.904554233, 166.2207555932, 822.2387640116,
644.8560420596, 187.5876711202, -6369.823761387, -67.14257070357,
94.07843520002, 830.6752266206, 368.576807058, 193.4436962257,
7828.239456118, -79.13811828727, 103.6561647363};
  cf_ = cfit(ft_,cv_{:});
end
h_ = plot(cf_,'fit',0.95);
legend off;
set(h_(1),'Color',[1 0 0],...
    'LineStyle','-', 'LineWidth',2,...
    'Marker','none', 'MarkerSize',6);
legh_(end+1) = h_(1);
legt_{end+1} = 'z-x拟合';
hold off;
legend(ax_,legh_, legt_);
```

5. Visual Basic program for 3D calculation

```
Dim x As Single: Dim y As Single: Dim i As Integer: Dim z1 As Single: Dim
z2 As Single: Dim z As Single
Dim p As Integer: Dim a As Integer: Dim b As Integer
Private Sub Command1_Click()
Text1.Text = "": Text2.Text = "": Text3.Text = ""
a = Text4.Text:
x = a: y = 1
For p = 1 To 10
For i = 1 To 60
z1 = 1.16 * 10 ^ (-10) * y ^ 4 - 1.57 * 10 ^ (-7) * y ^ 3 + 0.715 * 10
^ (-4) * y ^ 2 - 3 / 250 * y + 0.6151
z2 = 770 * (-1 / 13778 * x + 62 / 6889) * Exp(-(1 / 166 * x - 62 / 83)
```

```
^ 2) + 822 * (-1 / 17672 * x + 645 / 17672) * Exp(-(1 / 188 * x - 645 /
188) ^ 2) - 6370 * (-625 / 2765952 * x - 9325 / 614656) * Exp(-(25 / 2352
* x + 1119 / 1568) ^ 2) + 8307 / 10 * (-2 / 37249 * x + 738 / 37249) *
Exp(-(1 / 193 * x - 369 / 193) ^ 2) + 7828 * (-200 / 1075369 * x - 15880
/ 1075369) * Exp(-(10 / 1037 * x + 794 / 1037) ^ 2)
z = (Abs(z1) + Abs(z2)) / 2
Text1.Text = Text1.Text & vbCrLf & x
Text2.Text = Text2.Text & vbCrLf & y
Text3.Text = Text3.Text & vbCrLf & z
y = y + 10
Next i
x = x + 10
y = 1
Next p
End Sub

Private Sub Command2_Click()
Text1.Text = ""
Text2.Text = ""
Text3.Text = ""
End Sub
```