Application of Map Projection Transformation in Measurement of a Global Map

Team Members: Haocheng Ma, Yinxiao Li

School: Tsinghua High School

Location: P. R. China，Beijing

Academic Advisor: Jianjun Zhou

Application of Map Projection Transformation in Measurement of a Global Map

Abstract:

Cartographical projection is mathematical cartography. Map projection is the mathematical model of geoid. Gauss-Kruger projection is unable to realize seamless splicing between adjoining sheet maps. While Mercator projection, especially web Mercator projection, is able to realize seamless splicing but has larger measurement error. How to combine with the advantage of the two projections, apply the method of map projection transformation and realize accurate measurement under the framework of a global map, is the purpose of this article.

In order to demonstrate that the problem of measurement could be solved by map projection transformation, we studied the mathematical principle of Gauss - Kruger Projection and the Mercator Projection and designed the algorithm model based on characteristics of both projections. And established simulated data in ArcMap, calculated the error of the three indicators-the area, the length and the angle- under areas in different latitude(0°-4°、30°-34°、60°-64°)and longitude (120°-126°)range which validated the algorithm model. The conclusion suggests that it is feasible that using web Mercator Projection is capable to achieve a global map expression. Meanwhile, calculations of area, distance and angle using map projection transformation principle are feasible as well in Gauss - Kruger Projection. This algorithm combines the advantage of the two different kinds of projection, is able to satisfy uses' high demand and covers the shortage of measurement function of online map services in China. It has certain practical value.

CONTENT

1. Overview

Today, as closer interconnection between countries and regions, deep insight in earth that we are living on become an inevitable trend. A map becomes a way of understanding our surrounding conveniently and intuitively. Early in 1998, Gore, vice-president of U.S., had proposed the concept of digital earth. Emphasizing on integrating geographic data of global area, today, the concept has been widely accepted all around world. People's demand for a global map is increasing.

How do we get the map that we commonly use? When scientific and technological workers are surveying and mapping, they need to present the points of the ellipsoid onto the plane by mathematical methods and obtain a map. If the area for surveying is very small, when the radius is less than a certain range, we could not consider earth curvature. We could consider such a small curved surface as a plane directly. The map surveyed and made by this way is called planar graph. If the cartographic area is larger than any area that mentioned before even the global area, we must consider earth surface as ellipsoid. However, whether it is ellipsoid or sphere, it is non-developable surface. If a non-developable is forced to be flattened, just like a sliced and flattened ping-pong, breakage and overlapping must be generated. In this way, we could not obtain a continuous and complete plane figure of the surface of earth. Of course, users' requirement of the map could not be achieved. In order to solve the contradiction between the map plane and earth curved surface of earth, people finally have sought for this scientific method, map projection, through constant practicing. The surface of earth could be completely displayed on a plane by use of map projection. However, the realization is achieved by uniform extension of certain area in the range of projection and uniform reducing in another area.

With the rapid development of computer technology, the electronic map has already been deeply familiar to the public. On one screen, it has realized the function that displaying the whole world on one plane. Searching for the destination and calculating the distance before going out has become one of the applications of the electronic map. Occasionally, when I was browsing the map I found that there is no significant difference on map between area of South America and Greenland, which has a differential of nearly nine to one. So, we studied the well-known map service websites, such as Baidu, Google, Sogo, Tian Map and AMap. We found that display of global information on one map has been realized. As for public, to query a place or to browse things on map could be satisfied by these websites. But for some users, who have deep requirement of maps, it is hardly achieved, for example, the accurate area of this area. Here is the problem that how to integrate global spatial information onto one planimetric map and meanwhile complete accurate measurement under this framework.

Thus, the key issue of this article is to complete accurate measurement on the premise of realization of browsing a global map to satisfy users' deep demand. In other words, integrate global spatial information onto one planimetric map and meanwhile complete accurate measurement under this framework.

2. Basic Concepts

1. Map

Map is such a graph depending on certain mathematical laws, using cartographic language, presenting the spatial distribution, connection and changing development of time on some carrier by cartographic generalization.

2. Topographic Map

Topographic map is a kind of maps. It emphasizes on presenting the terrain of a region. It is a kind of projection map of fluctuation of earth surface and location and shape of surface features on horizontal surface.

The topographic map is widely used in the military and civil. At present, paper topographic map is generally used and adopts basic scales of China. (1:5000, 1:10000，1:25000, 1:50000, 1:100000, 1:250000, 1:500000, 1:1000000)

3. Map Projection

Map projection is the theory and method of converting latitude and longitude coordinate onto the plane by certain mathematical method. In general, it can be divided into geometric perspective method and mathematical analysis method. Map projection generally has deformations of area, length and angle.

Geometric perspective method is to make the projection of points on earth onto the projection plane by using perspectivity. This is an original projection method which has great limits, is hard to correct projection deformation and has low accuracy.

Mathematical analysis method is to establish the functional relationship among points between spherical surface and projection plane and determine the position of intersection between latitude and longitude by mathematical method.

Thus, mathematical analysis method is more accurate than geometric perspective method. Formulas of Gauss-Kruger projection and Mercator projection that applied below belong to mathematical analysis method.

4. Gauss-Kruger Projection

Gauss-Kruger Projection is also known as *Traverse Tangent Cylindrical Conformal Projection*. Assume that an elliptic cylinder traverses earth ellipsoid on a certain longitude, which is called central meridian. The central meridian is defined as axis of symmetry, and the projection is made on a cylinder within 3°– 6° of a central meridian ranging from meridian of 3^oE or W to 1.5^oW or E, according to mathematical principles and projection principles. Then it is unrolled into a plane.

5. Mercator Projection

Also called “cylindrical map projection”。Assume that a cylinder conforms to the direction of earth’s axis and transverses the earth. Under the conditions of equal angles, the graticule is projected on a cylinder. The cylinder version is unrolled into a plane. Mercator projection can keep the shape of projected object. Also the latitudes and longitudes are shown as parallel straight lines. Longitudes are perpendicular to latitudes.

6. Web Mercator Projection

The key difference between web Mercator projection and conventional Mercator projection is to simulate the earth as spheroid but not non-ellipsoid. It is mainly used for online GIS services, such as Google Maps, Virtual Earth. It has the advantage of simple and convenient calculation.

7. Map Projection Transformation

When data comes from map sheets of different projection, it is necessary to transform the input data as required. That is map projection transformation. That is to establish a corresponding relation between two coordinate systems.

8. ArcMap

ArcMap is one of the three user desktop components of ArcGIS Desktop. ArcGIS is the GIS that ESRI developed in 1978. ArcMap is an application program that could be used in data input, edit, query and analysis. It has all map based function, such as mapping, map editing and map analysis.

3. Mathematical Principles

The interpretations of positive solution and negative solution are necessary before start basic math research. Positive solution is to transform geographic coordinate (latitude and longitude) into mathematical plane coordinates (e.g. rectangular plane coordinate system).On the contrary, negative solution is to transform mathematical plane coordinates into geographic coordinates.

3.1General Theories of Map Projection

Map projection is to establish one-to-one functional relationship between points on the Earth ellipsoid and plane. The point on the Earth ellipsoid is represented in geographic coordinates as (B, L); the point on the plane is represented in Horizontal and vertical coordinates as (x, y).

Thus, the map projection equation is

$$\begin{aligned} x &= f_1(B,\ L) \\ y &= f_2(B,\ \mathrm{L}) \end{aligned} \qquad (3.1.1)$$

Projection deformation inevitably exists in map projections including area, length, angle deformation and etc.

The length ratio is the ratio of the differential line segment ds' on projection and the corresponding one ds on ellipsoid. That is defined as

$$\mu = \frac{ds'}{ds} \qquad (3.1.2)$$

The difference between the length ratio and 1 is relative length deformation. That is defined as

$$v = \mu - 1 \qquad (3.1.3)$$

Differential line segment ds' on projection is

$$ds'^2 = dx^2 + dy^2 \qquad (3.1.4)$$

From (3.1.1)

$$\left.\begin{aligned} dx &= x_B dB + x_l dl \\ dy &= y_B dB + y_l dl \end{aligned}\right\} \qquad (3.1.5)$$

$x_B,\ x_l, \cdots$ are short for partial derivative $\frac{\partial x}{\partial B}, \frac{\partial x}{\partial l}, \cdots$

Substituted in (3.1.4)and introduce signs.

$$\left.\begin{aligned} E &= {x_B}^2 + {y_l}^2 \\ F &= x_B x_l + y_B y_l \\ G &= {x_l}^2 + {y_l}^2 \end{aligned}\right\} \qquad (3.1.6)$$

Thus

$$ds'^2 = EdB^2 + 2FdBdl + Gdl^2 \qquad (3.1.7)$$

Differential line segment ds is

$$ds^2 = M^2 dB^2 + 2FdB^2 + r^2 dl^2 \qquad (3.1.8)$$

Equation (3.1.2)can be written as

$$\mu^2 = \frac{EdB^2 + 2FdBdl + Gdl^2}{M^2 dB^2 + r^2 dl^2} \qquad (3.1.9)$$

Azimuth of line segment AC in Diagram 3-1is a. Thus

$$\tan a = \frac{DC}{AD} \text{ or } \tan a = \frac{r}{M}\frac{dl}{dB}$$

Thus $$\frac{dl}{dB} = \frac{M}{r}\tan a \qquad (3.1.10)$$

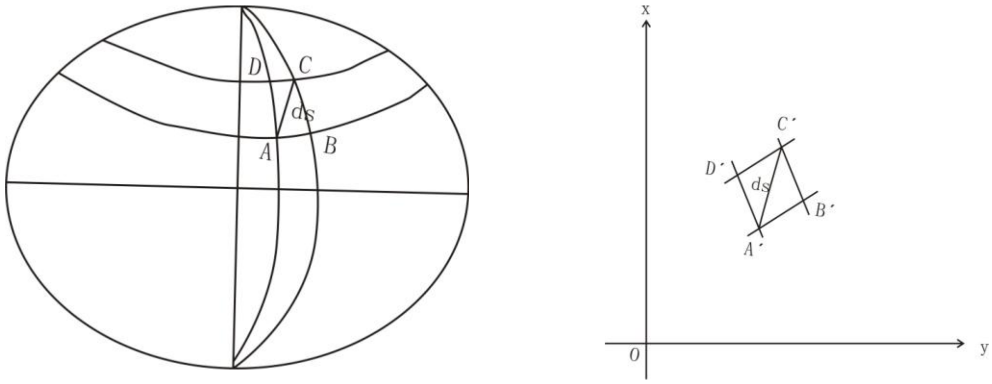


Diagram 3-1

Numerator and denominator on right of equation (3.1.9) divided by dB^2, and substitute (3.1.10) in it.

$$\mu^2 = \frac{E}{M^2}\cos^2 a + \frac{F}{Mr}\sin 2a + \frac{G}{r^2}\sin^2 a \qquad (3.1.11)$$

When a=0 °, the length ratio m along the meridian direction is

$$m = \frac{\sqrt{E}}{M} \qquad (3.1.12)$$

When a=0 °, the length ratio m along the parallel direction is

$$n = \frac{\sqrt{G}}{r} \qquad (3.1.13)$$

Deformation of angle of latitude and longitude lines: let angle of latitude and longitude lines on that projection as θ . Deformation of angle is $\varepsilon = \theta - 90°$. The calculation formula is

$$\tan \varepsilon = -\frac{F}{H} \tag{3.1.14}$$

$$H = x_B y_l - y_B x_l$$

Area ratio: Area ratio P is ratio of differential area ABCD and curved surface elements ABCD. The calculation formula is

$$P = \frac{1}{Mr}\left(x_B y_l - y_B x_l\right) \tag{3.1.15}$$

Let F be trapezoid area between parallels of equator B via a radian longitude difference on the ellipsoid.

$$dF = Mr\mathrm{dB} \tag{3.1.16}$$

Substitute variable F with latitude B， thus

$$P = x_F y_l - y_F x_l \tag{3.1.17}$$

Main direction: The length ratio along such direction has an extremum.

Let a and b be maximum and minimum length ratio, thus

$$a^2 + b^2 = m^2 + n^2, ab = mn\sin\theta = P \tag{3.1.18}$$

From above

$$\left.\begin{aligned} (\mathrm{a+b})^2 &= m^2 + 2mn\sin\theta + n^2 \\ (\mathrm{a-b})^2 &= m^2 - 2mn\sin\theta + n^2 \end{aligned}\right\} \tag{3.1.19}$$

The deformed ellipse: a differential circle on ellipsoid generally is an ellipse on projection. This ellipse is the deformed ellipse which is shown as Diagram 3-2.

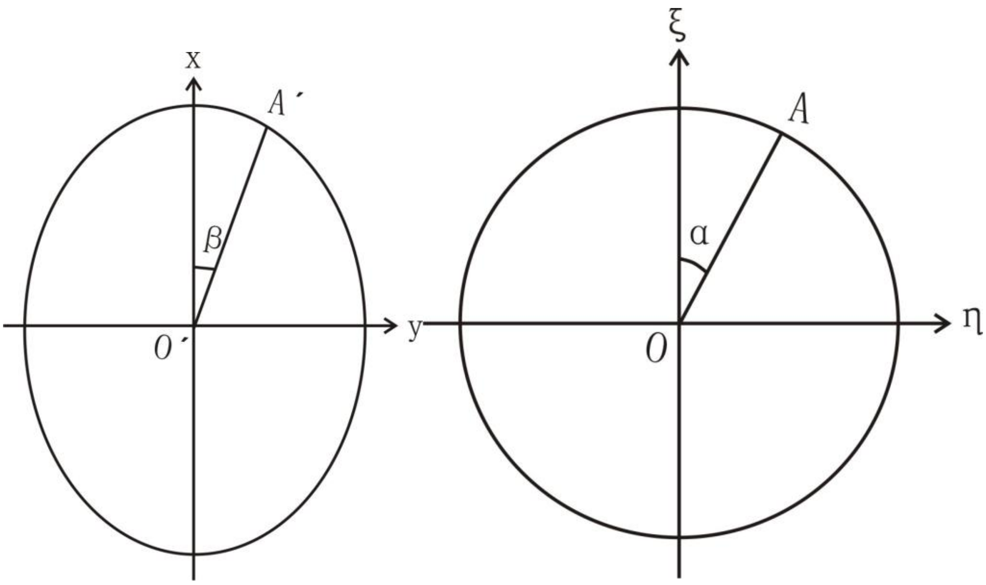


Diagram 3-2

Direction angle: Angle between main direction and other directions.

The direction angle on original surface is α. The corresponding direction angle on projection is β. The calculation formula is

$$\tan\beta = \frac{b}{a}\tan\alpha \qquad (3.1.20)$$

Maximum angle deformation: Angle between two maximum direction angle deformation lines has a maximum angle deformation ω. The calculation formula is

$$\sin\frac{\omega}{2} = \frac{a-b}{a+b} \qquad (3.1.21)$$

or

$$\tan\left(45^\circ + \frac{\omega}{4}\right) = \sqrt{\frac{a}{b}} \qquad (3.1.22)$$

$$\tan\left(45^\circ - \frac{\omega}{4}\right) = \sqrt{\frac{b}{a}} \qquad (3.1.23)$$

Equiangular projection conditions:

$$a = b \qquad (3.1.24)$$

or

$$\theta = 90^\circ,\ \ m = n \qquad (3.1.25)$$

or

$$\frac{\partial x}{\partial l} = -\frac{r}{M}\cdot\frac{\partial r}{\partial B}, \frac{\partial y}{\partial l} = +\frac{r}{M}\cdot\frac{\partial x}{\partial B} \qquad (3.1.26)$$

Equal area projection conditions:

$$P = ab = 1 \qquad (3.1.27)$$

or

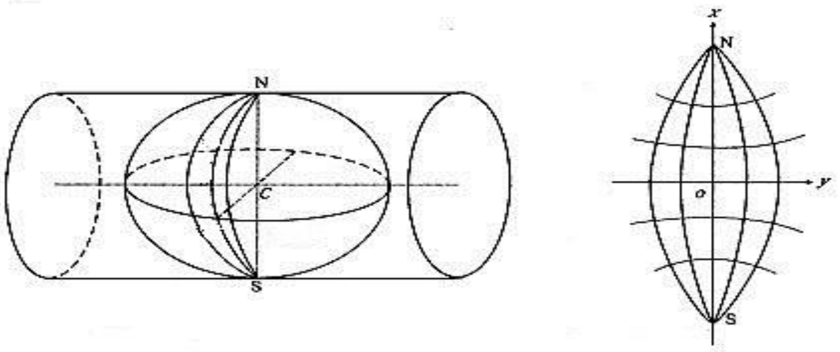
$$H = Mr \qquad (3.1.28)$$

Equidistant projection conditions:

$$\theta = 90^\circ,\ \ m = 1 \qquad (3.1.29)$$

3.2 Gauss-Kruger Projection

Gauss-Kruger Projection: Also known as Traverse *Tangent Cylindrical Conformal Projection*, it is an equiangular projection based on isometric conditions and mathematic analysis which assume that an elliptic cylinder traverses earth ellipsoid on a certain longitude.



3.2.1Mathematical Basis of Gauss-Kruger Projection

Conditions of Gauss-Kruger Projection are:

1. Latitude and longitude lines projections are curves symmetrical to the central meridian.

2.No Angle deformation after projection.

3.No length deformation of central meridian.

According to map projection theory, the general formula of equiangular projection is

$$x + iy = f(q + il) \qquad (3.2.1)$$

In above equation:q, l are isometric coordinate or Mercator coordinate.

$$q = \ln U = \ln\left[\tan\left(\frac{\pi}{4} + \frac{B}{2}\right)\left(\frac{1 - e\sin B}{1 + e\sin B}\right)^{\frac{e}{2}}\right] \qquad (3.2.2)$$

In Gauss-Kruger Projection,when l=y=0, x=S，according to condition 3.

$$S = f(q) \qquad (3.2.3)$$

For coordinate formula of Gauss-Kruger Projection, expand equation (3.2.1) in Taylor series at one certain point on central meridian.

$$\Delta B = B - B_0, \Delta x = x - x_0, \Delta q = q - q_0, l = L - L_0 \tag{3.2.4}$$

$$\Delta x + iy = a_1(\Delta q + il) + a_2(\Delta q + il)^2 + a_3(\Delta q + il)^3 + a_4(\Delta q + il)^4 + \cdots \tag{3.2.5}$$

$$\Delta q + il = b_1(\Delta x + iy) + b_2(\Delta x + iy)^2 + b_3(\Delta x + iy)^3 + b_4(\Delta x + iy)^4 + \cdots \tag{3.2.6}$$

In the equation

$$a_n = \frac{1}{n!}\left(\frac{d^n(x + iy)}{d(q + il)^n}\right)_0 \tag{3.2.7}$$

$$b_n = \frac{1}{n!}\left(\frac{d^n(q + il)}{d(x + iy)^n}\right)_0 \tag{3.2.8}$$

Notice that l =y=0,x=S,

$$a_n = \frac{1}{n!}\left(\frac{d^n S}{dq^n}\right)_0 \tag{3.2.9}$$

$$b_n = \frac{1}{n!}\left(\frac{d^n q}{dS^n}\right)_0 \tag{3.2.10}$$

Calculated

$$\left.\begin{aligned}
a_1 &= N_0 \cos B_0 \\
a_2 &= \frac{1}{2} N_0 t_0 (-1) \cos^2 B_0 \\
a_3 &= \frac{1}{6} N_0 \left(-1 + t_0{}^2 - \eta_0{}^2\right) \cos^3 B_0 \\
a_4 &= \frac{1}{24} N_0 t_0 \left(5 - t_0{}^2 + 9\eta_0{}^2 + 4\eta_0{}^4\right) \cos^4 B_0 \\
a_5 &= \frac{1}{120} N_0 \left(5 - 18t_0{}^2 + t_0{}^4 + 14\eta_0{}^2 - 58t_0{}^2\eta_0{}^2\right) \cos^5 B_0 \\
a_6 &= \frac{1}{720} N_0 t_0 \left(-61 + 58t_0{}^2 - t_0{}^4 - 270\eta_0{}^2 + 330t_0{}^2\eta_0{}^2\right) \cos^6 B_0
\end{aligned}\right\} \tag{3.2.11}$$

$$\left.\begin{aligned}
b_1 &= \frac{1}{N_0 \cos B_0} \\
b_2 &= \frac{1}{2N_0{}^2 \cos B_0} t_0 \\
b_3 &= \frac{1}{6N_0{}^3 \cos B_0}\left(1 + 2t_0{}^2 + \eta_0{}^2\right) \\
b_4 &= \frac{1}{24N_0{}^4 \cos B_0} t_0\left(5 + 6t_0{}^2 + \eta_0{}^2 - 4\eta_0{}^4\right) \\
b_5 &= \frac{1}{120N_0{}^5 \cos B_0}\left(5 + 28t_0{}^2 + 24t_0{}^4 + 6\eta_0{}^2 + 8t_0{}^2\eta_0{}^2\right) \\
b_6 &= \frac{1}{720N_0{}^6 \cos B_0} t_0\left(61 + 180t_0{}^2 + 120t_0{}^4 + 46\eta_0{}^2 + 48t_0{}^2\eta_0{}^2\right)
\end{aligned}\right\} \tag{3.2.12}$$

In equation: $t_0 = \tan B_0$, $\eta_0{}^2 = e'^2 \cos^2 B_0$

Positive solution of Gauss-Kruger Projection

Let point 0 and point Q be coincidence.

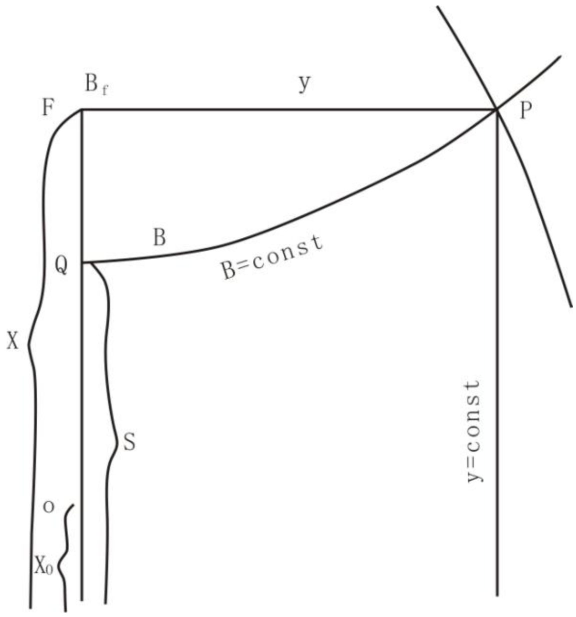


Diagram 3-3

$$\left.\begin{array}{l} B_0 = B，\ \Delta \mathrm{B} = 0 \\ x_0 = S，\ \Delta \mathrm{x} = x - S \\ q_0 = q，\ \Delta \mathrm{q} = 0 \end{array}\right\} \tag{3.2.13}$$

Separate equation (3.2.5)

$$\left.\begin{array}{l} x - S = -a_2 l^2 + a_4 l^4 - a_6 l^6 + \cdots \\ y = a_1 l - a_3 l^3 + a_5 l^5 - \cdots \end{array}\right\} \tag{3.2.14}$$

Notice that in (3.2.11)，using B instead of B_0

$$\left.\begin{array}{l} x = S + \frac{1}{2} Nt \cos^2 B \cdot l^2 + \frac{1}{24} Nt\left(5 - t^2 + 9\eta^2 + 4\eta^4\right)\cos^4 B \cdot l^4 \\ + \frac{1}{270} Nt\left(61 - 58t^2 + t^4 + 270\eta^2 - 330t^2\eta^2\right)\cos^6 B \cdot l^6 + \cdots \\ y = N \cos B \cdot l + \frac{1}{6} N\left(1 - t^2 + \eta^2\right)\cos^3 B \cdot l^3 + \frac{1}{120} N \\ \left(5 - 18t^2 + t^4 + 14\eta^2 - 58t^2\eta^2\right)\cos^5 B \cdot l^5 + \\ \frac{1}{5040} N\left(61 - 479t^2 + 179t^4 - t^6\right)\cos^7 B \cdot l^7 + \cdots \end{array}\right\} \tag{3.2.15}$$

In equation: $t = \tan B$, $\eta^2 = e'^2 \cos^2 B$, S is the meridian arc length from equator to latitude B.

Negative solution of Gauss-Kruger Projection

Let origin 0 and point F be coincidence.

$$\left.\begin{array}{l} B_0 = B_f，\ \Delta B = B - B_f \\ x_0 = x，\ \Delta x = 0 \\ q_0 = q_f，\ \Delta q = q - q_f \end{array}\right\} \tag{3.2.16}$$

Separate equation (3.2.6)

$$\left.\begin{aligned} q-q_f &= -(b_2)_f y^2 + (b_4)_f y^4 - (b_6)_f y^6 + \cdots \\ l &= (b_1)_f y - (b_3)_f y^3 + (b_5)_f y^5 - \cdots \end{aligned}\right\} \tag{3.2.17}$$

Notice that in (3.2.11)，using B_f instead of B_0

$$\begin{aligned} B &= B_f + \frac{1}{2N_f^2} t_f \left(-1-\eta_f^2\right) y^2 + \frac{1}{24N_f^4} t_f \left(5+3t_f^2+6\eta_f^2-6t_f^2\eta_f^2-3\eta_f^4-9t_f^2\eta_f^4\right) y^4 \\ &+ \frac{1}{720N_f^6} t_f \left(-61-90t_f^2-45t_f^4-107\eta_f^2+162t_f^2\eta_f^2+45t_f^4\eta_f^2\right) y^6 + \cdots \\ l &= \frac{1}{N_f \cos B_f} y + \frac{1}{6N_f^3 \cos B_f} \left(-1-2t_t^2-\eta_f^2\right) y^3 + \\ & \frac{1}{120N_f^5 \cos B_f} \left(5+28t_f^2+24t_f^4+6\eta_f^2+8t_2^2\eta_f^2\right) y^5 + \cdots \end{aligned} \tag{3.2.18}$$

In equation: B_f is latitude of pedal, thus $x = S_f$.

Above positive and negative solutions have a characteristic that expanding point is on central meridian. And it is a variable coefficient case which means variables an and bn are change with latitude B and Bf (or x).

3.2.2 Characteristics

Assume that an elliptic cylinder traverses earth ellipsoid on a certain longitude, which is called central meridian.And central axis of elliptic cylinder go through the center of the ellipsoid. In projection, central meridian is x-axis, equator projection is y-axis.

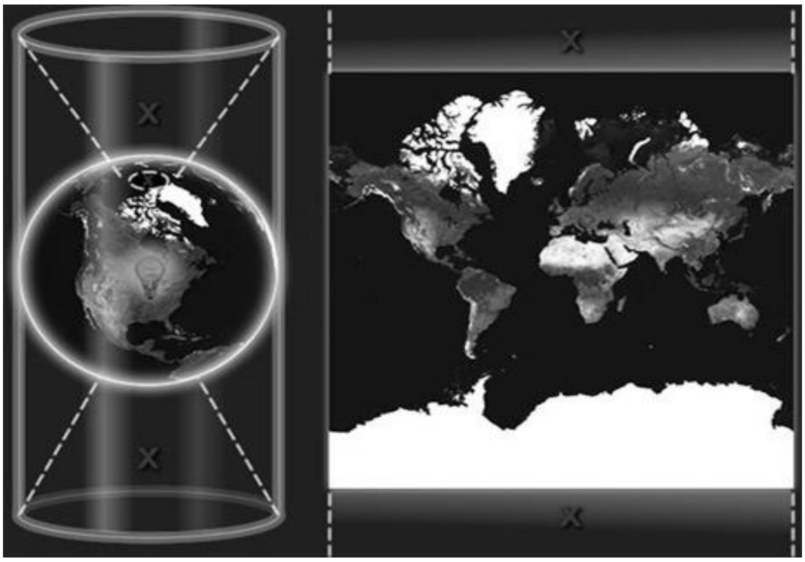
This projection does not have angle deformation. For limiting deformation of length and area, 3 ° or 6 °zoning methods are used which is made on a cylinder within 3 °– 6 ° of a central meridian ranging from meridian of 3°E or W to 1.5°W or E. However, when zone larger than 6 °, length and area deformation are increasing along with the distance to central meridian and cause high deformation ratio.

3.2.3Reasons Why Gauss-Kruger Projectionis Unable to Realize Seamless Splicing

Gauss-Kruger Projection has a character of “the farther form the central meridian, the greater the deformation”. We use 3°and 6°zoning method to control deformation in order to limit error. The strength of Gauss-Kruger Projection is minor deformation within 3°and 6°zone. However, it is also the key issue why Gauss-Kruger Projection unable to realize seamless splicing. Even the same point on the Earth may has different coordinates in Gauss-Kruger Projection by zoning with different central meridian. Because Gauss-Kruger Projection haven’t got a unified coordinate system which is the main defect of the projection.

3.3 Web Mercator Projection

Mercator Projection,also called “cylindrical map projection”, means longitude and latitude lines in geographic coordinate system are project asvertical and horizontal lines on the plane. And the interval between the two meridians is proportionate to the longitude difference.



3.3.1 Mathematical Basis of Web Mercator Projection

Web Mercator Projection is a simplified version of Mercator Projection which changed ellipsoid into sphere.

a. Positive solution of Web Mercator Projection

$$Y_E = F_E + R(L - L_0)$$

$$X_N = F_N + Rln[tan\,(\frac{\pi}{4} + \frac{B}{2})]$$

b. Negative solution of Web Mercator Projection

$$D = -(X_N - F_N)/R$$

$$B = \frac{\pi}{2} - 2arc\,tan\,e^D$$

$$L = [(Y_E - F_E)/R] + L_0$$

B,L-Latitude and Longitude coordinate of the calculated point（L:Longitude，B:Latitude）

YE,XN-Horizontal and Vertical coordinate of the calculated point

L0-Longitude of Standard Meridian

e-Natural Logarithm Base

R-Radius of sphere，take value of 6378137m（Length of Major semi-axis)

FE-False Easting

FN-False Northing

3.3.2Characteristics

Web Mercator Projectionis a simplified version of Mercator Projection which changes ellipsoid into sphere. It calculated easy but cause serious deformation.

Length and area deformations in Web Mercator Projection become more serious along with standard parallel approaching both poles and tending to infinity at poles. Therefore, different projection methods are generally used in pole areas which not discussed in this article. Mercator Projection is originally an equiangular projection. However angle deformation exists in Web Mercator Projection because it simplified the Earth as a sphere.

3.3.3Reasons Why Web Mercator Projectionis Able to Realize Seamless Splicing

Zoning is not necessary for Web Mercator Projection. No matter consider deformation parts at poles or not, Web Mercator Projection is definitely in one unified coordinate system which can perfectly realize seamless splicing.

3.4 Planar Polygon Area Calculation

Let Ωbe a polygon with m sides. Vertex $p_k(k=1,2,\ldots,m)$ ($p_{m+1}$is p_1) are positive arranged along the boundary. Coordinates of vertices are: $(x_1,y_1),(x_2,y_2),\ldots,(x_m,y_m)$, thus the area of this polygon is

$$S_\Omega = \sum_{k=1}^{m} S_{\Delta \mathrm{O} \mathrm{p}_k \mathrm{p}_{k+1}} = \frac{1}{2}\sum_{k=1}^{m}(x_k y_{k+1} - x_{k+1} y_k)$$

4. Algorithm Model

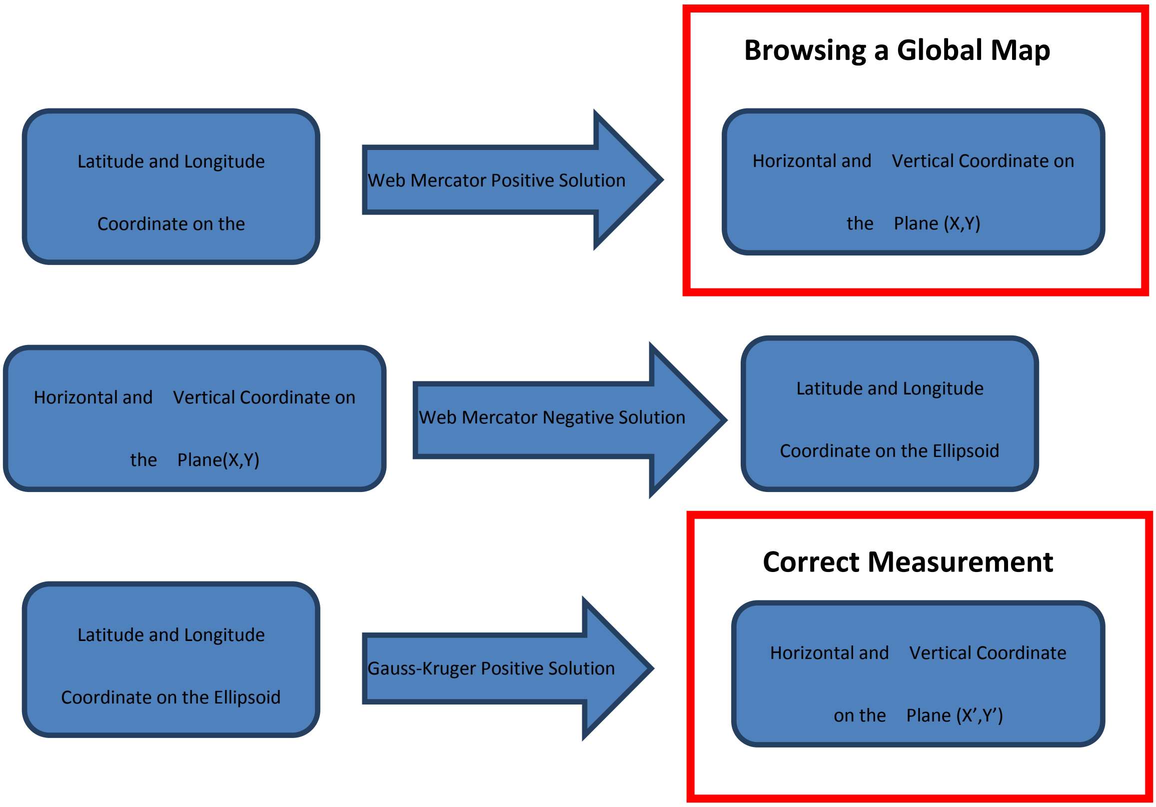
4.1Design of Algorithm Model

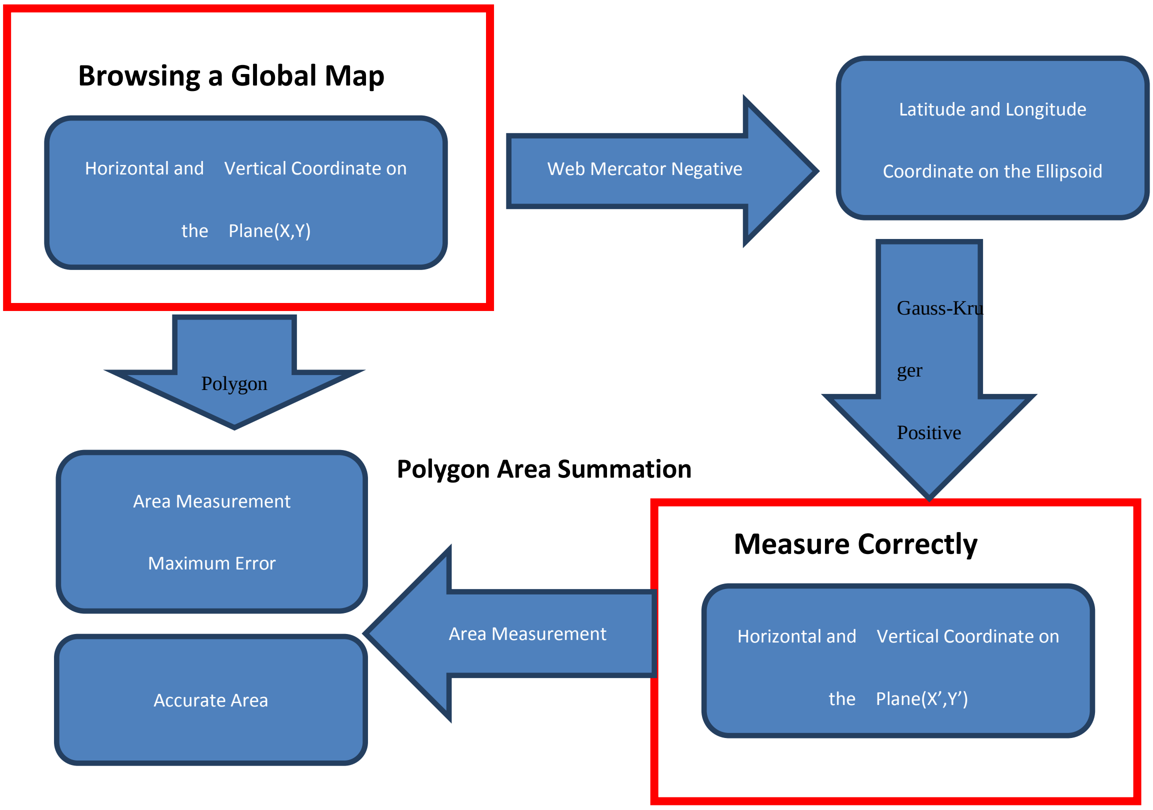
4.1.1 Design Ideas

After studying features of different kinds of projection, we proposed design ideas as follows:

Browse a global map on a flat map through the seamless splice of Web Mercator Projection. Meanwhile, the measurement of distance, area and angle are measured and calculated in the space of Gauss according to the principle of map projection transformation.

4.1.2Model Design





4.2Experimental Verification of Algorithm Model

4.2.1.Model Verification in Area Measurement

Experimental Environment:ArcMap9.0

Experimental Method:

Build three pieces of graticule in ArcMap9.0 with the range of (0° -4 °N, 120° -126 °E), (30° -34 °N, 120° -126 °E)and (60° -64° N，120 °-126°). Each cell represents 0.1° *0.1° .

As we already know, the graticule within 0.05 °of central meridian is regarded the same as earth surface under Gauss-Kruger projection.

Thus, Set 120.05 °, 120.15°, 120.25°……125.95°as central meridian, respectively. Calculate the area of each cell. This area is considered as the *actual area* of this region on earth.

Calculate the area of each cell of the graticule under Gauss-Kruger projection, taking central meridian at 123° and zoning by 6°. This area is considered as the *actual area* on *Gauss-Kruger projection* topographic map.

Calculate the area of each cell of the graticule under Web Mercator projection having equator as standard parallels. This area is considered as the *actual area* on *Web Mercator projection* topographic map.

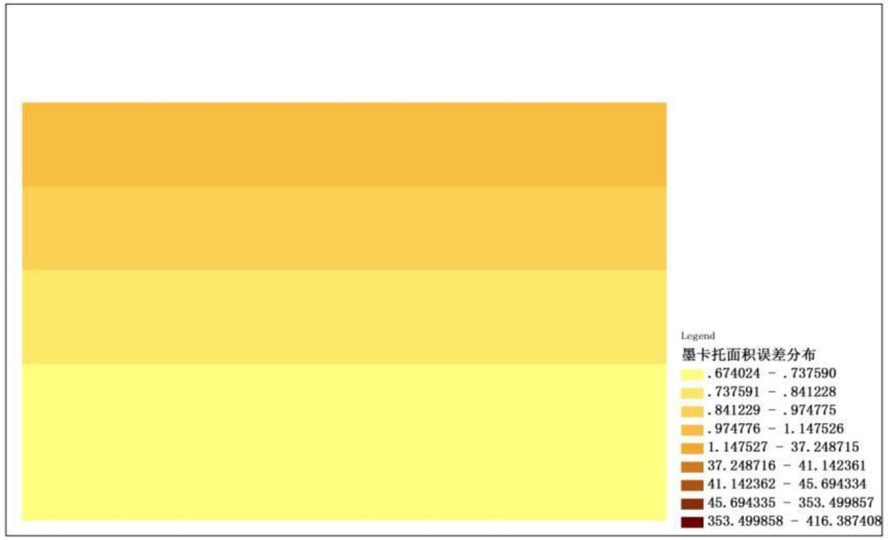
Experimental Result:

Data:经纬度格网_面积误差分布.xlsx

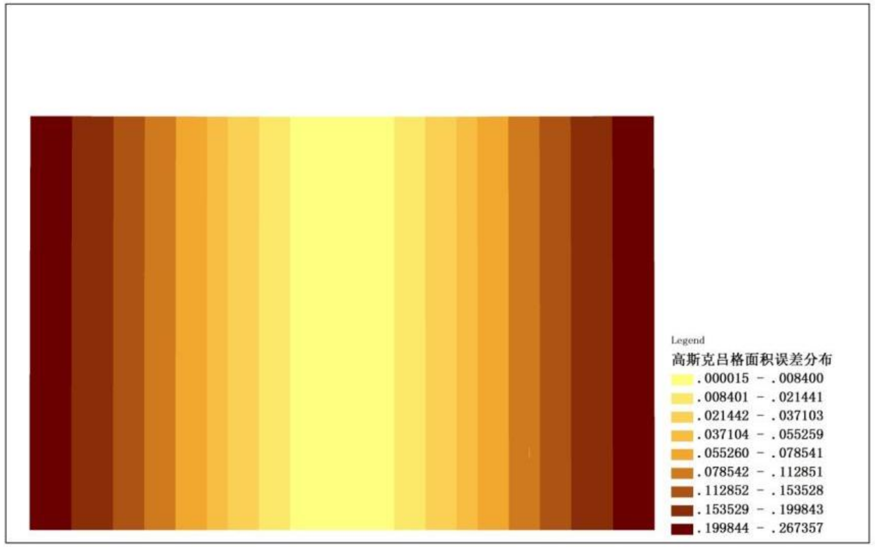
Area Measurement: Experimental Result			
Latitude	**0°-4°N**	**30°-34°N**	**60°-64°N**
Gauss-Kruger projection	0.000015%-0.267357%	0.000015%-0.199843%	0.000015%-0.078541%
Mercator projection	0.674024%-1.147526%	1.147527%-45.696334%	45.696335%-416.387408%

Visualization Image:

All the errors are relative errors. All error units are(%).

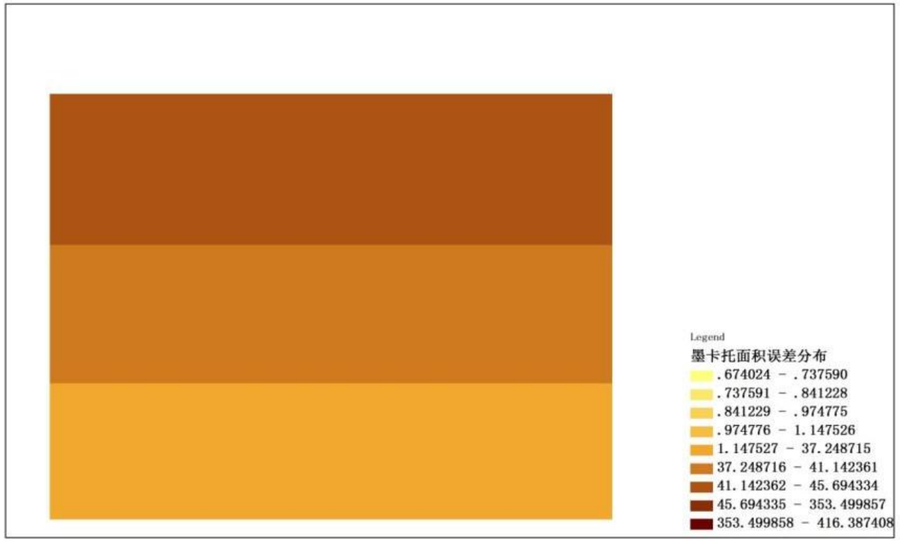


Area error distribution in low-latitude region under Web Mercator projection(0°-4°N)

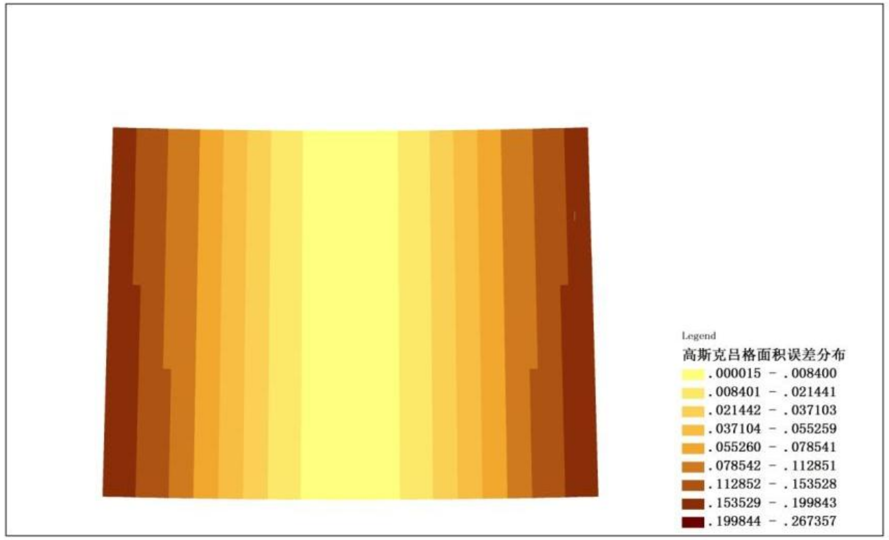


Area error distribution in low-latitude region under Gauss-Kruger projection (0 °-4 °N)

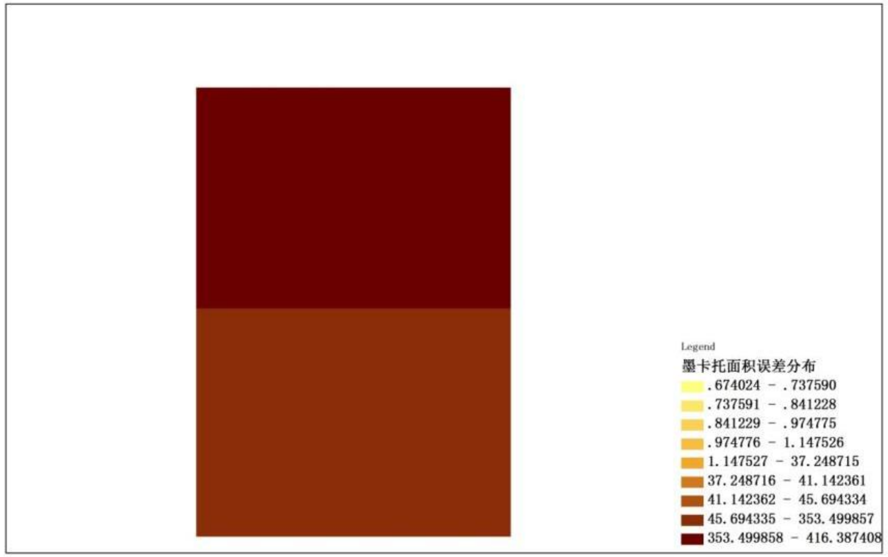
23



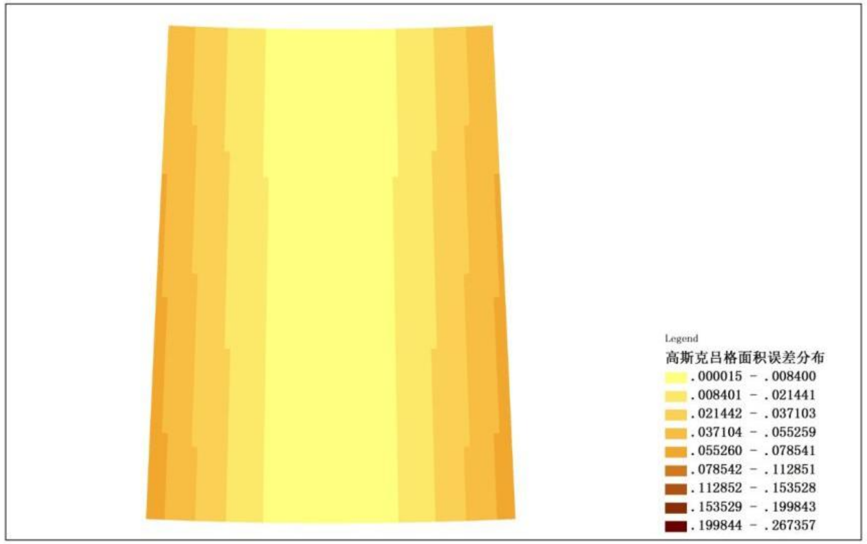
Area error distribution in middle-latitude region under Web Mercator projection (30 °-34 °N)



Area error distribution in middle-latitude region under Gauss-Kruger projection (30 °-34 °N)



Area error distribution in high-latitude region under Web Mercator projection (60 °-64 °N)



Area error distribution in high-latitude region under Gauss-Kruger projection (60 °-64 °N)

Brief Summary:

Obviously, the effect of reducing error of Gauss-Kruger projection is significant wherever in the region of low-, middle- or high-latitude. Especially in high-latitude region, the advantage of Gauss-Kruger projection is more prominent. The relative error of Web Mercator can be even larger than 300% in the same region but Gauss-Kruger projection is still effective in limiting error.

For the application for measurement on a global map, if the area measured covers longitude of less than 6°, it can be directly converted to 6°-zone or zones of less degrees of Gauss-Kruger projection for area measurement.If the area measured covers longitude of more than 6°, it should be divided into regions covering longitude of less than 6°. These regions are transformed using map projection respectively. After transforming web Mercator projection to Gauss-Kruger projection, these areas are measured and summed more precisely.

4.2.2. Model Verification in Angle Measurement

Experimental Environment: ArcMap9.0

Experimental Method:

Build three pieces of graticule in ArcMap9.0 with the range of (0° -4°N, 120° -126°E), (30° -34°N, 120° -126°E)and (60° -64° N，120°-126°). Each cell represents 0.1° *0.1° . And draw two diagonals in the cell of each graticule.

From the data we know, conventional Mercator projection is standard conformal cylindrical projection. We could consider that the angle formed between two lines measured on the topographic map under conventional Mercator projection is actually the angle formed between two lines on earth. In addition, under Mercator projection, web Mercator projection and Gauss-Kruger projection, the line must parallel or overlap with the horizontal axis of the projection plane.

Thus, calculate the angle formed between each diagonal drawn and the straight line projected by equator under Mercator projection having equator as standard parallel. In calculation, angles formed by part of diagonals (with negative slope)are considered as negative angles. So sum of absolute values of the two angles in the same cell could be presented the actual angle formed by two diagonals in the same cell on the surface of earth.

Then, calculate the angle formed between each diagonal drawn and the straight line projected by equator under Gauss-Kruger projection, taking central meridian at 123 °and zoning by 6 °. This angle is the angle presented on the topographic map under Gauss-Kruger projection.In calculation, angles formed by part of diagonals (with negative slope) are considered as negative angles. So sum of absolute values of the two angles in the same cell could be presented the actual angle formed by two diagonals in the same cell under Gauss-Kruger projection.

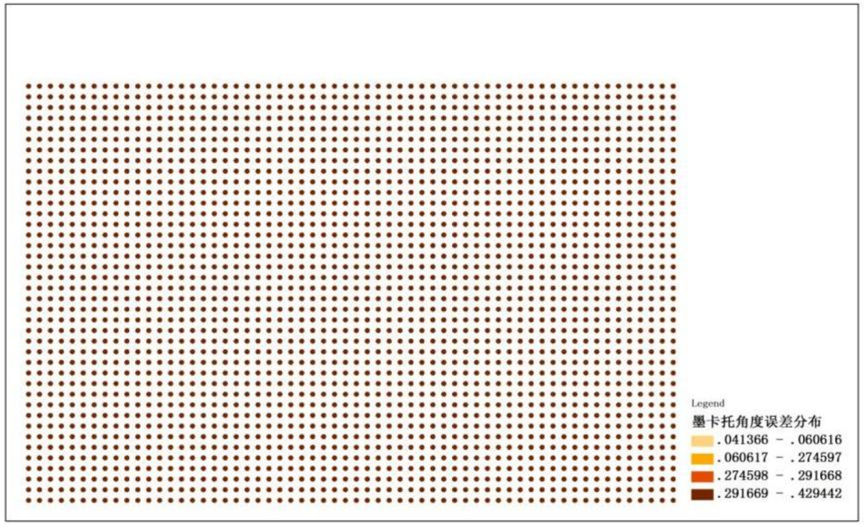
Similarly, calculate the angle formed between each diagonal drawn and the straight line projected by equator under Mercator projection having equator as standard parallel. In calculation, angles formed by part of diagonals (with negative slope) are considered as negative angles. So sum of absolute values of the two angles in the same cell could be presented the actual angle formed by two diagonals in the same cell under web Mercator projection.

Experimental Results: 经纬度格网_角度误差分布.xlsx

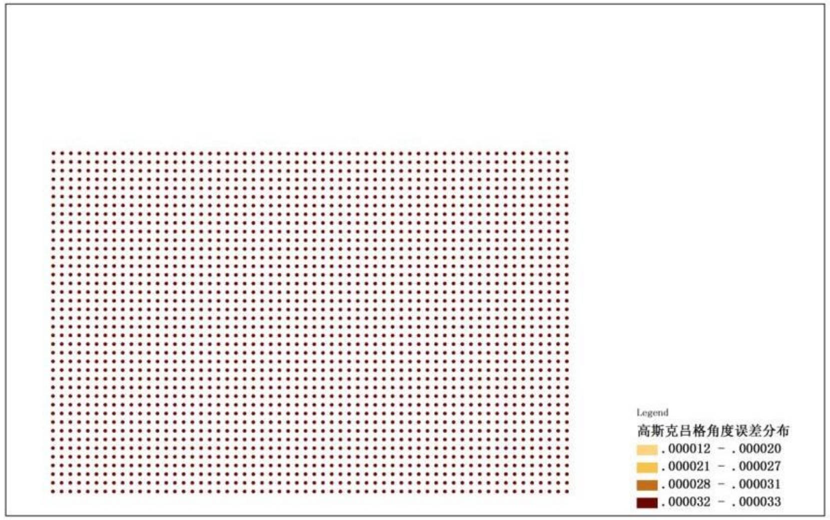
Angle Measurement: Experimental Results			
Latitude	**0 °-4 °N**	**30 °-34 °N**	**60 °-64 °N**
Gauss-Kruger projection	0.000032%-0.000033%	0.000021%-0.000027%	0.000012%-0.000027%
Mercator projection	0.291669%-0.429442%	0.060617%-0.291668%	0.041366%-0.060616%

Visualization Image:

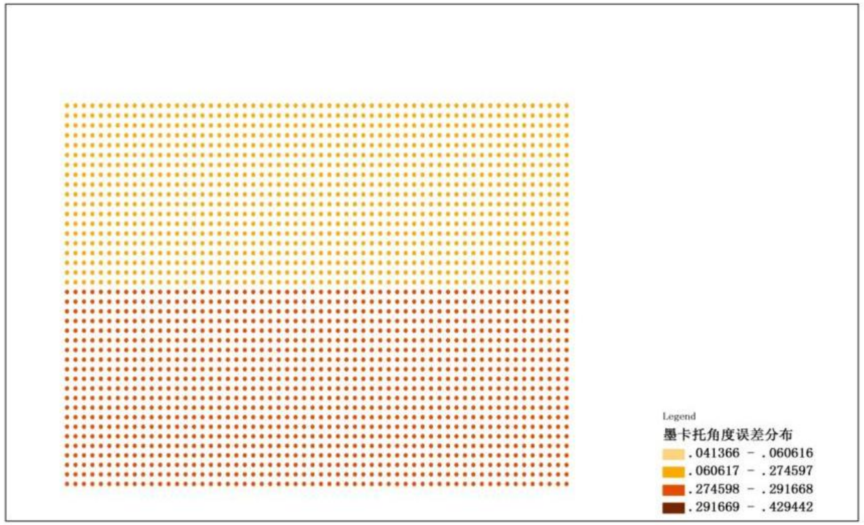
Each colored area in the image represents the relative error of the angle formed between two diagonals in that region. Different colors represent different ranges where relative errors stand. All error units are (%).



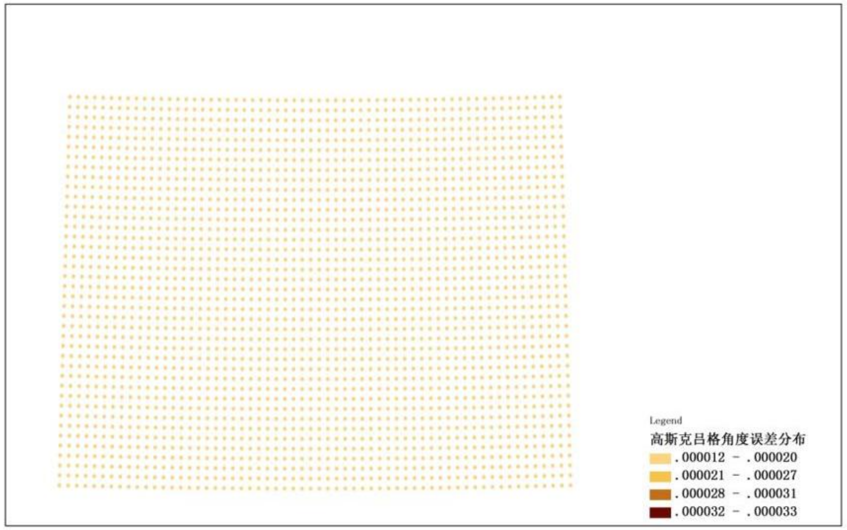
Angle error distribution in low-latitude region under Web Mercator projection(0 °-4 °N)



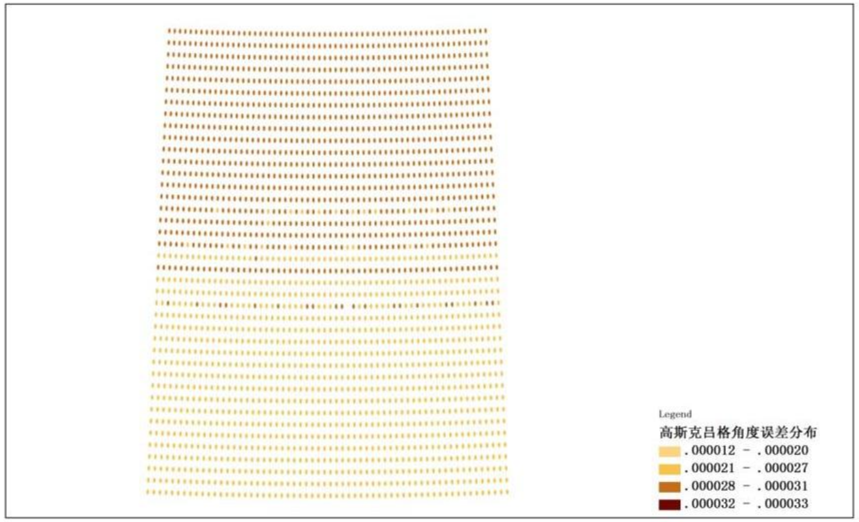
Angle error distribution in low-latitude region under Gauss-Kruger projection(0 °-4 °N)



Angle error distribution in middle-latitude region under Web Mercator projection(30 °-34 °N)



Angle error distribution in middle-latitude region under Gauss-Kruger projection(30 °-34 °N)



Angle error distribution in high-latitude region under Web Mercator projection(60 °-64 °N)

Angle error distribution in high-latitude region under Gauss-Kruger projection(60 °-64 °N)

Brief Summary:

From data above, it is easy to see that Gauss-Kruger projection has huge advantage of angle measurement. Error of 0.00001% of order of magnitude could almost be negligible.

In my opinion, the phenomenon of mixed color in the digital image in high-latitude area under Gauss-Kruger projection is normal. After all, the error of Gauss-Kruger projection is 0.00001% of order of magnitude. A minor variation, such as round-off, a tiny error in the program algorithm, may result in the phenomenon. Besides, the two colors represent two adjacent intervals and the position with mixed color locates the junction of the two colors. It is entirely possible that those minor changes lead to the error. But anyway, the error of 0.00001% of order of magnitude could be considered as no error.

On the measurement of a global map, mutual transformation between web Mercator projection and Gauss-Kruger projection could eliminate error. As for an angle under web Mercator projection, we could capture a line segment from the two radials of the angle separately and make the two line segments cover longitude of less than 6°. And then, transform it into Gauss-Kruger projection to make angle measurement in order to reduce angle error even to eliminate it.

4.2.3. Model Verification in Length Measurement

Experimental Environment: ArcMap9.0

Experimental Method:

Build three pieces of graticule in ArcMap9.0 with the range of (0°-4°N，120°-126°E)，(30°-34°N，120°-126°E)，(60°-64°N，120°-126°). Each cell represents 0.1°*0.1°.

As we already know, the graticule within 0.05°of central meridian is regarded the same as earth surface under Gauss-Kruger projection.

Set 120.05°, 120.15°, 120.25°……125.95°as central meridian, respectively. Calculate the length of each cell. This length is considered as the actual length of this region on earth.

Calculate the length of each cell of the graticule under Gauss-Kruger projection, taking central meridian at 123 ° and zoning by 6 °. This length is considered as the actual lengthon Gauss-Kruger projection topographic map.

Similarly, calculate the length of each cell of the graticule under Web Mercator projection having equator as standard parallels. This length is considered as the actual lengthon Web Mercator projection topographic map.

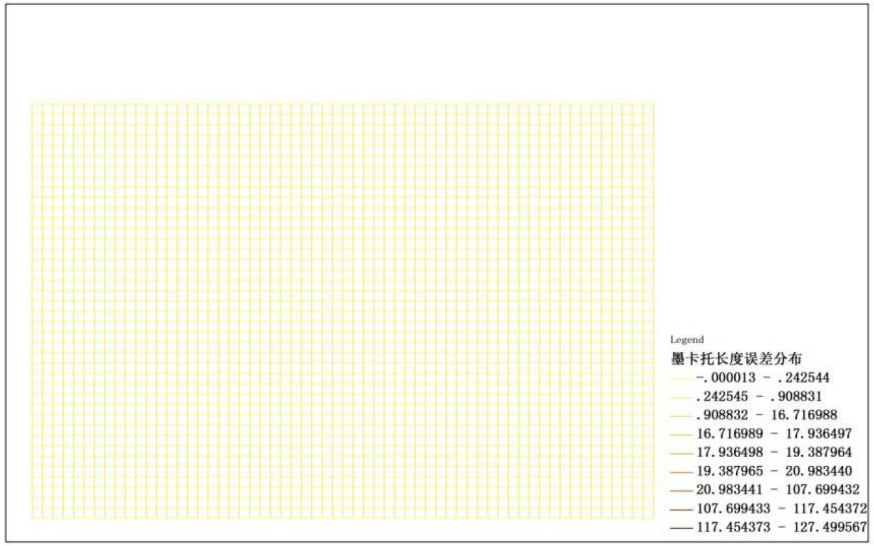
Experimental Results: 经纬度格网_长度误差分布.xlsx

Length Measurement: Experimental Results			
Latitude	**0 °-4 °N**	**30 °-34 °N**	**60 °-64 °N**
Gauss-Kruger projection	0.000038%-0.138123%	0.000038%-0.101993%	0.000038%-0.038350%
Mercator projection	0.000013%-0.242544%	16.716989%-20.983439%	20.983440%-127.499567%

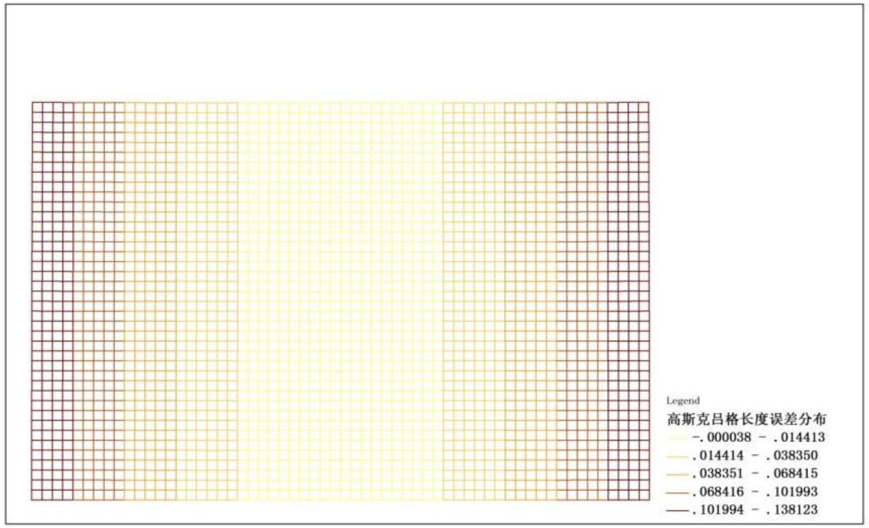
Visualization Image:

All the error units are (%)

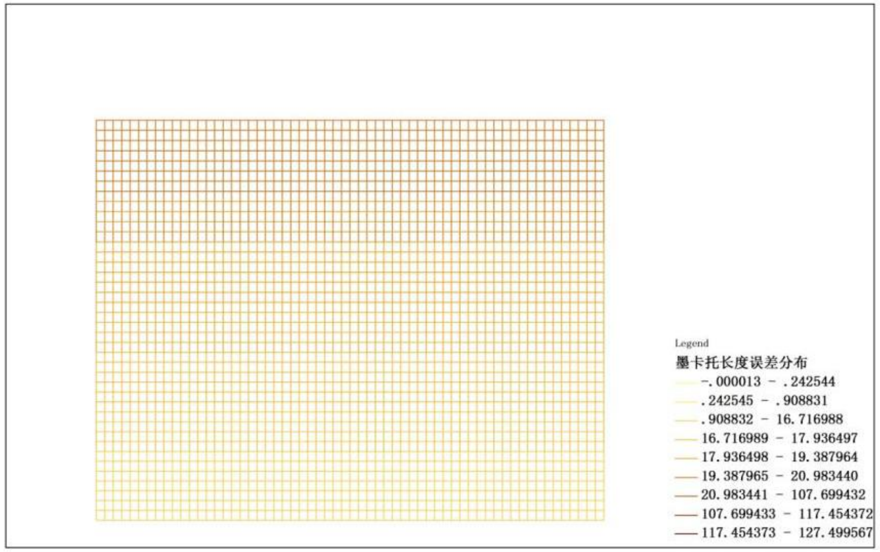
误差单位均为（%）



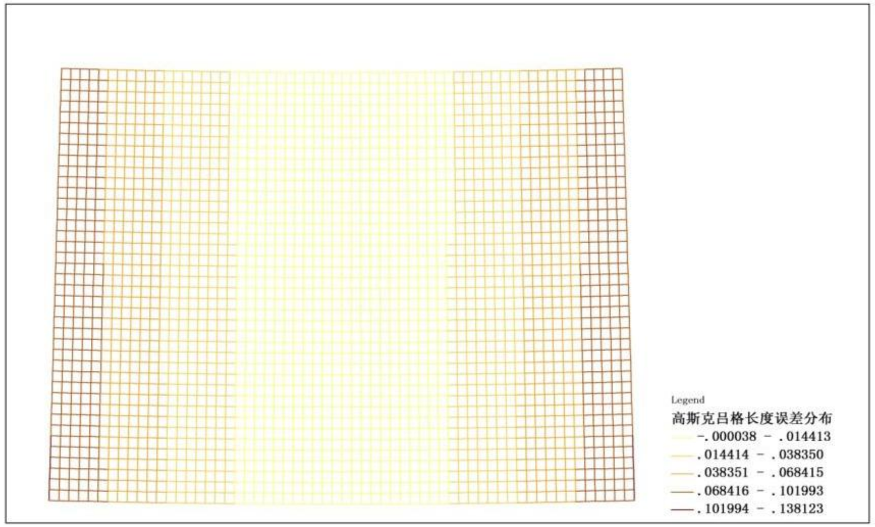
Length error distribution in low-latitude region under Web Mercator projection(0 °-4 °N)



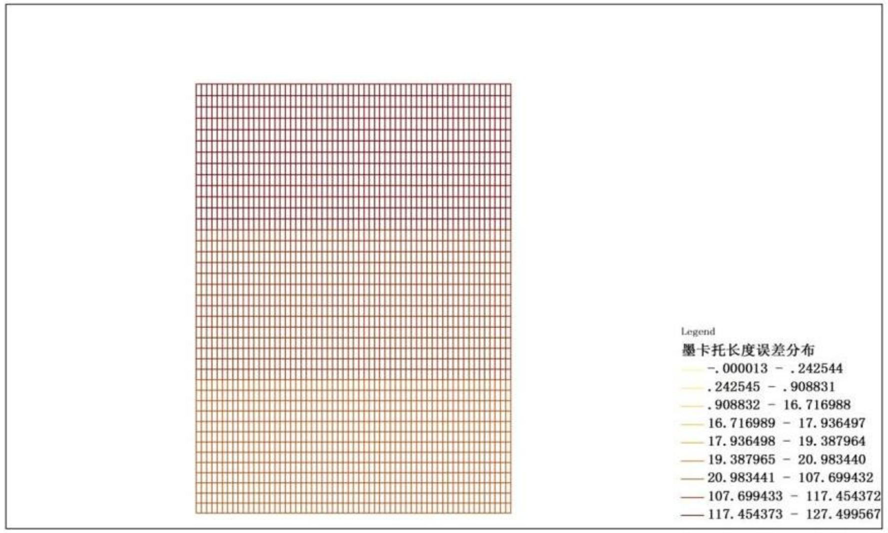
Length error distribution in low-latitude region under Gauss-Kruger projection(0 °-4 °N)



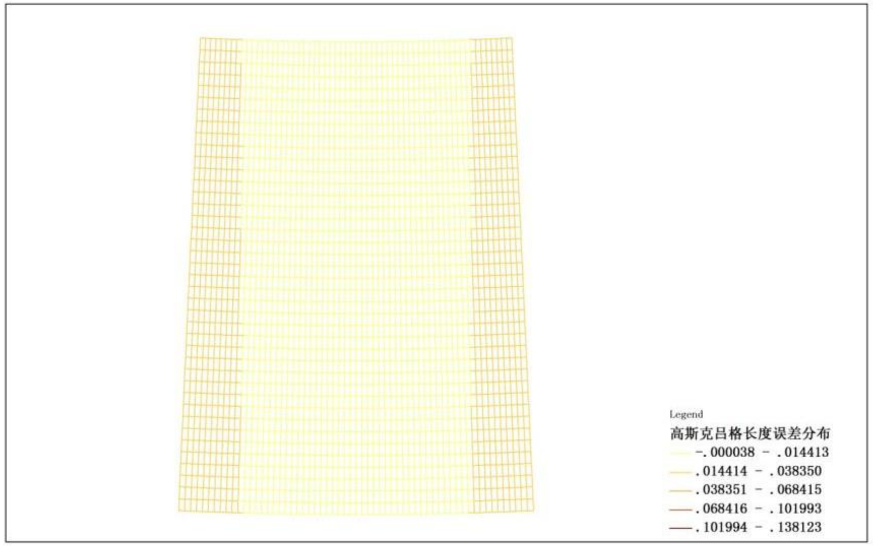
Length error distribution in mid-latitude region under Web Mercator projection(30 °-34 °N)



Length error distribution in middle-latitude region under Gauss-Kruger projection(30 °-34 °N)



Length error distribution in high-latitude region under Web Mercator projection(60 °-64 °N)



Length error distribution in high-latitude region under Gauss-Kruger projection(60 °-64 °N)

Brief Summary:

From the data and image we found that length and area distributions are extraordinarily similar.The deformation is directly proportional to the distance between Gauss-Kruger projection and the central meridian, but not longitudes.Higher the latitude of Web Mercator projection is, is greater the deformation.This also verifies the introduction about the two projections from the perspective of the experiment.

In high-latitude regions, the advantage of Gauss-Kruger projection is obvious. Comparing with length deformation of more than 100% of web Mercator projection, length deformation of of less than 0.14% of Gauss-Kruger projection is too accurate. This also demonstrates effectiveness of map projection transformation.

We considered the application method of length is similar as that of area because of similar laws. On a global map, if the length measured covers longitude of less than 6°, it can be directly converted to 6°-zone or zones of less degrees of Gauss-Kruger projection for length measurement.If the length measured covers longitude of more than 6°, it should be divided into regions covering longitude of less than 6°. These regions are transformed using map projection respectively. After performing Gauss-Kruger projection, these lengths are measured and summed more precisely.

Experimental Summarization:

From the data above, it is easy to see that result measured on the topographic map under Web Mercator is very inaccurate because of its deformation in all aspects. While using map projection transformation to make measurement could obtain more accurate result. Obviously map projection transformation can be also applied on a global map which obtained by Wed Mercator projection, as to obtain more accurate result. Therefore, the use of map projection transformation to solve the deformation of a global map obtained by Web Mercator is very feasible and effective.

5. Conclusion and Forward-looking

5.1 Conclusion and Innovation

Conclusions are as follows:

Map projection transformation can be also applied on a global map which obtained by Wed Mercator projection, as to obtain more accurate result. Therefore, the use of map projection transformation to solve the deformation of a global map obtained by Web Mercator is very feasible and effective.

Depending on the character of the two map projections, we propose that map projection transformation can be applied to solve the deformation of area, angle and length under Web Mercator.

Through the experiment under ArcMap, it is verified both the character of the two projections, and the feasibility of solving deformation of a global map under Web Mercator projection by the map projection transformation.

The experiment above verifies the correctness of the algorithm model that we designed. According to our limited information, currently, this algorithm model has not been applied in China. It could coverthe shortage of measurement function of online map services in China. It has certain practical value.

5.2 Forward-looking

With rapid development of computer technology, to complete large amount of projection coordinate transformation has not been a dream. Therefore, we could take advantage of each projection at the same time.When we are using web Mercator projection to browse a global map, we may measure area, length and angle under Gauss-Kruger projection. Even more, we could measure the geographical information of a certain area more accurately under other equidistant or equal-area projections. This process could be realized by writing software on the computer or tablet computer by application of the principle of map projection transformation. Thus, we could understand the selected area to the greatest extent by geographical information displayed by each projection from different perspectives (character of each projection). Meanwhile, the method of realization of measurement of a variety of geographical information on a global map by map projection transformation follows the concept of Digital Earth--integration of global geographical information. Since the hardware conditions have allowed us to implement map projection transformation, why not use such a convenient method that perfectly conforms to the concept of Digital Earth?

6. Main References:

1.Zhang Hong, Wen Yongning, Liu Aili. *Algorithm Basis of Geographic Information System*. Science Press Ltd. 2006

2. Dai Qinfen. *Google Earth and Open API and WEB Mercator Projection*. Qingdao Institute of Marine Geology. 2012

3. Li Guozao, Yang Qihe, Hu Dingquan. *Map Projection*. The people's Liberation Army Press. 1993

4. Yang Huan, *Design and Research on Map Projection Transformation*. Liaoning Technical University.Undergraduate Thesis. 2011

5. Mao Zanyou, Zhu Liang, Zhou Zhanao, Han Xuepei. *New Cartography Tutorial*. Higher Education Press. 2008

6.*ArcGIS Operating Manual* (Official Instruction)