The Application of the Modified Leslie Model in Predicting the Population Change of Nanjing

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Contents

Abstract: Based on Leslie Matrix, the paper tries to predict the population fluctuation of Nanjing by considering the regional distinction and latest policy impacts, and hereby comes up with four conclusions: a) The population of Nanjing will see a relatively rapid growth between 2013 and 2020; b) Given the situation, the elderly population of the city will experience a much faster increase in the next 40 years, causing an elderly bulge that not only poses heavy burden to social welfare but also gives rise to a disproportionate population structure; c) The proposed retirement age extension in theory, will reduce the pressure on pension system by retaining active labors, and thus alleviating the problems of aging population to some extent; d) The newly-issued Two-Child Policy will not change the trends in or transform the population structure, therefore its effects on population aging is rather limited.

Key Words: Population in Nanjing; Leslie Matrix; Gray Forecast Model; Population aging; Extension of Retirement Age; Two-Child Policy

I. Introduction

Population is an important factor in determining the comprehensive development of a country. Quality and size are the two basic elements of population; the latter directly impacts the economic and social development and influences resource allocation. Therefore, an accurate prediction of population fluctuation is a prerequisite to the formulation of economic and social policies.

While enjoying fast economic development as a first-tier city in Yangtze River Delta, Nanjing is facing intense pressure on environment and resources due to rapid growth of population. Therefore, an accurate prediction and analysis of its population is critical to the city's overall development. Compared with a forecast of the entire Chinese population, the prediction of Nanjing's population is a more complicated task, reason being China's migrant population is negligible to predicting the population dynamics of a country of 1.4 billion people, whereas Nanjing is going through the peak season of social, economic and population growth, and its migrant population cannot be safely ignored. Consequently, the difficulty of predicting demographic change greatly increases with such an open system.

Since Malthus, methods for population prediction have evolved through centuries, during which many unique methods were brought up, including the Malthusian Growth Model, Logistic Population Model, Gray Forecast GM (1,1) Model , Leslie Population Model, and Regression Analysis.

This paper on demographic change is based on a modified Leslie Matrix model. Moreover, it carries out further analysis of the population of Nanjing, including the overall trend in population change and the impact of certain social policies, like the extension of retirement age and two-child policy.

II. Construction of the model

i. Two assumptions concerning the model:

Do not consider the influence of unpredictable factors like natural disaster and wars, and take 90 years as the upper bound of a possible human age.

ii. Notation:

$x_i(t)$: Denotes the size of the i-year-old group in the year-t.

$d_i(t)$: Denotes the death rate of the i-year-old group in the year-t.

$s_i(t)$: Denotes the survival rate of the i-year-old group in the year-t. ($s_i(t) = 1 - d_i(t)$)

$w_i(t)$: Denotes the percentage of the i-year-old women-group in the year-t.

$b_i(t)$: Denotes the fertility rate of the i-year-old women-group in the year-t.

$x_0^{'}(t)$: Denotes the total number of the newly-born in the year-t.

$s_0^{'}(t)$: Denotes the survival rate of the newly-born in the year-t.

iii. The model

Our model consists of two major parts: one studies the population change of Nanjing as a closed system, while the other takes into account the impact of migrant population. To be concise, this paper first discusses each case individually and then combines the two to get the conclusion.

First Part: Consider Nanjing as a closed system

According to the parameters listed above, there is the following recursive formula:

$$x_{i+1}(t+1) = x_i(t)s_i(t) \quad \text{(i=0,1,2...n, t=0,1,2...)} \tag{1}$$

$$x_0'(t) = \sum_{i=i_1}^{i_2} w_i(t) b_i(t) x_i(t) \quad ([i_1, i_2]\text{denotes possible child-bearing age}) \tag{2}$$

$$x_0(t) = x_0'(t) s_0'(t) \tag{3}$$

Thus, we have the following equations:

$$\begin{cases} x_{i+1}(t+1) = x_i(t) s_i(t) \\ x_0(t) = s_0'(t) \sum_{i=i_1}^{i_2} w_i(t) b_i(t) x_i(t) \quad \text{(i=0,1,2...t=0,1,2...)} \end{cases} \tag{4}$$

Let $b_i'(t) = s_0'(t) w_i(t) b_i(t)$ be the modified fertility rate and consider population transformation vector $X(t) = [x_1(t)\ x_2(t) \cdots x_n(t)]^{\mathrm{T}}$. Therefore, $X(t+1) = L(t) X(t)$. (5)

$$L(t) = \begin{bmatrix} 0 & \cdots & b_{i_1}'(t) & \cdots & b_{i_2}'(t) & \cdots & 0 \\ s_0(t) & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & s_1(t) & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & s_{n-1}(t) & 0 \end{bmatrix}$$ is the Leslie Matrix.

Since the fertility rate of women in every age group and the survival rate of people in every group could be considered as approximately a constant in a short period, we simplify the Leslie as follows:

$$L = \begin{bmatrix} 0 & \cdots & b_{i_1}' & \cdots & b_{i_2}' & \cdots & 0 \\ s_0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & s_1 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \cdots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & s_{n-1} & 0 \end{bmatrix}. \tag{6}$$

Therefore, $X(t+1) = LX(t)$. (7)

Hence, $X(t) = L^t X(0)$. (8)

Take the year 2000 as the starting point: divide the ages from 0 to 90 into 91 one-year groups, take 15 to 49 as the possible child-bearing ages.

Use Matlab to calculate the Leslie Matrix iteration, we have the data as follows:

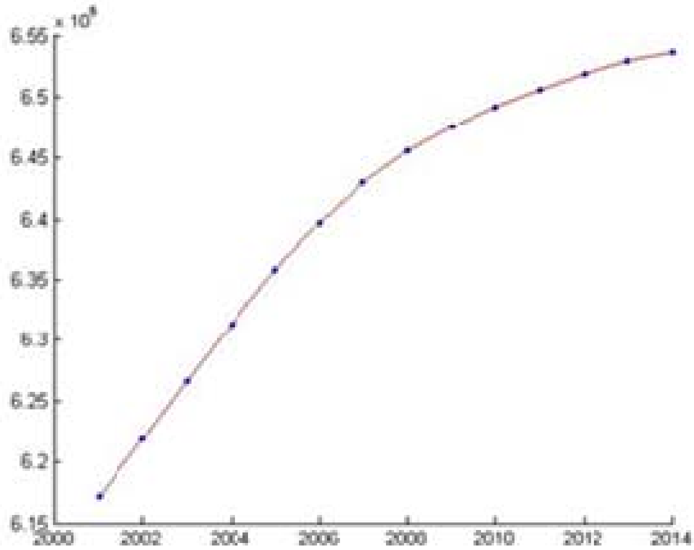

Figure 2.1 Prediction of Nanjing's total population as a closed system

Second Part: Consider Nanjing as an open system with migrant population

Since the population of Nanjing is a standard open system, migrant population is a crucial factor in the entire system. However, considering the incompleteness of certain data involving the migrant population, this paper uses Gray Forecast GM(1,1) Model to predict the entire migrant population.

GM(1,1) model is suitable for models with part of information known and is especially suitable for fast-growing small sample. The normal form of the model is:

Suppose the original non-negative sequence is $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots x^{(0)}(n)\}$. After one summation we have: $x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots x^{(1)}(n)\}$, where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(k)$.

Suppose $x^{(1)}$ satisfies the first-order Ordinary Differential Equation $\frac{dx^{(1)}}{dt} + ax^{(1)} = u$, where *a* is the development factor and *u* is gray actuating quantity.

Clearly, when satisfying the condition $t = t_0$, the equation has solution $x^{(1)}(t) = [x^{(1)}(t_0) - \frac{u}{a}]e^{-a(t-t_0)} + \frac{u}{a}$.

If we take discrete value for time, we then have $x^{(1)}(k+1)=[x^{(1)}(1)-\frac{u}{a}]e^{-ak}+\frac{u}{a}$. Put this discrete form into matrix, we get $y=BU$, where

$$y=(x^{(0)}(2),x^{(0)}(3),\cdots,x^{(0)}(N))^{\mathrm{T}}$$

$$B=\begin{bmatrix} -\frac{1}{2}[x^{(1)}(2)+x^{(1)}(1)] & 1 \\ -\frac{1}{2}[x^{(1)}(3)+x^{(1)}(2)] & 1 \\ \vdots & \\ -\frac{1}{2}[x^{(1)}(N)+x^{(1)}(N-1)] & 1 \end{bmatrix},\ U=\begin{bmatrix} a \\ u \end{bmatrix}$$

Applying least square estimation to the equation, we get $U^{'}=\begin{bmatrix} a^{'} \\ u^{'} \end{bmatrix}=(B^{T}B)^{-1}B^{T}y$.

Plug the estimated value $a^{'}$ and $u^{'}$ into the response equation:

$$\hat{x}^{(1)}(k+1)=\left[x^{(1)}(1)-\frac{\hat{u}}{\hat{a}}\right]e^{-\hat{a}k}+\frac{\hat{u}}{\hat{a}}$$

And then reduce by subtracting, we get the fitting value of original sequence $x^{(0)}$ when *k*=1,2...*n*-1. When $k\geq n$ we get the predicted value $\hat{x}^{(0)}(k+1)$.

According to the data of migrant population from year 2000 to 2008, we generate the following graph using Matlab:

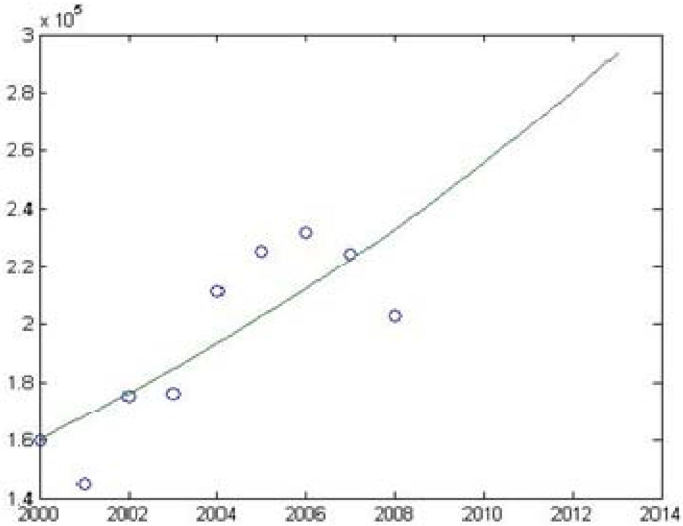

Figure 2.2 Prediction of Nanjing’s migrant population

In order to check the accuracy of Gray Forecast Model, we consider relation degree (r), posterior error ratio (c), small error possibility (p), and relative error (q). From the results of Matlab, we have r=0.6069, c=0.3466, p=1 and q=0.0075.

Table 2.1 Standard of prediction level

Level	Relative Error (q)	Posterior Error Ratio (C)	Small Error Possibility (P)
I	<0.01	<0.35	>0.95
II	<0.05	<0.50	<0.80
III	<0.10	<0.65	<0.70
IV	>0.20	>0.80	<0.60

From the table above, we conclude that our prediction is in level one, which means the prediction by Gray Forecast Model is quite accurate.

By synthesizing the first part and second part, we get the ultimate result of prediction:

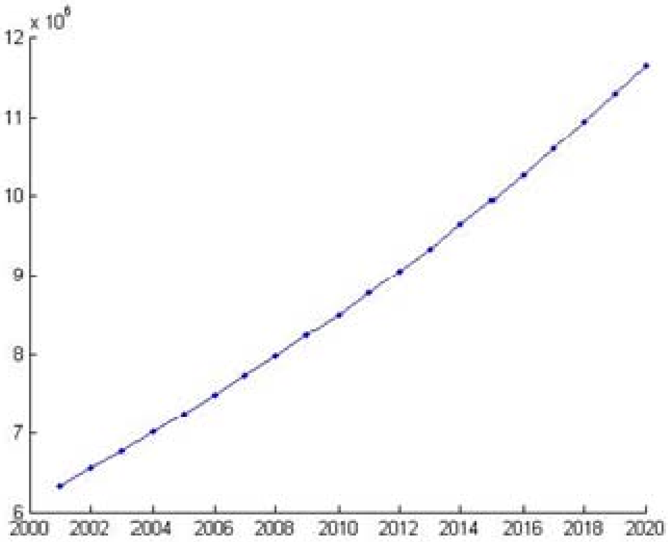

Figure 2.3 Prediction of Nanjing's total population as an open system

Table 2.2 Prediction of Nanjing's total population as an open system

Year	Total Population (10K)	Year	Total Population (10K)
2001	633.1	2011	875.9
2002	654.7	2012	904.0
2003	677.1	2013	933.1
2004	700.3	2014	963.2
2005	724.1	2015	994.2
2006	748.4	2016	1026.1
2007	772.9	2017	1059.0
2008	797.7	2018	1093.0
2009	823.0	2019	1128.2
2010	849.0	2020	1164.7

We then test the result using the Nanjing's total population for 2001-2012.

Table 2.3 Nanjing's total population for 2001-2012

Year	Total Population (10K)	Year	Total Population (10K)
2001	628.39	2007	741.30
2002	641.99	2008	758.89
2003	654.48	2009	771.31
2004	668.18	2010	800.76
2005	689.80	2011	810.91
2006	719.06	2012	816.10

(Data Source：Statistics Bureau of Nanjing）

After the calculation by Matlab, we get that relative error is q=0.0075, posterior error ratio c=0.1118. This shows that this model is relatively accurate.

III. Modification of the model

The previous discussion does not consider the impact of the change in women fertility rate when using the Leslie Matrix to predict the population dynamics, therefore we are modifying the model in this section for a more accurate forecast. Based on China's fifth and sixth population census, Nanjing's women fertility rate has changed a lot since the year 2000. The explicit data is as follows:

Table 3.1 2000.2010 Women fertility rate of every age group in Nanjing

fertility rate ‰	15-19	20-24	25-29	30-34	35-39	40-44	45-49
2000	2.76	119.59	58.21	9.36	2.50	0.64	0.25
2010	0.83	19.01	77.58	38.89	9.05	2.77	0.69

(Data Source：The fifth, the six population census）

It is conspicuous that with the social-economic development, more women choose to delay motherhood and "late marriage, late childbirth" has become more common in big cities like Nanjing. Therefore, it is necessary to consider the variation in women fertility rate when predicting the population growth.

Since the change is happening gradually, we use the function y=a*ln(x)+b to fit the change in women fertility rate. Here a and b are constants and x stands for year (starting from 2000).

Divide women aged from 15 to 49 into seven 5-year-groups, and denote the modified fertility rates as $y_1, y_2, y_3, y_4, y_5, y_6, y_7$. According to the data, we formulate the following fit function:

$$
\begin{aligned}
y_1 &= -0.0008049 \ln x + 0.00276 \\
y_2 &= -0.04195 \ln x + 0.1196 \\
y_3 &= 0.008078 \ln x + 0.05821 \\
y_4 &= 0.01231 \ln x + 0.00936 \\
y_5 &= 0.0002732 \ln x + 0.0025 \\
y_6 &= 0.0008883 \ln x + 0.00064 \\
y_7 &= 0.0001835 \ln x + 0.00025
\end{aligned}
$$

Plug in the modified fertility rate into Leslie Matrix, the estimate population are as follows:

Table 3.2 Nanjing's predicted population in 2001-2020 under modified women fertility rate

Year	Total Population(10K)	Year	Total Population(10K)
2001	634.6	2011	871.3
2002	656.9	2012	898.0
2003	679.8	2013	925.3
2004	702.9	2014	953.2
2005	725.9	2015	982.0
2006	749.0	2016	1011.7
2007	772.3	2017	1032.5
2008	795.9	2018	1044.3
2009	820.2	2019	1057.2
2010	845.3	2020	1064.3

Compared with the previous population estimation, the modified model might not be as effective as the previous one in predicting population change over a short period (since the change within fertility rates is almost negligible in the first few years). However, the modified model stands more accuracy in terms of a longer period of time, with the data from the previous model going against the actual regional stability as it still predicts an accelerated growth after year 2015. Therefore, the modified model has some advantages in long-term population estimation.

However, when examining the predicted data we find that the predicted value has an increasing deviation from the actual value after 2010. In retrospection, we find the deviation is due to the inaccuracy of Gray Forecast Model in long-time prediction because the value of Gray Forecast Model grows exponentially. To adjust the prediction, we decide to remodel the migrant population.

Considering that in such a fast-developing world, the migrant population tends to grow rapidly in a short period but will become relatively stable after a certain point, we use the following function to fit the growth of migrant population:

$$m(x) = ae^{-bx^2+cx+d}$$

Here $m(x)$ stands for the total migrant population while x stands for the year (starting from 2000) and a,b,c,d are constants.

Using matlab to fit the data we get a=6087000, b=0.004397, c=0.259184 and d=-3.819069. The R-square of the fitting is 0.9967, which means the fitting is accurate. Plug in these parameters to the modified model, we obtain the prediction of migrant population:

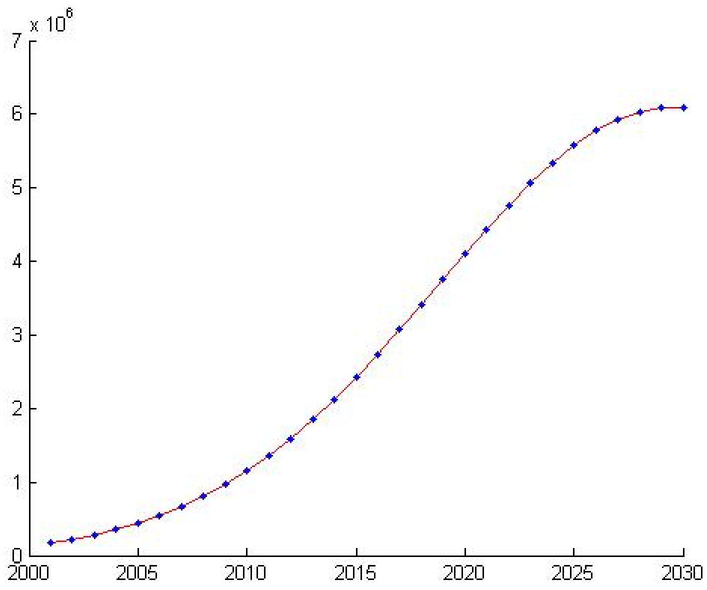

Figure3.1 Prediction of Nanjing's migrant population under modified model

From the prediction, it is clear that the migrant population will become saturated around 2030. For the migrant population, we assume it will be stable in a relatively long time after it reaches its peak value, which means we only consider the case where $m(x)$ is increasing. Therefore, we assume $m(x)$ to be a constant after 2030, which gives us the prediction of total population value:

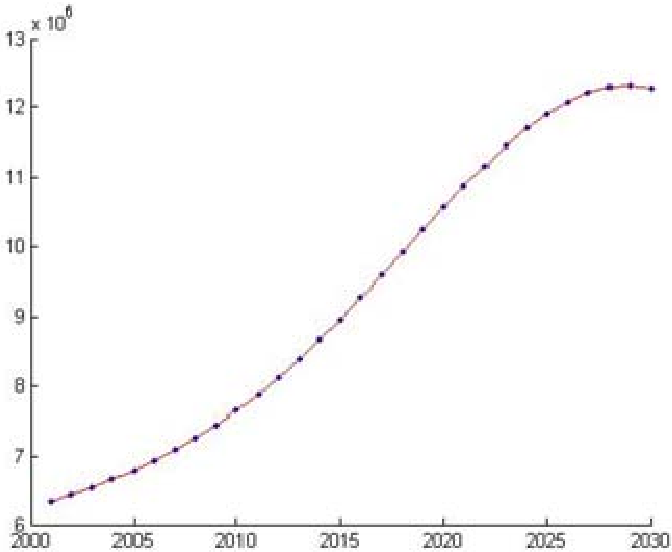

Figure 3.2 Prediction of Nanjing's total population under modified migrant population

Table 3.2 Prediction of Nanjing's total population under modified migrant population

Year	Total Population(10K)	Year	Total Population(10K)
2001	634.3	2010	764.1
2002	643.9	2011	786.4
2003	654.6	2012	810.9
2004	666.5	2013	837.6
2005	679.5	2014	866.1
2006	693.8	2015	896.2
2007	709.2	2016	927.6
2008	725.8	2017	959.9
2009	744.0	2018	992.6

After checking the predicted data, we find that the relative error is q=0.0094, posterior error ratio c=0.0528, and relation degree r=0.6286, which means the prediction accuracy improves.

IV. Study of the general tendency of population variation

As we try to study the population dynamics with the Leslie matrix, we limit our discussion to the closed system in this section, because the population in the closed system constitutes the majority of the population in Nanjing, and there are too many unpredictable factors influencing the migrant population.

Consider the original population equation $X(t+1) = LX(t)$ (5). For matrix *L*, we consider its characteristic equation:

$$|L-\lambda I| = \begin{bmatrix} -\lambda & \cdots & b'_{15} & \cdots & b'_{49} & \cdots & 0 \\ s_0 & -\lambda & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & s_1 & -\lambda & 0 & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & \cdots & 0 & s_{89} & -\lambda \end{bmatrix} = \sum_{i=1}^{35} b'_{50-i} \lambda^{40+i} \prod_{j=0}^{49-i} s_j - \lambda^{91}$$

Suppose $f(\lambda) = \frac{s_0 s_1 \cdots s_{48} b'_{49}}{\lambda^{50}} + \frac{s_0 s_1 \cdots s_{47} b'_{48}}{\lambda^{49}} \cdots + \frac{s_0 s_1 \cdots s_{14} b'_{15}}{\lambda^{16}}$ be the characteristic polynomial of *L*. Then the characteristic equation could be rewritten as $f(\lambda) = 1$. It is clear that when $\lambda > 0$, $f(\lambda)$ is continuous and decreasing. Furthermore, it is obvious that $\lim_{x \to 0^+} f(\lambda) = +\infty$ and $\lim_{x \to +\infty} f(\lambda) = 0$. Therefore, according to Intermediate Value Theorem, there is one and only one positive real number λ such that $f(\lambda) = 1$. Denote it as λ_0, then according to Perron-Frobenius Theorem, $\rho(L) = \lambda_0$. Here $\rho(L)$ is the spectrum radius of the matrix *L*. Moreover, from the corollary of Perron-Frobenius Theorem we can know that the population in this system is proportionate to λ_0^k in year *k*. Moreover, this can also be proved from another perspective: decompose $X(0)$ in the characteristic space of matrix L to get $X(0)=c_0v_0+c_1v_1+\cdots+c_{89}v_{89}$,where c_i are scalars and v_i are the eigenvectors of matrix L. Therefore, $X(t)=\lambda_0^t(c_0v_0+c_1\frac{\lambda_1^t}{\lambda_0^t}v_1+\cdots+c_{89}\frac{\lambda_{89}^t}{\lambda_0^t}v_{89})$

Thus, the population in this system in year t is proportionate to λ_0^t , therefore we get:

When $\lambda_0<1$, $\lim_{t\to+\infty}X(t)=0$;

When $\lambda_0=1$, $\lim_{t\to+\infty}X(t)=c_0v_0$;

When $\lambda_0>1$, $\lim_{t\to+\infty}X(t)=+\infty$.

From this we could see that, if λ_0 <1, then the closer λ_0 is to 1 and the more stable the population structure is. By plugging in data from Leslie Matrix and calculating by Matlab, we get that $\lambda_0=0.9883$. Thus, population structure in Nanjing will be stable in a long term.

V. Impact of social policies

Predictions and modifications above ignore the influence brought up by social policies. But in reality, public policies always have significant impact on demographic fluctuations. According to the model founded on Leslie Matrix, this paper further investigates the potential changes to be brought by the two policies, both in the context of Nanjing.

i. Extension of Retirement Age

Population aging is a phenomenon that occurs when the median age of a country or region increases due to rising life expectancy or declining fertility rates. This is usually reflected in an increase in the population's mean and median ages, a decline in the proportion of the population composed of children, and a rise in the proportion of the elderly population. According to the new UN standard, a region is categorized as ‘aging society’ if population aged 65 and over consists more than 7% of the total population. In China, this particular population reached 7.6% of the total population in 2005. By the end of 2008, population aged

65 and over reached 10,956 million, accounting for 8.3% of the total population. According to the statistics above, population in China has displayed a dramatic aging trend after years of Family Planning.

Population aging will cause severe obstacles to regional development. In addition to workforce reduction, downsizing of the domestic market, and increased burden on youth, population aging will also pose challenges to the current pension system. All these will create hardships for our social-economic development. Thus, how to alleviate the crisis caused by population aging has become a top priority for demographic development.

Based on the Leslie Model, this paper predicts the growth of elderly population (people aged 65 and over) in Nanjing from 2001 to 2030:

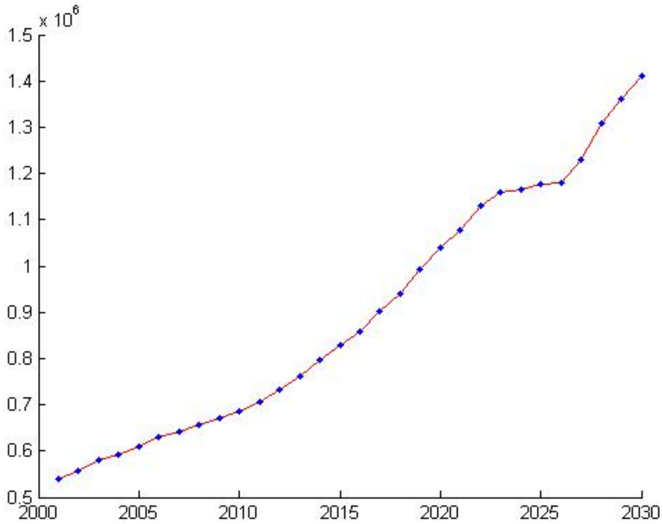

Figure 5.1 Prediction of elderly population in Nanjing

A closely related prediction is the proportion of elderly population, as shown below.

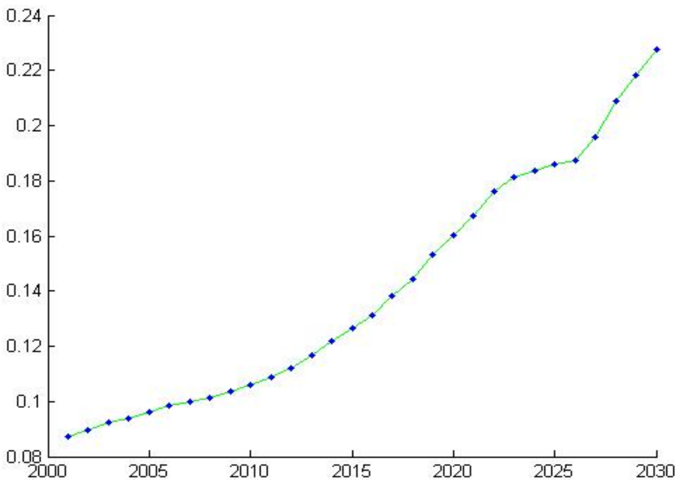

Figure 5.2 Prediction of elderly population's proportion in Nanjing

Based on our prediction, population aging in Nanjing will accelerate in the next 30 years. Proportion of the elderly population in Nanjing will reach 13.17% in 2015, while the same index nationwide is expected to be 9.8% over the same period, a proportion 3.37% lower than that of Nanjing. In 2030, the elderly population in Nanjing will reach 1,400,000, accounting for 22.75% of the municipal population and continue to grow, indicating that population aging is getting faster and even worse. On the other hand, the proportion of the juvenile population (0-15 years old) is shrinking rapidly:

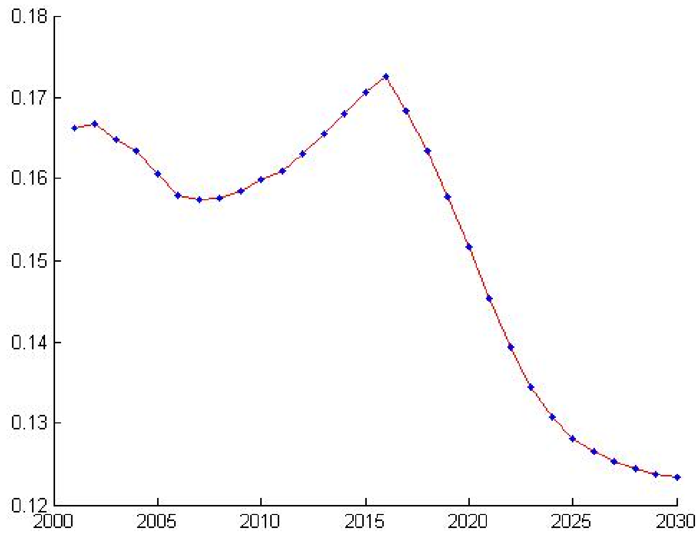

Figure 5.3 Prediction of juvenile population's proportion in Nanjing

According to our prediction, the total number of elderly population will exceed juvenile population in 2020, when the juvenile population percentage shrinks to 15.98% and the elderly population percentage rises to 16.04%. The pattern reveals a disproportionate population structure.

Population senility is also exacerbating. In demography, sexagenarians (60-69 years old) are regarded as younger elderly population, septuagenarians (70-79 years old) as middle-aged elderly population, and octogenarians (80 years of age or older) senior elderly population. Since the senior elderly are largely people of poor health and are advised against living by themselves, the more people enter into age of senior elderly would require more resources and support from their families and the society alike. The data shows, senior elderly population in Nanjing will be growing rapidly in the near future, and probably reach 230,000 by 2030.

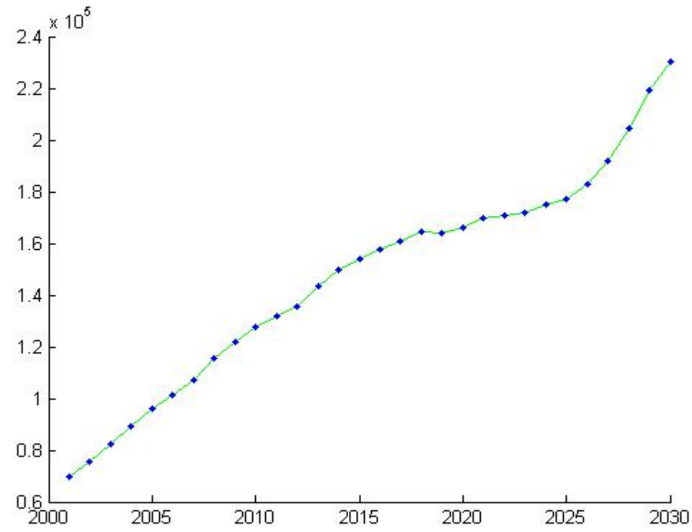

Figure 5.4 Prediction of senior elderly population in Nanjing

So increase in social supporting rate is anticipated to be another problem coming with population aging.

Denote the total population as $N(t)=\sum_{i=0}^{90} x_i(t)$, the labor population (population aged between 15 and 60) as $R(t)=\sum_{i=l_1}^{l_2} x_i(t)$, then let $l_1=15$ and $l_2=60$. Hence, social supporting rate is defined as $\rho(t)=\frac{N(t)-R(t)}{R(t)}$. A variation graph about social supporting rate is produced based on Leslie Model.

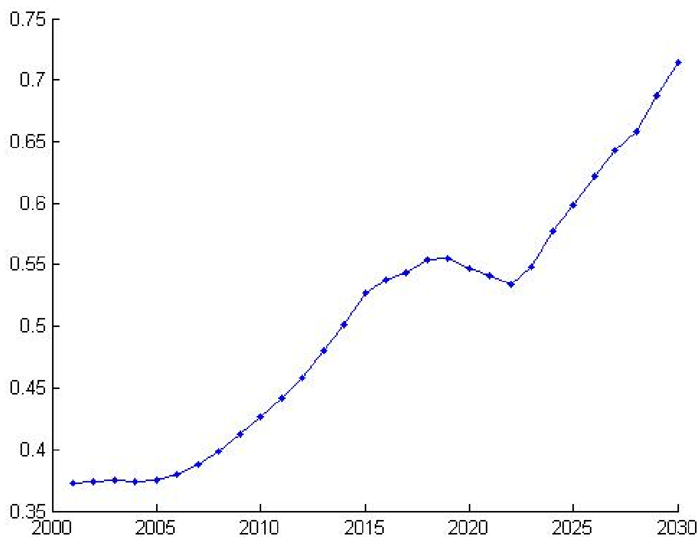

Figure 5.5 Nanjing's predicted social supporting rate

According to Figure 5.5, social supporting rate displays an increasing tendency, foreboding heavier social burdens and an unhealthy population structure in the city.

The ever deteriorating problem of population aging calls for efficient policies to alleviate pressures on social system.

In November 2013, the Ministry of Human Resources and Social Security came up with suggestions postponing the legal retirement age to 65 so as to save RMB 20 Billion in pension funds each year. According to the agency, ‘the economic and social development and prolongation of average life-span necessitate such an extension of retirement age.’

In response to the news, this paper compared the social supporting rate before and after the implementation of the retirement age extension.

Suppose that the legal retirement age will be extended from 60 to 65 since 2015. Consequently, labor population is adjusted to be the group aged between 15 and 65. A new variation graph is presented below:

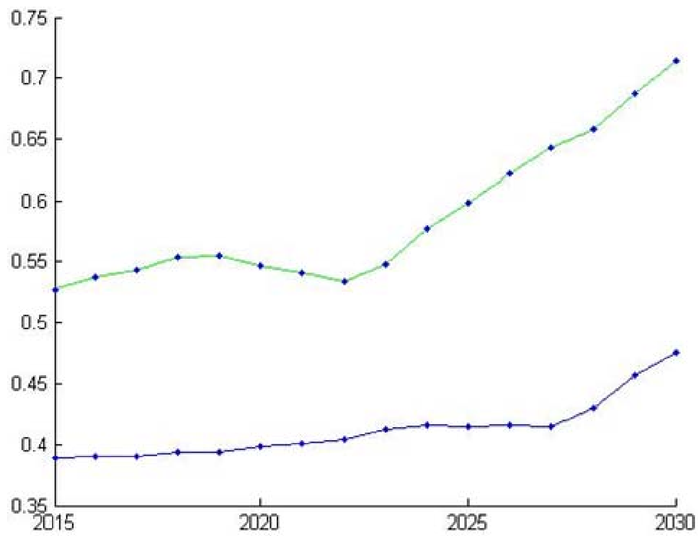

Figure 5.6 Nanjing’s predicted social supporting rate before and after the policy

According to the graph, the extension of retirement age will exert a remarkable impact on social supporting rate: in 2030, the social supporting rate will be kept at about 0.45, a relatively low level compared to the previous 0.7. Hence, postponing the legal retirement age will increase labor population and efficiently reduce social burden.

ii. Two-Child Policy(policy regulating the birth of the second child in china)

'Double One-Child' Policy: by 1999, Nanjing has implemented 'Double One-Child' Policy: couples are allowed to have two children in a household if both parents are born the only child of their original families.

'Single One-Child' Policy: in November 2013, China announced that it is planning to implement 'Single One-Child' Policy soon, which is to say couples are allowed to have two children if either parent is the only child of his or her original family.

Based on current reproductive age, marital status, and family-planning policies, the Population and Family Planning Committee in Nanjing estimated that, the number of One-Child family in Nanjing (including Double One-Child family and Single One-Child family) is estimated to be 270,000, including 100,000 Single One-Child families and 170,000 Double One-Child families. And the paper is taking an interest in the potential effects of the newly issued 'Single One-Child' policy.

However, due to the uncertainty of the future fertility rate, the paper considers the impact of Two-Child Policies in general and makes three hypothesis based on low, middle and high fertility rates.

Hypothesis A: Low fertility rate Hypothesis based on the assumption that future fertility rate will remain the same as the current fertility rate after the policy takes effect.

Hypothesis B: Middle fertility rate Hypothesis based on the assumption that 20% of the families with only one child intend to have a second child after the policy takes effect.

Hypothesis C: High fertility rate Hypothesis based on the assumption that 40% of the families with only one child intend to have a second child in the future after the policy takes effect.

Given the three situations, this paper predicts the total population from 2015 to 2030.

Table 5.1 Nanjing's predicted population for each hypothesis (partial)

Year	2026(10K)	2027(10K)	2028(10K)	2029(10K)	2030(10K)
Hypothesis A	1412.5	1466.2	1522.4	1581.3	1643.0
Hypothesis B	1451.5	1504.6	1560.4	1618.8	1679.9
Hypothesis C	1481.0	1536.2	1594.4	1655.2	1718.6

From the table we could see that both the Middle fertility rate hypothesis and the High fertility rate hypothesis show that the population presents a small increase under the encouragement of Two-Child Policy. Meanwhile, we also predict the impact of Two-Child Policy on the population structure, especially on the aging problem:

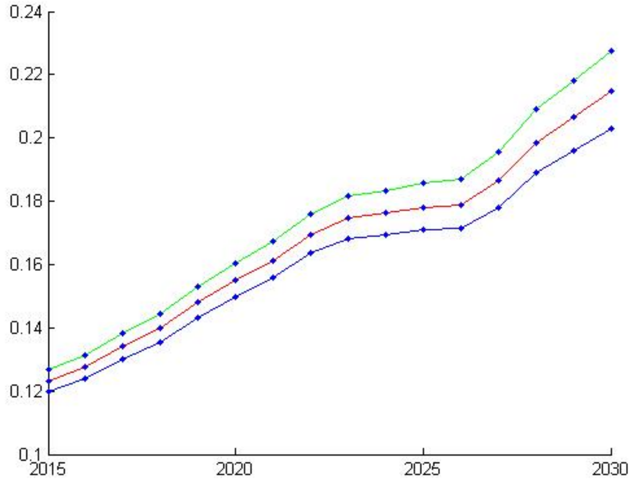

Figure 5.7 Nanjing’s elderly population’s proportion under each hypothesis

Compared to Low fertility rate Hypothesis, Middle fertility rate Hypothesis will bring down the proportion of elderly population by 1%, and High fertility rate Hypothesis will bring the proportion down by 2%. As a result, Two-Child Policy will only slightly curb the trends of population aging. But in the long run, with more and more child-bearing age women pursuing modern ideas in family and life, the low fertility rate resulted will offset any impacts intended by the Two-Child Policy, and thus shall not bring radical changes to the current population structure.

VI. Conclusion

This paper models the population fluctuation in Nanjing based on the famous Leslie Matrix and applies Grey Forecast Model to predicting the rural-to-urban migration. On top of it, the paper further modifies Leslie model by considering variations in women fertility rates. Through analysis we have reached following conclusions.

(1) The population in Nanjing will grow rapidly from 2013 to 2020. According to our prediction, the total population will break 10,000,000 in 2019, and keep on soaring afterwards. Specifically, migrant population will reach saturation in about 2030 and stabilize.

(2) Based on current policy, the elderly population in Nanjing will grow rapidly in the next 40 years, which will pose heavy burdens to society and give rise to a disproportionate population structure. The elderly population and its proportion will increase steeply, and the proportion will outnumber similar index nationwide. In 2030, the social supporting rate in Nanjing will reach 0.7, posing considerable social burdens. Consequently, the government should watch demographic changes closely and implement effective policies to contain the population growth.

(3) The extension of retirement age will effectively reduce social pressures and retain labors in workforce, and therefore, alleviate the crisis posed by population aging. Suppose the policy will be implemented in 2015, the social supporting rate in Nanjing will be kept at around 0.45.

(4) Two-Child Policy will relieve the pressures posed by aging only on a small scale, and it will not radically transform the population structure. Since women's future fertility intention is hard to estimate and quantify, this paper proposed three hypotheses, including Low, Middle and High fertility rate Hypothesis to project the population fluctuation. Compare with Low fertility rate Hypothesis, Middle fertility rate Hypothesis will bring down the proportion of elderly population by 1%, and High fertility rate Hypothesis will bring the proportion down by 2%. In the long run, the modernization of the fertility ideas and the low fertility rate resulted will weaken the intended impact posed by Two-Child Policy, and thus will not bring about radical changes to the current population structure.

Despite our best efforts, the paper has its limitations. For instance, it does not consider the possible change in death rates; the Gray Forecast Model has its restrictions when applied to long-term predictions. Besides, the paper might be too simplified on discussion and analysis of population structure in the eyes of demographers.

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Appendix

Matlab for prediction of total population:

```
X0=xlsread('population.xlsx');
L=xlsread('data.xlsx');
for k=1:20
    X=L^k*X0;
    Y=sum(X);
    hold on;
end
syms a b;
C=[a b];
A=[160085 144934 175223 176346 211428 225411 231863 224663 202741];
B=cumsum(A);
n=length(A);
for i=1:n-1
    C(i)=(B(i)+B(i+1))/2;
end
D=A;
D(1)=[];
D=D';
E=[-C; ones(1,n-1)];
C=inv(E*E')*E*D;
C=C';
a=C(1);
b=C(2);
F=[];F(1)=A(1);
for i=2:20
    F(i)=(A(1)-b/a)/exp(a*(i-1))+b/a;
end
G=[]; G(1)=A(1);
for i=2:20
    G(i)=F(i)-F(i-1);
end
for i=1:20
    P=sum(G(1:i));
    K=Y+P;
    T(i)=K;
    plot(i,K,'.')
end
disp(T)
```

Matlab for Gray Forecast Model

```
x0=[160085 144934 175223 176346 211428 225411 231863 224663 202741];
x1=G(1:9);
e0=x0-x1;
max1=max(abs(e0));
r=1;
for k=2:n
    r=r+0.5*max1/(abs(e0(k))+0.5*max1);
end
r=r/n;
q=e0/x0;
s1=var(x0);
s2=var(e0);
c=s2/s1;
len=length(e0);
p=0;
for i=1:len
    if (abs(e0(i))<0.6745*s1)
        p=p+1;
    end
end
p=p/len;
disp(r)
disp(c)
disp(p)
disp(q)
```

Mathlab for calculating the spectrum radius of the matrix L:

```
L=xlsread('data.xlsx');
p=poly(L);
r=roots(p);
r0=max(abs(r));
disp(r0)
```

Matlab for prediction of senior elderly population:

```
X0=xlsread('population.xlsx');
L=xlsread('data.xlsx');
for k=1:50
    X=L^k*X0;
    Y=sum(X(81:91));
    M(k)=Y;
    N=sum(X);
    N1=Y/N;
    A(k)=N1;
```

```
    plot(k,Y,'.');
    hold on;
end
```

Matlab for prediction of social supporting rate:

```
X0=xlsread('population.xlsx');
L=xlsread('data.xlsx');
for k=1:50
    X=L^k*X0;
    Y=sum(X);
    R=sum(X(16:61));
    S=(Y-R)/R;
    M(k)=S;
    plot(k,S,'.');
    hold on;
end
disp(M)
```