

Optimal Design for Window Ventilation

中国人民大学附属中学

赵昕玥 何博硕 李泰伯

ABSTRACT

This paper discusses the problem that making the air flowing into the room maximized by rotating the windows. Two models are built for rotary windows with 2 and 3 sashes respectively. Based on these models and the assumption of wind blowing from different directions with equal chance, the optimal way of opening windows is calculated by using Pascal programs on computer. Finally, the optimal design for window ventilation is found via comparing maximum value of wind quantity in two situations.

1. Introduction

Along with the improvement of living standard, people give more and more attention to their own health. Well housing environment is essential to people's health, as people spend most of their time indoors. Ventilation condition is a key factor of housing environment. Undoubtedly, appropriate ventilation through windows is contributive to clear indoor air condition either in summer or winter. Therefore, how to guarantee well ventilation condition is a problem which worthy to be discussed.



Figure 1-1

Figure 1-2

There are mainly two kinds of rotary window styles in common dwellings. One is the two-sash style, as shown in Figure 1-1; the other is three-sash style, as shown in Figure 1-2. Both styles have two sashes can be rotated. These sashes revolve around fixed axes. Each sash has a range of rotation angle from 0 degree to 90 degree, so that two movable sashes can form different combination modes which have different wind volume. Two movable sashes block a part of wind blowing into the room. That is why different combination modes have different wind volume. Appropriate combination mode can maximize the wind volume, so it is necessary to find this combination mode. In this paper, we will discuss this problem.

2. Model Assumption

Although different zone have different prevailing wind direction, many other factors can also affect the directions of wind, such as building, tall tree and facilities around window. So we can assume that wind blow from different directions with equal chance. Obviously, a window faced to south can not receive wind blowing from north, so we only consider wind blowing from window's obverse side with a range of 180 degree.

To simplify modeling, we ignore subordinate factors and assume that wind blow in a straight line like sunlight, and only have horizontal velocity. We also ignore the thickness of glass and assume that glass of sashes does not reflect wind. In this paper, we only discuss a single window with two or three sashes.

According to actually usage, we assume that the included angle between wind direction and windowsill has a variation range from 0 degree to 90 degree.

3. Building Model and Solution

We build models for each window styles respectively. We assume that the width of both kinds of windows is 1. And we use the length of intersecting line (in Figure 3-1, it is line EF) to represent the volume of wind blowing into the room. The intersecting line is generated by intersecting windowsill (in Figure 3-1, it is line AB) with parallel airflow (in Figure 3-1, it is represented by two dashed). Particularly, in three-sash style, the length of intersecting line should subtract the width of middle sash which can not be rotated, so that it can represent the wind volume correctly.

3.1 Model for two-sash style window

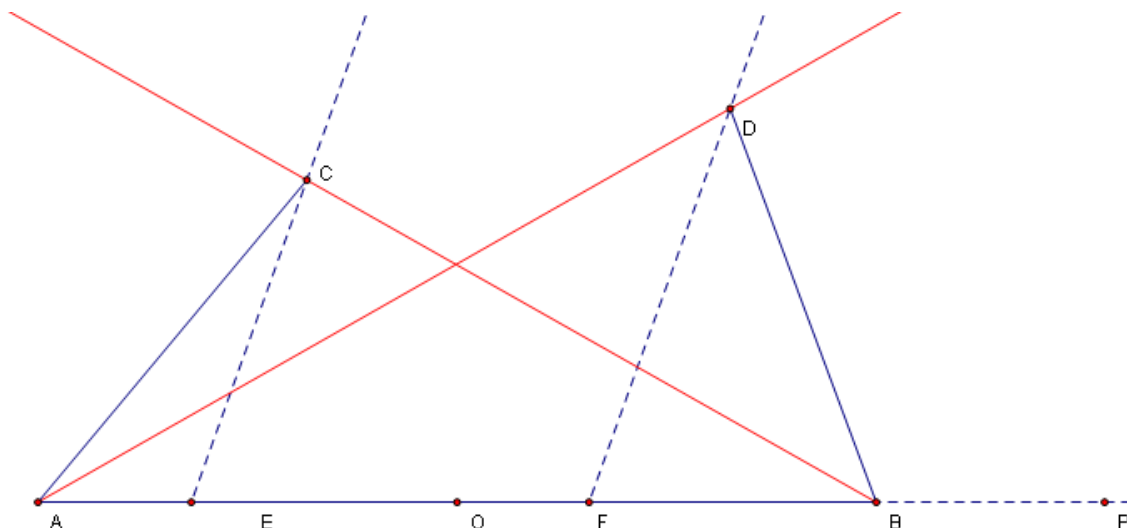


Figure 3-1

In Figure 3-1, the line AB represents the windowsill, and point O represents the middle arris of window.

Set $AB = 1$, then $AO = AC = BO = BD = \frac{1}{2}$. Let $\angle CAO = \alpha, \angle DBP = \beta$,

where $\alpha \in [0, \frac{\pi}{2}]$, $\beta \in [\frac{\pi}{2}, \pi]$; and define angle r is the intersect angle between wind direction and radial AP, where $r \in [0, \pi]$; then function $f(r)$ represents the length of

intersecting line EF.

We consider two different cases:

(1) When the difference between two sashes' rotary angle is small, as shown in Figure 3-1-1, we will have $\angle CAB \geq \angle DAB$, $\angle DBA \geq \angle CBA$. Under this condition, we can consider that two sashes have relative small influence on wind volume (in Figure 3-1-1, the red lines represent the critical status of wind direction).

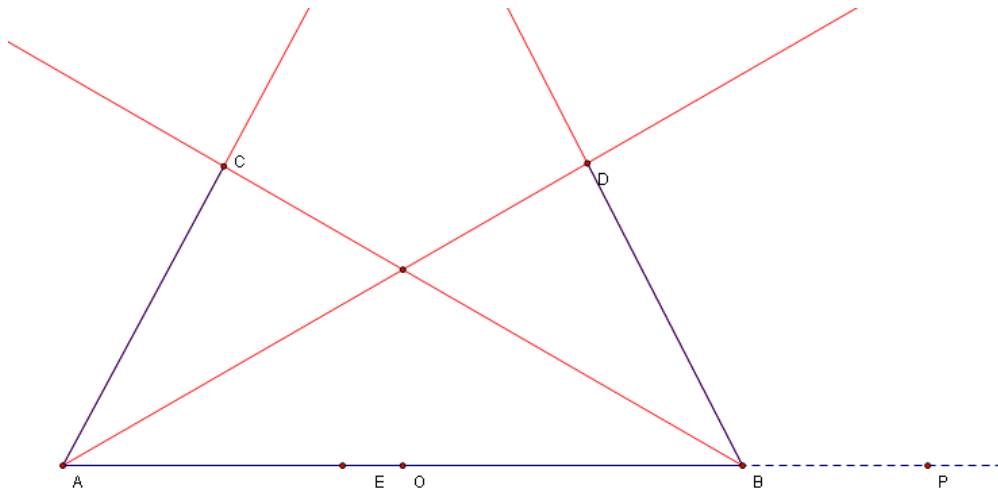


Figure 3-1-1

We further discuss five instances:

A) When $0 \leq r < \angle DAB$, wind is blocked by sash DB, we will have:

$$f(r) = 0$$

Let $\angle DAB = r_1$, which is the critical angle in this instance. Using sine's law in $\triangle ABD$, we will have

$$\frac{\sin \angle BDA}{\sin \angle DAB} = \frac{AB}{BD} = 2,$$

And $\angle BDA = \angle DBP - \angle DAB = \beta - r_1$

So,

$$2 = \frac{\sin(\beta - r_1)}{\sin r_1} = \frac{\sin \beta \cos r_1 - \sin r_1 \cos \beta}{\sin r_1},$$

Deduct that

$$\sin r_1 (2 + \cos \beta) = \cos r_1 \sin \beta ;$$

$$\therefore \tan r_1 = \frac{\sin \beta}{2 + \cos \beta} ,$$

Notice that $\beta \in [\frac{\pi}{2}, \pi]$, $\therefore \sin \beta \in [0, 1]$, $\cos \beta \in [-1, 0]$

$$\therefore \frac{\sin \beta}{2 + \cos \beta} \geq 0$$

Then, we can conclude that

$$r_1 = \arctan \frac{\sin \beta}{2 + \cos \beta} .$$

B) When $\angle DAB \leq r < \alpha$, sash AC has no influence on wind volume blowing into the room.

Let $r_2 = \alpha$, which is the critical angle in this instance.

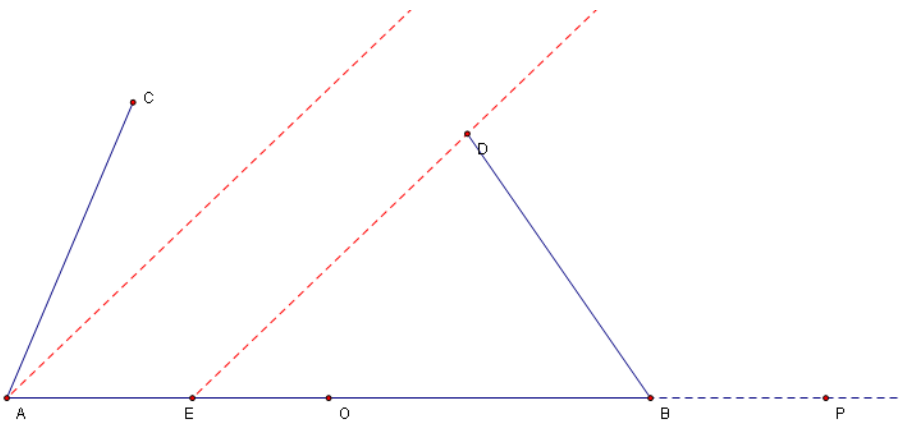


Figure 3-1-2

As shown in Figure 3-1-2, we will have

$$f(r) = AE = AB - BE .$$

Using sine's law in $\triangle EDB$, we will have

$$BE = \frac{\sin(\beta - r)}{\sin r} \square BD ,$$

Using given data in this formula, we will have

$$f(r) = AB - BE = 1 - \frac{1}{2} \cdot \frac{\sin(\beta - r)}{\sin r},$$

Rearranging this formula, we will have

$$f(r) = -\frac{1}{2} \sin \beta \cot r + \frac{1}{2} \cos \beta + 1;$$

C) When $\alpha \leq r < \beta$, let $r_3 = \beta$ is the critical angle in this instance.

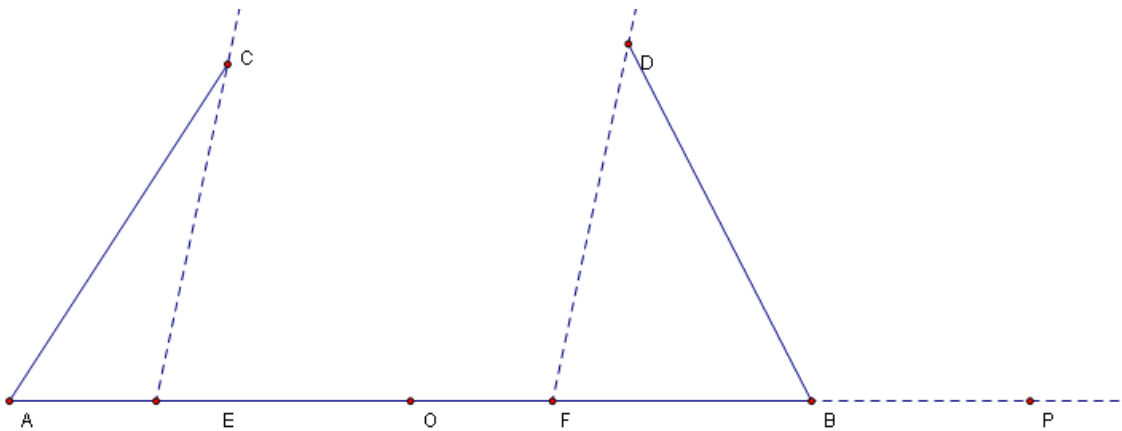


Figure 3-1-3

In Figure 3-1-3, we will have

$$f(r) = EF = AB - AE - BF.$$

Using sine's law in $\triangle CAE$ and $\triangle DFB$, we will have

$$AE = \frac{\sin(r - \alpha)}{\sin r} \square AC,$$

$$BF = \frac{\sin(\beta - r)}{\sin r} \square BD,$$

Using given data in this formula, then

$$f(r) = \frac{1}{2} \left[2 - \frac{\sin(r - \alpha)}{\sin r} - \frac{\sin(\beta - r)}{\sin r} \right],$$

Rearranging this formula, we will have

$$f(r) = \frac{1}{2}[(\sin \alpha - \sin \beta) \cot r + (2 - \cos \alpha + \cos \beta)];$$

D) When $\beta \leq \alpha < \angle CBP$, sash BD has no influence on wind volume blowing into the room.

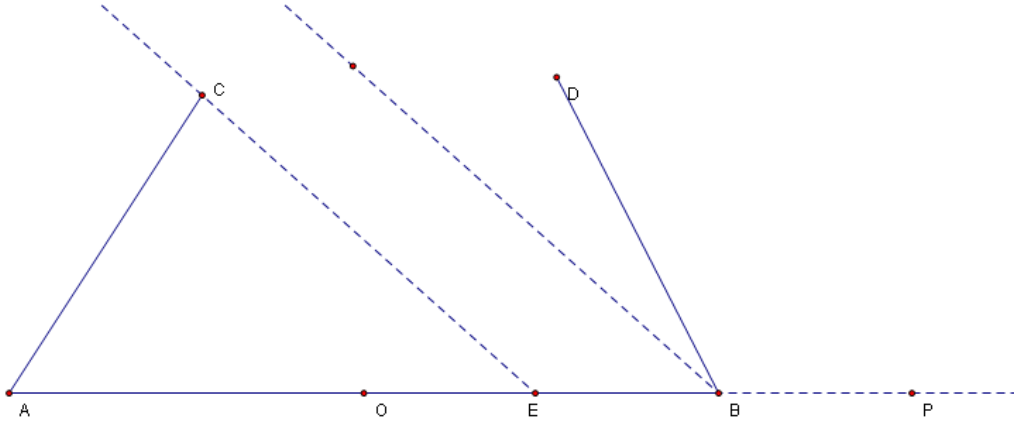


Figure 3-1-4

In Figure 3-1-4, $f(r) = BE = AB - AE$,

Using sine's law in $\triangle ACE$, we will have

$$AE = \frac{\sin(r - \alpha)}{\sin(\pi - r)} \square AC,$$

Using given data in this formula, then

$$f(r) = AB - AE = 1 - \frac{1}{2} \frac{\sin(r - \alpha)}{\sin r},$$

Rearranging this formula, we will have

$$f(r) = \frac{1}{2} \sin \alpha \cot r - \frac{1}{2} \cos \alpha + 1;$$

E) When $r \geq \angle CBE$, wind is blocked by sash AC, just like A), we will have

$$f(r) = 0,$$

Let $\angle CBE = r_4$ is the critical angle in this instance.

In Figure 3-1-1, using sine's law in $\triangle ABC$, we have

$$\frac{\sin \angle ACB}{\sin \angle CBA} = \frac{AB}{AC} = 2,$$

And $\angle ACB = \angle CBE - \angle CAB = r_2 - \alpha$, $\angle CBA = \pi - r_2$.

Then

$$2 = \frac{\sin(r_2 - \alpha)}{\sin(\pi - r_2)} = \frac{\sin r_2 \cos \alpha - \cos r_2 \sin \alpha}{\sin r_2},$$

We can conclude that

$$\sin r_2 (\cos \alpha - 2) = \sin \alpha \cos r_2;$$

$$\therefore \tan r_2 = \frac{\sin \alpha}{\cos \alpha - 2},$$

and $\alpha \in \left[0, \frac{\pi}{2}\right]$, $\therefore \sin \alpha \in [0,1]$, $\cos \alpha \in [0,1]$

$$\therefore \tan r_2 = \frac{\sin \alpha}{\cos \alpha - 2} \leq 0$$

$$\therefore r_2 = \arctan \frac{\sin \alpha}{\cos \alpha - 2} + \pi.$$

To sum up, we will have

$$f(r) = \begin{cases} 0, r \in [0, r_1) \\ -\frac{1}{2} \sin \beta \cot r + \frac{1}{2} \cos \beta + 1, r \in [r_1, r_2) \\ \frac{1}{2} [(\sin \alpha - \sin \beta) \cot r + (2 - \cos \alpha + \cos \beta)], r \in [r_2, r_3) \\ \frac{1}{2} \sin \alpha \cot r - \frac{1}{2} \cos \alpha + 1, [r_3, r_4) \\ 0, [r_4, \pi] \end{cases}$$

We use $F(\alpha, \beta)$ to denote the mean volume of wind blowing into the room, and let $r_5 = \pi$. Then we will have

$$F(\alpha, \beta) = \frac{\sum_{i=0}^4 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi - 0} = \frac{\sum_{i=0}^4 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi}.$$

Because of

$$\int \cot x dx = \ln |\sin x| + c \quad (C \text{ is a constant}),$$

So

$$\int_{r_1}^{r_2} f(r) dr = -\frac{1}{2} \sin \beta \ln \left| \frac{\sin r_2}{\sin r_1} \right| + \left(\frac{1}{2} \cos \beta + 1 \right) \bullet (r_2 - r_1)$$

$$\int_{r_2}^{r_3} f(r) dr = \frac{1}{2} (\sin \alpha - \sin \beta) \ln \left| \frac{\sin r_3}{\sin r_2} \right| + \frac{1}{2} (2 - \cos \alpha + \cos \beta) \square (r_3 - r_2)$$

$$\int_{r_3}^{r_4} f(r) dr = \frac{1}{2} \sin \alpha \ln \left| \frac{\sin r_4}{\sin r_3} \right| + \left(1 - \frac{1}{2} \cos \varepsilon \right) \bullet (r_4 - r_3)$$

We use PASCAL program to assign different value to α, β and calculate the value of integral expressions.

$$\text{When } \begin{cases} \alpha = \frac{\pi}{2} \\ \beta = \frac{\pi}{2} \end{cases},$$

$$\sum_{i=0}^4 \int_{r_i}^{r_{i+1}} f(r) dr \text{ has maximum value, it is } 0.4487.$$

$$\text{That is to say } F(\alpha, \beta) = \frac{\sum_{i=0}^4 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi} \text{ has maximum } 0.4487.$$

According to Figure 3-1-5, r_1 is an acute angle, so $\sin r_1 > 0$

$$r_1 = \arctan \frac{\sin \beta - \sin \alpha}{2 + \cos \beta - \cos \alpha}$$

B) When $\angle DQB \leq r < \beta$, as shown in Figure 3-1-6, let $\beta = r_2$ is the critical angle under this condition.

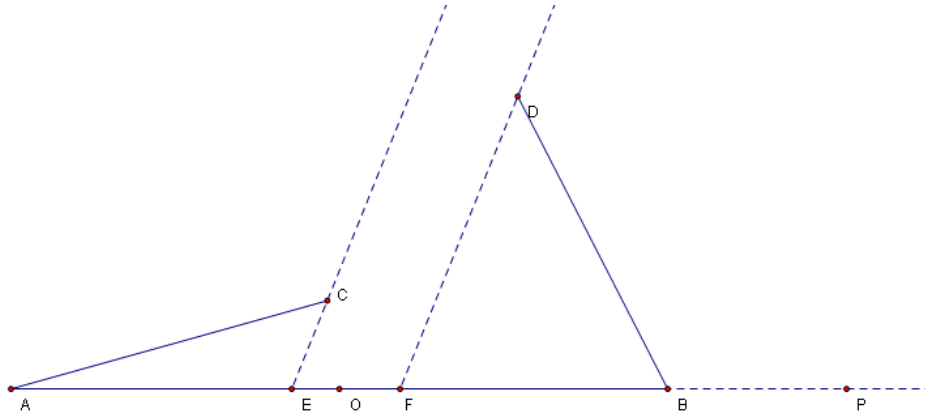


Figure 3-1-6

This instance is same as the instance C) in case (1). So we will have

$$f(r) = \frac{1}{2} [(\sin \alpha - \sin \beta) \cot r + (2 - \cos \alpha + \cos \beta)]$$

C) When $\beta \leq r < \angle CBP$, as shown in Figure 3-1-7, the instance is same as the instance D) in case 1.

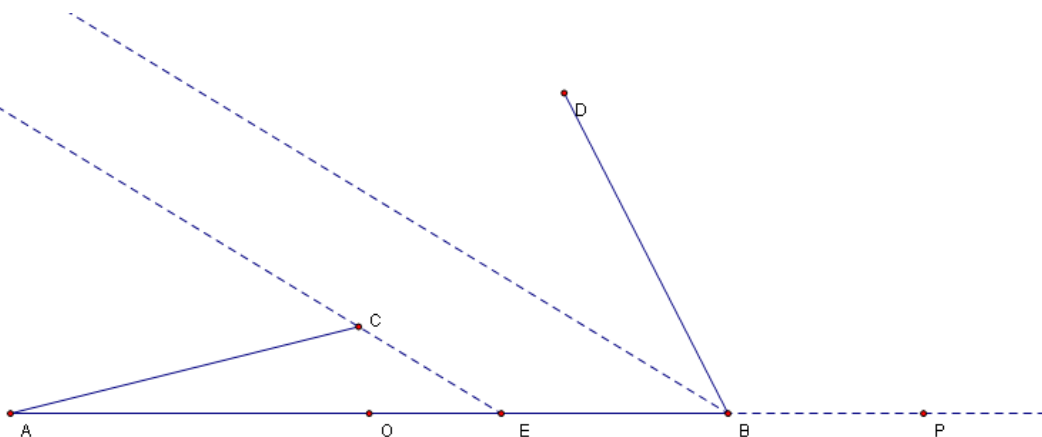


Figure 3-1-7

So we will have

$$f(r) = \frac{1}{2} \sin \alpha \cot r - \frac{1}{2} \cos \alpha + 1$$

D) When $\angle CBP \leq r \leq \pi$, wind is blocked by sash AC. So we will have

$$f(r) = 0.$$

And let $\angle CBP = r_3$ is the critical angle under this condition. This instance is like the instance E) in case1. So we will have

$$r_3 = \arctan \frac{\sin \alpha}{\cos \alpha - 2} + \pi$$

To sum up, we will have

$$f(r) = \begin{cases} 0, r \in [0, r_1) \\ \frac{1}{2} [(\sin \alpha - \sin \beta) \cot r + (2 - \cos \alpha + \cos \beta)], r \in [r_1, r_2) \\ \frac{1}{2} \sin \alpha \cot r - \frac{1}{2} \cos \alpha + 1, [r_2, r_3) \\ 0, [r_3, \pi] \end{cases}$$

So the mean volume of wind blowing into the room is ($F(\alpha, \beta)$ denotes to mean volume of wind, and $r_4 = \pi$)

$$F(\alpha, \beta) = \frac{\sum_{i=0}^3 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi - 0} = \frac{\sum_{i=0}^3 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi}$$

We use PASCAL program to assign different value to α, β and calculate the value of integral expressions.

$$\text{When } \begin{cases} \alpha = 0 \\ \beta = \frac{\pi}{2} \end{cases} \text{ (or symmetrical condition } \begin{cases} \alpha = \frac{\pi}{2} \\ \beta = \pi \end{cases}),$$

$$\sum_{i=0}^3 \int_{r_i}^{r_{i+1}} f(r) dr \text{ has maximum value, it is } 0.3079.$$

That is to say $F(\alpha, \beta) = \frac{\sum_{i=0}^3 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi}$ has maximum 0.3079.

To sum up two cases, we draw a conclusion that when two sash both have a 90 degree included angle with windowsill, we will get the maximum wind volume. The maximum volume is 0.4487

3.2 Model for three-sash style window

The model for three-sash style window is alike as two-sash model. Because of the middle non-moveable sash, there are more types of critical status in this model. And the deduction is more complex.

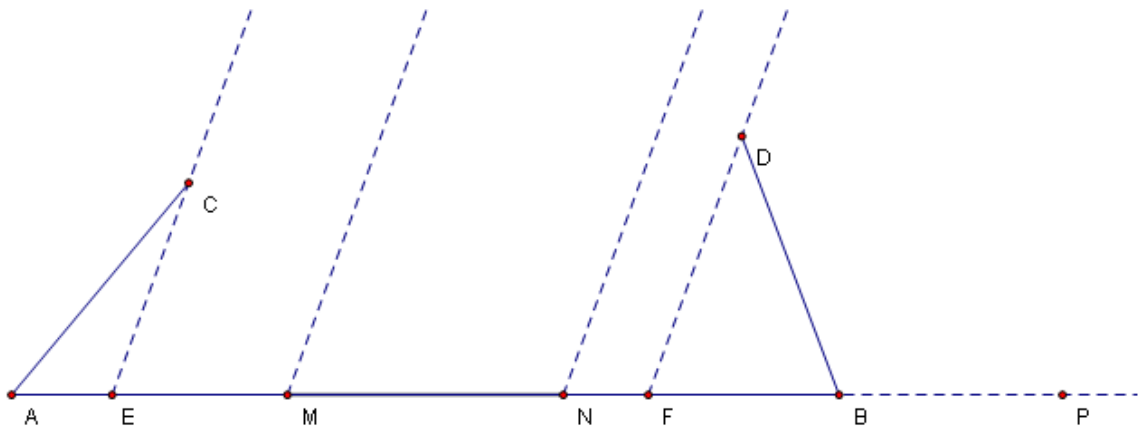


Figure 3-2

In Figure 3-2, let $AB = 1$, then $AC = AM = MN = BN = BD = \frac{1}{3}$ (the explanation is in section 5).

As the first model, let $\angle CAO = \alpha, \angle DBP = \beta$, where $\alpha \in [0, \frac{\pi}{2}], \beta \in [\frac{\pi}{2}, \pi]$;

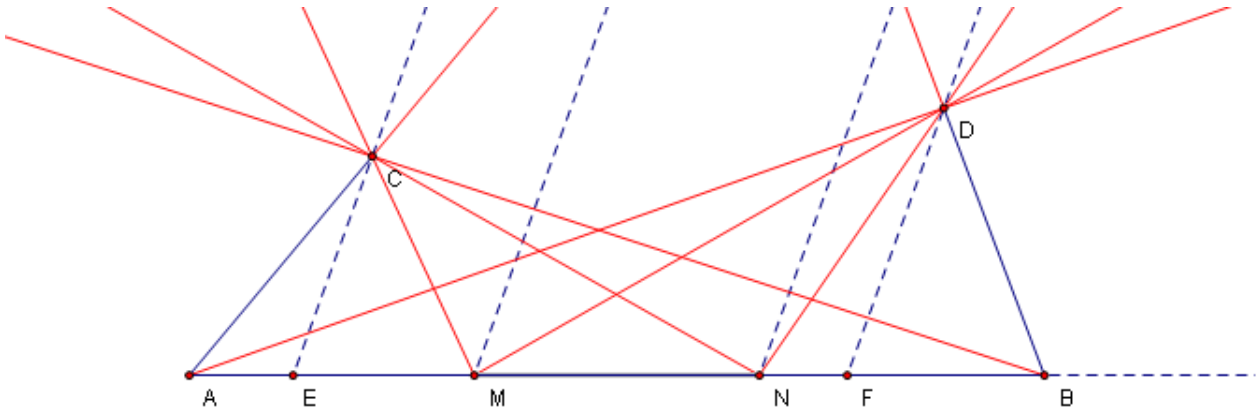


Figure 3-2-1

In Figure 3-2, we use red lines to denote eight critical angles, they are:

$$\angle DAB = \arctan \frac{\sin \beta}{3 + \cos \beta}$$

$$\angle DMB = \arctan \frac{\sin \beta}{2 + \cos \beta}$$

$$\angle DNE = \frac{\beta}{2}$$

$$\angle CAE = \alpha$$

$$\angle CME = \frac{\alpha + \pi}{2}$$

$$\angle DBE = \beta$$

$$\angle CNE = \pi + \arctan \frac{\sin \alpha}{\cos \alpha - 2}$$

$$\angle CBE = \pi + \arctan \frac{\sin \alpha}{\cos \alpha - 3}$$

We use $r_i (i=1,2,\dots,8)$ to denote these critical angles respectively, and $r_i < r_j (i, j=1,2,\dots,8, i < j)$. That is to say,

$$r_1 = \arctan \frac{\sin \beta}{3 + \cos \beta},$$

$$r_2 = \arctan \frac{\sin \beta}{2 + \cos \beta},$$

$$r_7 = \pi + \arctan \frac{\sin \alpha}{\cos \alpha - 2},$$

$$r_8 = \pi + \arctan \frac{\sin \alpha}{\cos \alpha - 3},$$

$$\{r_3, r_4\} = \left\{ \frac{\beta}{2}, \alpha \right\},$$

$$\{r_5, r_6\} = \left\{ \frac{\alpha + \pi}{2}, \beta \right\};$$

(α, β) has many kinds of combinations, we divide these combinations into four categories:

$$(1) \begin{cases} \alpha \geq \frac{\beta}{2} \\ \beta \geq \frac{\alpha + \pi}{2} \end{cases}, \quad (2) \begin{cases} \alpha \geq \frac{\beta}{2} \\ \beta < \frac{\alpha + \pi}{2} \end{cases}, \quad (3) \begin{cases} \alpha < \frac{\beta}{2} \\ \beta \geq \frac{\alpha + \pi}{2} \end{cases}, \quad (4) \begin{cases} \alpha < \frac{\beta}{2} \\ \beta < \frac{\alpha + \pi}{2} \end{cases}$$

Because of the symmetry, (1) is equivalent to (4), and (2) is equivalent to (3), so we only need to take combinations (1) and (2) into consideration.

$$(1) \text{ When } \begin{cases} \alpha \geq \frac{\beta}{2} \\ \beta \geq \frac{\alpha + \pi}{2} \end{cases}, \text{ we will have}$$

$$r_3 = \frac{\beta}{2}, r_4 = \alpha, r_5 = \frac{\alpha + \pi}{2}, r_6 = \beta,$$

Alike as the first model, we divide angle into nine ranges, they are listed below ($r_0 = 0, r_9 = \pi$):

Range	$[r_0, r_1)$	$[r_1, r_2)$
$f(r)$	0	$-\frac{1}{3} \sin \beta \cot r + \frac{1}{3} \cos \beta + 1$

$\int_{r_i}^{r_{i+1}} f(r) dr$	0	$-\frac{1}{3} \sin \beta \ln \left \frac{\sin r_2}{\sin r_1} \right + \left(\frac{1}{3} \cos \beta + 1 \right) (r_2 - r_1)$
Range	$[r_2, r_3)$	$[r_3, r_4)$
$f(r)$	x	$-\frac{1}{3} \sin \beta \cot r + \frac{1}{3} \cos \beta + \frac{2}{3}$
$\int_{r_i}^{r_{i+1}} f(r) dr$	$r_3 - r_2$	$-\frac{1}{3} \sin \beta \ln \left \frac{\sin r_4}{\sin r_3} \right + \left(\frac{1}{3} \cos \beta + \frac{2}{3} \right) (r_4 - r_3)$
Range	$[r_4, r_5)$	$[r_5, r_6)$
$f(r)$	$-\frac{2}{3} \sin \frac{\beta - \alpha}{2} \cot r + \frac{2}{3}$	$-\frac{1}{3} \sin \beta \cot r + \frac{1}{3} \cos \beta + \frac{1}{3}$
$\int_{r_i}^{r_{i+1}} f(r) dr$	$-\frac{2}{3} \sin \frac{\beta - \alpha}{2} \ln \left \frac{\sin r_5}{\sin r_4} \right + \frac{2}{3} (r_5 - r_4)$	$-\frac{2}{3} \sin \beta \ln \left \frac{\sin r_6}{\sin r_5} \right + \left(\frac{1}{3} \cos \beta + \frac{1}{3} \right) (r_6 - r_5)$
Range	$[r_6, r_7)$	$[r_7, r_8)$
$f(r)$	x	$\frac{1}{3} \sin \beta \cot r - \frac{1}{3} \cos \beta + 1$
$\int_{r_i}^{r_{i+1}} f(r) dr$	$r_7 - r_6$	$\sin \alpha \ln \left \frac{\sin r_8}{\sin r_7} \right + \left(1 - \frac{1}{3} \cos \alpha \right) (r_8 - r_7)$
Range	$[r_8, r_9]$	
$f(r)$	0	
$\int_{r_i}^{r_{i+1}} f(r) dr$	0	

So, under these conditions, we will have

$$F(\alpha, \beta) = \frac{\sum_{i=0}^8 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi} ;$$

$$(2) \text{ When } \begin{cases} \alpha \geq \frac{\beta}{2} \\ \beta < \frac{\alpha + \pi}{2} \end{cases}, \text{ we will have}$$

$$r_3 = \frac{\beta}{2}, r_4 = \alpha, r_5 = \beta, r_6 = \frac{\alpha + \pi}{2} ,$$

Alike as(1), the ranges are listed below ($r_0 = 0, r_9 = \pi$):

Range	$[r_0, r_1)$	$[r_1, r_2)$
$f(r)$	0	$-\frac{1}{3} \sin \beta \cot r + \frac{1}{3} \cos \beta + 1$
$\int_{r_i}^{r_{i+1}} f(r) dr$	0	$-\frac{1}{3} \sin \beta \ln \left \frac{\sin r_2}{\sin r_1} \right + \left(\frac{1}{3} \cos \beta + 1 \right) (r_2 - r_1)$
Range	$[r_2, r_3)$	$[r_3, r_4)$
$f(r)$	x	$-\frac{1}{3} \sin \beta \cot r + \frac{1}{3} \cos \beta + \frac{2}{3}$
$\int_{r_i}^{r_{i+1}} f(r) dr$	$r_3 - r_2$	$-\frac{1}{3} \sin \beta \ln \left \frac{\sin r_4}{\sin r_3} \right + \left(\frac{1}{3} \cos \beta + \frac{2}{3} \right) (r_4 - r_3)$
Range	$[r_4, r_5)$	$[r_5, r_6)$
$f(r)$	$-\frac{2}{3} \sin \frac{\beta - \alpha}{2} \cot r + \frac{2}{3}$	$-\frac{1}{3} \sin \alpha \cot r - \frac{1}{3} \cos \alpha + \frac{2}{3}$

$\int_{r_i}^{r_{i+1}} f(r) dr$	$-\frac{2}{3} \sin \frac{\beta - \alpha}{2} \ln \left \frac{\sin r_5}{\sin r_4} \right + \frac{2}{3} (r_5 - r_4)$	$\frac{1}{3} \sin \alpha \ln \left \frac{\sin r_6}{\sin r_5} \right + \left(\frac{2}{3} - \frac{1}{3} \cos \alpha \right) (r_6 - r_5)$
Range	$[r_6, r_7)$	$[r_7, r_8)$
$f(r)$	x	$\frac{1}{3} \sin \beta \cot r - \frac{1}{3} \cos \beta + 1$
$\int_{r_i}^{r_{i+1}} f(r) dr$	$r_7 - r_6$	$\sin \alpha \ln \left \frac{\sin r_8}{\sin r_7} \right + \left(1 - \frac{1}{3} \cos \alpha \right) (r_8 - r_7)$
Range	$[r_8, r_9]$	
$f(r)$	0	
$\int_{r_i}^{r_{i+1}} f(r) dr$	0	

So, under these conditions, we will have

$$F(\alpha, \beta) = \frac{\sum_{i=0}^8 \int_{r_i}^{r_{i+1}} f(r) dr}{\pi} ;$$

To sum up(1)(2), we use PASCAL program to assign different value to α, β and calculate the value of integral expressions. When $\alpha = 1.5708$ and $\beta = 3.0348$, we get the maximum value of $F(\alpha, \beta)$, which is 0.5956.

4. Conclusion and Application

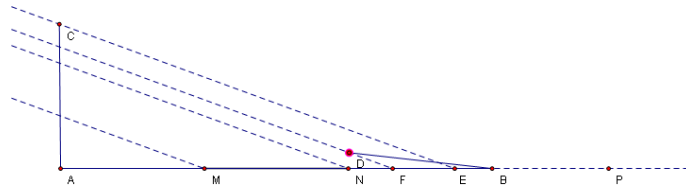


Figure 4-1

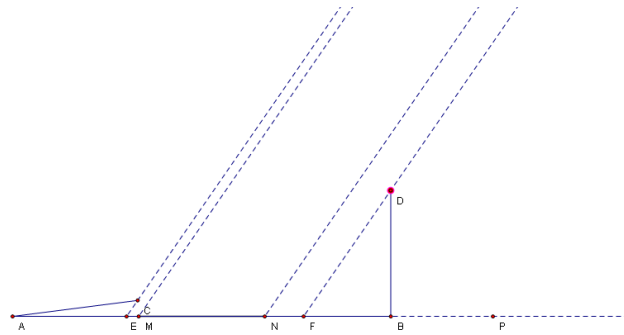


Figure 4-2

Referring to the anterior sections, we draw a conclusion that for two-sash windows, making both sashes perpendicular to windowsill can get maximum wind volume. And for three-sash windows, we should choose the combination mode as shown in Figure 4-1 and Figure 4-2. In this mode, one sash is perpendicular to windowsill, and the other has a 6.12 degree intersect angle with windowsill.

By comparing two styles of windows, we also find that three-sash windows can make more wind flow into the room. So we conclude that three-sash windows have better applying prospect than two-sash windows.

5. Further Optimization for Model

We have resolved an elementary model in this paper. Reviewing anterior sections, our models tally with practical situation very well. In the model, we ignore subordinate factors to simplify the real problem. But we clearly know that our research has some

points need improving, so that the model can fulfill more complex cases. We list these points below, and will go further into these points.

(1) Asymmetry three-sash window

The first one is what the wind volume of asymmetry three-sash window is. In our current model, we assume that three sashes have a same size. We draw this assumption according to the investigation from our daily life that most windows have sashes with the same size. And we also take factors of aesthetics into consideration. But asymmetry three-sash windows whose sashes have different sizes may have more wind volume blowing to the room. To affirm or overthrow our assumption that asymmetry styles are less likely to let more wind in, we should assign different sizes to sashes in our model so as to find the extremum by calculating partial derivative, and get the maximum by comparing them.

(2) The case that a room has several windows

The second one is that how to calculate wind volume when there are several windows in a wall. In our model, we only consider the case that a wall has a single window. In fact, a wall may have several windows, such as those of balconies, kitchens, and etc. The number of windows can influent the wind volume blowing into the room. So it is a valuable problem to research. The increase of windows' number will generate more complex cases. Critical conditions will increase with the number of walls in a wall, and need to be classified more systematically in detail. To resolve this problem, we could consider these windows as a whole. That is a window of n sashes. We have resolved the cases when $n=2$ and 3 . As for the next step, we will consider how to deduce this problem to n dimensionalities.

(3) Perpendicular velocity of airflow

The third one is that how to calculate wind volume when wind can blow in the vertical direction. To resolve this problem, firstly, we should investigate the changes of wind direction in the area nearing buildings. Then we should use aerodynamics knowledge to build a rational model. This is a very complex problem, and we spare no effort to resolve this problem in our next step of work.

6. Gains and Sentiment

Our three-person team began to cooperate from the moment that we decide to enter for the “Qiu Chengtong” cup competition. Information collecting, topic choosing, mathematics deduction, paper writing..., every step was hard for us. Thanks to the encouragement among each other of us, we have overcome all difficulties. In this progress, we realize the truth that detail decides the final result. When we were almost done with the paper, we suddenly found that we had ignored two important critical statuses. So we have to rework, and spend another 2 or 3 hours to remedy the mistake, which inspired us to understand the importance of details deeply.

When we accomplish total work by using collective wisdom, we all feel very excited. We learn a truth that mathematics is widely applied in daily life, that privities is generated from working together, and that success is approached by everybody's effort.

7. Acknowledgement

First of all, we want to thank our teacher Mr. Li and Mr. Tang for their important guidance and assistance. We are also profoundly grateful to our parents for their support.

Finally, we want to thank every judge in advance. We wish you could give us guidance about any leak or shortage in this paper.

Each of us contributed equally to the work.

Appendix: Pascal Program Code

(1) First Model

Case1:

```
program ss;

const
    pi=3.141592653589793238462643383279;

var
    r1,r2,r3,r4,r5,a,b,rc,max,max1,max2,aa,bb,c:double;
    i,j:longint;

begin
    assign(output,'qct.out');
    rewrite(output);
    aa:=0;
    bb:=pi/2;
    c:=pi/2/100;
    for i:=0 to 100 do
        for j:=0 to 100 do begin
            a:=aa+i*c;
            b:=bb+j*c;
```

```

r1:=arctan ( sin(b) / (2+cos(b)) );
r4:=arctan ( sin(a) / (cos(a)-2) )+pi;
r2:=a;
r3:=b;
if (a>=r1)and(b<=r2) then begin
    re:=
-1/2*sin(b)*ln(abs(sin(r2)/sin(r1)))+(1/2*cos(b)+1)*(r2-r1)+
1/2*(sin(a)-sin(b))*(ln(abs(sin(r3)))-ln(abs(sin(r2))))+1/2*(2-cos(a)+cos(b))*(r3
-r2)+
1/2*sin(a)*ln(abs(sin(r4)/sin(r3)))+(1-1/2*cos(a))*(r4-r3);
    writeln(a/pi*180:4:4,' ',b/pi*180:4:4,' ',re:4:4);
end;
if re>max then begin
    max:=re;
    max1:=a;
    max2:=b;
end;
end;
writeln;
writeln;
writeln(max1:4:4,' ',max2:4:4,' ',max:4:4);

```

```
writeln(max1/pi*180:4:4,' ',max2/pi*180:4:4,' ',max:4:4);  
close(output);
```

end.

Case2:

```
program ss;
```

```
const
```

```
    pi=3.141592653589793238462643383279;
```

```
var
```

```
    r1,r2,r3,r4,r5,a,b,rc,max,max1,max2,aa,bb,c:double;
```

```
    i,j:longint;
```

```
begin
```

```
    assign(output,'qct3.out');
```

```
    rewrite(output);
```

```
    aa:=0;
```

```
    bb:=pi/2;
```

```
    c:=pi/2/100;
```

```
    for i:=0 to 100 do
```

```
        for j:=0 to 100 do if (i<>0)and(j<>100) then begin
```

```
            a:=aa+i*c;
```

```
            b:=bb+j*c;
```

```
            r1:=arctan ( (sin(b) - sin(a)) / (2+cos(b)-cos(a)) );
```



```
r2:=b;
```

```
r3:=arctan(sin(a)/(cos(a)-2))+pi;
```

```
if a<arctan(sin(b)/(2+cos(b))) then begin
```

```
    re:=
```

```
1/2*(sin(a)-sin(b))*(ln(abs(sin(r2)))-ln(abs(sin(r1))))+1/2*(2-cos(a)+cos(b))*(r2  
-r1)+
```

```
1/2*sin(a)*ln(abs(sin(r3)/sin(r2)))+(1-1/2*cos(a))*(r3-r2);
```

```
    writeln(a/pi*180:4:4,' ',b/pi*180:4:4,' ',re:4:4);
```

```
end;
```

```
if re>max then begin
```

```
    max:=re;
```

```
    max1:=a;
```

```
    max2:=b;
```

```
end;
```

```
end;
```

```
writeln;
```

```
writeln;
```

```
writeln(max1:4:4,' ',max2:4:4,' ',max:4:4);
```

```
writeln(max1/pi*180:4:4,' ',max2/pi*180:4:4,' ',max:4:4);
```

```
close(output);
```

end.

(2) Second Model

```
program ss;
```

```
const
```

```
    pi=3.141592653589793238462643383279;
```

```
var
```

```
    a,b,rc,max,max1,max2,aa,bb,c:double;
```

```
    i,j:longint;
```

```
    r1,r2,r3,r4,r5,r6,r7,r8:double;
```

```
    re12,re23,re34,re45,re56,re67,re78:double;
```

```
begin
```

```
    assign(output,'qct2.out');
```

```
    rewrite(output);
```

```
    aa:=0;
```

```
    bb:=pi/2;
```

```
    c:=pi/2/500;
```

```
    for j:=0 to 500 do
```

```
        for i:=0 to 500 do begin
```

```
            a:=aa+i*c;
```

```
            b:=bb+j*c;
```

```
            if a>=b/2 then begin
```

$r1 := \arctan \left(\frac{\sin(b)}{3 + \cos(b)} \right);$

$r2 := \arctan \left(\frac{\sin(b)}{2 + \cos(b)} \right);$

$r7 := \pi + \arctan \left(\frac{\sin(a)}{\cos(a) - 2} \right);$

$r8 := \pi + \arctan \left(\frac{\sin(a)}{\cos(a) - 3} \right);$

if $b \geq (a + \pi)/2$ then begin

$r3 := b/2;$

$r4 := a;$

$r5 := (a + \pi)/2;$

$r6 := b;$

$re12 := -1/3 * \sin(b) * \ln(\text{abs}(\sin(r2)/\sin(r1))) + (1/3 * \cos(b) + 1) * (r2 - r1);$

$re23 := r3 - r2;$

$re34 := -1/3 * \sin(b) * \ln(\text{abs}(\sin(r4)/\sin(r3))) + (1/3 * \cos(b) + 2/3) * (r4 - r3);$

$re45 := -2/3 * (b - a)/2 * \ln(\text{abs}(\sin(r5)/\sin(r4))) + 2/3 * (r5 - r4);$

$re56 := -1/3 * \sin(b) * \ln(\text{abs}(\sin(r6)/\sin(r5))) + (1/3 * \cos(b) + 1/3) * (r6 - r5);$

$re67 := r7 - r6;$

$re78 := 1/3 * \sin(a) * \ln(\text{abs}(\sin(r8)/\sin(r7))) + (1 - 1/3 * \cos(a)) * (r8 - r7);$

re:=re12+re23+re34+re45+re56+re67+re78;

writeln(a/pi*180:8:4,b/pi*180:8:4,re:8:4);

if re>max then begin

max1:=a;

max2:=b;

max:=re;

end;

end else begin

r3:=b/2;

r4:=a;

r6:=(a+pi)/2;

r5:=b;

re12:=-1/3*sin(b)*ln(abs(sin(r2)/sin(r1)))+(1/3*cos(b)+1)*(r2-r1);

re23:=r3-r2;

re34:=-1/3*sin(b)*ln(abs(sin(r4)/sin(r3)))+(1/3*cos(b)+2/3)*(r4-r3);

re45:=-2/3*(b-a)/2*ln(abs(sin(r5)/sin(r4)))+2/3*(r5-r4);

```
re56:=1/3*sin(a)*ln(abs(sin(r6)/sin(r5)))+(2/3-1/3*cos(a))*(r6-r5);
```

```
re67:=r7-r6;
```

```
re78:=1/3*sin(a)*ln(abs(sin(r8)/sin(r7)))+(1-1/3*cos(a))*(r8-r7);
```

```
re:=re12+re23+re34+re45+re56+re67+re78;
```

```
writeln(a/pi*180:8:4,b/pi*180:8:4,re:8:4);
```

```
if re>max then begin
```

```
max1:=a;
```

```
max2:=b;
```

```
max:=re;
```

```
end;
```

```
end;
```

```
end;
```

```
end;
```

```
writeln;
```

```
writeln;
```

```
writeln(max1:10:4,' ',max2:10:4,' ',max:10:4);
```

```
writeln(max1/pi*180:4:4,' ',max2/pi*180:4:4,' ',max:4:4);
```

```
close(output);
```

```
end.
```

