OPTIMIZED METHODS OF THE EQUALIZED SPRINKLING IRRIGATION FOR GREENERY PATCHES

绿化喷灌中水量均衡的优化问题

English Version

Zhoujia Li, Yudi Fu

李周嘉，傅雨迪

Guided by：Mr. Yuanji Xu

指导老师：徐渊楫

Hangzhou Foreign Languages School

杭州外国语学校

No. 299 Liuhe Rd., Hangzhou, Zhejiang 310023

浙江省杭州市留和路 299 号 310023

August.2008

2008 年 8 月

Contents

Abstract

This paper mainly focuses on the optimized methods of the sprinkling irrigation for greenery patches, by maximally equalizing the amount of water sprayed on a certain area. Various models are being discussed, where the main mathematical tool is analytic geometry, employed to research the possible effects of different proposals.

Firstly, the simplest models are built based on a totally ideal situation. Assuming that sprinkling spouts are spinning over plain lawn with a set of specified radii, install them in arrangements of simple geometric figures. Areas of overlapping and blank parts are being calculated and the most reasonable arrangement of all that are studied is selected.

Secondly, real factors are taken into consideration separately as follows: 1. The disequilibrium of the water that drops in a line from the sprinkling center is transformed into a functional expression, whose graphs are drawn to show the water distributed over the area; 2. The plane models are changed into solid ones on the assumption that the sprinkling spouts are placed on slopes. Analytic geometry methods are employed to describe the range of sprayed water on the oblique surface. Through calculation and analysis, models can be adjusted to specific situations.

Finally, the boundary problems and landscape effects are involved.

Key words: Sprinkling irrigation; Optimizing; Spouts arrangements; Rate of covering; Rate of overlapping; Degree of equalizing on the planes; Degree of equalizing on the oblique planes

I. Introduction:

Mind on mathematics is a theoretical foundation, based on which real problems can be solved. Undoubtedly only with good model building, analyzing and calculating can various optimizing problems be worked out. As high school students, due to our limited mathematical knowledge, the methods we take to solve problems are not very far beyond the textbook. So we focused on a problem close to daily life as well as one we can more or less solve on our own. Our inspiration came from the small lawn near our classroom in junior school. Every time the spouts did the irrigation, there was a great amount of water spilled on the hallway, and even sometimes the water would went through the open windows into the classroom, which caused many inconveniences as well as much waste. In today's world where drinking water is badly in need, it is, with no doubt, very meaningful to maximally equalize the amount of water a certain area gets in order to decrease the amount of water wasted.Through the information we got, we found that there were three factors on which sprinkling irrigation mainly depends: intensity, equilibrium and diameter of water drops. We began to develop interests in the equilibrium factor and decided to make an optimized proposal by a series of mathematical methods. Thereupon this paper primarily focuses on the degree of equalizing in the sprinkling irrigation.

To equalize the water distribution through reasonable spout arrangements, researchers have studied several models. Some special irrigation tools have also been invented to solve this problem.

In this paper, we mainly focus on the most common spouts, in order to come up with an optimized plan that can be popularized. The two of the plane models are studied in a new aspect of analytic geometrey, combined with the degree of equalizing of the water in one single spout's covered range. Another creative point is the spout adjustment on slope surface. Adjust the angle of elevation to a certain slope angle, we can get an equalized water distribution. Thus the irrigation plan can be optimized in a practical way, without giving rise to the cost of equipment. The methods and models we used in this paper in equalizing the water in the sprinkling irrigation can also be employed to study some similar problems, such as network, radio or phone's signal covering. In addition, more solid models, like models on a sphere, can be further studied.

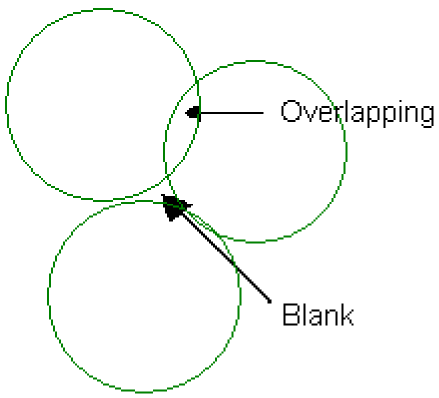
Before we began the study, we had found some information that primarily introduced the importance and advantages of the spinkling irration and some related knowledge about the irrigation technique. The information comes from Reference [1], [2] and [3].

II. Problem 1:

Efficiency of Irrigation in the Totally Ideal Situation

Assumption: Use the identical sprinkling spouts; the covered area of a single spout is an ordinary circle with an appointed radius; water is distributed evenly in each circle; the irrigation takes place on an infinite horizontal plane (namely this part only focuses on the central part of the irrigated area, regardless of the boundary problems).

In this simplified situation, we need to come up with the most water saving plan, namely the spout arrangement that brings about the most efficient irrigation effects. This arrangement must leave the least blank and overlapping areas.



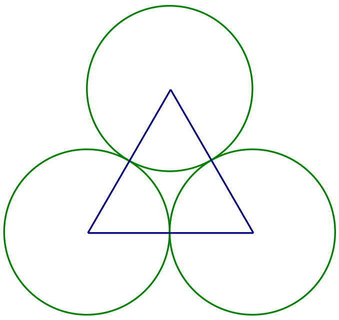
(A green circle encircles the covered area of a single spout, with following alike)

Therefore, we can transform this problem into a mathematical model: the irrigation area can be regarded as a plane with several circles on it, and the area of the blank and overlapping parts need to be minimized. This problem can be solved, depending on the paving of right polygons. Among all the regular polygons, there are three that can be paved on the plane:

equilateral triangle, square and right hexagon. Because a right hexagon is formed by six equilateral triangles, it has similar principles of paving to equilateral triangle, so here we only need to study equilateral triangles and squares. These two figures can be used as unit patterns to form other spout arrangements.

1. Spout Arrangements with Spaces Not Overlapping

(1) Equilateral Triangle Model



The centers of the circles are placed on the vertices of the triangle, respectively; and the radius is half the length of the side. Since the whole plane is formed by the unit triangles as is shown above , the percentage of the covered area of the whole plane is that of the triangle. We have:

$$c_T = \frac{S_c}{S_T}, \tag{1}$$

$$S_c = \frac{1}{2}\pi \cdot r^2, \tag{2}$$

where c_T denotes the rate of covering;

S_c denotes the area of the covered parts;

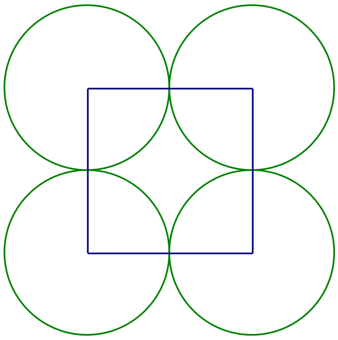
S_T denotes the area of the unit triangle;

r denotes the radius.

We can get

$$c_T = \frac{S_c}{S_T} = \frac{\frac{1}{2}\pi \cdot r^2}{\frac{1}{2}(2r)^2 \sin 60°} = \frac{\pi}{2\sqrt{3}}. \tag{3}$$

(2) Square Model



The centers of the circles are placed on the vertices of the square, respectively; and the radius is half the length of the side of the square. Since the whole plane is formed by the unit squares as is shown above, the percentage of the covered area of the whole plane is that of the square. We have:

$$c_S = \frac{S_c}{S_S}, \tag{4}$$

$$S_c = \pi \cdot r^2, \tag{5}$$

where c_S denotes the rate of covering;

S_c denotes the area of the covered parts;

S_S denotes the area of the unit square.

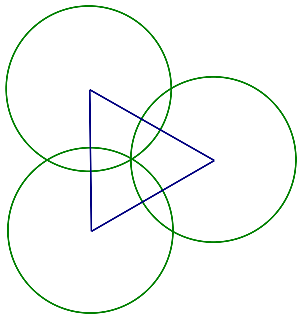
We can get

$$c_S = \frac{S_c}{S_S} = \frac{\pi \cdot r^2}{(2r)^2} = \frac{\pi}{4}. \tag{6}$$

For $\frac{\pi}{2\sqrt{3}} > \frac{\pi}{4}$, we can tell $c_T > c_S$, which means in the situation without overlapping areas, the Equilateral Triangle Model has a higher density of covering and thus is more economical. Therefore we adopt the Equilateral Triangle Model.

2. Spout Arrangements without Blank Areas

(1) Equilateral Triangle Model



The centers of the circles are placed on the vertices of the triangle, respectively; and the radius is the distance between one of the vertices and the centroid of the triangle. Since the whole plane is formed by the unit triangles as is shown above, the percentage of the overlapping area of the whole plane is that of the triangle. We have

$$o_T = \frac{S_o}{S_T}, \tag{7}$$

$$S_o = \frac{3}{2}\left(\frac{2}{6}\pi \cdot r^2 - r^2 \sin 60°\right) = \left(\frac{\pi}{2} - \frac{3\sqrt{3}}{4}\right) r^2, \tag{8}$$

where o_T denotes the rate of overlapping;

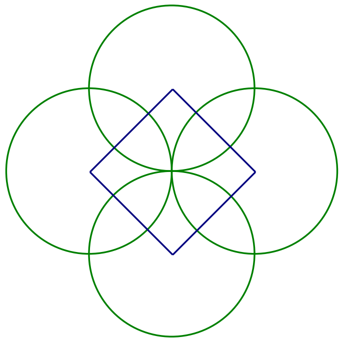
S_o denotes the area of the overlapping parts;

S_T denotes the area of the unit triangle.

We can get

$$o_T = \frac{S_o}{S_T} = \frac{(\frac{\pi}{2} - \frac{3\sqrt{3}}{4})r^2}{\frac{1}{2}(2\times\frac{\sqrt{3}}{2}r)^2 \sin 60°} = \frac{\frac{\pi}{2} - \frac{3\sqrt{3}}{4}}{\frac{3\sqrt{3}}{4}} = \frac{2\sqrt{3}\pi}{9} - 1. \tag{9}$$

(2) Square Model



The centers of the circles are placed on the vertices of the square, respectively; and the radius is the distance between one of the vertices and the center of the mass of the square. Since the whole plane is formed by the unit squares as is shown above, the percentage of the overlapping area of the whole plane is that of the square. We have

$$o_S = \frac{S_o}{S_S}, \tag{10}$$

$$S_o = 2(\frac{2}{4}\pi \cdot r^2 - r^2) = (\pi - 2)r^2, \tag{11}$$

where o_S denotes the rate of overlapping;

S_o denotes the area of the overlapping parts;

S_S denotes the area of the unit square.

We can get

$$o_S = \frac{S_o}{S_S} = \frac{(\pi - 2)r^2}{(\sqrt{2}r)^2} = \frac{\pi}{2} - 1. \tag{12}$$

For $\frac{2\sqrt{3}\pi}{9} - 1 < \frac{\pi}{2} - 1,$ we can tell $o_T < o_S$, which means in the situation without blank areas, the Equilateral Triangle Model has a lower rate of overlapping and is more economical. Therefore we adopt Equilateral Triangle Model.

3. Brief Summary

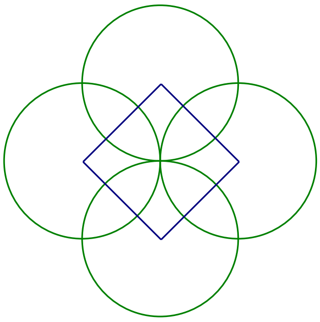
On the assumption that all the factors are ideal, the Equilateral Triangle Model reaches a superior irrigation effect to the others in a situation whether there are blank or overlapping areas. So in the simple arrangement, it is more economical to apply the Equilateral Triangle Model into practice. Drought tolerant plants or stones, walking paths and statues can fill up the blanks, or, the areas out of the irrigated range, to make the landscape more beautiful. As for the areas where much water is distributed, also called overlapping areas, some moisture tolerant plant can be planted, to make full use of the extra water. Thus, in addition to increasing the efficiency to the highest it can reach, this spout arrangement also brings some changes to the monotonous lawn. When needed, blanks and overlapping areas can appear alternately, and the irrigation efficiency can be even higher.

III. Problem 2: Boundary Problems

Problem 1 has been discussed without considering the boundaries, but in reality, boundary problems must be taken into consideration, for beyond the edges there are things that cannot be sprayed on, like buildings and roads etc.. As in the case of our school campus, the water would go onto the walkways and even into classrooms.

1. Adjusting Boundaries to the Spouts Arranged

Example: in the Square Model in Problem 1, the covered area is a figure encircled by four arcs.

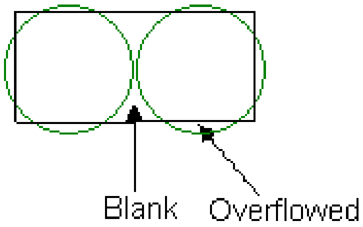


The planted area can be also made into this figure, and considering the entire visual effect, fences, statues and stone roads can be built along the boundaries.

But this method has its limitation. It needs irrigation to be under consideration at the very beginning of the design for greenery patches. It is not very changeable. Additionally, generally speaking, irrigation problems are discussed after a piece of land is planted, so this method is not so practical.

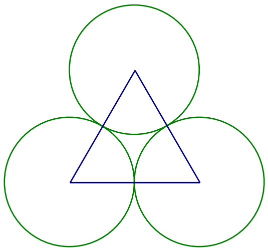
2. Compromise Pattern

Coordinate the blank with the overflowed areas, leaving some parts not watered while some water is spilled out.

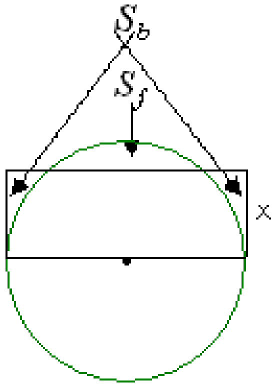


To get a better arrangement, balance the blank with the overflowed areas. Following are discussions respectively about the boundary problems in situations without overlapping or blanks (not considering the inner blanks and overlapping as was discussed in Problem 1). The former part has concluded that using the Equilateral Triangle Model is more efficient, so following questions are only discussed in the Equilateral Triangle Model.

(1) No Overlapping



Suppose the irrigated area is big enough, so the irregular parts such as a corner can be neglected. The boundaries of the whole plane can be regarded as the unit figure as is shown below. Calculate the value of x when $S_b = S_f$, where S_b denotes the area of blank parts and S_f denotes the area of overflowed parts.



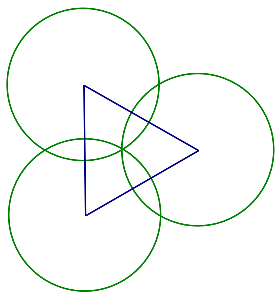
(The upper side of the rectangle refers to the edge of a lawn, with following alike)

Assume the radius of the circle above is r, we have

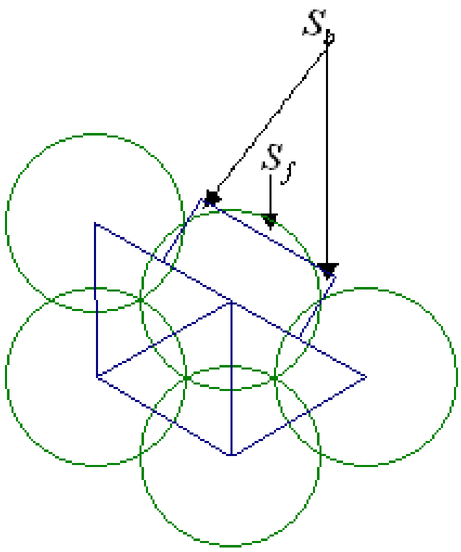
$$S_b = 2rx - (\frac{1}{2}\pi \cdot r^2 - S_f) = S_f + 2rx - \frac{1}{2}\pi \cdot r^2, \tag{13}$$

$S_b = S_f$, namely $S_f + 2rx - \frac{1}{2}\pi \cdot r^2 = S_f$, so $x = \frac{1}{4}\pi \cdot r$. That is to say, when $x = \frac{1}{4}\pi \cdot r$, $S_b = S_f$, the overflowed and blank areas equal each other, and the effect of the irrigation reaches its best.

(2) No Blanks



Suppose the irrigated area is big enough, so the irregular parts such as a corner can be neglected. Calculate the value of x when $S_b = S_f$.



Assume the radius of the circles is r, we have

$$\begin{aligned} S_b &= x\cdot\sqrt{3}r-[\frac{1}{2}\pi\cdot r^2-S_f-\frac{1}{2}(\frac{2}{6}\pi\cdot r^2-r^2\sin 60°)] \\ &= x\cdot\sqrt{3}r-\frac{1}{3}\pi\cdot r^2-\frac{\sqrt{3}}{4}r^2+S_f \end{aligned} \tag{14}$$

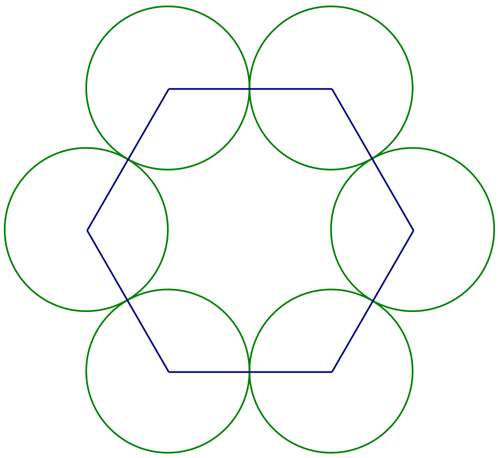
$S_b = S_f$, namely $x\cdot\sqrt{3}r-\frac{1}{3}\pi\cdot r^2-\frac{\sqrt{3}}{4}r^2=0$, so $x=(\frac{\sqrt{3}}{9}\pi+\frac{1}{4})r$. That is to say, when $x=(\frac{\sqrt{3}}{9}\pi+\frac{1}{4})r$, $S_b = S_f$, the overflowed and blank areas equal each other, and the effect of the irrigation reaches its best.

It is a compromise, which can be selected when there is little impact on the walkways around. Though there is a certain amount of water wasted, it is a good arrangement, thanks to its simplicity. However, it has its limitation: if the water that overflows may cause huge inconveniences, spouts need to be set leaving a large blank, in order not to spill the water out, which may be disadvantageous for plants' growth.

3. Use of Angle-limited Spinning Spouts

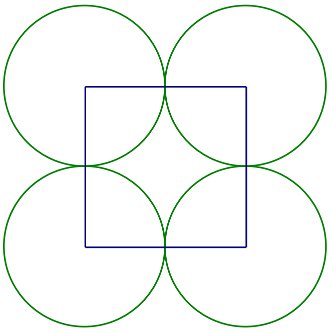
Now spouts that have appointed spinning angles are available. Some spouts can spin in angle ranges up to 90°, 120°, 180°, 270° or even any degree. But compared to common fully circling spouts, they may cost more, so they are not widely used for central irrigation but at boundaries.

For example, the boundaries of the irrigated area is a right hexagon, as is shown below (here we neglect the central part).



Use fully circling spouts in the inner areas, and when meeting boundaries, set spouts that spin limited angles identical to the interior angles of the right hexagon. As is shown in the figure above, we can use 6 spouts with a limited spinning angle of 120° to fulfill this task.

Notice that the use of spinning angle-limited spouts is only suitable for large areas where several fully circling spouts can be placed in the center while angle-limited spouts are placed along the sides for boundary irrigation. It is not suitable for very small irrigation areas.



For example, in the case of the small square area whose lengths of sides are equal to the diameter of the covered area of one spout, as is shown above, using a fully circling spout is as effective as using 4 spouts with limited spinning angles of 90°. They both leave a blank area of $(2r)^2 - \pi \cdot r^2 = (4-\pi)r^2$. The former only needs one spout, but the latter needs four that each is even more expensive, which is obviously not economical.

4. Use of Specially Designed Spouts

Example: the spout as is shown below.

Mechanical force rotates the pipe round the axis, so that it can distribute water in a rectangle with an area of $10 \sim 40m \times 20m$. Meanwhile this sort of spouts has a beautiful visual effect, which makes it appropriate for gardening irrigation.

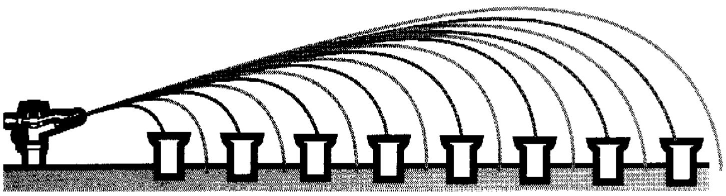
However, compared to the common spouts mentioned before, these spouts seem costly, so here this part is only a brief introduction of it. The special spouts are not considered in the following discussions.

IV. Problem 3:

Degree of Equalizing of Water Unequally Distributed

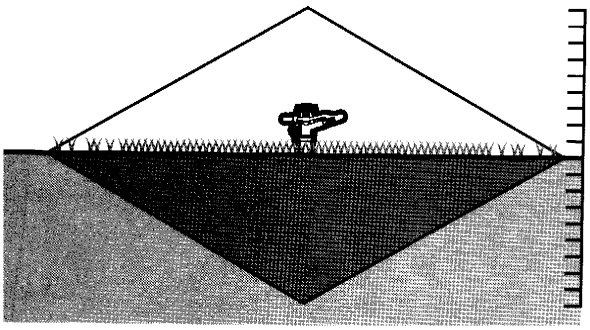
Problem 1 discussed the best spout arrangement in the situation that the water drops equally in a ray from the sprinkling center. But in reality, there exists disequilibrium of water distribution in the covered area of a single spout, which means, some parts of the circular area that a spout irrigates get more water than other parts. Following is a discussion about how to get the water well distributed on the horizontal plane in this situation.

To determine the water distribution in the line that stretches from a single spout, we can place identical containers that each keeps the same distance with its neighbors. After the spout sprinkles water for some time, measure the depth of the water that each container has collected and then draw the graph of the water distribution.



Reference [1]

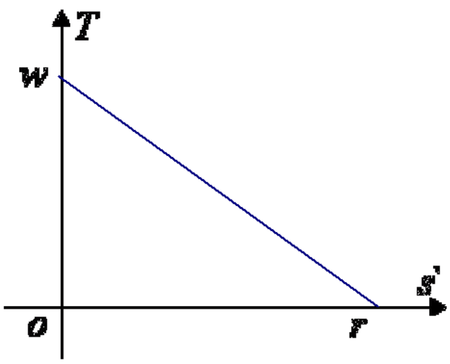
The graph of the water distribution in the covered area of a single spout can be offered as basic information by its manufacturer, commonly as is shown below:



Reference [1]

Apparently, the closer it is to the spout, the more water a certain point gets. As its distance from the spout increases, the container gets less and less water. In the container that is the farthest from the center, there is virtually no water.

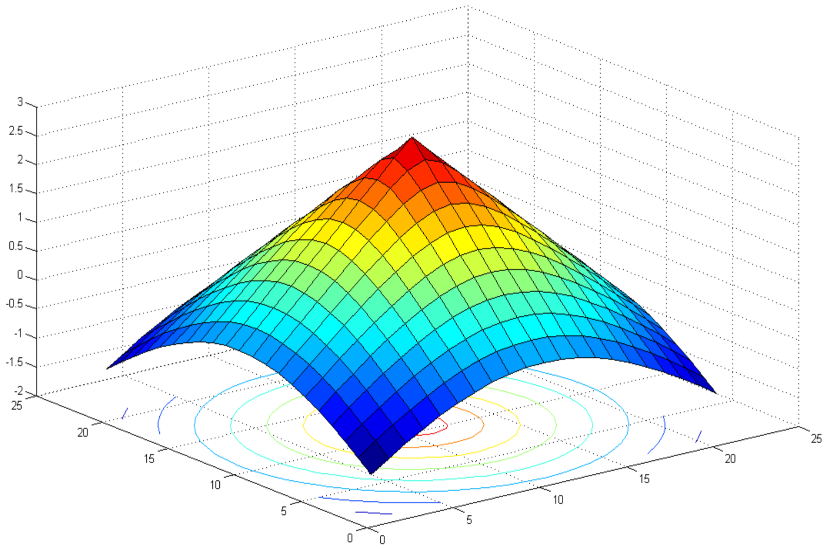
According to the picture above, we can draw the graph of the function that shows the relationship between T and s (T denotes the amount of water the ground gets from a single spout, s denotes the distance between the spout and an arbitrary point within each covered area; meanwhile use constants w, r: w is the water amount that the spout gets, and r is the radius of the covered circle):



Thus we have

$$T = -\frac{w}{r} \cdot s + w, \quad 0 \le s \le r \,. \tag{15}$$

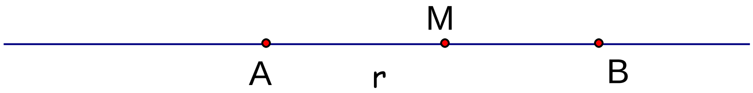
Using the mathematical software MATLAB, we can generate the graph of the water distribution produced by a single spout. As is presented, apparently the area closer to the spout gets more water.



1. Degree of Equalizing in One-dimensional Lines

Due to the difficulty of directing the two-dimensional plane, we first focus on the water distribution on the one-dimensional line for reference.

The easiest way to equalize the water amount all over the irrigated area on a straight line is to set the distance between two neighboring spouts same as the radius r of the covered circle. In this way, every point on the line only gets watered by two spouts.



Let A, B represent two neighboring identical spouts, and M is an arbitrary point on line segment AB; s denotes the distance between M and A, so the distance between M and B is $r-s$. We have

$$T_M = (-\frac{w}{r} \cdot s + w) + [-\frac{w}{r}(r - s) + w] = w\,, \tag{16}$$

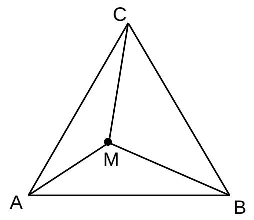
being is a constant; namely on a one-dimensional line this model can reach an ideal irrigation effect, with water equally distributed over the irrigated area.

2. Degree of Equalizing on Two-dimensional Planes

On a two-dimensional plane, because the covered area of a single spout is circular, we cannot simply use the model for the one-dimensional lines to simulate its effect but can model on its researching method. As is concluded in the one-dimensional situation, when the neighboring spouts keep a distance same as the radius of the covered area of a spout, the water is well distributed. So here we can cite this arranging plan in the following discussion.

Based on Problem 1, we consider using the Equilateral Triangle Model or the Square Model to fill the plane by paving. In the case of the Square Model, though the lengths of the sides are the same, the diagonal lengths are longer, so it is difficult to make the distances between every neighboring spouts equal to each other. So there is possibly an area in the center that lacks of water; but in the Equilateral Triangle Model, this problem can be avoided. So here we mainly consider the Equilateral Triangle Model.

First we focus on the area of a single equilateral triangle. Suppose that an arbitrary point in the considered area only gets irrigated from the spouts placed on the three vertices of the triangle.



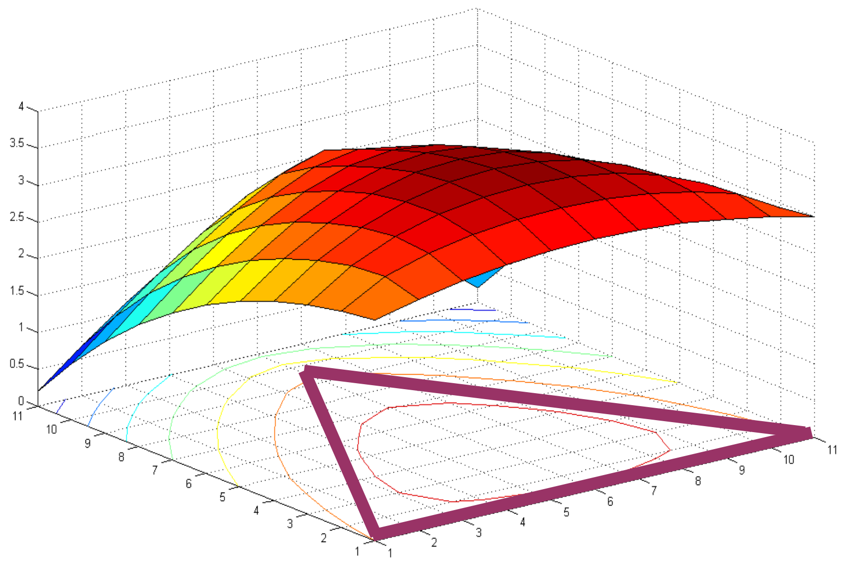
As is shown above, the equilateral triangle ABC is the considered area; A,B,C are the spouts placed on the three vertices of the triangle; M is an arbitrary point in the triangle and a,b,c denote the lengths of line segments MA,MB,MC ,respectively, so the total amount of water that point M gets is

$$T_M = (-\frac{w}{r}\cdot a + w) + (-\frac{w}{r}\cdot b + w) + (-\frac{w}{r}\cdot c + w) = (3 - \frac{a+b+c}{r})w\,. \tag{17}$$

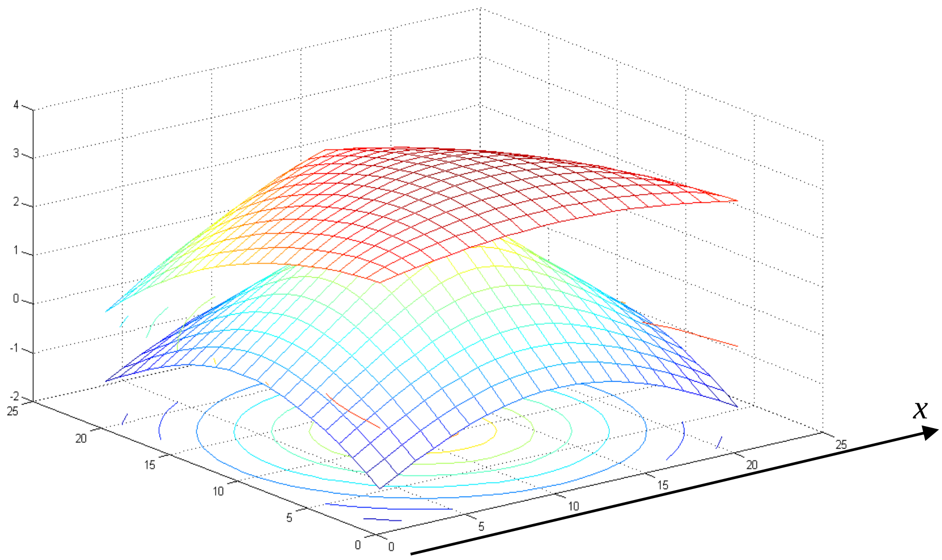
Due to the uncertainty of the value $a+b+c$, the received water differs as the position of M changes.

It's easy to prove that the most possible minimum value of the sum of the distances from one specific point in an equilateral triangle to three vertices is attained in the centroid of the triangle, which means the centroid of the triangle gets the maximum amount of water, and the amount of water reduces gradually as M moves to the boundary.

From the graph generated by MATLAB, we can conclude the same. However, generally speaking, the contour lines of water amount are sparsely distributed, so the water are relatively well-distributed. That is to say, this kind of spout arrangement is quite good.

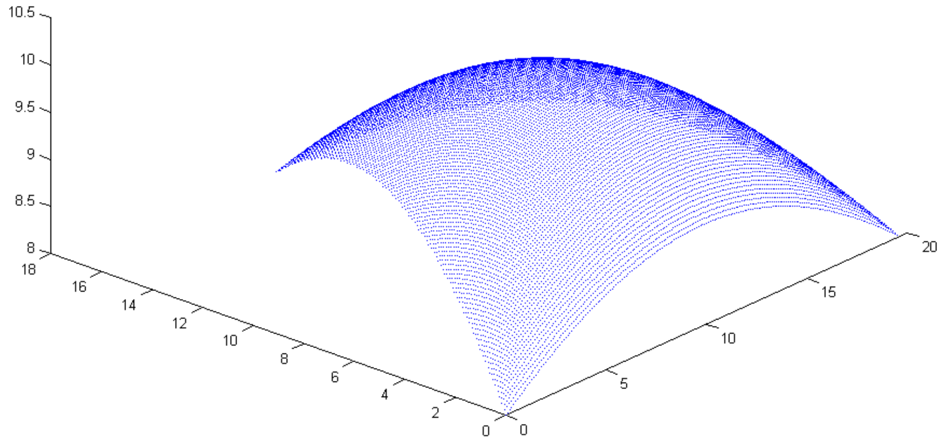


Put this graph together with the previous graph of water distribution produced by a single spout, apparently we made a big progress.



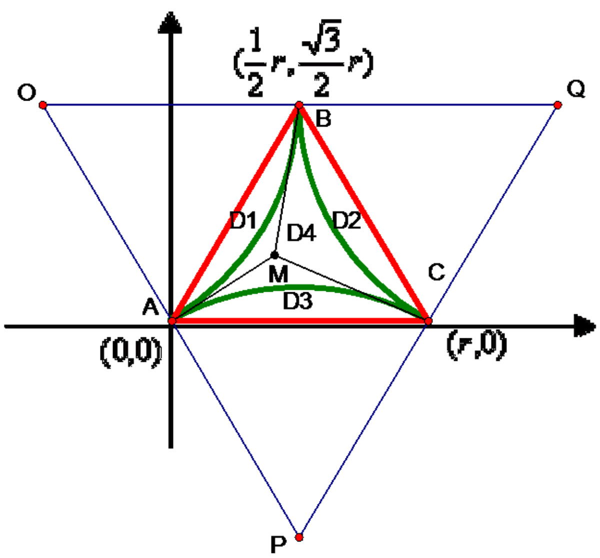
(Due to the difficulty of generating a graph with range of a triangle, the graph above is the smallest square that contains the considered equilateral triangle; one of the vertices and one side of the considered triangle are concurrent with the origin and the x-axis, respectively)

Since the functional expressions established to simulate the water distribution graphs in the following paragraphs are much more complex, the graphing method above will not be used again. So here its graph is drawn by listplotting, for further reference.



As a matter of fact, the length of side of an equilateral triangle is longer than the median, so some parts in an equilateral triangle as is considered before is sure to be irrigated by spouts from nearby triangles. We divide that triangle into 4 parts: $D1, D2, D3, D4$, in which $D1, D2, D3$ get the water not only from spout A, B, C, but also from nearby spout O, P, Q; $D4$ only gets the water from A, B, C. For sure we need to rebuild the function.

Establish the Cartesian coordinate system as is shown in the graph below:



The coordinates of point A, B, C, O, P, Q are $(0,0)$, $(\frac{1}{2}r,\frac{\sqrt{3}}{2}r)$, $(r,0)$, $(-\frac{1}{2}r,\frac{\sqrt{3}}{2}r)$, $(\frac{1}{2}r,-\frac{\sqrt{3}}{2}r)$, $(\frac{3}{2}r,\frac{\sqrt{3}}{2}r)$, respectively.

Assume that the coordinates of point M are (x_M, y_M), then

$$\begin{aligned}
MO &= \sqrt{(x_M+\frac{1}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}, \\
MP &= \sqrt{(x_M-\frac{1}{2}r)^2+(y_M+\frac{\sqrt{3}}{2}r)^2}, \\
MQ &= \sqrt{(x_M-\frac{3}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}, \\
MA &= \sqrt{x_M{}^2+y_M{}^2}, \\
MB &= \sqrt{(x_M-\frac{1}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}, \\
MC &= \sqrt{(x_M+r)^2+y_M{}^2}.
\end{aligned} \tag{18}$$

If point M belongs to district $D1$, then

$$T_M = (4-\frac{\sqrt{(x_M+\frac{1}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}+\sqrt{x_M{}^2+y_M{}^2}+\sqrt{(x_M-\frac{1}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}+\sqrt{(x_M+r)^2+y_M{}^2}}{r})w; \tag{19}$$

if M belongs to $D2$, then

$$T_M = (4-\frac{\sqrt{(x_M-\frac{3}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}+\sqrt{x_M{}^2+y_M{}^2}+\sqrt{(x_M-\frac{1}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}+\sqrt{(x_M+r)^2+y_M{}^2}}{r})w; \tag{20}$$

if M belongs to $D3$, then

$$T_M = (4-\frac{\sqrt{(x_M-\frac{1}{2}r)^2+(y_M+\frac{\sqrt{3}}{2}r)^2}+\sqrt{x_M{}^2+y_M{}^2}+\sqrt{(x_M-\frac{1}{2}r)^2+(y_M-\frac{\sqrt{3}}{2}r)^2}+\sqrt{(x_M+r)^2+y_M{}^2}}{r})w; \tag{21}$$

if M belongs to $D4$, then

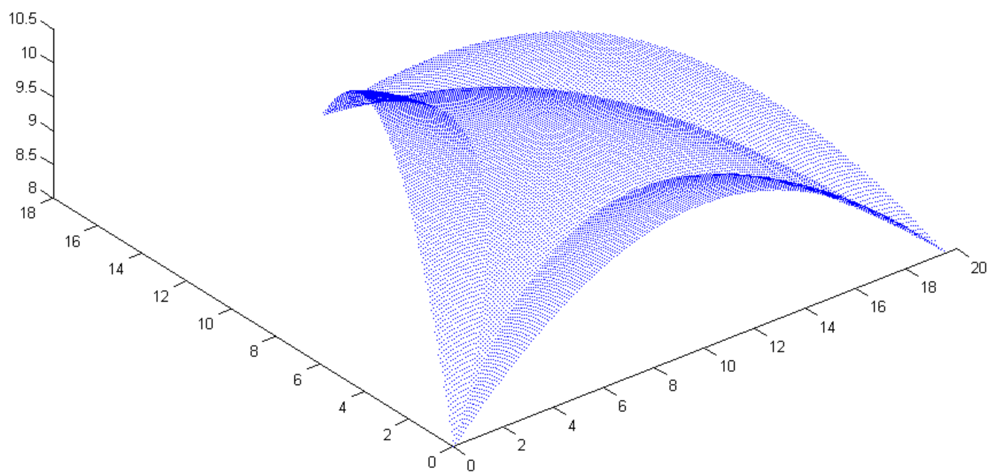
$$T_M = (3 - \frac{\sqrt{x_M^{\ 2} + y_M^{\ 2}} + \sqrt{(x_M - \frac{1}{2}r)^2 + (y_M - \frac{\sqrt{3}}{2}r)^2} + \sqrt{(x_M + r)^2 + y_M^{\ 2}}}{r})w. \quad (22)$$

So we get the function of water amount that different parts of the triangle get:

$$T_M = \begin{cases} (4 - \frac{\sqrt{(x_M + \frac{1}{2}r)^2 + (y_M - \frac{\sqrt{3}}{2}r)^2} + \sqrt{x_M^{\ 2} + y_M^{\ 2}} + \sqrt{(x_M - \frac{1}{2}r)^2 + (y_M - \frac{\sqrt{3}}{2}r)^2} + \sqrt{(x_M + r)^2 + y_M^{\ 2}}}{r})w, M \in D1; \\ (4 - \frac{\sqrt{(x_M - \frac{3}{2}r)^2 + (y_M - \frac{\sqrt{3}}{2}r)^2} + \sqrt{x_M^{\ 2} + y_M^{\ 2}} + \sqrt{(x_M - \frac{1}{2}r)^2 + (y_M - \frac{\sqrt{3}}{2}r)^2} + \sqrt{(x_M + r)^2 + y_M^{\ 2}}}{r})w, M \in D2; \\ (4 - \frac{\sqrt{(x_M - \frac{1}{2}r)^2 + (y_M + \frac{\sqrt{3}}{2}r)^2} + \sqrt{x_M^{\ 2} + y_M^{\ 2}} + \sqrt{(x_M - \frac{1}{2}r)^2 + (y_M - \frac{\sqrt{3}}{2}r)^2} + \sqrt{(x_M + r)^2 + y_M^{\ 2}}}{r})w, M \in D3; \\ (3 - \frac{\sqrt{x_M^{\ 2} + y_M^{\ 2}} + \sqrt{(x_M - \frac{1}{2}r)^2 + (y_M - \frac{\sqrt{3}}{2}r)^2} + \sqrt{(x_M + r)^2 + y_M^{\ 2}}}{r})w, \quad M \in D4. \end{cases}$$

(23)

Due to the difficulty of generating a piecewise function directly using MATLAB, we consider plotting the graph of the function by means of MATLAB programming:



The figure implies that the water from nearby triangles in district

$D1, D2, D3$ to some extent makes up the lack of water in periphery area of the triangle and as well even the water distribution out, making the relative difference (defined as $\frac{\max - \min}{average}$, where max, min and average represent the maximum, minimum and average water amount in the covered area, respectively, with following alike) smaller than 25%. Also, the districts with less water are mostly near the spouts where few plants grow. This spout arrangement turns out to be acceptable with water relatively well distributed.

3. Brief Summary

In this problem, by calculating the water distribution, we conclude that the Equilateral Triangle Model is an efficient way of irrigation on the horizontal plane, while water drops are unequally distributed in the covered area of a single spout.

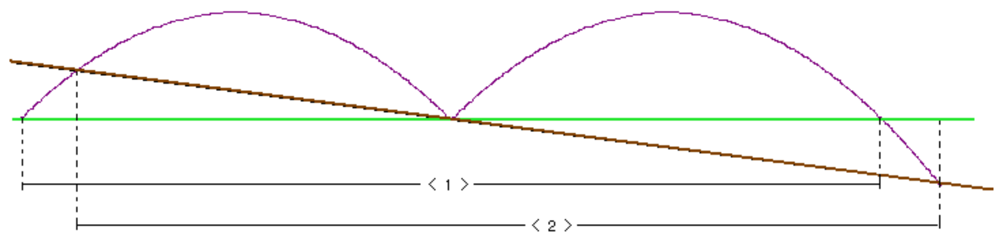
V. Problem 4: Degree of Equalizing on Slope Surface

Problem 3 discussed the reasonable spout arrangements in the sake of the equilibrium of water distribution, in a situation that the irrigation takes place on a horizontal plane. But in real irrigation, the water amount in the soil can be greatly affected by the slope of the irrigated surface..

Because the technique of irrigation requires the water which drops onto the irrigated surface to immediately percolate down into the soil, without producing runoff and puddles, in order not to cause soil erosion and too much evaporation. So the runoff brought about by the slope is not considered. What we need to focus on is the changes of area brought forth by the gradient of the irrigated area.

1. Area Changes from Planes to Slopes

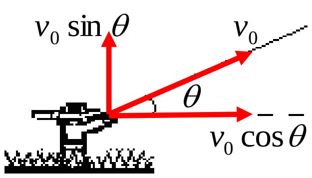
As is shown below: it is a figure of side-glance of the spout and the slope; the purple parabola represents the current coming out of one single spout; the green line represents a horizontal plane; the brown line represents a slope; <1> means the covered range on the horizontal plane; <2> means the covered range on the slope.



Obviously, from the horizontal plane to the slope, the covered area of a single spout changes. As the spout arrangement designed for the horizontal plane is no longer suitable, the previous function needs to be revised.

First we need to analyze the how the spout sprays water: water drops go out of the nozzle at a certain angle according to the spout's angle of elevation, and it has a initial velocity that is determined by the water pressure and the radius of the spray nozzle; water drops, after going out of the nozzle, fall and follow the rule of the projectile motion, leaving a locus that is a parabola. In rough calculation, the aero resistance can be neglected, and the winds are not considered in the condition.

Taking the spout as the origin and the x-axis parallel to the horizontal plane, establish the Cartesian coordinate system xOy on the vertical plane in which the position of the water drops can be described as (y,z). θ denotes the angle of elevation (The spouts are installed perpendicular to the horizontal plane, though, in reality, spouts are always set perpendicular to the slope. However, because water drops may be greatly influenced by the gravity if the spouts are perpendicular to the oblique plane, and when the gradient is very small, there are few differences between the two ways of installing the spouts. In order to get a simple equation of the locus of the water drops' motion, here we keep the spout perpendicular to the horizontal plane. Notice that now the angle of elevation θ must be smaller than the slope angle α); v_0 denotes the initial velocity (so $v_0 \cos\theta$ denotes its component horizontal velocity and $v_0 \sin\theta$ denotes its component vertical velocity), g denotes the acceleration due to the gravity



which directs vertically downwards to the horizontal plane, and t denotes the time. The relationship between them can be determined by the rule of the projectile motion in physics:

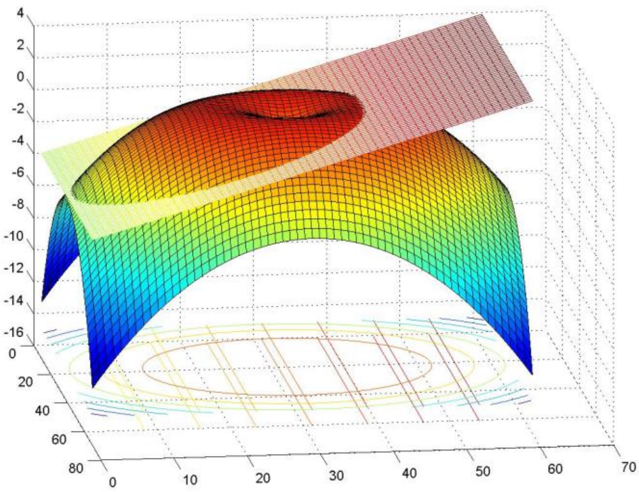
$$\begin{aligned} y &= v_0 \cos\theta \cdot t, \\ z &= v_0 \sin\theta \cdot t + \frac{1}{2}(-g)t^2. \end{aligned} \tag{24}$$

Eliminate t from the two equations above，and we have:

$$z = v_0 \sin\theta \cdot \frac{y}{v_0 \cos\theta} - \frac{1}{2} g\left(\frac{y}{v_0 \cos\theta}\right)^2 = \tan\theta \cdot y - \frac{g}{2v_0^2 \cos^2\theta} y^2. \tag{25}$$

Establish the three-dimensional Cartesian coordinate system $Oxyz$, in which the spout is taken as origin. A slope of $\alpha (0 \le \alpha < \frac{\pi}{2})$ can be expressed as $z = \tan\alpha \cdot y \quad x \in R$, and the locus of the movement of the water, in the space, can be regarded as the curved surface formed by the parabola (25) on plane yOz, which rotates around the z-axis:

$$\begin{aligned} z &= \tan\theta \cdot \sqrt{x^2+y^2} - \frac{g}{2v_0^2 \cos^2\theta}(\sqrt{x^2+y^2})^2 \\ &= \tan\theta \cdot \sqrt{x^2+y^2} - \frac{g}{2v_0^2 \cos^2\theta}(x^2+y^2). \end{aligned} \tag{26}$$



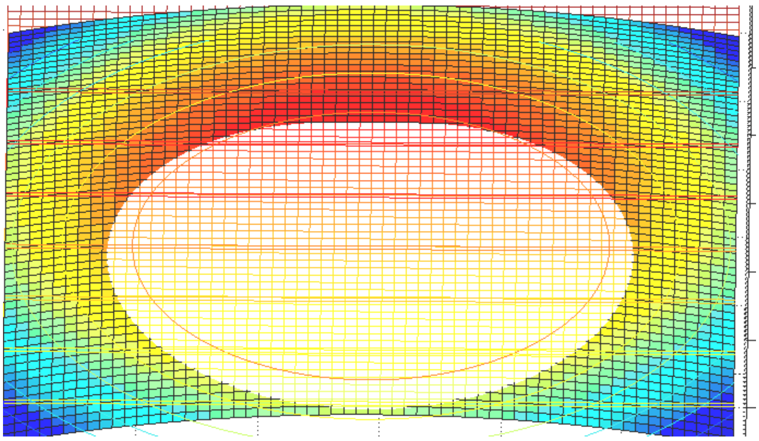
(The colored curved surface is the locus of the movement of water drops; the plane uncolored is the slope)

The curve that encircles the covered area can be regarded as the intersection line of plane $z = \tan\alpha \cdot y \quad (0 \le \alpha < \frac{\pi}{2})$ and the curved surface (26) so the equation of the intersection line is:

$$\begin{cases} z = \tan\alpha \cdot y & 0 \le \alpha < \dfrac{\pi}{2} \\ z = \tan\theta \cdot \sqrt{x^2 + y^2} - \dfrac{g}{2v_0^2 \cos^2\theta}(x^2 + y^2) \end{cases}. \tag{27}$$

On the horizontal plane, that is to say, $\alpha = 0$, we have the equation of the intersection line which is $x^2 + y^2 = (\frac{2v_0^2 \cos\theta \sin\theta}{g})^2$, namely a circle with the radius $\frac{2v_0^2 \cos\theta \sin\theta}{g}$.

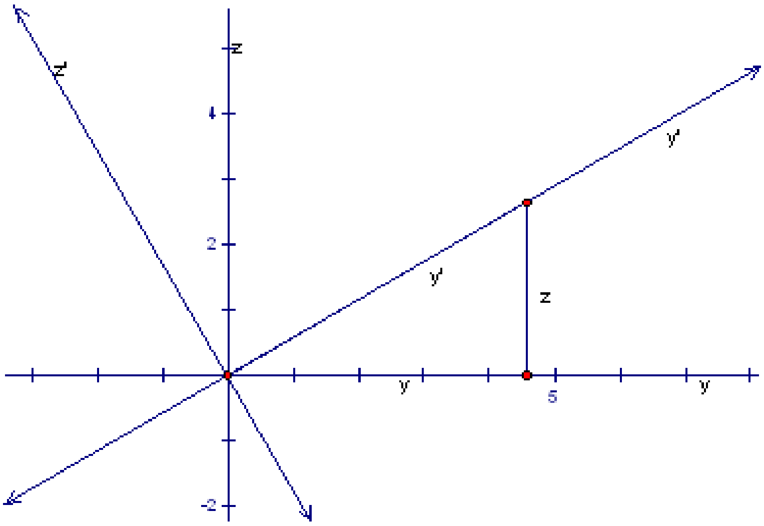
On a slope of $\alpha (0 \le \alpha < \frac{\pi}{2})$, because the intersection line is in the space, which makes it hard to deal with, we rotate it to the horizontal plane:



(The light colored part in the midst is the covered area as it is rotated to the horizontal plane. Marked as Graph #)

The rotation through an angle $\alpha (0 \le \alpha < \frac{\pi}{2})$ (with the positive direction taken anticlockwise) carries the plane $z = \tan\alpha \cdot y \quad (0 \le \alpha < \frac{\pi}{2})$ in the previous coordinate system to the plane xOy. From the graph shown below we learn that the old and new coordinates are related by

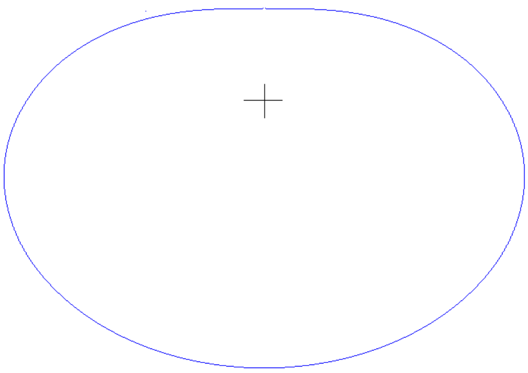
$$\begin{cases} x = x' \\ y = y'\cos\alpha \\ z = y'\sin\alpha \end{cases} \tag{28}$$



Substitute (28) into (27), and we have the equation of the intersection line on the plane:

$$y'\sin\alpha = \tan\theta \cdot \sqrt{x'^2 + y'^2\cos^2\alpha} - \frac{g}{2v_0^2\cos^2\theta}(x'^2 + y'^2\cos^2\alpha) \tag{29}$$

That is to say, the circular covered area changes into the curve we can draw according to the equation (29). Using MATLAB (The program is presented in Appendix 1), we have the graph of the curve (the position where the cross stands is the position of the spout):



Corresponding well with Graph #, it proves the correctness of the equation.

2. Degree of Equalizing with Water Distributed on Slopes

From Problem 3, we can see the amount of water (T) that a specific point M gets from a single spout O can be uniquely determined by the length of the directed line-segment $\overrightarrow{OM}$ as well as the sprinkling radius in the direction of $\overrightarrow{OM}$:

$$T = -\frac{w}{d} \cdot s + w, 0 \le s \le d, \tag{30}$$

where w is a constant, representing the water amount that the spout gets, namely the maximum amount of water; d is the sprinkling radius in the direction of $\overrightarrow{OM}$ (in order to distinguish it from the radius r on the horizontal plane, here we use d to represent the similar status in function (15).

For an arbitrary spout $P(a,b)$ on the plane, the equation of the boundary of the covered area can be obtained by translation of axes:

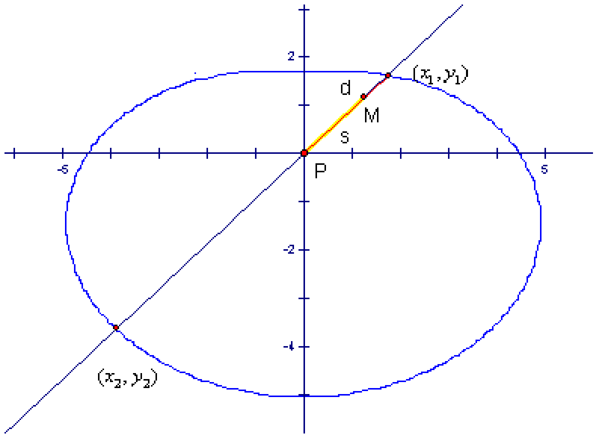
$$(y-b) \cdot \sin\alpha = \tan\theta \cdot \sqrt{(x-a)^2 + (y-b)^2 \cos^2\alpha} - \frac{g}{2v_0^2 \cos^2\theta}((x-a)^2 + (y-b)^2 \cos^2\alpha). \tag{31}$$

We focus on an arbitrary point $M(x_M, y_M)$ on the plane (within the covered area of the spout $P(a,b)$ but not concurrent with it, namely $\begin{cases} x_M \ne a \\ y_M \ne b \end{cases}$), the distance between point M and the spout P is

$$s = |MP| = \sqrt{(x_M - a)^2 + (y_M - b)^2} \tag{32}$$

To calculate the radius d in the direction of $\overrightarrow{PM}$, we get the system of equations for the straight line MP and equation (31). Solving the equations, we can get the coordinates of the intersect points $P_1(x_1,y_1), P_2(x_2,y_2)$, so

$$d=\begin{cases}|PP_1|, & y_M>b\\ |PP_2|, & y_M<b\\ |x_1-a|, & y_M=b\end{cases}.$$



The equation of line MP can be written as

$$l_{MP}:(y-b)(x_M-a)=(y_M-b)(x-a)\,, \tag{33}$$

so we can get

$$x-a=\frac{(y-b)(x_M-a)}{y_M-b}. \tag{34}$$

Substituting (34) into (31), we have:

$$(y-b)\cdot\sin\alpha=\tan\theta\cdot\sqrt{(\frac{(y-b)(x_M-a)}{y_M-b})^2+(y-b)^2\cos^2\alpha}-\frac{g}{2v_0^2\cos^2\theta}((\frac{(y-b)(x_M-a)}{y_M-b})^2+(y-b)^2\cos^2\alpha), \tag{35}$$

then we have the three roots of the equation y_1, y_2, y_3:

$$\begin{cases} y_1 - b = \dfrac{\tan\theta \cdot \sqrt{\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha} - \sin\alpha}{\dfrac{g}{2v_0^2\cos^2\theta}\left(\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha\right)}, \quad y > b; \\[2em] y_2 - b = \dfrac{-\tan\theta \cdot \sqrt{\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha} - \sin\alpha}{\dfrac{g}{2v_0^2\cos^2\theta}\left(\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha\right)}, \quad y < b; \\ y_3 = b. \end{cases} \tag{36}$$

Case I. If $y_M > b$, then the radius d in the direction of $\overrightarrow{PM}$ equals $|PP_1|$:

$$\begin{aligned} d &= |PP_1| = \sqrt{(x_1-a)^2 + (y_1-b)^2} \\ &= \sqrt{\frac{(x_M-a)^2}{(y_M-b)^2} + 1} \cdot (y_1 - b) \\ &= \sqrt{(x_M-a)^2 + (y_M-b)^2} \cdot \frac{\tan\theta \cdot \sqrt{\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha} - \sin\alpha}{\dfrac{g}{2v_0^2\cos^2\theta}\left(\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha\right)\cdot(y_M-b)} \\ &= \sqrt{(x_M-a)^2 + (y_M-b)^2} \cdot \frac{2v_0^2\cos\theta}{g} \cdot \frac{\sin\theta \cdot \sqrt{(x_M-a)^2 + \cos^2\alpha\cdot(y_M-b)^2} - \cos\theta\sin\alpha\cdot(y_M-b)}{(x_M-a)^2 + \cos^2\alpha\cdot(y_M-b)^2}; \end{aligned} \tag{37}$$

Case II. If $y_M < b$, then d equals $|PP_2|$:

$$\begin{aligned} d &= |PP_2| = \sqrt{(x_2-a)^2 + (y_2-b)^2} \\ &= \sqrt{\frac{(x_M-a)^2}{(y_M-b)^2} + 1} \cdot (y_2 - b) \\ &= \sqrt{(x_M-a)^2 + (y_M-b)^2} \cdot \frac{-\tan\theta \cdot \sqrt{\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha} - \sin\alpha}{\dfrac{g}{2v_0^2\cos^2\theta}\left(\dfrac{(x_M-a)^2}{(y_M-b)^2} + \cos^2\alpha\right)\cdot(y_M-b)} \\ &= \sqrt{(x_M-a)^2 + (y_M-b)^2} \cdot \frac{2v_0^2\cos\theta}{g} \cdot \frac{\sin\theta \cdot \sqrt{(x_M-a)^2 + \cos^2\alpha\cdot(y_M-b)^2} - \cos\theta\sin\alpha\cdot(y_M-b)}{(x_M-a)^2 + \cos^2\alpha\cdot(y_M-b)^2}; \end{aligned} \tag{38}$$

Case III. If $y_M = b$, then d equals $|x_1 - a|$ and the equation of line MP can be written as $l_{MP} : y = b$.

Together with (31), we have: $|x-a| = \frac{2v_0^2 \cos\theta \sin\theta}{g}$, so

$$d = |x-a| = \frac{2v_0^2 \cos\theta \sin\theta}{g}. \tag{39}$$

As we can see from (37), (38) and (39), the expression of sprinkling radius d in the direction of $\overrightarrow{PM}$, which has nothing to do with the relationship between y_M and b, is

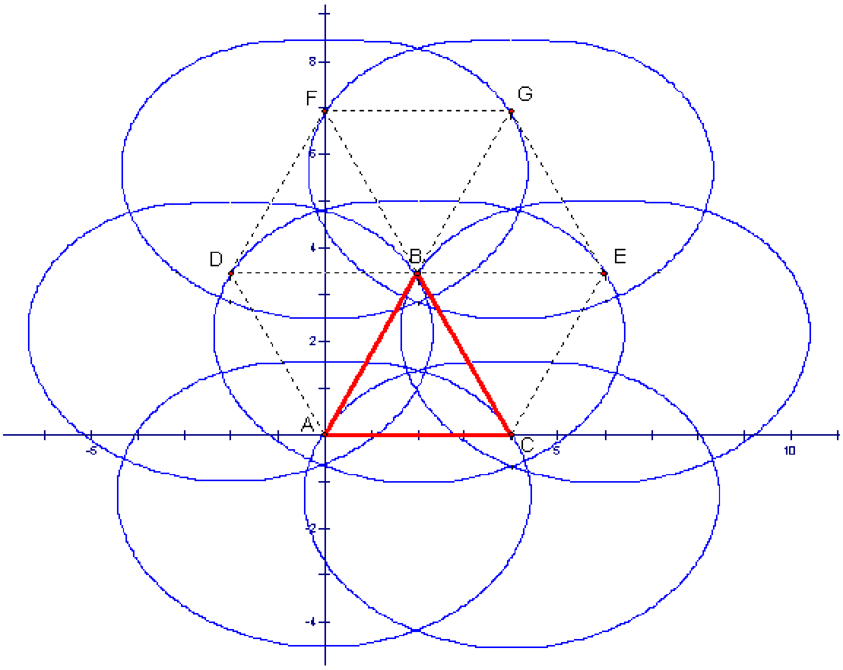
$$d = \sqrt{(x_M - a)^2 + (y_M - b)^2} \cdot \frac{2v_0^2 \cos\theta}{g} \cdot \frac{\sin\theta \cdot \sqrt{(x_M - a)^2 + \cos^2\alpha \cdot (y_M - b)^2} - \cos\theta \sin\alpha \cdot (y_M - b)}{(x_M - a)^2 + \cos^2\alpha \cdot (y_M - b)^2}. \tag{40}$$

Substitute (40) and (32) into function (30):

$$\begin{aligned} T &= -\frac{w}{d} \cdot s + w \\ &= \frac{-w\sqrt{(x_M - a)^2 + (y_M - b)^2}}{\sqrt{(x_M - a)^2 + (y_M - b)^2} \cdot \frac{2v_0^2 \cos\theta}{g} \cdot \frac{\sin\theta \cdot \sqrt{(x_M - a)^2 + \cos^2\alpha \cdot (y_M - b)^2} - \cos\theta \sin\alpha \cdot (y_M - b)}{(x_M - a)^2 + \cos^2\alpha \cdot (y_M - b)^2}} + w \\ &= \frac{-w \cdot g}{2v_0^2 \cos\theta} \cdot \frac{(x_M - a)^2 + \cos^2\alpha \cdot (y_M - b)^2}{\sin\theta \cdot \sqrt{(x_M - a)^2 + \cos^2\alpha \cdot (y_M - b)^2} - \cos\theta \sin\alpha \cdot (y_M - b)} + w. \end{aligned} \tag{41}$$

When the point $M(x_M, y_M)$ that we are studying is concurrent with the spout, namely $\begin{cases} x_M = a \\ y_M = b \end{cases}$, there does not exist a d, so we cannot use function (41) directly to calculate the water amount. However, as we assumed w is the water amount that the spout gets, that is to say when M is concurrent with the spout, the water amount (T) that M gets equals w, it can be classified in function (41) with $s=0$, with the water distribution not influenced.

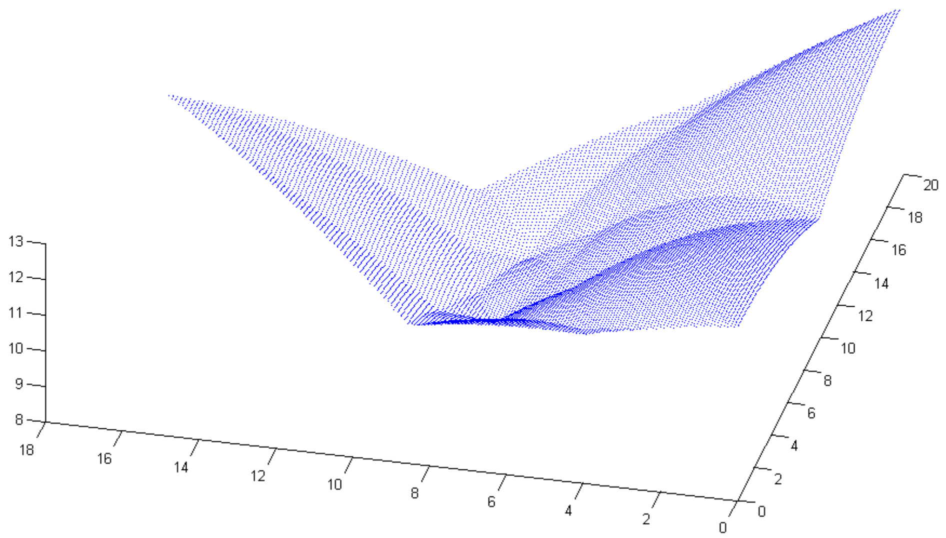
Apply the relatively ideal spout arrangement designed for horizontal plane (pave the plane with equilateral triangles whose lengths of sides are equal to the radius r of the covered area of a single spout on the horizontal plane, and the spouts are placed on the vertices) to the oblique plane. Simulate the water distribution in this situation:



The graph implies that the district of a unit triangle only receives water from 7 spouts placed in the position $A(0,0)$, $B(\frac{1}{2}r,\frac{\sqrt{3}}{2}r)$, $C(r,0)$, $D(-\frac{1}{2}r,\frac{\sqrt{3}}{2}r)$, $E(\frac{3}{2}r,\frac{\sqrt{3}}{2}r)$, $F(0,\sqrt{3}r)$, $G(r,\sqrt{3}r)$.

Substitute $\begin{cases} a=0 \\ b=0 \end{cases}$, $\begin{cases} a=\frac{1}{2}r \\ b=\frac{\sqrt{3}}{2}r \end{cases}$, $\begin{cases} a=r \\ b=0 \end{cases}$, $\begin{cases} a=-\frac{1}{2}r \\ b=\frac{\sqrt{3}}{2}r \end{cases}$, $\begin{cases} a=\frac{3}{2}r \\ b=\frac{\sqrt{3}}{2}r \end{cases}$, $\begin{cases} a=0 \\ b=\sqrt{3}r \end{cases}$, and $\begin{cases} a = r \\ b = \sqrt{3}r \end{cases}$ into (41), respectively. We get 7 functions that show the relationship between the coordinates of an arbitrary point in the irrigated area and the amount of water it gets.

However the functional expression we get is not only complex with lots of sections but also indirect, which makes the water distribution hard to be evaluated. So we state the graph directly instead of showing the complete expression of the function (The program is presented in Appendix 2):



The graph gives a general idea of the water distribution that the nearer a point is to a spout, the more water it gets. The relative difference $\frac{\max - \min}{average}$, compared to the same spout arrangement on the horizontal plane, is bigger.

However, in reality, the spouts' angles of elevation θ and the slope angle α in (41) are variable. Different angles of elevation θ and slope angles α may cause differences in degree of equalizing of the water distribution (While drawing the graphs above, we appointed that

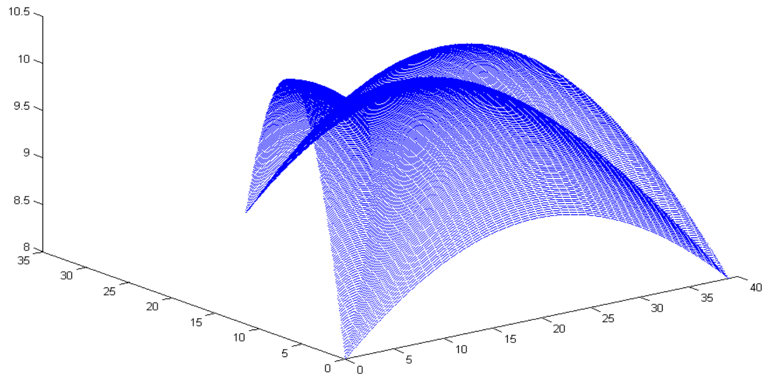
$\theta = 15^\circ, \alpha = 7.5^\circ$). So for a certain slope angle, we can adjust the angle of elevation of the spouts to make the water best distributed, namely to make the relative difference minimum.

Write programs through MATLAB (The program is presented in Appendix 3), we can get a spouts' angle of elevation θ for each set slope α, when the relative difference of the water amount reaches its minimum. Because there is little influence brought about by the different slope angles when they differ by less than 5°, and in order to reduce the frequency of the computer calculation, the slope angle α is set one value every 5°, from 0° to 40°; and due to the limitation of the adopted model (in which spouts are installed perpendicular to the horizontal plane), the slope angles α cannot be bigger than the spouts' angles of elevation θ.

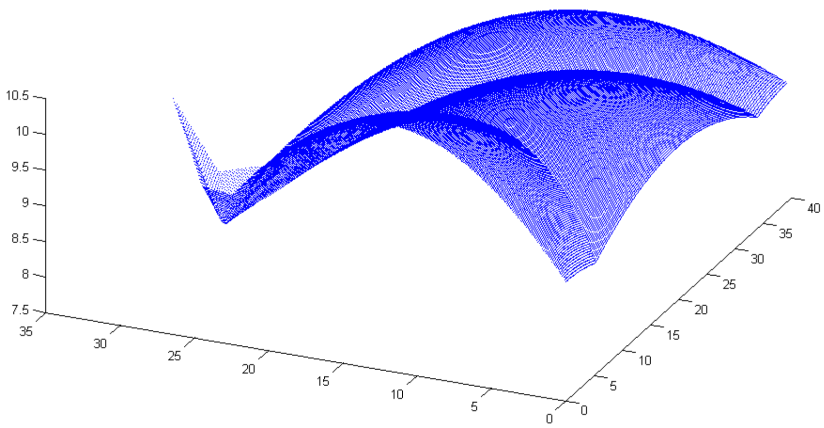
	Slope angle $\alpha(rad)$	Angle of elevation $\theta(rad)$	Relative difference %
1	0	0.7505	22.37
2	0.0873($\frac{\pi}{36}$)	0.7854	28.45
3	0.1745($\frac{\pi}{18}$)	0.7854	32.48
4	0.2618($\frac{\pi}{12}$)	0.7854	33.30
5	0.3491($\frac{\pi}{9}$)	0.5236	30.91
6	0.4363($\frac{5\pi}{36}$)	0.6894	27.90
7	0.5236($\frac{\pi}{6}$)	0.7854	25.47
8	0.6109($\frac{7\pi}{36}$)	0.7854	25.24
9	0.6981($\frac{2\pi}{9}$)	0.7854	31.92

We plot the graph of the water distribution based on the data listed above to see a direct visual impact:

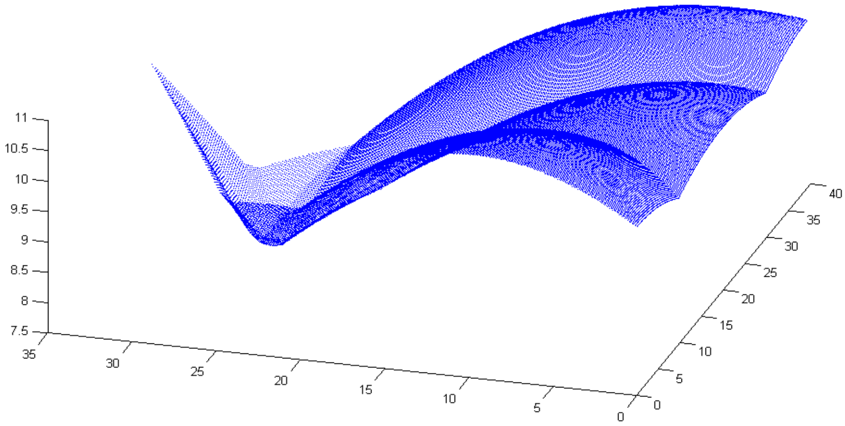
1.



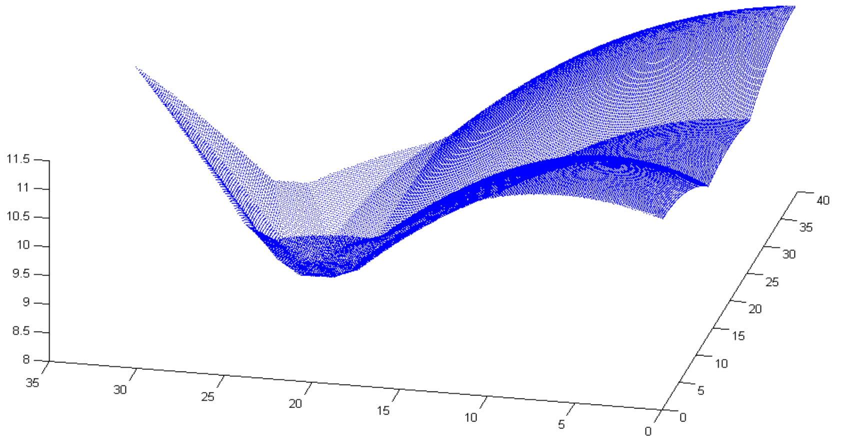
2.



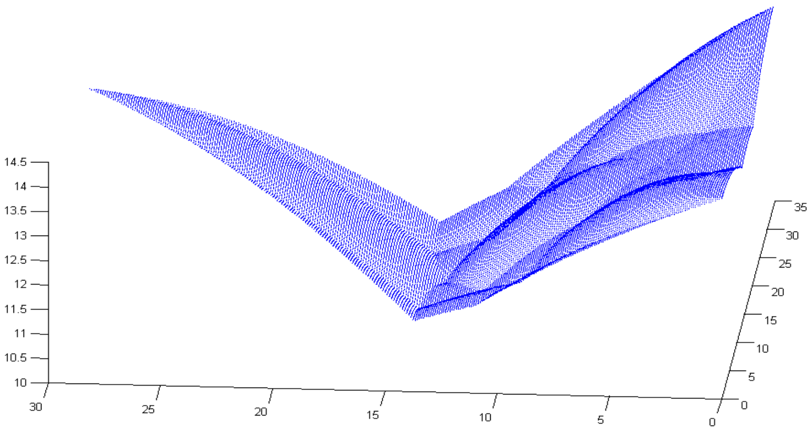
3.



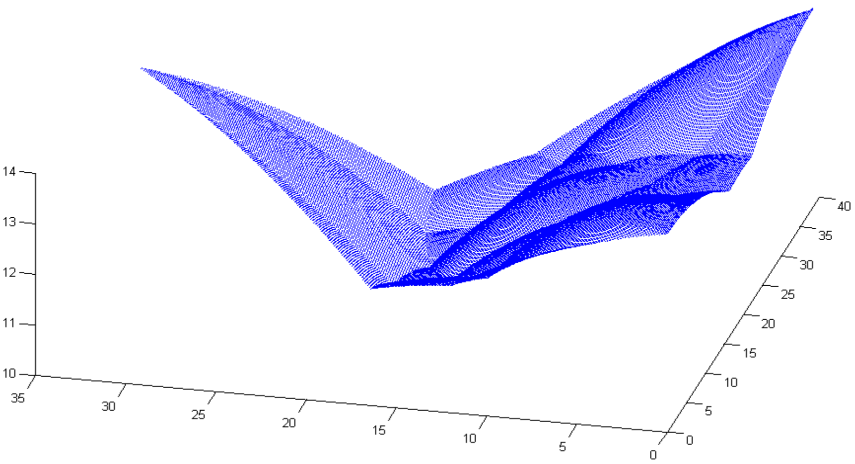
4.


11.5
11
10.5
10
9.5
9
8.5
8
35
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
40

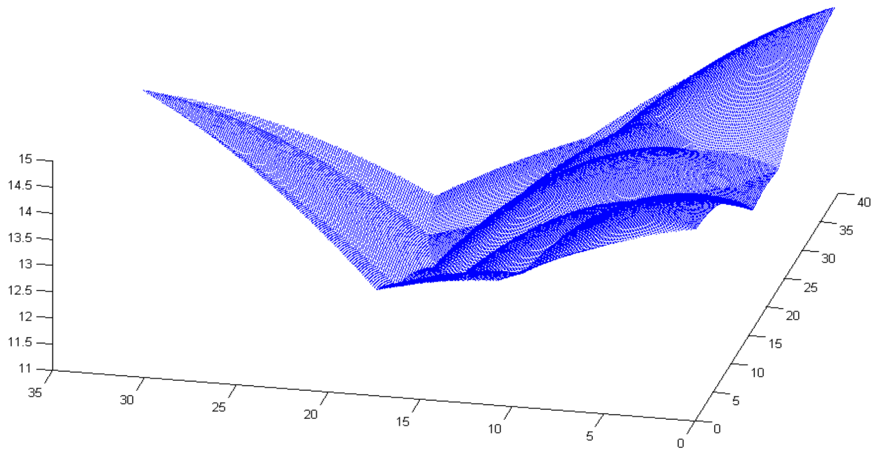
5.


14.5
14
13.5
13
12.5
12
11.5
11
10.5
10
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35

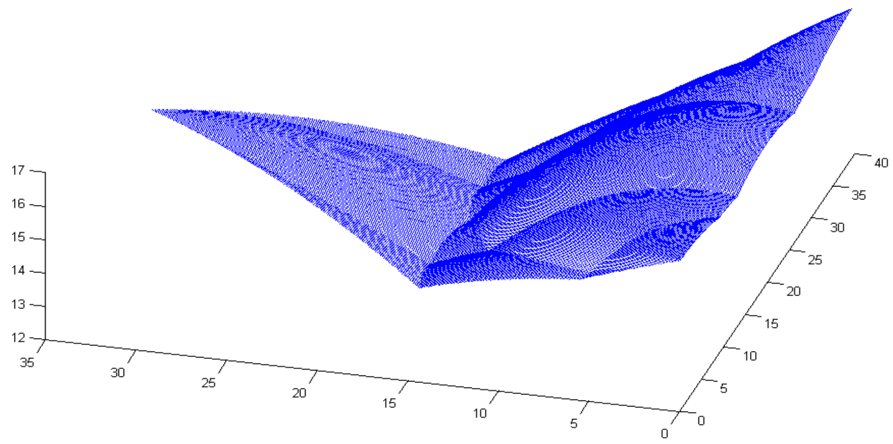
6.


14
13
12
11
10
35
30
25
20
15
10
5
0
0
5
10
15
20
25
30
35
40

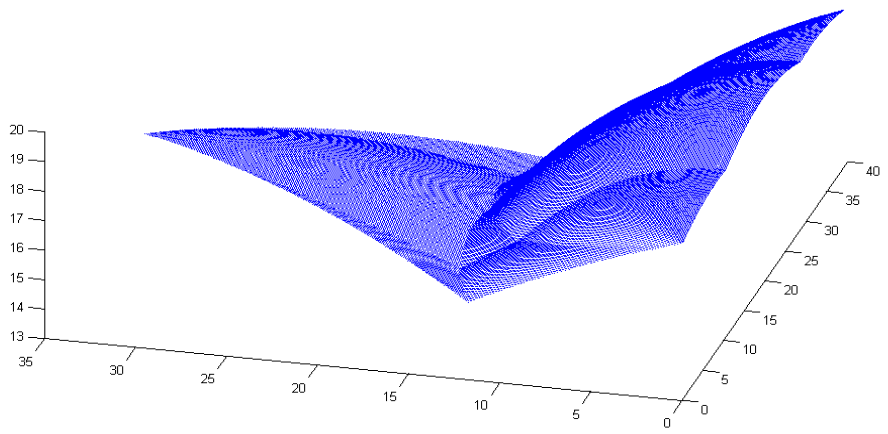
7.



8.



9.



While drawing the graph, we found that our previous analysis of the

Equilateral Triangle Model has some shortcomings. When we considered the spouts that might influence the water distribution in a unit triangle, we only drew a graph, appointing that $\theta = 15^\circ, \alpha = 7.5^\circ$. That is worth referring to but cannot involve all the possible situations. When $\alpha = 0^\circ$ in the case of the horizontal plane model in Problem 3, for instance, a unit triangle not only gets irrigated by the mentioned 7 spouts but also the spouts below the x-axis. But when $\alpha = 7.5^\circ$ it doesn't need to be considered. So we need to consider more spouts that may influence the water distribution, and make a further discussion.

Meanwhile, the term "relative difference" ($\frac{\max - \min}{average}$), which we employed to describe the degree of equalizing and got the spout's angle of elevation that made the water best distributed for an appointed slope angle, is not persuasive enough. It doesn't include the influence on water distribution at all the points in the calculation. Maybe adopting terms like standard deviation will be more comprehensive, where we can make further improvement.

3. Brief Summary

In this part, for the irrigation on the slope surface, we adopted the Equilateral Triangle Model that gives the most efficient irrigation in the studied situation, and for a certain slope angle, we calculated the exact angle of elevation of the spouts that makes the water best distributed.

VI. Design of Irrigative Landscape Effects

Irrigation technology is mainly used for plantation maintaining，but meanwhile it helps to create better visual effects. On the premise that it does not influence the irrigative equilibrium, the design of the irrigative landscape effects is as well one part a optimized project for the sprinkling irrigation.

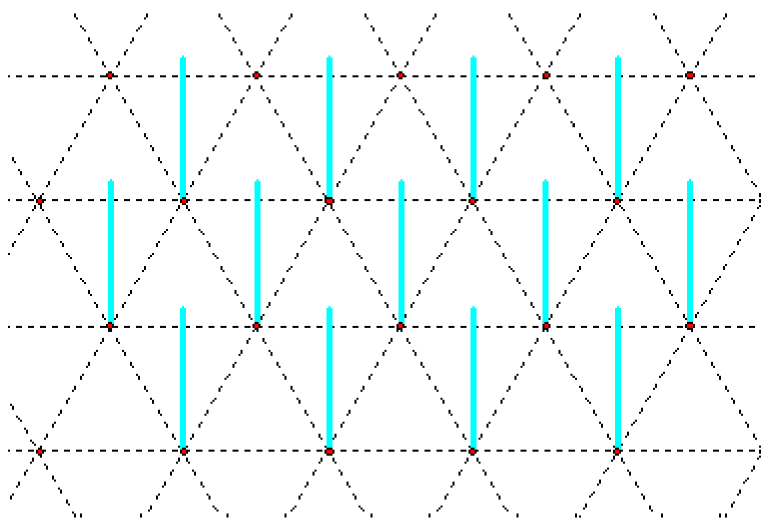
Taking the viability of the design and the general esthetics into consideration, this study puts forward two projects, utilizing the geometric symmetry, in order to reach a simple, regular and harmonious visual effect.

To avoid disorderly effects, here the spouts are set to be all spinning clockwise, at the same angular velocities. Now the origin spout exposure is under discussion.

Project 1: Total Synchronism

All the spouts are set to start in the same direction and spin slowly at the same angular velocities.

The effective image is presented below (the radius and the arrangement takes the best ones concluded in the former parts):

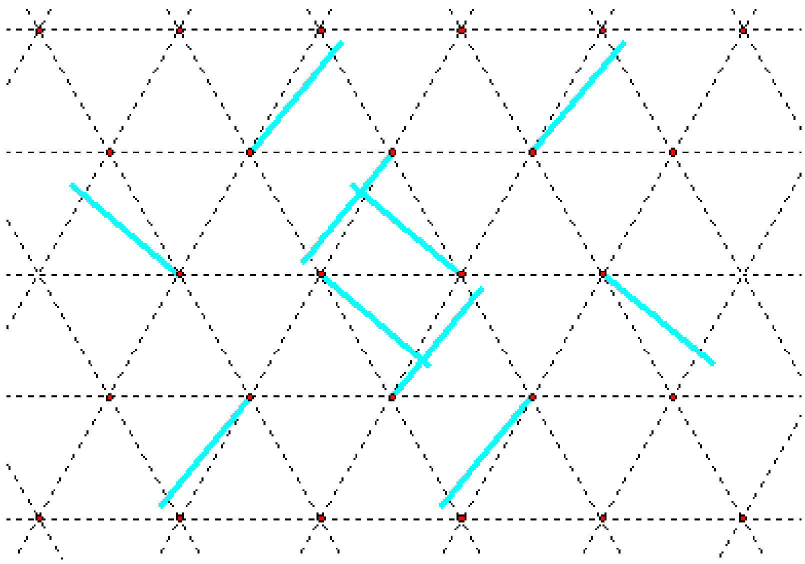


The advantage of this design is its simplicity. It is easy to control the irrigation system, and the sightseers can easily avoid being spilled over. It is applicable to parks and some other places where tourists can step on the grass.

Project 2： Extreme Symmetry

All the spouts are set to start in different but regular-patterned directions. Here they are designed to be, alternately, outwardly radiating and inwardly radiating, horizontally and longitudinally.

The effective image is presented below:



The advantage of this design is that it has a fancier visual effect. This project is suitable for large areas of greenery patches for sight seeing, especially for a bird’s-eye view.

The dynamically effective images are up loaded adhering. Please check them in the accessory.

VII. Retrospect

1. Sketch

This paper goes from simple to complicated conditions and from ideal to realistic situations. It starts in an totally ideal situation, gradually developing deeper as factors are in succession taken into consideration.

Firstly, the simplest models are built based on a totally ideal situation. Assumed to spin over plain lawn with radii appointed, sprinkling spouts are installed in arrangements of simple geometric figures. Areas of overlapping and blank spaces are calculated and the most reasonable arrangement of all that are suggested is selected.

Secondly, real factors are taken into consideration: 1. The disequilibrium of the water that drops in a line from the sprinkling center is transformed into a function, whose graphs are drawn to show the water distributed over the area; 2. The plane models are changed into solid ones when the sprinkling spouts are assumed to be placed on slopes. Analytic geometry methods are employed to deduce the range of sprayed water on the oblique planes. Through calculation and analysis, models can be adjusted to specific situations.

Finally, the boundary problems and landscape effects are also involved.

Although the models are still a little too idealized, during the process of researching, we got through the challenges while our mathematical abilities got more excellent.

2. Difficulty Overcoming in the Studying Process

Information acquiring. Searching for helpful information took us a good deal of time, and selecting useful things by comparing what we had got was also difficult to some extent. Due to the lack of professional knowledge, we met with some trouble. However, as we were getting to know more, we finally got accurate results.

Calculating and data sorting. Since the functional expressions were complex, simplifying and calculating required a lot of patience as well as a reliable calculating ability. As for the complex results, we redid the calculation to guarantee their correctness.

Applying mathematical knowledge and ability. Problems were even more complex than we had expected. Relying on the high school math and some extracurricular knowledge, we spent a lot of time establishing appropriate functions. As the research went further, we were faced with many barriers. To get the equation of the figure formed by the water sprayed on a slope, for instance, we made efforts that did not all helped. What we needed to do first was to draw the traces of the water drops that fell onto the ground as a spout finished spinning a circuit (just like being continually exposed by a camera). Then we had to cut through the three-dimensional figure with an oblique plane, get the graph of the section on that plane, deduce its function and, at last, put it into a plane xOy coordinate. That might be a piece of cake to those who could master more mathematical techniques, yet to us, it was a logical and intellectual challenge. We had discussed the problem with three of the math teachers at school who gave us some directive advice but failed to provide a specific means to work it out, before we raised the questions to a university instructor and a graduate, who, later, explained some further knowledge related to analytic geometry for us. However, new problems emerged: because the ways of thinking differed, they misunderstood our train of thought, and the function we got by methods they provided was not what we had expected it to be. Finally we decided to solve the problem ourselves, using exactly what we had learned in high school——somehow we had a premonition that we could make it. After a hard time's work, we eventually got the correct functional expressions through consumption, deduction and examination, which, to some extent, resulted in our solid mathematical and physical abilities. After the graph was drawn through mathematical software, we were delighted to find that it tallied with what we had imagined through common sense. Before we set the seal on its correctness, we had repeatedly examined every step during the deduction. Maybe the method we took seems a little bit awkward, but it enabled us to realize the practicality of math. There are other examples like that, and even though they slowed up our progress, they seem valuable because of all the sparks of thought produced as the conclusion came to light.

Graph showing. For it was geometry problems that we were trying to figure out, we felt obliged to use a series of graphs to show the effects our proposal brought forth. We mainly employed The Geometer's Sketchpad and MATLAB. The former was used for some basic geometric drawing and was employed to simulate the landscape effects when for different spout arrangement. The latter was much harder to master for it acquired more professional skills. We input our functions in MATLAB language and then got the images directly showing the irrigation effects. It was difficult at first, for correctly inputting the functions as well as checking the correctness of the graphs need a lot of repeating. Usually, a graph was perfectly drawn after much examination and correction. To better master this skill, we read books on the MATLAB and consulted our teachers. We got satisfactory results, ultimately, after so many attempts.

References

[1] Guangyong Li, Xingfa Huang; [M]Primary Design for Plantation Sprinkling Irrigation, China Plants Publisher, 2001.7 (李光永, 黄兴法；[M]园林草坪灌溉设计基础, 中国花卉出版)

[2] Xi Liu; Brief Introduction to the Sprinkling Irrigation on Greenary Patches (刘熹；浅谈绿地灌溉问题)

[3] One Factor of Sprinkling Irrigation: Itensity, Linhai Agricultural Web, 2001.10.11 (喷灌技术要素之一——喷灌强度，临海农网)

Appendix 1

```
theta=pi/12;
g=9.8;
v0=19.6;
alpha=pi/24;
x1=2*v0^2*cos(theta)^2*(tan(theta)-tan(alpha))/g;
y1=tan(alpha)*x1;
ymax=sqrt(x1^2+y1^2);
x2=-2*v0^2*cos(theta)^2*(tan(theta)+tan(alpha))/g;
y2=tan(alpha)*x2;
a=g/(2*v0^2*cos(theta)^2);
b=tan(theta);
ymin=-sqrt(x2^2+y2^2);
y=ymin:.00001:ymax;
x=sqrt(((b+sqrt(b.^2-4*a*y*sin(alpha)))/(2*a)).^2-y.^2*cos(alpha).^2);
plot(x,y);
hold on
plot(-x,y);
plot(0,0)
```

Appendix 2

```
theta=pi/12;
g=9.8;
v0=19.6;
alpha=pi/24;
r=2*v0^2*cos(theta)*sin(theta)/g;
w=8;
a=[0,1/2*r,r,-1/2*r,3/2*r,0,r];
b=[0,sqrt(3)/2*r,0,sqrt(3)/2*r,sqrt(3)/2*r,sqrt(3)*r,sqrt(3)*r];
ym=0;
xm=0;
while xm<=r
    ym=0;
    if xm<=r/2
        while ym<=sqrt(3)*xm
            z=0;
            for i=1:7
                M=sqrt((xm-a(i))^2+(ym-b(i))^2);
                N=(xm-a(i))^2+cos(alpha)^2*(ym-b(i))^2;
    d=M*2*v0^2*cos(theta)/g*(sin(theta)*sqrt(N)-cos(theta)*sin(alpha)*(ym-b(i)))/N;
                s=sqrt((xm-a(i))^2+(ym-b(i))^2);
                if s>d
                    T=0;
                else
                    T=-w/d*s+w;
                end
                z=z+T;
            end
            plot3(xm,ym,z);
            hold on
            ym=ym+0.1;
        end
    else
        while ym<=sqrt(3)*(r-xm)
            z=0;
            for i=1:7
                M=sqrt((xm-a(i))^2+(ym-b(i))^2);
                N=(xm-a(i))^2+cos(alpha)^2*(ym-b(i))^2;
    d=M*2*v0^2*cos(theta)/g*(sin(theta)*sqrt(N)-cos(theta)*sin(alpha)*(ym-b(i)))/N;
                s=sqrt((xm-a(i))^2+(ym-b(i))^2);
                if s>d
                    T=0;
                else
```

```
                T=-w/d*s+w;
            end
            z=z+T;
        end
        plot3(xm,ym,z);
        hold on
        ym=ym+0.1;
    end
  end
  xm=xm+0.1;
end
```

Appendix 3

```
g=9.8;
v0=19.6;
w=8;
alpha=0;
while alpha<=2*pi/9
    theta=pi/4;
    p=10;
    while theta>alpha
        r=2*v0^2*cos(theta)*sin(theta)/g;
        a=[0,1/2*r,r,-1/2*r,3/2*r,0,r];
        b=[0,sqrt(3)/2*r,0,sqrt(3)/2*r,sqrt(3)/2*r,sqrt(3)*r,sqrt(3)*r];
        xm=0;
        max=0;
        min=100;
        k=0;
        Q=0;
        while xm<=r
            ym=0;
            if xm<=r/2
                while ym<=sqrt(3)*xm
                    z=0;
                    for i=1:7
                        M=sqrt((xm-a(i))^2+(ym-b(i))^2);
                        N=(xm-a(i))^2+cos(alpha)^2*(ym-b(i))^2;
                        if N==0
                            T=w;
                        else
    d=M*2*v0^2*cos(theta)/g*(sin(theta)*sqrt(N)-cos(theta)*sin(alpha)*(ym-b(i)))/N;
                            s=sqrt((xm-a(i))^2+(ym-b(i))^2);
                            if s>d
                                T=0;
                            else
                                T=-w/d*s+w;
                            end
                        end
                        z=z+T;
                    end
                    if z>max
                        max=z;
                    end
                    if z<min
                        min=z;
```

```
                end
                Q=Q+z;
                k=k+1;
                ym=ym+0.1;
            end
        else
            while ym<=sqrt(3)*(r-xm)
                z=0;
                for i=1:7
                    M=sqrt((xm-a(i))^2+(ym-b(i))^2);
                    N=(xm-a(i))^2+cos(alpha)^2*(ym-b(i))^2;
                    if N==0
                        T=w;
                    else
d=M*2*v0^2*cos(theta)/g*(sin(theta)*sqrt(N)-cos(theta)*sin(alpha)*(ym-b(i)))/N;
                        s=sqrt((xm-a(i))^2+(ym-b(i))^2);
                        if s>d
                            T=0;
                        else
                            T=-w/d*s+w;
                        end
                    end
                    z=z+T;
                end
                if z>max
                    max=z;
                end
                if z<min
                    min=z;
                end
                Q=Q+z;
                k=k+1;
                ym=ym+0.1;
            end
        end
        xm=xm+0.1;
    end
    ave=Q/k;
    c=(max-min)/ave;
    if c<p
        p=c;
        the=theta;
    end
    theta=theta-pi/360;
```

```
        end
        p
        the
        alpha
        alpha=alpha+pi/36;
end
```