A Research On a Kind of Special Points Inside Convex

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Abstract

In this paper, we study the point set consisting of the centers of all inscribed central-symmetric convex polygons of a convex. We prove that for any convex, the area of the point set consisting of the centers of its inscribed central-symmetric convex polygons is not greater than $\frac{1}{4}$ of the area of the convex (the equality holds if and only if the convex is a triangle). This conclusion can provide us a method to measure the extent of central symmetry of a planar figure.

Key words: plane convex set, central-symmetric figure

1 Introduction

For a convex, every point inside the convex is the midpoint of a certain chord. What about the points that are the common midpoint of at least two chords? Such points are the centers of inscribed central-symmetric convex polygons of the convex. In this paper, we will study the properties of the point set of all these centers.

Definition 1.1 *For two arbitrary points A, B of point set M, we call M a convex set if all points on segment AB belong to M.*

Definition 1.2 *We call M a convex if convex set M is a bounded closed set.*

Let Ω be a convex in the plane. Let T be the set consisting of the centers of all inscribed central-symmetric convex polygons of Ω. Denote the area of T by $S(T)$.

We have the following theorem:

Theorem 1.3 *Let Ω be a convex in the plane, then*

$$0 \leq S(T) \leq S(\Omega),$$

the left equality holds if and only if Ω is a central-symmetric figure. The right equality holds if and only if Ω is a triangle.

In the following of the paper, if there is no special illustration, a convex or a central-symmetric figure only refers to a figure in the plane. A convex or a central-symmetric figure cannot be a line or part of a line.

2 Preliminary discussion

2.1 Definitions and Notations

Definition 2.1 *Let M be a convex. We shall call l a support line of convex M if:*

(i) the line l has at least one point in common with convex M.

(ii) all points of convex M lie either on one side of line l or on the line l.

Thus, when given a certain direction, convex M has two support lines both parallel to the direction; when given a certain point, there must exist at least one support line of convex M passing through the point.