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A characterization of compact complex tori via automorphism groups

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Abstract We show that a compact Kähler manifold X is a complex torus if both the continuous part and discrete part of some automorphism group G of X are infinite groups, unless X is bimeromorphic to a non-trivial G-equivariant fibration. Some applications to dynamics are given.

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1 Introduction

We work over the field $\mathbb{C}$ of complex numbers. Let X be a compact Kähler manifold. Denote by $\mathrm{Aut}(X)$ the automorphism group of X and by $\mathrm{Aut}_0(X)$ the identity connected component of $\mathrm{Aut}(X)$. By [7], $\mathrm{Aut}_0(X)$ has a natural meromorphic group structure. Further there exists a unique meromorphic subgroup, say $L(X)$, of $\mathrm{Aut}_0(X)$, which is meromorphically isomorphic to a linear algebraic group and such that the quotient $\mathrm{Aut}_0(X)/L(X)$ is a complex torus. In the following, by a subgroup of $\mathrm{Aut}_0(X)$ we always mean a meromorphic subgroup and by a linear algebraic subgroup of $\mathrm{Aut}_0(X)$ we mean a Zariski closed meromorphic subgroup contained in $L(X)$.

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For a subgroup $G \leq \mathrm{Aut}(X)$, the pair (X, G) is called *strongly primitive* if for every finite-index subgroup G_1 of G, X is not bimeromorphic to a non-trivial G_1-equivariant fibration, i.e., there does not exist any compact Kähler manifold X' bimeromorphic to X, such that X' admits a G_1-equivariant holomorphic map $X' \to Y$ with $0 < \dim Y < \dim X$ and $G_1 \leq \mathrm{Aut}(X')$. From the dynamical point of view, these manifolds are essential. Our main result Theorem 1.1 says that for these manifolds, unless it is a complex torus, there is no interesting dynamics if its automorphism group has positive dimension.

Theorem 1.1 *Let X be a compact Kähler manifold and $G \leq \mathrm{Aut}(X)$ a subgroup of automorphisms. Assume the following three conditions.*

(1) $G_0 := G \cap \mathrm{Aut}_0(X)$ is infinite.
(2) $|G : G_0| = \infty$.
(3) The pair (X, G) is strongly primitive.

Then X is a complex torus.

As a key step towards Theorem 1.1, we prove the following result. A proof for Theorem 1.2(2) is long overdue (and we do it geometrically via 1.2(1)), but the authors could not find it in any literature, even after consulting many experts across the continents.

Theorem 1.2 *Let X be a compact Kähler manifold and $G_0 \subset \mathrm{Aut}_0(X)$ a linear algebraic subgroup. Assume that G_0 acts on X with a Zariski open dense orbit. Then we have:*

(1) X is projective; the anti canonical divisor $-K_X$ is big, i.e. $\kappa(X, -K_X) = \dim X$.
(2) $\mathrm{Aut}(X)/\mathrm{Aut}_0(X)$ is finite.

Remark 1.3 (i) The condition (2) in Theorem 1.1 is satisfied if G acts on $H^2(X, \mathbb{C})$ as an infinite group, or if G has an element of positive entropy (cf. Section 3.2 for the definition).
(ii) Theorem 1.2 implies that when $\dim X \geq 3$ the case(4) in [18, Theorem 1.2] does not occur, hence it can be removed from the statement.
(iii) Theorem 1.1 generalizes [2, Theorem 1.2], where it is proven for $\dim X = 3$ and under an additional assumption.
(iv) A similar problem for endomorphisms of homogeneous varieties has been studied by Cantat in [4].

Two applications are given. The first one generalizes the following result due to Harbourne [9, Corollary (1.4)] to higher dimension: if X is a smooth projective rational surface with $\mathrm{Aut}_0(X) \neq (1)$, then $\mathrm{Aut}(X)/\mathrm{Aut}_0(X)$ is finite. To state it, recall ([7, Theorem 4.1]) that for any connected subgroup $H \leq \mathrm{Aut}(X)$, there exist a quotient space X/H and an H-equivariant dominant meromoprhic map $X \dashrightarrow X/H$, which satisfies certain universal property.

Application 1.4 *Let X be a compact Kähler manifold with irregularity $q(X) = 0$. Suppose that the quotient space $X/\mathrm{Aut}_0(X)$ has dimension ≤ 1. Then X is projective and $\mathrm{Aut}(X)/\mathrm{Aut}_0(X)$ is finite.*

The second application essentially says that when we study dynamics of a compact Kähler manifold X, we may assume that $L(X) = (1)$.

Application 1.5 *Let X be a smooth projective variety and $G_0 \lhd G \leq \mathrm{Aut}(X)$. Suppose that G_0 is a connected linear closed subgroup of $\mathrm{Aut}_0(X)$. Let Y be a G-equivariant resolution of the quotient space X/G_0 and replace X by a G-equivariant resolution so that the natural map $\pi : X \to Y$ is holomorphic. Then for any $g \in G$, we have the equality of the first dynamical degrees:*

$$d_1(g_{|X}) = d_1(g_{|Y}),$$

where $d_1(g_{|X}) := \max\{|\lambda| \,;\, \lambda$ is an eigenvalue of $g^* \mid H^{1,1}(X)\}$.

In particular, $G_{|X}$ is of null entropy if and only if so is $G_{|Y}$ (cf. 3.2).

If $q(X) = 0$, then $\mathrm{Aut}_0(X)$ (hence G_0) is always a linear algebraic group. On the other hand, if G_0 is not linear, then Application 1.5 does not hold (cf. Section 3.4).

2 Proof of Theorems

2.1 For a compact Kähler manifold X, denote by $\mathrm{NS}_{\mathbb{R}}(X)$ its Neron-Severi group. For an element $[A] \in \mathrm{NS}_{\mathbb{R}}(X)$, let $\mathrm{Aut}_{[A]}(X) := \{\sigma \in \mathrm{Aut}(X) \mid \sigma^*[A] = [A]$ in $\mathrm{NS}_{\mathbb{R}}(X)\}$.

Lemma 2.1 (cf. [10, ChII, Propositions 3.1 and 6.1]) *Let X be a projective variety and U an affine open subset of X. Then $D = X \backslash U$ is of pure codimension 1 and further, when D is $\mathbb{Q}$-Cartier, it is a big divisor, i.e. the Iitaka D-dimension $\kappa(X, D) = \dim X$.*

Lemma 2.2 *Let X be a compact Kähler manifold and B a big Cartier divisor. Then X is projective. Let $G \leq \mathrm{Aut}(X)$ be a subgroup such that $g^*B \sim B$ for every $g \in G$. Then $|G : G \cap \mathrm{Aut}_0(X)| < \infty$.*

Proof The existence of a big divisor on X implies that X is Moishezon, so X is projective since it is also Kähler (cf. [14]). Replacing B by a multiple, we may assume that the complete linear system $|B|$ gives rise to a birational map $\Phi_{|B|} : X \dashrightarrow Y$. Take a G-equivariant blowup $\pi : X' \to X$ such that $|\pi^*B| = |M| + F$ where $|M|$ is base point free and hence nef and big. Then $\mathrm{Aut}_{[M]}(X')$ is a finite extension of $\mathrm{Aut}_0(X')$ (cf. [17, Lemma 2.23], [15, Proposition 2.2]). By the assumption, $G \leq \mathrm{Aut}_{[M]}(X')$. Set $G_0 := G \cap \mathrm{Aut}_0(X')$. Then $|G : G_0| \leq |\mathrm{Aut}_{[M]}(X') : \mathrm{Aut}_0(X')| < \infty$. Take an ample divisor A on X. Then $G_{0|X'}$ fixes the class $[\pi^*A]$ and hence $G_{0|X} \leq \mathrm{Aut}_{[A]}(X)$. As $\mathrm{Aut}_{[A]}(X)$ is a finite extension of $\mathrm{Aut}_0(X)$, $G_{0|X}$ is a finite extension of $G_0 \cap \mathrm{Aut}_0(X)$. Now the lemma follows. □

Lemma 2.3 *Let X be a compact Kähler manifold and $G_0 \subset \mathrm{Aut}_0(X)$ a linear algebraic subgroup. Assume that G_0 acts on X with a Zariski open dense orbit. Then X is projective. Assume furthermore that the open G_0-orbit U is isomorphic to G_0/Γ, for some finite group Γ. Then $-K_X$ is big.*

Proof By a classical result of Chevalley, a connected linear algebraic group is a rational variety. By [7], G_0 has a compactification G_0^* such that the map $G_0 \times X \to X$ extends to a meromorphic map $G_0^* \times X \dashrightarrow X$. Hence if G_0 is a linear algebraic group and has a Zariski dense orbit in X, then X is meromorphically dominated by a rational variety G_0^* and is unirational. Hence X is Moishezon and also Kähler. Thus X is projective.

Let $f : X \to Y$ be a G_0-equivariant compactification of the quotient map $G_0 \to G_0/\Gamma$. By the ramification divisor formula $K_X = f^*K_Y + R_f$ with R_f effective, to say $-K_Y$ is big, it suffices to say the same for $-f^*K_Y$ or $-K_X$. So we may assume that $\Gamma = (1)$.

Let $D = \sum_i D_i$ be the irreducible decomposition of $D := X \setminus U$, which is G_0-stable and of pure codimension one since U is affine and by Lemma 2.1. Furthermore, Lemma 2.1 implies that D is big. By [11, Theorem 2.7], we have $-K_X = \sum_i a_i D_i$ for some integers $a_i \geq 1$. Thus $\kappa(X, -K_X) = \kappa(X, D) = \dim X$. □

2.2 Proof of Theorem 1.2 The assertion (2) follows from (1) and Lemma 2.2 since every automorphism of X preserves the divisor class $[-K_X]$. We now prove the assertion (1). Replacing G_0 by its connected component, we may assume that G_0 is connected. Let $U = G_0/H$ be the open G_0-orbit in X, where $H = (G_0)_{x_0}$ is the stabilizer subgroup of G_0 at a point $x_0 \in U$. First we show that $-K_X$ is effective. Let $\mathfrak{g}$ be the Lie algebra of G_0. As X is almost homogeneous, we can take $n = \dim X$ elements $v_1, \cdots, v_n$ in $\mathfrak{g}$ such that $\sigma = \tilde{v_1} \wedge \cdots \wedge \tilde{v_n}$ is not identically zero on X, where $\tilde{v_i}$ is the vector field corresponding to v_i via the isomorphism $\mathfrak{g} \simeq H^0(X, T_X)$. Then σ gives a non-zero section of $-K_X$. Hence $-K_X$ is effective.

Let H_0 be the identity connected component of H and $N(H_0)$ its normalizer in G_0. We consider the Tits fibration $X \dashrightarrow Y$ which on the open orbit is the G_0-equivariant quotient map $U = G_0/H \to G_0/N(H_0)$ with respect to the natural actions of G_0 on G_0/H and $G_0/N(H_0)$ (cf. [12, Propositions 1 and 6, page 61 and 65], or [1, Section 1.3]).

If the Tits fibration is trivial, i.e., its image is a point, then $G_0 = N(H_0)$. Hence H_0 is a normal subgroup of G_0 and the quotient G_0/H_0 is a connected linear algebraic group. Thus $-K_X$ is big by Lemma 2.3.

Now assume that the Tits fibration $X \dashrightarrow Y$ is non-trivial, i.e., $\dim G_0/N(H_0) > 0$. Taking G_0-equivariant blowups $\pi : X' \to X$ and $Y' \to Y$, we may assume that $\pi^*(-K_X) = L + E$ and a base point free linear system $\Lambda \subseteq |L|$ gives rise to the Tits fibration $f : X' \to Y'$ (cf. [1,12]). Write $L = f^*A$ with A very ample. Write $K_{X'} = \pi^*K_X + E'$ with E' effective. Let F be a general fibre of f. Then F is almost homogeneous under the action of the linear algebraic group $N(H_0)/H_0$ and the latter is isomorphic to the open orbit of F. Hence $-K_F$ is big by Lemma 2.3.

So $-\pi^*K_X|F = -K_{X'}|F + E'|F = -K_F + E'|F$ is big i.e., $-\pi^*K_X$ is relatively big over Y', which is also effective by the discussion above. By [3, Lemma 2.5], the divisor $-\pi^*K_X + f^*A = L + E + f^*A = 2L + E$ is big. Thus $\kappa(X', -\pi^*K_X) = \kappa(X', L + E) = \kappa(X', 2L + E) = \dim X'$. So $-\pi^*K_X$ and hence $-K_X$ are both big. This proves Theorem 1.2(1).

2.3 Proof of Theorem 1.1 Since (X, G) is strongly primitive, there is no non-trivial G-equivariant fibration. In particular, the Kodaira dimension $\kappa(X) \leq 0$, noting that $|G| = \infty$ and that a variety Y of general type is known to have finite $\mathrm{Aut}(Y)$. Let

$$\bar{G}_0 \leq \mathrm{Aut}_0(X)$$

be the Zariski-closure of G_0 which is normalized by G and of dimension ≥ 1 by our assumption on G_0, and let $\bar{G}_{00} := (\bar{G}_0)_0$ be its connected component.

If $\bar{G}_0$ does not have a Zariski-dense open orbit in X, then as in [18, Lemma 2.14] with $H := \bar{G}_{00}$ which is normalized by a finite-index subgroup G_1 of G, there is a G_1-equivariant non-trivial fibration, contradicting the strong primitivity of (X, G). So we may assume that X is almost homogeneous under the action of the algebraic group $\bar{G}_0$.

Suppose that $q(X) = 0$. Then $\mathrm{Aut}_0(X)$ is a linear algebraic group (cf. [15, Theorem 3.12]). The natural composition $G \to G.\bar{G}_0 \to (G.\bar{G}_0)/((G.\bar{G}_0) \cap \mathrm{Aut}_0(X))$ induces the first injective homomorphism below while the middle one is due to the second group isomorphism theorem:

$$\begin{aligned} G/G_0 \hookrightarrow (G.\bar{G}_0)/((G.\bar{G}_0) \cap \mathrm{Aut}_0(X)) &\cong ((G.\bar{G}_0).\mathrm{Aut}_0(X))/\mathrm{Aut}_0(X) \\ &\leq \mathrm{Aut}(X)/\mathrm{Aut}_0(X) \end{aligned}$$

where the last group is finite by Theorem 1.2. This contradicts the assumption.

Suppose now that $q(X) > 0$. Let $\mathrm{alb}_X : X \to A := \mathrm{Alb}(X)$ be the Albanese map which is automatically $\mathrm{Aut}(X)$- and hence G-equivariant, and which must be generically finite onto the image $\mathrm{alb}_X(X)$ by the strong primitivity of (X, G). Hence $\kappa(X) \geq \kappa(\mathrm{alb}_X(X)) \geq 0$. Thus $\kappa(X) = 0$. So alb_X is a bimeromorphic and surjective morphism (cf. [13, Theorem 24]).

Since X is almost homogeneous under the action of $\bar{G}_0$ and also of $\bar{G}_{00}$, so is A under the action of $\bar{G}_{00|A}$. Hence $\bar{G}_{00|A} = \mathrm{Aut}_0(A) = A$. We still need to show that $\mathrm{alb}_X : X \to A$ is an isomorphism. Suppose the contrary that we have a non-empty exceptional locus $E \subset X$ over which alb_X is not an isomorphism. Then both E and $F := \mathrm{alb}_X(E)$ are stable under the actions of $\bar{G}_{00}$, and $\bar{G}_{00|A} = A$, respectively. Hence $\dim F \geq \dim A$, contradicting the fact that alb_X is a bimeromorphic map. Theorem 1.1 is proved.

3 Proof of Applications

3.1 **Proof of Application 1.4** Set $G := \mathrm{Aut}(X)$ and $G_0 := \mathrm{Aut}_0(X)$. Since $q(X) = 0$, G_0 is a linear algebraic group (cf. [15, Theorem 3.12]). By [7, Lemma 4.2] and since $G_0 \lhd G$, there is a quotient map $X \dashrightarrow Y = X/G_0$ such that the action of G on X descends to a (not necessarily faithful) action of G on Y. Taking a G-equivariant resolution $Y' \to Y$ and a G-equivariant resolution $X' \to \Gamma_{X/Y'}$ of the graph of the composition $X \dashrightarrow Y \dashrightarrow Y'$, the natural map $f : X' \to Y'$ is holomorphic and G-equivariant. A general fibre F of f is almost homogeneous under the action of G_0. If $\dim Y' = 0$, then Application 1.4 follows from Theorem 1.2.

Suppose that $\dim Y' = 1$. Since $q(Y') \leq q(X') = q(X) = 0$, $Y' \cong \mathbb{P}^1$. So $-K_{Y'}$ is ample. By Theorem 1.2, $-K_{X'}|F = -K_F$ is big, i.e., $-K_{X'}$ is relatively big over Y'. So $B' := -K_{X'} + mf^*(-K_{Y'})$, with $m >> 1$, is a big divisor (cf. Proof of [3, Lemma 2.5]), whose class is stabilized by G. Now $B = \pi_* B'$, with $\pi : X' \to X$ the natural birational morphism, is a big divisor on X whose class is stabilized by G. Thus Application 1.4 follows from Lemma 2.2.

3.2 We recall some basic notions from dynamics. Let X be a compact Kähler manifold. For an automorphism $g \in \mathrm{Aut}(X)$, its (topological) *entropy* $h(g) = \log \rho(g)$ is defined as the logarithm of the *spectral radius* $\rho(g)$, where

$$\rho(g) := \max\{|\lambda|\,;\ \lambda \text{ is an eigenvalue of } g^*|\oplus_{i\geq 0} H^i(X,\mathbb{C})\}.$$

By the fundamental result of Gromov and Yomdin, the above definition is equivalent to the original dynamical definition of entropy (cf. [8,16]).

An element $g \in \mathrm{Aut}(X)$ is of *null entropy* if its (topological) entropy $h(g)$ equals 0. For a subgroup G of $\mathrm{Aut}(X)$, we define the *null subset* of G as

$$N(G) := \{g \in G \mid g \text{ is of null entropy, i.e., } h(g) = 0\}$$

which may *not* be a subgroup. A group $G \leq \mathrm{Aut}(X)$ is of *null entropy* if every $g \in G$ is of null entropy, i.e., if G equals $N(G)$.

For $g \in \mathrm{Aut}(X)$, let

$$d_1(g) := \max\{|\lambda|\,;\ \lambda \text{ is an eigenvalue of } g^* \mid H^{1,1}(X)\}$$

be the *first dynamical degree* of g (cf. [6, Section 2.2]), which is ≥ 1.

Let X and Y be compact Kähler manifolds of dimension n and l, with Kähler forms (or ample divisors when X and Y are projective) ω_X and ω_Y, respectively. For a surjective holomorphic map $\pi : X \to Y$ and an automorphism $g \in \mathrm{Aut}(X)$, the first relative dynamical degree of g (cf. [5, Section 3]) is defined as $d_1(g|\pi) = \lim_{s\to\infty} \lambda_1(g^s|\pi)^{1/s}$; here

$$\lambda_1(g^s|\pi) = \langle (g^s)^*\omega_X \wedge \pi^*\omega_Y^l, \omega_X^{n-1-l}\rangle;$$

it depends on the choice of Kähler forms, but $d_1(g|\pi)$ does not, because of sandwich-type inequalities between positive multiples of any two Kähler forms.

3.3 Proof of Application 1.5 By the Khovanskii-Tessier inequality, the first dynamical degree $d_1(g) = 1$ if and only if the topological entropy $h(g) = 0$ (cf. [6, Corollaire 2.2]), hence the second claim follows from the first one. By [5, Theorem 1.1], $d_1(g_{|X}) = \max\{d_1(g_{|Y}), d_1(g|\pi)\}$. Thus, to prove Application 1.5, it suffices to show $d_1(g|\pi) \leq 1$.

Let F be a general fibre of $\pi : X \to Y$. Then F is almost homogeneous under the action of G_0. Hence $-K_X|F = -K_F$ is big by Theorem 1.2, i.e., $-K_X$ is relatively big over Y. Let A be an ample divisor on Y such that $-K_X + \pi^*A$ is a big divisor on X (cf. Proof of [3, Lemma 2.5]) and is hence equal to $L + E$ for an ample $\mathbb{Q}$-divisor L and an effective $\mathbb{Q}$-divisor E.

Set $n := \dim X$ and $\ell := \dim Y$. Noting that $\pi^*A \cdot F = 0$, we have

$$\begin{aligned}\lambda_1(g^s|\pi) &:= (g^s)^*(L) \cdot L^{n-1-\ell} \cdot F \leq (g^s)^*(L+E) \cdot L^{n-1-\ell} \cdot F \\ &= (g^s)^*(-K_X) \cdot L^{n-1-\ell} \cdot F = (-K_X) \cdot L^{n-1-\ell} \cdot F =: c\end{aligned}$$

where the last term is a positive number independent of s. Hence

$$d_1(g|\pi) = \dim_{s\to\infty} (\lambda_1(g^s|\pi))^{1/s} \leq \dim_{s\to\infty} c^{1/s} = 1.$$

This proves Application 1.5.

Remark 3.1 (i) If we denote by K the kernel of the natural surjective homomorphism $G_{|X} \to G_{|Y}$, then we have an equality of null sets (as sets of cosets):

$$N(G_{|X})/K = N(G_{|Y}).$$

(ii) If the irregularity $q(X) = 0$, then $\mathrm{Aut}_0(X)$ and hence G_0 are always linear algebraic groups (cf. [15, Theorem 3.12]). In this case, Application 1.5 implies that we may assume that $\mathrm{Aut}_0(X) = (1)$ when studying dynamics.

(iii) Application 1.5 does not hold if G_0 is not linear. For example, let $X = T_1 \times T_2$ be the product of two complex tori and $G_0 = \mathrm{Aut}_0(T_1)$. The quotient space $Y := X/G_0$ is T_2. Suppose that $g \in \mathrm{Aut}(T_1)$ is an element of positive entropy which acts trivially on T_2. Then we have $d_1(g_{|X}) > d_1(g_{|Y}) = 1$.

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