

Entanglement and mixed states of Young tableau states in gauge/gravity correspondence

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Abstract

We use entangled multimode coherent states to produce entangled giant graviton states, in the context of gauge/gravity duality. We make a smeared distribution of the entangled multimode coherent states on the circle, or on the five-sphere, in the higher dimensional view. In gauge/gravity duality, we analyze the superposition of giant graviton states, and the entangled pairs of giant graviton states. We map a class of angular distribution functions to unitary operations on the pairs. We also use Young tableau states to construct cat states and qudit states. Various bipartite quantum states involving Young tableau states are analyzed, including micro-macro entangled states. Mixed states of Young tableau states are generated, by using ensemble mixing using angular distribution functions, and also by going through noisy quantum channels. We then produce mixed entangled pair of giant graviton states, by including interaction with the environment and using noisy quantum channels.

1 Introduction

The gauge/gravity correspondence [1, 2, 3] is a remarkable correspondence between a quantum system without gravity on the boundary and a quantum theory with gravity in the bulk. The correspondence reveals the nature of the emergent spacetime [4, 5, 6, 7]. The bulk emerges dynamically from the quantum mechanical description that lives in fewer dimensions on the boundary. The boundary system is described by a quantum field theory or quantum mechanics with a well-defined global time near the boundary. Hence, the boundary theory has a well-defined Hilbert space. There is a correspondence between observables of the bulk spacetime and observables of the boundary.

This correspondence provides a method for working on quantum gravity by quantum field theory on the boundary of the spacetime. The correspondence allows us to perform calculations related to string theory and quantum gravity from working on the quantum field theory side. On the other hand, the string theory provides the ultraviolet completion of supergravity, and is hence a ultraviolet-complete quantum gravity.

In the gauge/gravity correspondence, the quantum field theory side of the duality is an example of a quantum system with many degrees of freedom. In such a many-body quantum system, quantum correlations and quantum entanglement are generic [8]. Quantum entanglement are important resources for quantum information processing. Many tools and methods in quantum information are also useful for working on quantum gravity, since quantum gravity is also described by quantum mechanical rules. We can perform superpositions and quantum operations such as unitary transformations on these quantum states for the gravity side.

We analyze states which have interesting gravitational properties. There are states that are of interest both in the quantum gravity side and in quantum information theory, such as coherent states and their superpositions and entanglement. Moreover, there are Young tableau states [9, 10, 7], which are also entangled states. In gauge/string correspondence, there are backreacted geometries that correspond to highly excited states in the quantum field theory side, such as the bubbling geometries. The states in the Hilbert space of the quantum field theory are explicitly mapped to the gravity side. Since they live in the same Hilbert space, one can perform quantum superpositions and quantum operations on these states.

On the quantum field theory side, there are interesting coherent states [6, 11]. These are the superpositions of multi-trace states. In the context of gauge/gravity correspondence, BPS coherent states have gravity dual descriptions. They are important ingredients in the superposition-induced topology change in quantum gravity [11]. The transition between Young tableau (YT) states and coherent states involves a topology change in the dual spacetime. The superposition formula that gives a Young tableau state or a Brauer state by superposing coherent states, have been computed [11, 12, 13]. The superposition induced topology changes in the gravity side have been observed in [11, 12, 14, 15].

We also focus on Young tableau states. The YT states, which are labeled by Young tableaux, are linear combinations of multi-trace states. The single trace states include the descriptions of close strings, and similarly, the YT states also describe brane states. The YT states are also dual to giant gravitons on the gravity side. The giant gravitons can be viewed as polarized from point gravitons, under the influence of the anti-symmetric form-fields.

These YT states are entangled states in the product space of the multi-trace Hilbert spaces [11, 12], especially when they contain a big number of boxes. These states have nontrivial entanglement stored between different multi-trace Hilbert spaces. The entanglement entropy of these states, entangled in the multi-trace Hilbert space, have been computed in [12, 11]. These states can have different bases or labelings, and different bases can be transformed into each other, e.g. [12, 13]. The transformation between multi-traces and coherent states are also computed, e.g. [13]. For more details on the mathematics of the underlying symmetric groups and Young tableaux, see e.g. [16, 17, 18] and [19].

Among other things, we analyze bipartite quantum states involving YT states. We constructed bipartite entangled states involving YT, which describe entangled pairs of giant gravitons. We also use YT states to construct cat states and qudit states. The bipartite entangled states are important in gauge/gravity duality and quantum information. We further produced mixed states of YT and mixed entangled pair of YT.

The organization of this paper is as follows. In Sec. 2, we use entangled multimode coherent states to produce entangled giant graviton states. This involves entanglement from two portions of the internal five-sphere. In Sec. 3, we produce entangled states, which are entangled between Young tableau states and trace states. In Sec. 4, we generate mixed states of Young tableau states, by using ensemble mixing and then by using noisy quantum channels. In Sec. 5, we produce mixed or noisy entangled pair of giant gravitons, by including interaction with the environment and by using noisy quantum channels. Finally, we discuss our results and draw some conclusions in Sec. 6. In Appendix A, we briefly overview multimode coherent states and Young tableau states for the convenience of readers.

2 Entangled YT states

2.1 Entangled bipartite states and entanglement between two halves of the five-sphere

In this section, we consider a class of multimode coherent states and Young tableau states. More details of these two types of states are in Appendix A. This class of multimode coherent states was constructed in [11], and analyzed in further details in

[12, 15]. This multi-mode coherent state is

$$|Coh(\Lambda)\rangle = \prod_{k=1}^{\infty} \exp(\Lambda^k \frac{a_k^\dagger}{k}) |0\rangle_k \quad (2.1)$$

with normalization $\|Coh(\Lambda)\| = \sqrt{\mathcal{N}(\Lambda)}$ where

$$\mathcal{N}(\Lambda) = \exp\left(\sum_{k=1}^{\infty} \frac{|\Lambda|^{2k}}{k}\right) = \frac{1}{(1 - |\Lambda|^2)}. \quad (2.2)$$

The complex parameter is $\Lambda = |\Lambda|e^{i\theta_0}$. The shape of the multimode coherent states is determined by the expectation value of the chiral field evaluated on the states [11]. The chiral field describes the dynamics of the boundary between black and white droplet regions. The shape is a bump along the droplet boundary [20] located at $\theta_0 = \arg(\Lambda)$ with magnitude $|\Lambda|$. The shape of the bump is calculated by the expectation value of the chiral field $\langle \hat{\phi}(\theta) \rangle_{|\Psi\rangle}$, where $\hat{\phi}(\theta) = \sum_{k>0} (a_k \exp(-ik\theta) + a_k^\dagger \exp(ik\theta))$ [11]. See also [12] for detailed analysis. Note that $|Coh(\Lambda)\rangle$ is a multimode coherent state, and not a single mode coherent state. This state has a gravity dual in terms of a bump on the five-sphere.

The one-row Young tableau state is [11, 12]

$$|\Delta_n\rangle = \frac{1}{2\pi i} \oint_{\mathcal{C}} \frac{dw}{w} w^{-n} |Coh(w)\rangle, \quad (2.3)$$

where \mathcal{C} is a path that encloses 0. The $|\Delta_n\rangle$ has unit norm, i.e. $\|\Delta_n\| = 1$. It has property

$$|Coh(w)\rangle = \sum_{n=0}^{\infty} w^n |\Delta_n\rangle, \quad \langle \Delta_n | Coh(w) \rangle = w^n. \quad (2.4)$$

Hence

$$|\Delta_n\rangle = \frac{1}{2\pi} |\Lambda|^{-n} \int_0^{2\pi} d\theta e^{-in\theta} |Coh(\Lambda e^{i\theta})\rangle. \quad (2.5)$$

$|\Delta_n\rangle$ is a state with n boxes. They describe the giant gravitons. There are two different physical interpretations of the states. The giant graviton is a brane wrapping submanifolds of internal dimensions and the dual giant graviton is a brane wrapping submanifolds of the AdS space. One interpretation is that, it can be interpreted as a state of n Kaluza-Klein (KK) gravitons. Another interpretation is that, it can also be interpreted as a state of a dual giant graviton with n units of angular momentum, [21, 22, 23]. These giant gravitons travel along the circle of the five-sphere.

Now we use entangled multimode coherent states to produce entangled giant graviton states, or in other words, entangled YT states. Let $\Psi \in \mathcal{H}_A \otimes \mathcal{H}_B$ and let $\Lambda_1 \Lambda_2 = \Lambda$. We consider a generic quantum superposition of angularly-correlated multimode coherent

states, with a generic complex distribution function $\alpha(\theta) \in \mathbb{C}$. Depending on the angular correlations of the two multimode coherent states, there are different types of entangled states between the two. Generally, the following is a class of bipartite entangled states,

$$\begin{aligned}\Psi &= \Psi[\alpha(\theta)] := \frac{1}{\sqrt{\mathcal{N}(\Lambda)}} \int_0^{2\pi} \frac{d\theta}{2\pi} \alpha(\theta) |Coh(\Lambda_1 e^{i\theta})\rangle |Coh(\Lambda_2 e^{-i\theta})\rangle \\ &= \frac{1}{\sqrt{\mathcal{N}(\Lambda)}} \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \int_0^{2\pi} \frac{d\theta}{2\pi} \alpha(\theta) e^{i(n_1-n_2)\theta} \Lambda_1^{n_1} \Lambda_2^{n_2} |\Delta_{n_1}\rangle |\Delta_{n_2}\rangle.\end{aligned}\quad (2.6)$$

Here $\alpha(\theta)$ is a general complex coefficient, i.e. $\alpha(\theta) \in \mathbb{C}$. The two angles are oppositely correlated. The angular distribution function gives rise to superposition of the bipartite states. We have expanded the entangled states in YT basis which is the giant graviton basis.

Now we consider a special case when $\Lambda_2/\Lambda_1 \in \mathbb{R}_{>0}$ for (2.6); In this convention of the parameters, the state (2.6) describes entanglement between two halves of the circle, since the angles are oppositely correlated, and in the higher dimensional view, between two halves of the five-sphere. We have the inclusion $i : S^1 \hookrightarrow S^5$ and the projection $p : S^5 \rightarrow S^1$. As we show, they are also equivalent to entangled giant graviton states. The states of bubbling geometries and the states of giant gravitons are dual to each other. This includes the case when both sides are entangled states. Hence we can also interpret A and B as two subsystems corresponding to the two halves of five-spheres. Hence, this state can also be viewed as an entangled state between excitations on the two halves of the circle, or on the two halves of the five-sphere in the higher dimensional view. These entangled configurations have also been considered in [14]. Moreover, quantum states of giant gravitons, e.g. [6],[24],[25]–[41], can also be enumerated by states of multi-dimensional quantum harmonic oscillators.

For $\alpha(\theta) = 1$, since $\int_0^{2\pi} \frac{d\theta}{2\pi} e^{i(n_1-n_2)\theta} = \delta_{n_1, n_2}$,

$$\begin{aligned}\Psi &= \frac{1}{\sqrt{\mathcal{N}(\Lambda)}} \int_0^{2\pi} \frac{d\theta}{2\pi} |Coh(\Lambda_1 e^{i\theta})\rangle |Coh(\Lambda_2 e^{-i\theta})\rangle \\ &= \sqrt{1 - |\Lambda|^2} \sum_{n=0}^{\infty} \Lambda^n |\Delta_n\rangle |\Delta_n\rangle.\end{aligned}\quad (2.7)$$

We have a more general coefficient $\alpha(\theta)$ by the expansion,

$$\alpha(\theta) = \sum_l \alpha_l e^{il\theta}, \quad (2.8)$$

and $\alpha_l = \frac{1}{2\pi} \int_0^{2\pi} d\theta \alpha(\theta) e^{-il\theta}$. For this case,

$$\begin{aligned}
\Psi &= \frac{1}{\sqrt{\mathcal{N}(\Lambda)}} \int_0^{2\pi} \frac{d\theta}{2\pi} \left(\sum_l \alpha_l e^{il\theta} \right) |Coh(\Lambda_1 e^{i\theta})\rangle |Coh(\Lambda_2 e^{-i\theta})\rangle \\
&= \frac{1}{\sqrt{\mathcal{N}(\Lambda)}} \sum_l \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \left(\int_0^{2\pi} \frac{d\theta}{2\pi} \alpha_l e^{i(n_1-n_2+l)\theta} \right) \Lambda_1^{n_1} \Lambda_2^{n_2} |\Delta_{n_1}\rangle |\Delta_{n_2}\rangle \\
&= \frac{1}{\sqrt{\mathcal{N}(\Lambda)}} \sum_{n=0}^{\infty} \Lambda^n |\Delta_n\rangle \left(\sum_l \alpha_l \Lambda_2^l |\Delta_{n+l}\rangle \right). \tag{2.9}
\end{aligned}$$

Eq. (2.7, 2.9) is the production of the entangled pair of giant gravitons over the five-spheres.

The reduced density matrix $\rho_A = \text{tr}_{\mathcal{H}_B} |\Psi\rangle\langle\Psi|$ is

$$\rho_A = (1 - |\Lambda|^2) \sum_{n=0}^{\infty} |\Lambda|^{2n} |\Delta_n\rangle\langle\Delta_n|. \tag{2.10}$$

If we make an identification $e^{-\beta} = |\Lambda|^2$, $\beta = 1/T_{\text{eff}}$, the thermal mixed YT state is

$$(1 - e^{-\beta}) \sum_{n=0}^{\infty} e^{-\beta n} |\Delta_n\rangle\langle\Delta_n|, \quad \beta = 1/T. \tag{2.11}$$

2.2 Finite D case

In Sec 2.1, we have a continuous superposition using an angular distribution function $\alpha(\theta)$, which is a continuous function. In this section, we make a discretization of the distribution function in Sec 2.1. In other words, we use a discrete distribution function in this section. In order to have a finite D version, we use a large D approximation of the YT states $|\Delta_n\rangle$ by the regularized YT states $|\tilde{\Delta}_n\rangle$ as we will discuss.

The regularized YT state $|\tilde{\Delta}_n\rangle$ is as follows. In order to have a finite D version of the Hilbert space, we use a large D approximation of the YT states by using the large D cat states [15]:

$$\Phi_{D,n}(\Lambda) = \frac{1}{\sqrt{\mathcal{N}_{D,n}}} \sum_{m=0}^{D-1} |Coh(\Lambda e^{i\frac{2\pi}{D}m})\rangle e^{-i\frac{2\pi}{D}mn}, \tag{2.12}$$

for $n = 0, \dots, D-1$, and $\mathcal{N}_{D,n} = \frac{D^2 |\Lambda|^{2(n-D[n/D])}}{1-|\Lambda|^{2D}}$ where $[n/D]$ denotes the integer part. We have that $\|\Phi_{D,n}(\Lambda)\| = 1$. It is pointed out in [15] that $\Phi_{D,n}(\Lambda)$, $n = 0, \dots, D-1$, corresponds to the n -th irreducible representation of the group Z_D , and hence they are mutually orthogonal to each other [15]. The large D cat states converge to the YT

states in the large D limit [15]. These Schrodinger cat states approach the one-row Young tableau states, with fidelity between them asymptotically reaches 1 at large D . These cat states are defined for n smaller than D , primarily. However, we can extend the definition of the cat states for n bigger or equal to D by performing a shift symmetry identification, $\Phi_{D,n+D}(\Lambda) = \Phi_{D,n}(\Lambda)$.

We define the finite D regularized YT states $|\tilde{\Delta}_n\rangle$ for large D ,

$$|\tilde{\Delta}_n\rangle := \lim_{\Lambda \rightarrow 0, \Lambda \neq 0} \frac{|\Lambda|^n}{\Lambda^n} \Phi_{D,n}(\Lambda). \quad (2.13)$$

Here $\frac{|\Lambda|^n}{\Lambda^n} = e^{-in\theta_0}$ is an overall constant phase factor. The norm is $\|\tilde{\Delta}_n\| = 1$. The inner product of YT and cat state [15] is $\langle \Delta_n | \Phi_{D,n}(\Lambda) \rangle = \frac{\Lambda^n}{|\Lambda|^n} \sqrt{1 - |\Lambda|^{2D}}$, $0 < |\Lambda| < 1$. It is proved in [15] that

$$\lim_{D \rightarrow \infty} |\tilde{\Delta}_n\rangle = |\Delta_n\rangle. \quad (2.14)$$

We call $|\tilde{\Delta}_n\rangle$ the regularized version of the state $|\Delta_n\rangle$. We define the regularized multimode coherent states $|Coh_D(w)\rangle$ as follows,

$$|Coh_D(w)\rangle := \sum_{n=0}^{D-1} w^n |\tilde{\Delta}_n\rangle, \quad (2.15)$$

for any $w \in \mathbb{C}$ and $0 < |w| < 1$. The expansion of it contains only finite order terms. We call (2.15) the regularized state or the finite D version of the state $|Coh(w)\rangle$.

We have the following proposition:

Proposition 2.1. *The state $|Coh_D(w)\rangle$ converges to $|Coh(w)\rangle$ in the infinite D limit, i.e.,*

$$\lim_{D \rightarrow \infty} |Coh_D(w)\rangle = |Coh(w)\rangle. \quad (2.16)$$

Proof. We have that $|Coh_D(w)\rangle = \sum_{n=0}^{D-1} w^n |\tilde{\Delta}_n\rangle$. We also have the relation $\lim_{D \rightarrow \infty} |\tilde{\Delta}_n\rangle = |\Delta_n\rangle$. Hence, $\lim_{D \rightarrow \infty} |Coh_D(w)\rangle = \sum_{n=0}^{\infty} w^n |\Delta_n\rangle = |Coh(w)\rangle$. \square

Now we use the multimode coherent states to produce the entangled regularized YT states, as the finite D version of Sec. 2.1. Let $\Psi \in \mathcal{H}_A \otimes \mathcal{H}_B$ and let $\Lambda_1 \Lambda_2 = \Lambda$. We denote $\Lambda_1 = |\Lambda_1| e^{i\varphi_1}$, $\Lambda_2 = |\Lambda_2| e^{i\varphi_2}$, and $\arg(\Lambda_2/\Lambda_1) = \varphi_2 - \varphi_1$. In the finite D version, we have that $\theta_m = \frac{2\pi}{D}m$ where D is a positive integer, and $\lim_{D \rightarrow \infty} \frac{1}{D} \sum_{m=0}^{D-1} = \frac{1}{2\pi} \int_0^{2\pi} d\theta$.

The finite D version of the entangled multimode coherent states is

$$\Psi = \frac{1}{\sqrt{\mathcal{N}_D(\Lambda)}} \frac{1}{D} \sum_{m=0}^{D-1} \tilde{\alpha}(m) |Coh_D(\Lambda_1 e^{i\frac{2\pi}{D}m})\rangle |Coh_D(\Lambda_2 e^{-i\frac{2\pi}{D}m})\rangle, \quad (2.17)$$

where $\mathcal{N}_D(\Lambda) = \frac{1-|\Lambda|^{2D}}{1-|\Lambda|^2}$. For $\tilde{\alpha}(m) = 1$,

$$\Psi = \frac{1}{\sqrt{\mathcal{N}_D(\Lambda)}} \sum_{n=0}^{D-1} \Lambda^n |\tilde{\Delta}_n\rangle |\tilde{\Delta}_n\rangle. \quad (2.18)$$

This has a natural $D \rightarrow \infty$ limit to (2.7). For $\tilde{\alpha}(m) = \sum_{l=0}^{D-1} \alpha_l e^{il\frac{2\pi}{D}m}$,

$$\Psi = \frac{1}{\sqrt{\mathcal{N}_D(\Lambda)}} \sum_{n=0}^{D-1} \Lambda^n |\tilde{\Delta}_n\rangle \left(\sum_l \alpha_l \Lambda_2^l |\tilde{\Delta}_{n+l}\rangle \right). \quad (2.19)$$

The reduced density matrix $\rho_A = \text{tr}_{\mathcal{H}_B} |\Psi\rangle\langle\Psi|$ is

$$\rho_A = \left(\frac{1-|\Lambda|^2}{1-|\Lambda|^{2D}} \right) \sum_{n=0}^{D-1} |\Lambda|^{2n} |\tilde{\Delta}_n\rangle\langle\tilde{\Delta}_n|. \quad (2.20)$$

Now we consider the simplification of the states in the $|\Lambda| \rightarrow 1$ limit. In this limit, by L'Hospital's rule, $\frac{(1-|\Lambda|^2)}{(1-|\Lambda|^{2D})} \rightarrow \frac{1}{D}$, and then (2.18) is $\Psi = \frac{1}{\sqrt{D}} \sum_{n=0}^{D-1} |\tilde{\Delta}_n\rangle |\tilde{\Delta}_n\rangle$, which is a maximally entangled state or EPR state [42]. For more general $|\Lambda|$, the states are non-maximally entangled. The limit $|\Lambda| \rightarrow 1$ is equivalent to $|\Lambda_1| \rightarrow 1, |\Lambda_2| \rightarrow 1$. Eq. (2.19) is then

$$\Psi = \frac{1}{\sqrt{D}} \sum_{n=0}^{D-1} e^{in\varphi_1} |\tilde{\Delta}_n\rangle \left(\sum_l \alpha_l e^{i(n+l)\varphi_2} |\tilde{\Delta}_{n+l}\rangle \right), \quad (2.21)$$

and $|\Psi| = 1, \sum_l |\alpha_l|^2 = 1$. We can redefine the basis $e^{in\varphi_1} |\tilde{\Delta}_n\rangle \rightarrow |\tilde{\Delta}_n\rangle$. And then,

$$\Psi = \frac{1}{\sqrt{D}} (I \otimes U) \sum_{n=0}^{D-1} |\tilde{\Delta}_n\rangle |\tilde{\Delta}_n\rangle, \quad (2.22)$$

which is in the form of Choi state. U is a unitary matrix of order D ,

$$U = \sum_{n',n} U_{n',n} |\tilde{\Delta}_{n'}\rangle\langle\tilde{\Delta}_n| = \sum_{n',n} \sum_l \alpha_l e^{i(n+l)(\varphi_2-\varphi_1)} \delta_{n',n+l} |\tilde{\Delta}_{n'}\rangle\langle\tilde{\Delta}_n|. \quad (2.23)$$

The norm of Ψ is 1, meaning that $\sum_{n=0}^{D-1} U_{n',n} U_{n,n'}^* = 1, \forall n'$, or equivalently $UU^\dagger = I$. There is a shift symmetry identification $|\tilde{\Delta}_{n+D}\rangle \rightarrow |\tilde{\Delta}_n\rangle$ for the states $|\tilde{\Delta}_{n+D}\rangle$ where $n+D > D$.

In the case $\varphi_2 - \varphi_1 = 0$, define the matrix

$$X := \sum_{n',n} \delta_{n',n+1} |\tilde{\Delta}_{n'}\rangle\langle\tilde{\Delta}_n|, \quad (2.24)$$

with the shift symmetry identification $|\tilde{\Delta}_{n+D}\rangle = |\tilde{\Delta}_n\rangle$. X is a cyclic permutation sending $(\tilde{\Delta}_0, \tilde{\Delta}_1, \dots, \tilde{\Delta}_{D-2}, \tilde{\Delta}_{D-1})$ to $(\tilde{\Delta}_1, \tilde{\Delta}_2, \dots, \tilde{\Delta}_{D-1}, \tilde{\Delta}_0)$. Then

$$X_l := X^l = \sum_{n',n} \delta_{n',n+l} |\tilde{\Delta}_{n'}\rangle \langle \tilde{\Delta}_n|, \quad (2.25)$$

$l = 0, \dots, D-1$. X_l are also cyclic permutations. The X_l are in fact circulant matrices, and they form a group ring, in which $X_{l_1} X_{l_2} = X_{l_1+l_2}$. We include $X_0 = I$ which is the identity. Hence, $U = \sum_l \alpha_l X_l$. The above U is unitary since $\sum_l |\alpha_l|^2 = 1$.

For $\varphi_2 - \varphi_1 \neq 0$, the factor $e^{i(n+l)(\varphi_2 - \varphi_1)} = \omega^{(n+l)}$ corresponds to multiplying a diagonal unitary matrix $Y = \text{diag}(1, \omega, \omega^2, \dots, \omega^{D-1})$, where $\omega = e^{i(\varphi_2 - \varphi_1)}$. Hence,

$$U = Y \sum_l \alpha_l X_l. \quad (2.26)$$

Define $W = \text{diag}(1, \omega, \omega^2, \dots, \omega^{D-1})$, where $\omega = e^{i\frac{2\pi}{D}}$. We have that

$$W := \sum_n e^{i\frac{2\pi}{D}n} |\tilde{\Delta}_n\rangle \langle \tilde{\Delta}_n|, \quad W_j := W^j = \sum_n e^{i\frac{2\pi}{D}nj} |\tilde{\Delta}_n\rangle \langle \tilde{\Delta}_n|. \quad (2.27)$$

We include $W_0 = I$ which is the identity.

The advantages and motivations of the discretization is as follows. First, it is motivated by and corresponds to a discrete distribution in the coherent state superposition. In some situations, the discrete distribution gives rise to a finite group symmetry such as Z_D which makes computations simpler. Second, the discrete case or the finite D case, enables a large D limit to recover the infinite D case. The finite D case is a regularization for the infinite D case. Third, it makes the Hilbert spaces of these subsystems finite dimensional and hence simpler for computations. These states become discrete-variable quantum states, for example, the qudits with dimensionality D .

For the finite D states, we can take two limits of them, one is the infinite D limit, and another is the $|\Lambda| \rightarrow 1$ limit. This can be illustrated as follows, with the example of $\frac{1-|\Lambda|^2}{1-|\Lambda|^{2D}}$. For finite D , $|\Lambda| \rightarrow 1$, $\frac{1-|\Lambda|^2}{1-|\Lambda|^{2D}} \rightarrow \frac{1}{D}$. For $D \rightarrow \infty$, $|\Lambda| < 1$, $\frac{1-|\Lambda|^2}{1-|\Lambda|^{2D}} \rightarrow 1 - |\Lambda|^2$. In the joint limit, $D \rightarrow \infty$, $|\Lambda| \rightarrow 1$, $\frac{1-|\Lambda|^2}{1-|\Lambda|^{2D}} \rightarrow 0$.

2.3 More general cases of angular distributions

In this section, we focus on more general angular distributions, than that of Sec 2.2. Consider that the distribution function $\alpha(\theta, \tilde{\theta})$ has two angular variables $(\theta, \tilde{\theta})$ corresponding to two independent angles of the pair of multi-mode coherent states. The distribution function $\alpha(\theta, \tilde{\theta})$ is

$$\alpha(\theta, \tilde{\theta}) = \sum_l \alpha_l(\tilde{\theta}) e^{il\theta}, \quad \alpha_l(\tilde{\theta}) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \alpha(\theta, \tilde{\theta}) e^{-il\theta} \quad (2.28)$$

and

$$\Psi = \frac{1}{\sqrt{\mathcal{N}(\Lambda)}} \int_0^{2\pi} \int_0^{2\pi} \frac{d\theta d\tilde{\theta}}{(2\pi)^2} \left(\sum_l \alpha_l(\tilde{\theta}) e^{il\theta} \right) |Coh(|\Lambda_1|e^{i\theta})\rangle |Coh(|\Lambda_2|e^{i(\tilde{\theta}-\theta)})\rangle. \quad (2.29)$$

We also have $\arg(\Lambda_2/\Lambda_1) = \tilde{\theta}$. Hence, in the finite D version, $\tilde{\theta}_j = \frac{2\pi}{D}j$, and $\lim_{D \rightarrow \infty} \frac{1}{D} \sum_{j=0}^{D-1} = \frac{1}{2\pi} \int_0^{2\pi} d\tilde{\theta}$. Hence,

$$\alpha_{l,j} = \alpha_l \left(\frac{2\pi}{D} j \right). \quad (2.30)$$

We then obtain the finite D version (2.31) in the following proposition 2.2.

Proposition 2.2. *The finite D version of (2.29), i.e.,*

$$\Psi = \frac{1}{\sqrt{\mathcal{N}_D(\Lambda)}} \frac{1}{D^2} \sum_{m=0}^{D-1} \sum_{j=0}^{D-1} \left(\sum_l \alpha_{l,j} e^{il \frac{2\pi}{D} m} \right) |Coh_D(|\Lambda_1|e^{i \frac{2\pi}{D} m})\rangle |Coh_D(|\Lambda_2|e^{i \frac{2\pi}{D} (j-m)})\rangle \quad (2.31)$$

in the $|\Lambda_1 \Lambda_2| = |\Lambda| \rightarrow 1$ limit, can be written as

$$\Psi = \frac{1}{\sqrt{D}} \sum_{n=0}^{D-1} \sum_{l,j} \alpha_{l,j} (I \otimes U_{l,j}) |\tilde{\Delta}_n\rangle |\tilde{\Delta}_n\rangle, \quad (2.32)$$

where $U_{l,j}$ is an unitary matrix, which is $U_{l,j} = W_j X_l$.

Proof.

$$\Psi = \frac{1}{\sqrt{\mathcal{N}_D(\Lambda)}} \sum_{n=0}^{D-1} |\Lambda_1|^n |\tilde{\Delta}_n\rangle \left(\sum_{j=0}^{D-1} \sum_l \alpha_{l,j} |\Lambda_2|^{n+l} e^{i(n+l) \frac{2\pi}{D} j} |\tilde{\Delta}_{n+l}\rangle \right). \quad (2.33)$$

We then take $|\Lambda| \rightarrow 1$ limit. Hence define

$$\begin{aligned} U &= \sum_{l,j} \alpha_{l,j} W_j X_l = \sum_{l,j} \alpha_{l,j} U_{l,j} \\ &= \sum_{l,j} \alpha_{l,j} \sum_{n',n} e^{i \frac{2\pi}{D} n' j} \delta_{n',n+l} |\tilde{\Delta}_{n'}\rangle \langle \tilde{\Delta}_n|. \end{aligned} \quad (2.34)$$

We have that

$$\Psi = \frac{1}{\sqrt{D}} \sum_{n=0}^{D-1} \sum_{l,j} \alpha_{l,j} (I \otimes U_{l,j}) |\tilde{\Delta}_n\rangle |\tilde{\Delta}_n\rangle. \quad (2.35)$$

$U_{l,j}$ form a complete basis for unitary matrices of order D . \square

Due to the orthogonality of the generators $U_{l,j}$, the normalization condition here is $\sum_{l,j} |\alpha_{l,j}|^2 = 1$. These states are maximally entangled EPR states [42]. They form basis states for entangled pair of qudits. For more general $|\Lambda\rangle$, the states are non-maximally entangled. We can also write

$$\begin{aligned}\Psi &= \sum_{l,j} \alpha_{l,j} \Psi_{l,j}, \\ \Psi_{l,j} &= (I \otimes U_{l,j}) \Psi_0,\end{aligned}\tag{2.36}$$

where $\Psi_0 = \frac{1}{\sqrt{D}} \sum_{n=0}^{D-1} |\tilde{\Delta}_n\rangle |\tilde{\Delta}_n\rangle$. We include $U_{0,0} = I$ which is the identity.

The states $\sum_n s_n |\tilde{\Delta}_n\rangle$, with $\sum_n |s_n|^2 = 1$ can be viewed as cat qudits, with qudit dimensionality D . The $D = 2$ case and $D = 4$ case are particularly interesting. Since $D = 2$ are qubits and can be realized by ordinary cat qubits, parametrized by amplitudes $\Lambda, -\Lambda$. The $D = 4$ can be realized by superpositions of two different cat qubits, parametrized together by amplitudes $\Lambda, \Lambda e^{i\frac{\pi}{2}}, -\Lambda, -\Lambda e^{i\frac{\pi}{2}}$.

We have that $\sum_{l,j} \alpha_{l,j} U_{l,j}$ can be viewed as Choi matrices for the pair of qudits. For example,

$$D = 2, \quad X = \sigma_X, \quad W = \sigma_Z,\tag{2.37}$$

and

$$D = 3, \quad X = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{2\pi}{3}} & 0 \\ 0 & 0 & e^{i\frac{4\pi}{3}} \end{bmatrix}.\tag{2.38}$$

Note that the $D = 2$ case is Schrodinger cat states with two components, i.e. ordinary cat qubits. We can perform unitary operations and quantum operations on these qudits.

On the other hand, the summation (2.31) goes over to an integration (2.29). We can view (2.29) as the large D limit of (2.31). The finite-dimension D can be viewed as a regularization for a finite dimension of the Hilbert space. In the context of gauge/gravity duality, we can naturally generate entangled pairs of giant graviton qudits and coherent states, as in Sec. 2. The state $\sum_n (I \otimes U) |\Delta_n\rangle |\Delta_n\rangle$ is the state of a pair of entangled fluctuating giant gravitons. The superposition of bipartite entangled multi-mode coherent states gives rise to Choi states in the Young tableau basis, with a general unitary U . This U can be viewed as a quantum operation, by the channel/state correspondence [43, 44, 45].

After the interaction of the entangled giant graviton states with the background closed string states, the entangled pair becomes mixed entangled pair, as we shall discuss in Sec. 5. Now we turn to the relation to Sec. 2.1. Consider the mixed-state entangled state

$$\rho_{AB} = \sum_{l,j} p_{l,j} |\Psi_{l,j}\rangle \langle \Psi_{l,j}|,\tag{2.39}$$

where $\sum_{l,j} p_{l,j} = 1$. For ρ_{AB} as pure states, either maximally entangled or non-maximally entangled, the entanglement entropy between the two halves of the five-sphere is $S(\rho_A)$ or $S(\rho_B)$. For ρ_{AB} as mixed states, the mutual information $I(A, B)$ between the two halves of the five-sphere is $S(\rho_A) + S(\rho_B) + \sum_{l,j} p_{l,j} \ln p_{l,j}$. We also consider the case when ρ_{AB} are mixed states, under noisy channels due to interactions with background fluctuation modes or environment states. These mixed entangled states can be generated by noisy channels, as we discuss in Sec. 5.

3 Bipartite entangled states with YT states

In this section, we consider entangled states which are entangled between trace states and YT states. We produce entanglement between trace states, which are dual to closed string states, and YT coherent states. We consider unitary operations and entangling gates by composite operations of squeezer, beam splitter, displacer, as well as other operations.

The $\Delta_{n,k}$ is a Young tableau with n columns each with a column-length k . Consider the creation and annihilation operators A_k^\dagger, A_k on the multi-row Young tableau states $|\Delta_{n,k}\rangle$,

$$A_k^\dagger |\Delta_{n,k}\rangle = \sqrt{k(n+1)} |\Delta_{n+1,k}\rangle, \quad A_k |\Delta_{n,k}\rangle = \sqrt{kn} |\Delta_{n-1,k}\rangle, \quad (3.1)$$

where $\frac{1}{k}[A_k, A_k^\dagger] = 1$ and $A_k |\Delta_{0,k}\rangle = 0$. The first equation in (3.1) can also be written as

$$A_k^\dagger |\Delta_{n-1,k}\rangle = \sqrt{kn} |\Delta_{n,k}\rangle. \quad (3.2)$$

In other words, $\frac{1}{\sqrt{k}} A_k^\dagger$ and $\frac{1}{\sqrt{k}} A_k$ play the role of ordinary creation and annihilation operators, with the $\frac{1}{\sqrt{k}}$ factor due to our particular convention of the definition. This is in the large N limit. Hence $|\Delta_{n,k}\rangle = \frac{(A_k^\dagger)^n}{\sqrt{k^n n!}} |\Delta_{0,k}\rangle$ and $\|\Delta_{n,k}\| = 1$. The action of A_k^\dagger is adding one column of length- k on the Young tableau. The action of A_k is removing one column of length- k from the Young tableau. These creation and annihilation operators are derived from correlation functions in the large N quantum field theory, e.g. [11, 46, 47, 15, 48] and references therein.

The coherent states of multi-row (k -row) Young tableaux are,

$$|\Lambda\rangle_k = e^{-\frac{|\Lambda|^2}{2k}} e^{\frac{1}{k} \Lambda A_k^\dagger} |\Delta_{0,k}\rangle = e^{-\frac{|\Lambda|^2}{2k}} \sum_{n=0}^{\infty} \frac{1}{\sqrt{k^n n!}} (\Lambda)^n |\Delta_{n,k}\rangle. \quad (3.3)$$

We have that $A_k |\Lambda\rangle_k = \Lambda |\Lambda\rangle_k$ and $\| |\Lambda\rangle_k \| = 1$. Here the range of Λ is $0 < |\Lambda| < \infty$. The mean column length is $\langle \hat{N}_k \rangle_{|\Lambda\rangle_k} = \frac{|\Lambda|^2}{k}$, and the variance of the column length is $(\Delta N)_{|\Lambda\rangle_k} = \frac{|\Lambda|}{\sqrt{k}}$. Hence it is a blob with area $|\Lambda|^2/k$, in the unit of $2\pi\hbar$, saturating the Heisenberg uncertainty relation. $|\Lambda|^2$ measures the number of boxes of the Young

tableau. It is a blob with k fermions occupied in that droplet in the phase space plane [20, 10, 9, 6, 49, 50, 11].

For a general k , we have the cat state

$$|cat_{\pm}(\Lambda)\rangle_k = \frac{1}{\sqrt{N_{k,\pm}}}(|\Lambda\rangle_k \pm |-\Lambda\rangle_k) \quad (3.4)$$

where $|cat_{\pm}(\Lambda)\rangle_k$ has unit norm and here $N_{k,\pm} = 2(1 \pm e^{-\frac{2}{k}|\Lambda|^2})$. It is a cat that lives simultaneously in two different locations on the five-sphere. The interpretation of the cat state (3.4) is that it is a superposition of two blobs in the gravity side. This parallels the $D = 2$ situation in Sec. 2.3. The two states can be easily distinguished by a parity measurement, since they have distinct eigenvalues of parity.

The cat states can be approximated by squeezed states after subtraction of even and odd number of particles:

$$\begin{aligned} |cat_+(\Lambda)\rangle_k &\sim \hat{S}(r)|\Delta_{0,k}\rangle, \\ |cat_-(\Lambda)\rangle_k &\sim \hat{S}(r)|\Delta_{1,k}\rangle, \end{aligned} \quad (3.5)$$

where in this case the squeeze operator is $\hat{S}(r) = e^{\frac{r}{2k}(A_k^\dagger A_k^\dagger - A_k A_k)}$. We have generalized them to new states defined using Young tableaux, from those happen in quantum information theory [51, 52] and references therein. Since $\hat{S}(r)^\dagger \hat{S}(r) = I$, the two states (3.5) are orthogonal. One can approximate the odd cat as $|cat_-(\Lambda)\rangle \propto A_k \hat{S}(r)|\Delta_{0,k}\rangle \propto \hat{S}(r)|\Delta_{1,k}\rangle$. There are multiple ways to approximate the even cat states. One way is by (3.5). Another way is to approximate the even cat as $|cat_+(\Lambda)\rangle \propto A_k |cat_-(\Lambda)\rangle \propto A_k^2 \hat{S}(r)|\Delta_{0,k}\rangle$ and $A_k^2 \hat{S}(r)|\Delta_{0,k}\rangle \propto \hat{S}(r)|\Delta_{0,k}\rangle + \sqrt{2} \tanh(r) \hat{S}(r)|\Delta_{2,k}\rangle$. These approximations of states have high fidelity [51, 52].

We use $\hat{S}(r)^\dagger A_k \hat{S}(r) = \cosh(r) A_k + \sinh(r) A_k^\dagger$ and $\hat{S}(r)^\dagger A_k^\dagger \hat{S}(r) = \cosh(r) A_k^\dagger + \sinh(r) A_k$. This is also a form of Bogouliubov transformation. We have

$$\Lambda^{-1} A_k |cat_{\pm}(\Lambda)\rangle = \sqrt{\frac{N_{k,\mp}}{N_{k,\pm}}} |cat_{\mp}(\Lambda)\rangle, \quad (3.6)$$

$$\hat{P} |cat_{\pm}(\Lambda)\rangle = \pm |cat_{\pm}(\Lambda)\rangle, \quad (3.7)$$

where $\hat{P} = \exp(i\frac{\pi}{k} A_k^\dagger A_k)$. We have that

$$A_k \hat{S}(r)|\Delta_{0,k}\rangle = \sqrt{k} \sinh(r) \hat{S}(r)|\Delta_{1,k}\rangle. \quad (3.8)$$

$$A_k^\dagger \hat{S}(r)|\Delta_{0,k}\rangle = \sqrt{k} \cosh(r) \hat{S}(r)|\Delta_{1,k}\rangle. \quad (3.9)$$

For the simplicity of the calculation, we approximate the even cat state as (3.5). In the approximation for the states, $\sinh(r) \simeq \sqrt{\frac{N_{k,-}}{N_{k,+}}} \frac{\Lambda}{\sqrt{k}}$. These approximations simplify calculations and are practically useful.

The beam-splitter operator describes the absorption and emission of closed strings $|t_k\rangle$ by giant gravitons with column length k . It is

$$\hat{U}(c) = \exp(c(a_k^\dagger A_k - a_k A_k^\dagger)) \simeq 1 + c(a_k^\dagger A_k - a_k A_k^\dagger). \quad (3.10)$$

c is proportional to the probability amplitude of the absorption and emission. The a_k is defined in Appendix A for the trace states. The second equation is when c is small. $a_k A_k^\dagger$ describes the absorption and $a_k^\dagger A_k$ describes the emission.

We use the operation of $\hat{U}(c)\hat{S}(r)$. The state before the action of the unitary operation is

$$|t_k^0\rangle\hat{S}(r)|\Delta_{0,k}\rangle. \quad (3.11)$$

After the unitary operation, where c is small,

$$\begin{aligned} & \hat{U}(c)(|t_k^0\rangle\hat{S}(r)|\Delta_{0,k}\rangle) \\ & \simeq \frac{1}{\sqrt{\mathcal{N}}} \left(|t_k^0\rangle\hat{S}(r)|\Delta_{0,k}\rangle + c\sqrt{k} \sinh(r) |t_k^1\rangle\hat{S}(r)|\Delta_{1,k}\rangle \right) \end{aligned} \quad (3.12)$$

$$\simeq \frac{1}{\sqrt{\mathcal{N}}} \left(|t_k^0\rangle|cat_+(\Lambda)\rangle + ck \sinh(r) \frac{|t_k^1\rangle}{\sqrt{k}} |cat_-(\Lambda)\rangle \right). \quad (3.13)$$

Here $\frac{1}{\sqrt{\mathcal{N}}}$ is a normalization factor. The two terms in the superposition are orthogonal to each other. Hence we produce entangled state

$$c_1 |t_k^0\rangle|cat_+(\Lambda)\rangle + c_2 \frac{|t_k^1\rangle}{\sqrt{k}} |cat_-(\Lambda)\rangle. \quad (3.14)$$

We can tune the parameters $ck \sinh(r)$ to obtain arbitrary $\frac{c_2}{c_1} = ck \sinh(r)$.

There are also other methods of producing the entangled states, such as by the operation of $\hat{S}(r)\hat{U}(c)$. We have another method:

$$\begin{aligned} \hat{S}(r)\hat{U}(c)\left(\frac{1}{\sqrt{k}}|t_k^1\rangle|\Delta_{0,k}\rangle\right) & \simeq \frac{1}{\sqrt{\mathcal{N}}} \left(\frac{1}{\sqrt{k}}|t_k^1\rangle\hat{S}(r)|\Delta_{0,k}\rangle - ck|t_k^0\rangle\hat{S}(r)|\Delta_{1,k}\rangle \right) \\ & \simeq \frac{1}{\sqrt{\mathcal{N}}} \left(\frac{|t_k^1\rangle}{\sqrt{k}}|cat_+(\Lambda)\rangle - ck|t_k^0\rangle|cat_-(\Lambda)\rangle \right). \end{aligned} \quad (3.15)$$

Similarly, we can produce the states for general $l \geq 1$, where $\frac{|t_k^l\rangle}{\sqrt{l!k^l}}$ has a unit norm.

Denote $|\Psi_0\rangle = \frac{|t_k^{l_1}\rangle}{\sqrt{l_1!k^{l_1}}}|l_1, k\rangle$. We can produce the entangled states

$$\begin{aligned} & \hat{S}(r)\hat{U}(c)|\Psi_0\rangle \\ & \simeq \frac{1}{\sqrt{\mathcal{N}}} \left(\frac{|t_k^{l_1}\rangle}{\sqrt{l_1!k^{l_1}}}|cat_+(\Lambda)\rangle - ck \frac{|t_k^{l_2}\rangle}{\sqrt{l_2!k^{l_2}}}|cat_-(\Lambda)\rangle \right), \end{aligned} \quad (3.16)$$

where $l_2 = l_1 - 1$. Hence we can produce the entangled states of the form

$$c_1|\phi_+\rangle|cat_+(\Lambda)\rangle + c_2|\phi_-\rangle|cat_-(\Lambda)\rangle. \quad (3.17)$$

The two terms involve opposite parity.

Starting from the $|cat_-(\Lambda)\rangle$ state, by acting the single subtraction operator $\Lambda^{-1}A_k$ on it, we could produce the $|cat_+(\Lambda)\rangle$ state. The big coherent state amplitude limit is particularly interesting, since for $|\Lambda| \gg 1$, $\frac{N_{k,\mp}}{N_{k,\pm}} \simeq 1$, and the calculations can be simplified. We have the Pauli operators X, Y, Z acting on the cat qubits, $X := \Lambda^{-1}A_k$, $Y := i\Lambda^{-1}A_k \exp(i\frac{\pi}{k}A_k^\dagger A_k)$, and $Z := \exp(i\frac{\pi}{k}A_k^\dagger A_k)$. Here, Z coincides with the parity operator, since $\hat{P}|cat_\pm(\Lambda)\rangle = \pm|cat_\pm(\Lambda)\rangle$. X and Z are bit-flip and phase-flip operators acting on the cat qubits. They, together with the identity, generates the Pauli group.

The displacer is $\hat{D}(\alpha) = \exp(\frac{1}{k}(\alpha A_k^\dagger - \bar{\alpha} A_k))$. We have $\hat{D}(\alpha)^\dagger A_k \hat{D}(\alpha) = A_k + \alpha$ and $\hat{D}(\alpha)^\dagger A_k^\dagger \hat{D}(\alpha) = A_k^\dagger + \bar{\alpha}$. Moreover, there is a qubit phase rotation gate. We generalize slightly the gate in [53]. We use displaced-two-photon-subtractor

$$\hat{R} = \frac{A_k}{\Lambda} \hat{D}(\alpha)^\dagger \frac{A_k}{\Lambda} \hat{D}(\alpha) \quad (3.18)$$

instead, and

$$\hat{R} (c_1|cat_+(\Lambda)\rangle + c_2|cat_-(\Lambda)\rangle) = c'_1|cat_+(\Lambda)\rangle + c'_2|cat_-(\Lambda)\rangle, \quad (3.19)$$

where

$$\frac{c'_2}{c'_1} = \frac{c_2}{c_1} \frac{1 + \frac{c_1}{c_2} \alpha / \Lambda}{1 + \frac{c_2}{c_1} \alpha / \Lambda}. \quad (3.20)$$

These operations realize phase rotations of the cat qubits.

We can also produce entanglement between trace states and YT coherent states. We can use the operation $\hat{D}(\Lambda)\hat{U}(c)$. We have $A_k^\dagger \hat{D}(\Lambda)|\Delta_{0,k}\rangle = \bar{\Lambda} \hat{D}(\Lambda)|\Delta_{0,k}\rangle + \hat{D}(\Lambda)|\Delta_{1,k}\rangle$. Denote $|\Psi_0\rangle = \frac{|t_k^{l_1}\rangle}{\sqrt{l_1!k^{l_1}}}| \Delta_{0,k}\rangle$. We can also produce the entangled states:

$$\begin{aligned} & \hat{D}(\Lambda)\hat{U}(c)|\Psi_0\rangle \\ & \simeq \frac{1}{\sqrt{\mathcal{N}}} \left(\frac{|t_k^{l_1}\rangle}{\sqrt{l_1!k^{l_1}}} \hat{D}(\Lambda)|\Delta_{0,k}\rangle - ck \frac{|t_k^{l_2}\rangle}{\sqrt{l_2!k^{l_2}}} \hat{D}(\Lambda)|\Delta_{1,k}\rangle \right), \end{aligned} \quad (3.21)$$

where $l_1 - l_2 = 1$. Since $\hat{D}(\Lambda)^\dagger \hat{D}(\Lambda) = I$, $\hat{D}(\Lambda)|\Delta_{0,k}\rangle$ and $\hat{D}(\Lambda)|\Delta_{1,k}\rangle$ are orthogonal. This type of micro-macro entangled states can be experimentally realized using photons and coherent states of photons, e.g. [54, 55]. Here, these can be interpreted as the entangled states of closed string states and the droplets.

Both (3.17) and (3.21) are methods creating micro-macro entangled states. These types of cat states and other macro states as well as their entangled states, have been realized experimentally in the context of photonic coherent states. The photon subtraction techniques are crucial in quantum state preparations, e.g. [51, 52, 56].

4 Mixed YT states and noisy YT states

4.1 Mixed states of YT

Now in this section, we consider mixed states of giant gravitons arising from ensemble of pure states. Now we consider mixed states of multimode coherent states and YT basis. The mixed states of multi-mode coherent states is

$$\rho = \rho[p(\theta)] := \frac{1}{\mathcal{N}_D(\Lambda)} \int_0^{2\pi} d\theta p(\theta) |Coh(\Lambda e^{i\theta})\rangle\langle Coh(\Lambda e^{i\theta})|, \quad (4.1)$$

where $p(\theta)$ is a distribution function, in the angular direction. We have $p(\theta) \geq 0$ and $\int_0^{2\pi} d\theta p(\theta) = 1$, and hence $\text{tr } \rho = 1$. $\rho[p(\theta)]$ is a functional of $p(\theta)$. The coefficient $(1 - |\Lambda|^2)$ is a normalization factor for the un-normalized multi-mode coherent states (2.1). We will expand it in the Young tableau basis. We will show that angular superpositions of multi-mode coherent state density matrices, give rise to mixed states in the Young tableau basis.

In [15] superposition pure states of the multi-mode coherent states were analyzed, while here we analyze mixed states of the ensemble of the multi-mode coherent states, by superimposing density matrices.

For $p(\theta) = \frac{1}{2\pi}$,

$$\begin{aligned} \rho_0 &= (1 - |\Lambda|^2) \frac{1}{2\pi} \int_0^{2\pi} d\theta \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} e^{i(n_1-n_2)\theta} |\Lambda|^{n_1+n_2} |\Delta_{n_1}\rangle\langle\Delta_{n_2}| \\ &= (1 - |\Lambda|^2) \sum_{n=0}^{\infty} |\Lambda|^{2n} |\Delta_n\rangle\langle\Delta_n|. \end{aligned} \quad (4.2)$$

It is a mixed state with ‘thermal’-like distribution. It can be viewed as a Gibbs state. If we make an identification $e^{-\beta} = |\Lambda|^2$, $\beta = 1/T_{\text{eff}}$, the thermal mixed YT state is

$$(1 - e^{-\beta}) \sum_{n=0}^{\infty} e^{-\beta n} |\Delta_n\rangle\langle\Delta_n|, \quad \beta = 1/T. \quad (4.3)$$

More generally, we have $p(\theta) = c_0 \frac{1}{2\pi} + \sum_{l \geq 1} c_l \frac{1}{\pi} \cos^2 \frac{1}{2} l\theta = \frac{1}{2\pi} + \sum_{l \geq 1} \frac{c_l}{2\pi} \cos l\theta$, where $\sum_{l \geq 0} c_l = 1$ and $c_l \geq 0, \forall l \in \mathbb{Z}_{>0}$. The above distribution functions are all linear combinations of periodic functions, with periods $\frac{2\pi}{l}$, along the circle. This is a deformed distribution with respect to the constant distribution $p(\theta) = \frac{1}{2\pi}$. From (4.1) we have that

$$\rho = (1 - |\Lambda|^2) \sum_{n=0}^{\infty} |\Lambda|^{2n} (|\Delta_n\rangle\langle\Delta_n| + \sum_{l \geq 1} \frac{c_l}{2} (|\Delta_{n+l}\rangle\langle\Delta_n| + |\Delta_n\rangle\langle\Delta_{n+l}|)). \quad (4.4)$$

The second piece are off-diagonal components of the density matrix and $\text{tr } \rho = 1$. The number of non-zero c_l can be finite.

These mixed states are smeared distributions of giant gravitons over the five-sphere. Mixed states of giant gravitons and/or droplet configurations, and their related aspects, were also considered in e.g. [50],[14],[15],[57]–[61].

4.2 Finite D case

In Sec 4.1, we have a continuous distribution function $p(\theta)$ to generate the mixed states. In this section, we make a discretization of the distribution function in Sec 4.1. In other words, we use a discrete distribution function in this section. We have that $\theta_m = \frac{2\pi}{D}m$ where D is a positive integer, and $\lim_{D \rightarrow \infty} \frac{1}{D} \sum_{m=0}^{D-1} = \frac{1}{2\pi} \int_0^{2\pi} d\theta$. The density matrix of the mixed state is

$$\rho = \rho[\{p_m\}] := \frac{1}{\mathcal{N}_D(\Lambda)} \frac{2\pi}{D} \sum_{m=0}^{D-1} p_m |Coh_D(\Lambda e^{i\frac{2\pi}{D}m})\rangle \langle Coh_D(\Lambda e^{i\frac{2\pi}{D}m})|, \quad (4.5)$$

where $\mathcal{N}_D(\Lambda) = \frac{(1-|\Lambda|^2)}{(1-|\Lambda|^{2D})}$. We have that $\frac{2\pi}{D} \sum_{m=0}^{D-1} p_m = 1$.

For $p_m = \frac{1}{2\pi}$,

$$\rho_0 = \frac{(1-|\Lambda|^2)}{(1-|\Lambda|^{2D})} \sum_{n=0}^{D-1} |\Lambda|^{2n} |\tilde{\Delta}_n\rangle \langle \tilde{\Delta}_n| \quad (4.6)$$

and $\text{tr } \rho_0 = 1$. This is the finite D version of (4.2), and has a natural $D \rightarrow \infty$ limit to (4.2). Eq. (4.6) can also be obtained by partial tracing as in (2.20).

For $p_m = c_0 \frac{1}{2\pi} + \sum_{l=1}^{\lfloor \frac{D}{2} \rfloor} c_l \frac{1}{\pi} \cos^2 l \frac{\pi}{D} m$, where $\sum_{l \geq 0} c_l = 1$ and $c_l \geq 0, \forall l$,

$$\begin{aligned} \rho &= \rho_0 + \frac{1}{\mathcal{N}_D(\Lambda)D} \left(\sum_{l=1}^{\lfloor \frac{D}{2} \rfloor} \sum_{m=0}^{D-1} \sum_{n_1, n_2} \frac{c_l}{2} e^{i(n_1 - n_2 - l) \frac{2\pi}{D} m} |\Lambda|^{n_1 + n_2} |\Delta_{n_1}\rangle \langle \Delta_{n_2}| + \text{h.c.} \right) \\ &= \rho_0 + \frac{(1-|\Lambda|^2)}{(1-|\Lambda|^{2D})} \sum_{l=1}^{\lfloor \frac{D}{2} \rfloor} \frac{c_l}{2} \sum_{n=0}^{D-1} |\Lambda|^{2n} (|\tilde{\Delta}_{n+l}\rangle \langle \tilde{\Delta}_n| + |\tilde{\Delta}_n\rangle \langle \tilde{\Delta}_{n+l}|) \\ &= \rho_0 + \sum_{l=1}^{\lfloor \frac{D}{2} \rfloor} \frac{c_l}{2} (X_l \rho_0 + \rho_0 X_l^\dagger). \end{aligned} \quad (4.7)$$

We have included all periodic functions, and have set $c_l = 0$ for $l > \lfloor \frac{D}{2} \rfloor$. This is the finite D version of (4.4). In the limit $|\Lambda| \rightarrow 1$, $\rho_0 = \frac{I}{D}$ and $\rho = \frac{1}{D} (I + \sum_{l=1}^{\lfloor \frac{D}{2} \rfloor} \frac{c_l}{2} (X_l + X_l^\dagger))$. X_l is (2.25) and they are traceless for $1 \leq l < D$.

The second Renyi entropy $S^{(2)}(\rho) = -\log(\text{tr } \rho^2)$ and the entropy $S(\rho) = -\text{tr}(\rho \log \rho)$ of (4.6) and (4.7) are:

$$S^{(2)}(\rho) = -\log \left(\frac{(1 - |\Lambda|^2)(1 + |\Lambda|^{2D})}{(1 - |\Lambda|^{2D})(1 + |\Lambda|^2)} \left(1 + \sum_{1 \leq l \leq \lfloor \frac{D-1}{2} \rfloor} \frac{c_l^2}{2} + \frac{(1 + |\Lambda|^D)^2}{(1 + |\Lambda|^{2D})} \sum_{\lfloor \frac{D-1}{2} \rfloor < l \leq \lfloor \frac{D}{2} \rfloor} \frac{c_l^2}{2} \right) \right) \quad (4.8)$$

and

$$S(\rho) = -\text{tr}(\rho_0 \log \rho_0) - 2D \left(\sum_{1 \leq l \leq \lfloor \frac{D-1}{2} \rfloor} \frac{c_l}{2D} \log \frac{c_l}{2D} + 2 \sum_{\lfloor \frac{D-1}{2} \rfloor < l \leq \lfloor \frac{D}{2} \rfloor} \frac{c_l}{2D} \log \frac{c_l}{2D} \right), \quad (4.9)$$

where $-\text{tr}(\rho_0 \log \rho_0) = \log D$, for the $|\Lambda| \rightarrow 1$ case. The second term in (4.9) also describes the entropy change $\Delta S = S(\rho) - S(\rho_0)$. With the finite D results, we can take an infinite D limit to obtain the expressions for Sec 4.1.

The density matrices (4.7) are deformations of the density matrix (4.6). They can be viewed as reduced density matrices by partial tracing of one subspace of the Hilbert spaces, and hence the above also describe entanglement entropies. As entanglement entropy is an important observable in the usual gauge/gravity correspondence, it might also be interesting to produce these entropies from gravity computations by entangled giant gravitons and for example along the lines of [11, 12, 14, 15].

We make an identification $e^{-\beta} = |\Lambda|^2$, $\beta = 1/T_{\text{eff}}$, the thermal mixed YT state with finite D is

$$\frac{(1 - e^{-\beta})}{(1 - e^{-\beta D})} \sum_{n=0}^{D-1} e^{-\beta n} |\tilde{\Delta}_n\rangle \langle \tilde{\Delta}_n|, \quad \beta = 1/T. \quad (4.10)$$

This is a Gibbs state with a finite dimension. Its infinite D limit goes over to (4.3).

4.3 Noisy YT channels and noisy YT states

The mixed noisy YT states can be produced by interactions with background fluctuation modes which we view as environment states. Now in this section, we consider producing these mixed states by noisy quantum channels. The finite dimensional case can be considered as an approximation of the infinite dimensional case. For finite dimensional cases, we would use the regularized YT states $|\tilde{\Delta}_n\rangle$ to approximate the YT states $|\Delta_n\rangle$. The infinite D limit of the former gives the later. We denote A the system and B the environment, in this section.

The state $|\Psi_{AB}^0\rangle = \sum_n s_n |\Delta_n\rangle |\phi_0\rangle$, where $\sum_n |s_n|^2 = 1$, evolves to

$$\begin{aligned} |\Psi_{AB}\rangle &= U^{AB} \sum_n s_n |\Delta_n\rangle |\phi_0\rangle \\ &= \sum_{n,n',i} s_n c_{n',n}^i |\Delta_{n'}\rangle |\phi_i\rangle, \end{aligned} \quad (4.11)$$

in which $|\phi_i\rangle$ are environment states of B and

$$c_{n',n}^{i,j} = \langle \phi_i | \langle \Delta_{n'} | U^{AB} | \Delta_n \rangle | \phi_j \rangle, \quad c_{n',n}^i = c_{n',n}^{i,0}, \quad (4.12)$$

where $U^{AB} \in U(\mathcal{H}_{AB})$.

Here, $c_{n',n}^i$ encodes the information of the interaction between the states and the environment. (4.12) is a four-point function, and in the special case when $|\phi_0\rangle$ is the empty vacuum state, is a three-point function. For example, we view $|\Delta_n\rangle$ as giant graviton states and $|\phi_i\rangle$ as close string states. We interpret the coefficients as four-point functions. The correlation functions from the field theory side describing closed strings interacting with giant gravitons were considered in e.g. [62]–[69] and references therein.

Such interactions and correlators can be described by the absorption and emission of closed strings by giant gravitons, e.g. [70]–[75], [65] and their related discussion, in the context of string theory and gauge-string duality. These coefficients $c_{n',n}^i$ are determined by the Hamiltonian of the dual CFT or dual gauge theory. The unitary matrix depends on the details of the Hamiltonian of the dual field theory.

Eq. (4.11) are entangled states. Note that $\sum_n s_n |\Delta_n\rangle$ is a more general states like the macro state in (3.17) in Sec. 3, and $|\phi_i\rangle$ is a more general closed string states like the micro state in (3.17). U is a more general evolution operator, like the one in (3.10). The U in Sec. 3 is a special case. More generally,

$$U^{AB} = \sum_{i,j,n',n} c_{n',n}^{i,j} |\Delta_{n'}\rangle |\phi_i\rangle \langle \phi_j| \langle \Delta_n|. \quad (4.13)$$

Since $\sum_{n',i} c_{n,n'}^{j,i} c_{n',l}^{i,k} = \delta_{n,l} \delta_{j,k}$, hence for the $j = 0, k = 0$ component, $\sum_{n',i} c_{n,n'}^{i,*} c_{n',l}^i = \delta_{n,l}$.

The system of giant gravitons can be viewed as an open quantum system, since they can interact with background closed string states. AB is a closed subsystem of the Hilbert space of the dual CFT. States in A can interact with states in B , hence A can be viewed as an open quantum system, and B viewed as environment [76]. The evolution in the close system AB is unitary [76]. The dimension of the environment states is d_B . Taking trace over B is the same as summing over the basis of environment states $|\phi_i\rangle$ in \mathcal{H}_B .

The noisy YT state can be viewed as the transformation of a pure YT state by a noisy quantum channel defined by a linear completely-positive trace-preserving map. The noisy YT channel is \mathcal{E} ,

$$\rho_A = \mathcal{E}(\rho_A^0) = \text{tr}_B U^{AB} (\rho_A^0 \otimes \rho_B^0) U^{AB\dagger}. \quad (4.14)$$

In the case that the input are pure states, we have that $\rho_A^0 = \sum_{n,l} s_n \bar{s}_l |\Delta_n\rangle \langle \Delta_l|$ and $\rho_B^0 = |\phi_0\rangle \langle \phi_0|$. The output state is

$$\rho_A = \sum_{n',l'} \sum_{n,l,i} s_n \bar{s}_l c_{n',n}^i c_{l',n'}^{i,*} |\Delta_{n'}\rangle \langle \Delta_{l'}|. \quad (4.15)$$

It can be written as $\rho_A = \sum_i E_i \rho_A^0 E_i^\dagger$, where the Kraus operator in this case is $E_i = \sum_{n',n} c_{n',n}^{i,0} |\Delta_{n'}\rangle\langle\Delta_n|$, or $(E_i)_{n',n} = c_{n',n}^{i,0}$. On the other hand, $\rho_B = \text{tr}_A U^{AB}(\rho_A^0 \otimes \rho_B^0)U^{AB\dagger}$ and

$$\rho_B = \sum_{i,j} \sum_{n,l,n'} s_n \bar{s}_l c_{n',n}^i c_{l,n'}^{j*} |\phi_i\rangle\langle\phi_j|. \quad (4.16)$$

It can also be written as $\rho_B = \sum_{i,j} M_{ij} |\phi_i\rangle\langle\phi_j|$, where the matrix elements of M is $M_{ij} = \text{tr}(\rho_A^0 E^{j\dagger} E^i)$. We have also

$$\rho_{AB} = \sum_{i,j} (E^i \rho_A^0 E^{j\dagger}) |\phi_i\rangle\langle\phi_j|. \quad (4.17)$$

In this case, ρ_B may be understood as being through a conjugate channel [77, 78, 79].

The change of the entropy of A is

$$\Delta S_A = S(\rho_A) - S(\rho_A^0) = -\text{tr}(M \log M). \quad (4.18)$$

The entropy production, in other words the increase of the entropy of A , after the interaction is ΔS_A . This is the entropy production of the system, after the interaction of the system with the environment. This is also proportional to the change of the mutual information between A and B , after the interaction. We have that

$$I(A, B) - I(A, B)^0 = S(\rho_{AB} || \rho_A \otimes \rho_B) = -2 \text{tr}(M \log M). \quad (4.19)$$

We can have these interaction processes occur multiple times, and the evolution forms a semigroup evolution along the time direction.

The above derivation assumes that ρ_A^0 is pure. The definition of the channel (4.14) is good for both the cases when ρ_A^0 is pure or mixed, since (4.14) is a linear map of the input density matrix. If we trace out the environment states, we hence define a noisy quantum channel.

Some examples of noisy YT states are as follows. First, the pure YT state is

$$|\Delta_n\rangle\langle\Delta_n|. \quad (4.20)$$

The noisy YT state is

$$\alpha |\Delta_n\rangle\langle\Delta_n| + \sum_{n' \neq n} p_{n'} |\Delta_{n'}\rangle\langle\Delta_{n'}|, \quad \sum_{n' \neq n} p_{n'} = 1 - \alpha, \quad 0 < \alpha < 1. \quad (4.21)$$

The thermal mixed YT state is

$$\rho_0 = (1 - e^{-\beta}) \sum_n e^{-\beta n} |\Delta_n\rangle\langle\Delta_n|, \quad \beta = 1/T, \quad (4.22)$$

which is also (4.3) in Sec. 4.1. The noisy YT state with a thermal noise is

$$\alpha |\Delta_n\rangle\langle\Delta_n| + (1 - \alpha) \rho_0, \quad 0 < \alpha < 1, \quad (4.23)$$

where ρ_0 is (4.22) and the second term in (4.23) represents the thermal noise. Some of these noisy states include those mixed states discussed in Sec. 4.1 and 4.2.

5 Noisy entangled YT states

In this section, we produce mixed entangled pair by a noisy channel. The pure entangled pair becomes mixed entangled pair after interaction with background environment states. We generalize the situation of Sec 4.3, from single partite states interacting with the environment states, to bipartite entangled states interacting with the environment states. We denote AB the entangled pair and C the environment, in this section.

Consider the states having interactions with background fluctuation modes, which we model as noise. The evolution of the total system takes into account the interaction hamiltonian and the unitary evolution operators. The background fluctuation modes are environment states, for example a background of random closed strings.

The initial state in AB and in C are $|\Psi_{AB}^0\rangle = \sum_n \sqrt{p_n} |\Delta_n\rangle |\Delta_n\rangle$ and $|\Psi_C^0\rangle = |\phi_0\rangle$, where $\sum_n p_n = 1$. The initial state of the entangled pair (AB) and the environment (C) is

$$|\Psi_{ABC}^0\rangle = \sum_n \sqrt{p_n} |\Delta_n\rangle |\Delta_n\rangle |\phi_0\rangle. \quad (5.1)$$

We consider interaction between B and C . Under the interaction, the state $|\Psi_{ABC}^0\rangle$ evolves to $|\Psi_{ABC}\rangle$. We denote the initial states by a subscript ‘0’, such as $\rho_{AB}^0, \rho_C^0, \rho_{ABC}^0$, and after the evolution, the states are $\rho_{AB}, \rho_C, \rho_{ABC}$. The initial state is a direct product state between AB and C , but not a direct product state between A and B , i.e. $\rho_{ABC}^0 = \rho_{AB}^0 \otimes \rho_C^0$. We have that $\rho_A^0 = \sum_n p_n |\Delta_n\rangle \langle \Delta_n| = \rho_B^0$ are mixed states. After the interaction between AB and C , the state after evolution ρ_{ABC} , is not a direct product state of AB and C .

Due to the interaction with the environment,

$$\begin{aligned} |\Psi_{ABC}\rangle &= (I \otimes U^{BC}) \sum_n \sqrt{p_n} |\Delta_n\rangle |\Delta_n\rangle |\phi_0\rangle \\ &= \sum_{n,n',i} \sqrt{p_n} A_{n',n}^i |\Delta_n\rangle |\Delta_{n'}\rangle |\phi_i\rangle, \end{aligned} \quad (5.2)$$

in which $|\phi_i\rangle$ is the orthonormal basis of the environment states and $U^{BC} \in U(\mathcal{H}_{BC})$. We have that $|\Psi_{ABC}\rangle$ has a unit norm. Here,

$$A_{n',n}^i = \langle \phi_i | \langle \Delta_{n'} | U^{BC} | \Delta_n \rangle | \phi_0 \rangle. \quad (5.3)$$

This is equivalent to scattering matrix elements. p_n encodes the information of the bipartite states and $A_{n',n}^i$ encodes the information of the interaction between the bipartite states and the environment. (5.3) is a four-point function, and in the special case when $|\phi_0\rangle$ is the empty vacuum state, is a three-point function. We view $|\Delta_n\rangle$ as giant graviton states and $|\phi_i\rangle$ as close string states. In the above,

$$U^{BC} = \sum_{i,j,n',n} A_{n',n}^{i,j} |\Delta_{n'}\rangle |\phi_i\rangle \langle \phi_j| \langle \Delta_n|, \quad A_{n',n}^i = A_{n',n}^{i,0}. \quad (5.4)$$

The interaction details are similar to Sec 4.3.

Tracing C , the superoperator \mathcal{E} is

$$\rho_{AB} = \mathcal{E}(\rho_{AB}^0) = \text{tr}_C (I \otimes U^{BC})(\rho_{AB}^0 \otimes \rho_C^0)(I \otimes U^{BC})^\dagger, \quad (5.5)$$

where $\rho_C^0 = |\phi_0\rangle\langle\phi_0|$. We have that

$$\rho_{AB} = \sum_{n,n',l,l'} \sum_i \sqrt{p_n p_l} A_{n',n}^i A_{l,l'}^{i*} |\Delta_n\rangle |\Delta_{n'}\rangle \langle\Delta_{l'}| \langle\Delta_l|. \quad (5.6)$$

It can also be written as $\rho_{AB} = \sum_i E_i \rho_{AB}^0 E_i^\dagger$ and the Kraus operator is $E_i = I \otimes A^i$, where $A^i = \sum_{n',n} A_{n',n}^i |\Delta_{n'}\rangle \langle\Delta_n|$ and $\sum_i E_i^\dagger E_i = I$. This is equivalent to $\sum_{n',i} A_{n,n'}^{i*} A_{n',l}^i = \delta_{n,l}$. These are also equivalent to that U^{BC} is unitary.

Tracing BC , $\rho_A = \sum_i (\rho_A^0)^{1/2} A^{i\dagger} A^i (\rho_A^0)^{1/2} = \rho_A^0$, and hence $S(\rho_A) = S(\rho_A^0)$. This is because that the interaction does not involve A . Tracing AC ,

$$\rho_B = \sum_{n',l'} G_{n',l'} |\Delta_{n'}\rangle \langle\Delta_{l'}| = \sum_i A^i \rho_B^0 A^{i\dagger}, \quad (5.7)$$

where the matrix elements of G is

$$G_{n',l'} = \langle\Delta_{n'}|\rho_B|\Delta_{l'}\rangle = \sum_{i,n} p_n A_{n',n}^i A_{n,l'}^{i*}. \quad (5.8)$$

Define $\rho_B = \mathcal{E}^B(\rho_B^0) = \sum_i A^i \rho_B^0 A^{i\dagger}$. Hence $\rho_{AB} = \mathcal{E}(\rho_{AB}^0) = (I \otimes \mathcal{E}^B)(\rho_{AB}^0)$. Tracing A ,

$$\begin{aligned} \rho_{BC} &= \sum_{n',i,l',j} \sum_n p_n A_{n',n}^i A_{n,l'}^{j*} |\Delta_{n'}\rangle |\phi_i\rangle \langle\phi_j| \langle\Delta_{l'}| \\ &= \sum_{i,j} (A^i \rho_B^0 A^{j\dagger}) |\phi_i\rangle \langle\phi_j|. \end{aligned} \quad (5.9)$$

Tracing AB , $\rho_C = \sum_{i,j} M_{ij} |\phi_i\rangle \langle\phi_j|$, where the matrix elements of M is

$$M_{ij} = \langle\phi_i|\rho_C|\phi_j\rangle = \sum_{n,n'} p_n A_{n',n}^i A_{n,n'}^{j*} = \text{tr}(\rho_B^0 A^{j\dagger} A^i). \quad (5.10)$$

The mutual information between A and B is

$$I(A, B) = - \sum_n p_n \log p_n - \text{tr}(G \log G) + \text{tr}(M \log M). \quad (5.11)$$

The change of the mutual information between A and B is

$$I(A, B) - I(A, B)^0 = \sum_n p_n \log p_n - \text{tr}(G \log G) + \text{tr}(M \log M). \quad (5.12)$$

Since the right side of (5.12) is also equal to $-I(A, C)$, and $I(A, C) = S(\rho_A) + S(\rho_A) - S(\rho_{AC}) \geq 0$ which is always non-negative, we have that $I(A, B) - I(A, B)^0 \leq 0$ holds. The mutual information between the entangled pair is non-increasing after the interaction with the environment.

On the other hand,

$$I(A, B) - I(A, C) = 2I^{coh} = S(\rho_B) - S(\rho_{AB}) \quad (5.13)$$

where I^{coh} is coherent information. The coherent information has deduced the amount of information transmitted to or leaked to the environment [80, 8]. The detailed methods for computing the coherent information are in [81, 80, 8]. We also have that $I(A, B) + I(A, C) = 2S(\rho_A^0) = -2 \sum_n p_n \log p_n$ is conserved.

The change of the entropy is

$$\Delta S_{AB} = S(\rho_{AB}) - S(\rho_{AB}^0) = -\text{tr}(M \log M), \quad (5.14)$$

where M is (5.10). Starting from a pure state ρ_{AB}^0 , if there is no interaction of AB with the environment, the change of entropy after the evolution would be zero. Hence, the change of entropy $S(\rho_{AB}) - S(\rho_{AB}^0)$ is related to the amount of interaction between AB and the environment. This is the entropy production of the system, after the interaction of the system with the environment.

There are two important regimes. In the regime that $S(\rho_C) = S(\rho_{AB})$ is small, the interaction of AB with the environment is small, and the coherent information is relatively bigger. On the other hand, in the regime that $S(\rho_C) = S(\rho_{AB})$ is big, the interaction of AB with environment is big, and the coherent information is relatively smaller.

In the more general case, interactions are between AB and C . Due to the interaction of AB with environment,

$$\begin{aligned} |\Psi_{ABC}\rangle &= U^{ABC} \sum_n \sqrt{p_n} |\Delta_n\rangle |\Delta_n\rangle |\phi_0\rangle \\ &= \sum_{n,n',n'',i} \sqrt{p_n} A_{n'',n',n}^i |\Delta_{n''}\rangle |\Delta_{n'}\rangle |\phi_i\rangle. \end{aligned} \quad (5.15)$$

Here,

$$A_{n'',n',n}^i = \langle \phi_i | \langle \Delta_{n'} | \langle \Delta_{n''} | U^{ABC} | \Delta_n \rangle | \Delta_n \rangle | \phi_0 \rangle. \quad (5.16)$$

This involves six-point functions which may be decomposable in terms of products of lower-point functions, in the context of dual conformal field theory.

Tracing C , the superoperator is

$$\rho_{AB} = \mathcal{E}(\rho_{AB}^0) = \text{tr}_C (U^{ABC})(\rho_{AB}^0 \otimes \rho_C^0)(U^{ABC})^\dagger. \quad (5.17)$$

Here $\rho_C^0 = |\phi_0\rangle\langle\phi_0|$. We have that

$$\rho_{AB} = \sum_{n',n'',l',l''} \sum_{i,n,l} \sqrt{p_n p_l} A_{n'',n',n}^i A_{l',l'',l}^{i*} |\Delta_{n''}\rangle |\Delta_{n'}\rangle \langle\Delta_{l'}| \langle\Delta_{l''}|. \quad (5.18)$$

Under the special case $U^{ABC} = I \otimes U^{BC}$, the scattering processes in A are completely disconnected with the scattering processes in BC , and hence $A_{n'',n',n}^i = A_{n',n}^i \delta_{n'',n}$, this recovers (5.3).

6 Discussion

In this paper, we used entangled multimode coherent states to produce entangled giant graviton states. This multimode coherent state has a gravity dual in terms of a bump on the five-sphere. We make a smeared distribution of the entangled multimode coherent states on the circle, or on the five-sphere, in the higher dimensional view. By the linear transformation from multimode coherent states to giant graviton states, these states become entangled giant graviton states. The distribution functions play the role of the deformation of the circular edge of the droplets and are closely related to the chiral field describing the edges of the droplets. In the context of gauge/gravity duality, we analyzed the superposition of giant graviton states, and the entangled pairs of giant graviton states.

We use regularized YT states and further make use of them to analyze superpositions and entanglement. This regularization or discretization is motivated by a discrete distribution in the coherent state superposition and makes simpler the computations. These states can then become discrete-variable quantum states, for example, the cat qudits. We use the regularized YT states as qudits. There are many unitary quantum operations acting on them as we analyzed in Sec 2.3 and Sec 3. The entangled giant graviton states can also be described by Choi states, parameterized by unitary quantum operations, in which the unitary operations are mapped from the angular distribution functions in Prop. 2.2. The micro-macro entangled states between YT states and trace states are also produced in Sec. 3. We analyzed various quantum operations, such as beam splitters, squeezers and displacers, on bipartite states involving the YT states. We have seen that the particle subtraction and particle addition operations are very useful in increasing the entanglement in bipartite states. In the context of gauge/gravity duality, the YT states describe the giant graviton states.

We then produced mixed states of YT states and computed their entropies. They can be produced by ensemble mixing of pure YT states. We use angular distribution functions, and make a smeared distribution of giant gravitons on the circle, or on the five-sphere, in the higher dimensional view, to produce the mixed states. We also use noisy quantum channels, in which, by partial tracing after the interaction with the environment, to produce the noisy YT states. We also use noisy quantum channels to

produce mixed entangled YT states, from pure entangled YT states going through the channel. We analyzed observables of quantum information, such as the entropies of the subsystems and the mutual informations between the system and environment, between the subsystems, and between the mixed entangled pair.

The gauge/gravity duality enables us to analyze the aspects of superposition and entanglement for the quantum gravity side. The ideas of superposition of states on the gravity side, in the nonperturbative and global sense, have been considered in [11, 82, 83, 84, 85, 12]. The gravitational aspects of the gravitational superposition states have been discussed in [11, 82, 83, 84]. The cat states in four dimensional gravity have been discussed in [84]. It is useful to explore these ideas with the setup of this paper. These geometries are very explicit and they serve as a good laboratory to perform quantitative computations. Moreover, the system is ultraviolet finite, since it has the ultraviolet completion in string theory.

Our results may provide further insights into other interesting related phenomena in gauge/gravity correspondence. Various other similar spacetime geometries, in the context of string theory and quantum gravity have been analyzed, see for example [88, 86, 87] and their related discussion. Our methods and discussions are also related to fuzzball proposal [88]–[93]. There are also scrambling behaviors in heavy and excited states in the gauge theory duals [94, 95]. On the other hand, scrambling behaviors have also been observed in fuzzball geometries [93].

The approach of correlation and entanglement in phase space [96] is convenient for studying questions with gravitational degrees of freedom using different regions of the phase space [14, 15]. The setup here naturally includes the phase space in the gravitational system, and provides a laboratory for studying these quantum gravitational questions. It is convenient to analyze the correlation and entanglement between gravitational degrees of freedom using different regions of the phase space plane in bubbling AdS. Inspired by previous works [11, 12, 14], it is very interesting to further study the relation between entanglement and the dual spacetime physics.

Entanglement between different parts of internal five-spheres were also discussed in [14, 97, 15, 11, 57, 98]. In addition to different parts of the coordinate five-sphere, [14, 15] also used the different parts of phase space [20, 96] to describe the entanglement. The internal five-sphere can be generalized to more general five-manifolds and the corresponding state space is similar, e.g. [31]. Entanglement between different parts of the internal dimensions or extra dimensions, and from string and brane degrees of freedom in spacetime, have also been considered in [99, 100, 101]. These important insights are closely related to our scenarios. It would be interesting to see more detailed relations between these insights and the discussions here.

This analysis could possibly be generalized to the analysis of quarter BPS states. There are quarter BPS giant gravitons. The quarter BPS sector has representation bases which can be labeled by multiple Young tableaux. There are droplet descriptions in which the quarter BPS sector has a four dimensional droplet space. The analysis

for quarter BPS states would involve entanglement between degrees of freedom on two halves of five-spheres, which upon projecting to the droplet space, correspond to two halves of the droplet boundaries. Moreover, transformation between a class of quarter BPS representation bases and of quarter BPS coherent states, has been constructed [13].

The approach here is interesting for understanding the emergence of spacetime geometries, see for example [5, 102, 4, 7]. These geometries are very explicit and they serve as a good laboratory to perform quantitative calculations and predictions. It would also be good to understand in more detail the relation to the scenarios of building-up spacetime geometries, as proposed in for example [102, 103, 5]. Viewing the spacetime as a quantum error correction code, is a very remarkable insight [104, 105, 106, 107] for emergent spacetime, and see also, related insights in [108, 109]. It would be good to understand these related aspects better, in the context of emergent spacetime and gauge/gravity duality.

Acknowledgments

We would like to thank B. Czech, R. de Mello Koch and J. Simon for communications or discussions. The work was supported in part by National Key R&D Program of China grant No. 2020YFA0713000, by grant TH-533310008 of Tsinghua University (to H.L.), and by Yau Mathematical Sciences Center, Tsinghua University.

A Multimode coherent states and Young tableau states

In this appendix, we overview a class of single mode and multi mode coherent states. This class was constructed in [11], and analyzed in further details in [12, 15]. We then briefly overview the Young tableau states.

Consider the Hilbert space factorizes as $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots = \otimes_k \mathcal{H}_k$. Here \mathcal{H}_k is the Hilbert space for mode $k \in \mathbb{Z}_{>0}$. The creation and annihilation operators for mode k are a_k^\dagger and a_k . The state in mode k , with occupation number l , is

$$t_k^l = (a_k^\dagger)^l |0\rangle_k, \quad (\text{A.1})$$

where $|0\rangle_k$ is the vacuum of \mathcal{H}_k and $t_k^0 = |0\rangle_k$. In our convention, the normalization is $\frac{1}{k} [a_k, a_k^\dagger] = 1$. The state $\frac{1}{\sqrt{l!k^l}} (a_k^\dagger)^l |0\rangle_k$ has unit norm.

This construction works generally for systems having a similar Hilbert space, and in the context of the gauge theory in which gauge invariant observables can be constructed from a complex matrix Y , the t_k corresponds to $\text{Tr } Y^k$. In that case,

$$a_k^\dagger |0\rangle_k = \frac{1}{\sqrt{N^k}} \text{Tr } Y^k, \quad (\text{A.2})$$

and $t_k^l = (a_k^\dagger)^l |0\rangle_k = (\frac{1}{\sqrt{N^k}} \text{Tr}(Y^k))^l$. In the context of gauge theory, the prefactors involving N can be calculated by gauge theory computations, e.g. [111, 112, 47, 11]. Here, we work in the large N limit of gauge theory, because in this limit, the multi traces provide good orthogonality property. In the context of gauge/string duality, t_k is also a closed string state.

We consider coherent states generalized from the coherent states of photons [110]. A general multi-mode coherent state can be written as [11],

$$|Coh(\{\Lambda_k\})\rangle = \prod_k \exp(\Lambda_k \frac{a_k^\dagger}{k}) |0\rangle_k \quad (\text{A.3})$$

where $\Lambda_k \in \mathbb{C}$. The parameters $\{\Lambda_k\}$ in $Coh(\{\Lambda_k\})$ is a family of complex parameters for the modes $k \in \mathbb{Z}_{>0}$. These coherent states are at the level of multi-mode coherent states in the Hilbert space $\otimes_k \mathcal{H}_k$.

These are pure coherent states. They are the eigenstates of the annihilation operator a_k with eigenvalue Λ_k ,

$$a_k |Coh(\{\Lambda_k\})\rangle = \Lambda_k |Coh(\{\Lambda_k\})\rangle. \quad (\text{A.4})$$

The single-mode coherent state is a special case. We denote $|Coh(\{\Lambda_k\})\rangle$ for the multi-mode one, and $|coh(\Lambda_k)\rangle_k = \exp(\Lambda_k \frac{a_k^\dagger}{k}) |0\rangle_k$ for the single-mode one in mode k . The subscript k denotes that the state is in mode- k subspace \mathcal{H}_k . A special multi-mode coherent state is when $\Lambda_k := (\Lambda)^k$, in which $\Lambda \in \mathbb{C}$. The amplitude in each mode is correlated, since $\Lambda_k = \Lambda^k$. This multi-mode state is

$$|Coh(\Lambda)\rangle = \prod_{k=1}^{\infty} \exp(\Lambda^k \frac{a_k^\dagger}{k}) |0\rangle_k. \quad (\text{A.5})$$

The normalization for $|Coh(\Lambda)\rangle$ is

$$\mathcal{N}(\Lambda) = \langle Coh(\Lambda) | Coh(\Lambda) \rangle = \exp\left(\sum_{k=1}^{\infty} \frac{|\Lambda|^{2k}}{k}\right) = \frac{1}{(1 - |\Lambda|^2)}. \quad (\text{A.6})$$

Hence the inner product of two unit-norm multi-mode coherent states is

$$\begin{aligned} & \frac{1}{\sqrt{\mathcal{N}(\Lambda_{(0)})\mathcal{N}(\Lambda_{(1)})}} \langle Coh(\Lambda_{(0)}) | Coh(\Lambda_{(1)}) \rangle \\ &= \frac{(1 - |\Lambda_{(0)}|^2)^{1/2} (1 - |\Lambda_{(1)}|^2)^{1/2}}{(1 - \bar{\Lambda}_{(0)}\Lambda_{(1)})}. \end{aligned} \quad (\text{A.7})$$

In addition to the single-parameter multimode coherent states, there are also multi-parameter multimode coherent states constructed in [11], and further analyzed in [12, 15].

For a Young tableau λ , we write $\lambda \vdash n$ to mean that λ corresponds to a partition of n . From the representation theory of symmetric group, we know that each Young tableau λ is associated with an irreducible representation of the symmetric group S_n . We define a Young tableau state that is associated with the Young tableau λ by

$$|\lambda\rangle = \sum_{\vec{w} \in p(n)} \chi_\lambda(\vec{w}) \prod_k \frac{1}{k^{w_k} w_k!} (t_k)^{w_k}, \quad (\text{A.8})$$

where $p(n)$ is the set of all partitions of n , which is all such \vec{w} that $\sum k w_k = w_1 + 2w_2 + 3w_3 + \dots = n$. Here \vec{w} denotes a partition and also a conjugacy class [17, 16, 18]. Here χ_λ is the character of the irreducible representation associated with λ , and $\chi_\lambda(\vec{w})$ means the value of the character on the conjugacy class \vec{w} , or $\chi_\lambda(\prod_k t_k^{w_k})$. The norm is $\|\lambda\| = 1$. These Young tableau states were constructed in [9] and analyzed in details in [10, 11, 47]. We can view a general Young tableau as a multipartite system [11, 12], as it can be expanded by different conjugacy classes of cycles with various lengths. They can also be expanded as linear combinations of the multi-traces.

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