CLASSIFICATION AND NON-DEGENERACY OF $SU(n+1)$ TODA SYSTEM WITH SINGULAR SOURCES

CHANG-SHOU LIN

ABSTRACT. We will consider the $SU(N+1)$ Toda system with singular sources:

$$\Delta u_i + \sum_{j=1}^{n} a_{ij} e^{u_j} = 4\pi\gamma_i\delta_0 \quad \text{in } \mathbb{R}^2, \quad \forall 1 \le i \le N,$$

where $\gamma_i > -1$ and δ_0 is the Dirac measure at 0 and (a_{ij}) is the Cartan matrix:

$$(a_{ij}) = \begin{pmatrix} 2 & -1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & 0 & \cdots & -1 & 2 \end{pmatrix}. \tag{0.1}$$

We want to classify all the entire solutions $u = (u_1, u_2, \cdots, u_N)$ such that $e^{u_i} \in L^1(\mathbb{R}^2)$, and to prove the non-degeneracy of the linearized equation at any entire solution.