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Paper Title: A study of geometry lecturing tool based on GeoGebra automatic plotting

A study of geometry lecturing tool based on GeoGebra automatic plotting

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Abstract: In view of such problems as lacking interaction between figures and text and difficulty in simultaneous positioning of explanation with figures in chinese geometry exercise solution, a Win32 C++ voice lecturing tool integrated with HTML5 dynamic geometry software is designed to realize free control of lecturing progress, random skipping through double-click, simultaneous interaction and highlighting of figure and text, and automatic synchronized reading. On the basis of JavaScript regular expression, the semantic comprehension of geometry exercise text is realized and the production rule set of automatic plotting is designed. During voice synthesising line by line, the mathematic language undergoes normalization processing and figure highlighting information extraction. The study can shed some light on the GeoGebra integrated development and semantic comprehension-based automatic plotting application, and it can be applied to innovative geometry teaching and knowledge base structuring fields.

Keywords: geometry teaching; semantic recognition; automatic plotting; text-to-speech; GeoGebra

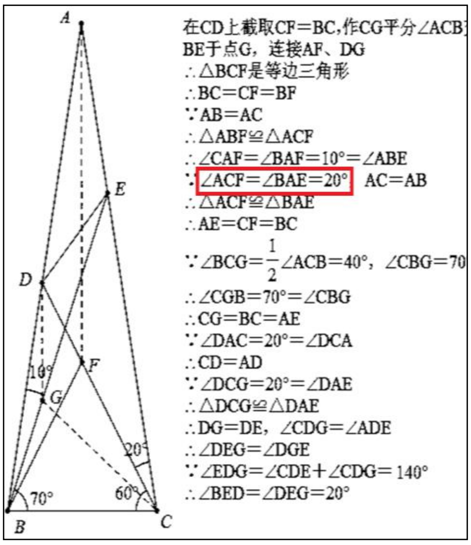
Contents

Declaration

The paper sets to study the geometry exercises in Chinese. Significant difference can be found between English and Chinese, for they are from different families of language. The geometry exercises in English haven't been deeply analyzed due to limitation of time. After being translated into English, the paper will inevitably contain some Chinese characters in some chapters or sections (e.g. geometry exercise text in Chinese, EBNF mode matching description, optimized syntax diagram, and constructed regular expression rules). For those contents in Chinese, the paper processes them in following ways: (1). provide word-for-word translated version in italics, see Chapter 4.2 and 4.3; (2). add Fig. 6(b) and 7(b) as well as Table 3(b) after studying specifically the semantic comprehension of parallel relationship in geometry exercises in English recently; (3). offer expected English effect for the parts that haven't been researched presently, see Table 4 and 5. A lot of work needs to be done in order to enable the software to fully support the lecturing of geometry exercises in English, which will be discussed in the outlook part. In the future, I will further study the geometry exercises in English and write new papers.

1 Introduction

The geometry exercises in middle school usually contain question text, solution text and geometric figures and involve a lot of geometric elements. It is, thus, necessary to display corresponding geometric figures in lecturing.



The "static figure-text explanation" on testbooks and exercise books is both primitive and simple, but it lacks interaction between the text explanation and presented figures and appears to be difficult in achieving simultaneous positioning. In addition, it is difficult to display the dynamic evolution process of a figure. As shown in the right figure, when reading the solution of the geometry exercise, we have to switch time and again between the solution text and geometric figures in order to find corresponding figure elements. For example, after reading the text "∠ACF=∠BAE=20°", we need to locate "∠ACF" , "∠BAE" and make a mark of 20° on each of them; then the similar process is repeated until the text is read through. This process is very complicated.

The figures drawn by teachers on blackboard in traditional class are usually non-standard and unsuitable for sharing and producing dynamic change. The dynamic geometry software such as Geometer's Sketchpad is ideal for geometry teaching due to their dynamic and interactive features. Many studies concentrate on how to utilize dynamic geometry software to prepare courseware and reform the lecturing process, but such courseware-preparing cost remains high and the explanation and figures can hardly be simultaneously positioned and freely skipped. In addition, aforesaid ways of geometry exercises lecturing either have no voice or need dubbing by teachers which could be non-standard. Both effect and cost should be taken into consideration.

The test of geometry in China occupies an important position in senior high school and university entrance examinations as well as math Olympiad. How to teach geometry more efficiently and directly is a problem worthy of further discussion. Therefore, to develop a geometry lecturing tool equipped with automatic text reading, automatic figure plotting and simultaneous figure-text highlighting can help to improve the teaching efficiency and enhance the teaching effect. It can also adjust to the internet teaching mode in this age of AI.

The dynamic geometry software that can plot figures with the command we input and combine numbers with figures is able to meet the figure demand of geometry lecturing tools. Among such software, GeoGebra (or GGB) appears to be an ideal choice with its powerful functions (in terms of geometry, algebra, symbolic computation and form), open source, and cross-platform. A comparative analysis is made in Table 1.

Table 1 Comparison of some common dynamic geometry software

	Geometer's Sketchpad	**Super Sketchpad**	**Netpad**	**GeoGebra**	**JSXGraph**
Country of royalty	America	China	China	Austria/IGI	German
Functional scale	Large	Relatively large	Moderate	Large	Small
Supporting platform	Windows	Windows	Browser	Cross-platform /browser	Browser
Open source	Closed source	Closed source	Closed source	Open source	Open source
Secondary development	Integrated ware library	Customized within software	Unknown	Java/JavaScript	JavaScript
Charge	Charged	Partly charged	Free	Free	Free

The JavaScript (or JS) transplanted version (Math Apps Bundle) of GGB is proper for secondary development, and a few teaching websites and geometric knowledge bases have applied it to practice [1-2]. Most of traditional teaching software belongs to Windows desktop application, whereas integration with WebKit kernel (having open source implementation versions like WKE and CEF) is helpful to utilize HTML5 in existing software and avoid IE control so that the C++ and JS codes are more easily called by each other [3]. It is feasible to use C++ and JS on the basis of GGB to realize development of geometry teaching software.

Automatic plotting and simultaneous highlighting demand the natural language of geometry exercises to be converted into the formalized language so as to realize deep semantic comprehension by computer. The approaches for coping with text of semantic comprehension include Chinese words segmentation, rule matching, ontology matching, and deep learning [4-5]. The results can be expressed and stewarded with finite state machine (FSM), production, predicate logic, semantic net, and ontology [6], among which the production and dictate logic get widely applied due to easier comprehension and implementation. The middle school geometry exercises and corresponding analysis are provided with relatively standard mathematical symbolic language, so they can be comprehended and expressed through rule matching, production and dictate logic [2, 4-5]. The mapping of semantic comprehension results into GGB plotting command realizes automatic plotting in geometric language [2].

Based on the summary of existing research, the paper sets to convert natural language of Chinese geometry exercises to GGB automatic plotting through semantic comprehension, and integrate GGB software of HTML5 with the text-to-speech software developed by VC++/MFC so as to develop the geometry lecturing tool that has both voice and simultaneous text-figure highlighting functions.

Through the investigation, it is found that most of the studies are focused on how to improve the dynamic geometric lecturing courseware and automatic plotting. At present, there is no research similar to the research in this paper.

2 Application effect

To apply the geometry lecturing tool, the user should edit following two files for each geometric question:

(1) Save the question and solution text into text file (e.g. Question 1. txt);

(2) Use GGB software to plot the figures involved in the question and solution, and save the figures under the same category and file name (e.g. Question 1. ggb), and set corresponding starting line number as the precondition for the display of figure in solution.

Then click “Open File” button in the lecturing tool, and choose the corresponding text file, the GGB geometric figures and question and solution text will be displayed as shown in Fig. 1 in such case, the lecturing state is under free control.

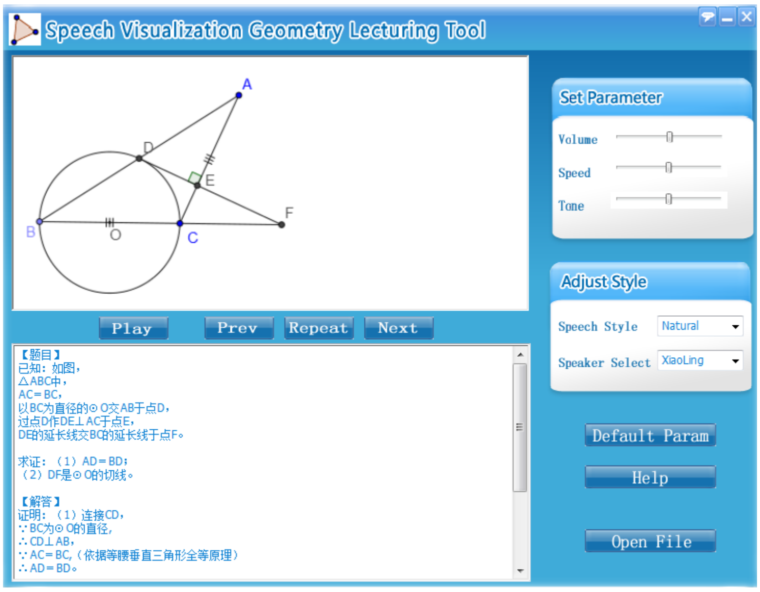


Fig. 1　Lecturing interface

The speech is synchronized with figure highlighting. The speech progress highlighting in the text area and synchronized highlighting effect in the figure area are as shown in Fig. 2 and Fig. 3. Fig. 2 illustrates serial highlighting of Question 1: triangle highlighting, special highlighting of vertical relation and parallel relation; while Fig. 3 is about the serial highlighting of Question 2, including the highlighting of circle and equal angles.

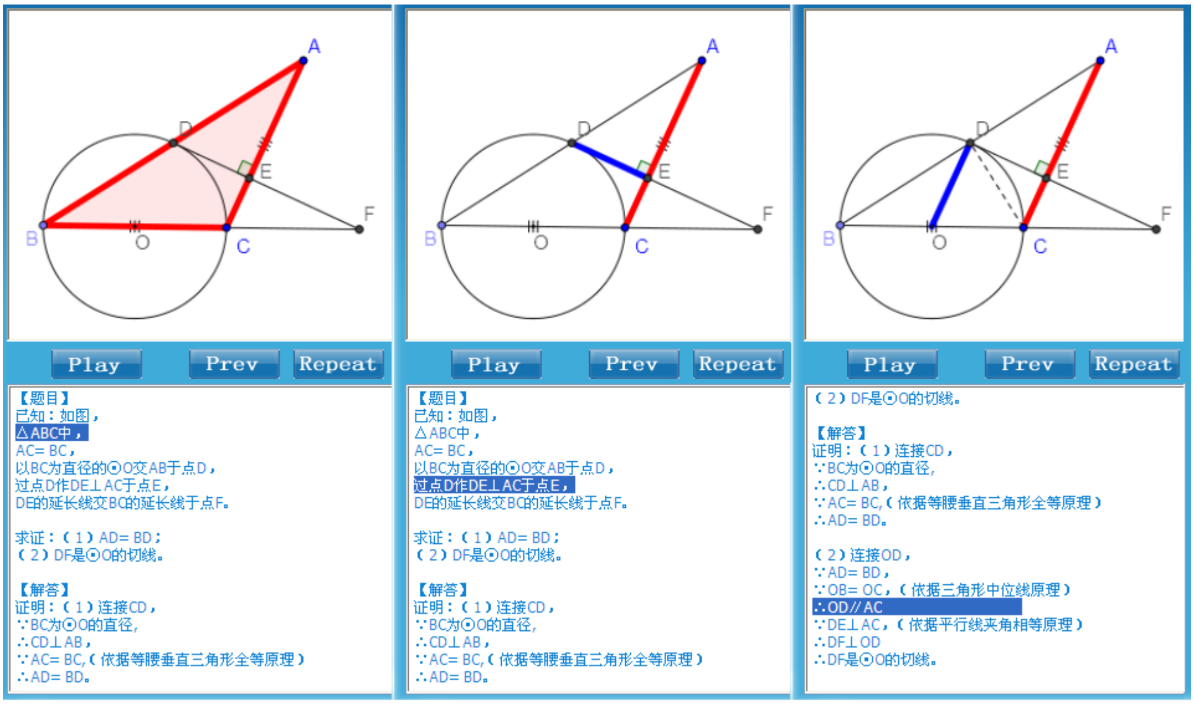


Fig. 2 Interface of synchronized highlighting of the text and figure with the speech (Question 1)

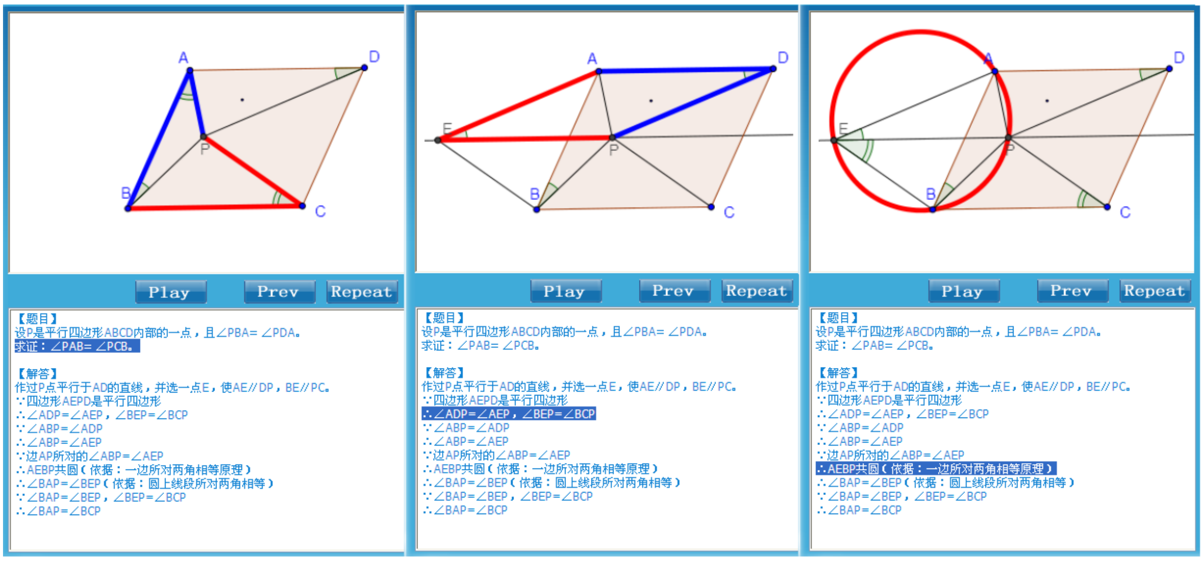


Fig. 3 Interface of synchronized highlighting of the text and figure with the speech (Question 2)

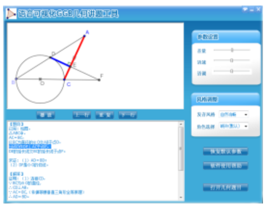
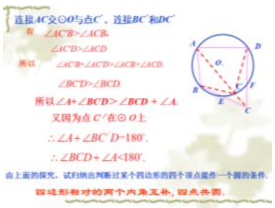
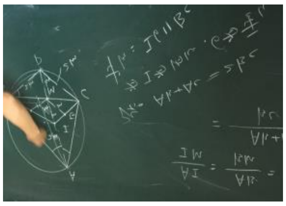
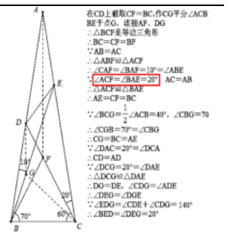
In Table 2, this lecturing mode (1st) is compared with the common lecturing modes. The results are as follows:

(1) The 4th lecturing mode is the most primitive and simple one as well as the most frequently adopted one in present exercise book application. It contains no interaction at all;

(2) The 3rd lecturing mode which is widely used in present classroom teaching is a kind of non-repeatable lecturing. It is vivid and widely accepted, but requires artificial lecturing with poor repeatability and low transmission possibility. In addition, it also has such problems as non-standard plotting and dialect accent in lecturing. The teacher has to do a lot of repetitive work in the teaching process. It can hardly adapt to the demand of AI age;

(3) The second is through dynamic courseware. This method is dynamic to some degree. Nevertheless, it still requires artificial lecturing, and the lecturing progress can't be controlled when necessary.

Table 2 Comparison of merits and demerits of four geometry lecturing modes

Way of lecturing	Innovative		Traditional	
	the 1st mode **GGB synchronic automatic lecturing**	the 2nd mode **Dynamic courseware teacher lecturing**	the 3rd mode **Blackboard plotting teacher lecturing**	the 4th mode **Textbook exercises and solutions**
View				
Standard figure	Standard	Standard	Non-standard	Standard
Vivid presentation	Very vivid	Quite vivid	Very vivid	Bad vividness
Text-to-speech	Supported	Not supported, teacher lecturing	Not supported, teacher lecturing	Not supported
Repeated lecturing	Able	Able	Disable	Able
Skipping to any part	Direct skipping through double-clicking	Not supported, only next and previous step is supported	Not supported	Supported
Convenient revision	Very convenient	Convenient	Inconvenient	Convenient
Dynamic geometry	Realizable	Realizable	Unrealizable	Unrealizable
Internet transmission	Transmittable	Transmittable	Inconvenient transmission	Transmittable

Note: there are three dominating ways of dynamic geometric courseware: PPT(often made into the video), Geometer's Sketchpad and GGB geometry courseware.

(4) The 1st lecturing mode is what we discuss in the paper, which can remedy the

shortcomings of aforesaid modes. It can supplement the 3rd blackboard lecturing mode. This method featuring simultaneous interaction of figures, text and speech is very vivid and especially suitable for the students to check the solution of the geometry exercises and learn how to solve the geometry problems when being at home, during a trip or searching solutions through the answer-searchng APP on mobile phone.

(5) The commonly used answer-searching APPs include Xuebajun, Zuoyebang and Xiaoyuansouti. Such APPs have accumulated a lot of exercises and solutions. In the solutions, besides the conventional solving process, there are also problem-solving ideas and evaluation about the core of the problem which can be used as the fundamental materials for the 1st lecturing mode. The answer-searching APPs have already adopted the lecturing through video in some of the exercises. And this lecturing mode can bring a revolutionary change to them.

3 System architecture

The geometry lecturing tool adopts the Hybrid architecture design as shown in Fig. 4. Within C++ desktop application, HTML5 dynamic geometry software and geometric semantic recognition modules are imbedded.

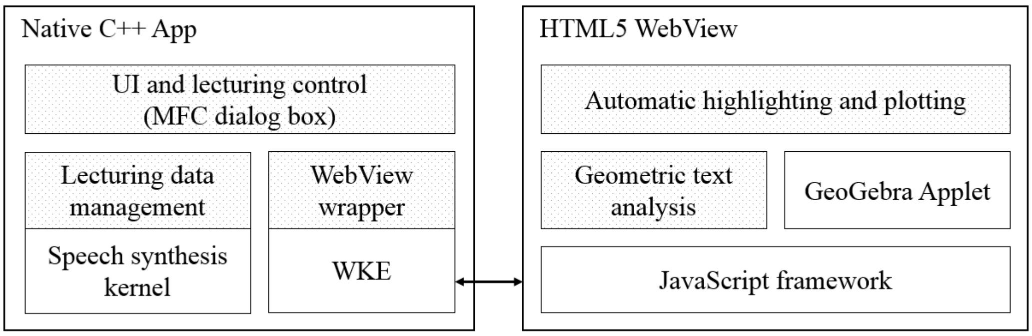


Fig. 4 Hybrid architecture of lecturing tool

Developed on the basis of VC++2008 and MFC framework, the main program integrates text-to-speech (TTS) module and WKE browser kernel module. HTML5 application is developed on the basis of JS/HTML language and GGB applet interface, containing GGB offline operation file (Math Apps Bundle) [1]. HTML5 application is imbedded into the major interface of the dialog box as Web View and realizes such functions as window life cycle, window area, interface display and interaction message delivery through "Web View wrapper". HTML5 through WKE interface: the local application invokes "run JS(expression)" function of WKE interface to execute JS statement. The two core modules, namely geometric text analyzing module and automatic highlighting and plotting module, are also developed with JS as the basis.

As for the HTML browser engine model selection, the WKE kernel (based on WebKit starter edition branch open source program) is small in size (with dynamic depot being about 11MB) and convenient in use, whereas CEF and Blink has large size and are difficult to encode. After some experiments, the present lecturing tool chooses the WKE kernel that can meet the integration demand of GGB.

4 Realization of several key technologies

4.1 Lecturing interaction control

Apart from some basic control functions such as line-by-line sequential speech and speech pausing and recovery, the lecturing tool also enables the teachers to skip some steps in the lecturing process. As shown in Fig.5, the users can freely control the lecturing process by clicking “previous line” and “next line” to skip one or more steps and “repeat” to repeat the speech and highlight present line. In addition, the students can also change the lecturing process by double-clicking any line in the solution text area. In such case, synchronized highlighting can be found in the figure area and synchronized speech of the line can be heard. The underlying principle is that the speech is synthesized with each line as the fundamental unit, and keywords are dynamically extracted and transmitted during the speech to synchronize the speech with figure highlighting. The users can also control the lecturing through keyboard instead of the mouse: “space” for pausing the play, “left” for executing the previous line, “right” for executing the next line “down” for repeating the present line so that the lecturing is under free control.

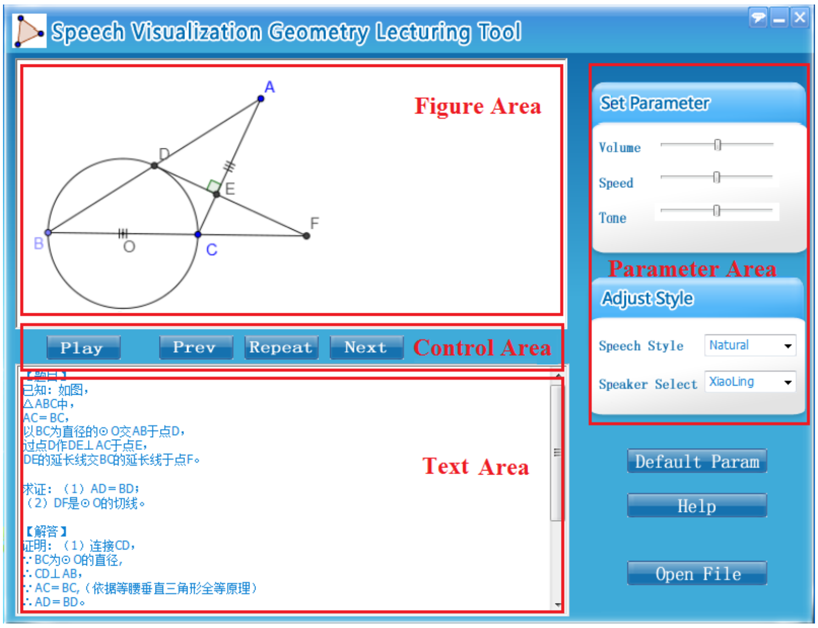


Fig. 5 Interaction control interface of geometry lecturing tool

In Fig.5, there are following four primary interaction areas on the lecturing interface.

(1) Figure area: it displays GGB figures and highlights corresponding figure elements according to the speech progress;

(2) Text area: it shows the question text and solution text. The line being spoken will be highlighted accordingly.

(3) Control area: it enables the users to control the speech progress through such functions as play, pause, previous line, next line and repeat. Double clicking a line in the text area will result in direct reading of that line. In the meantime, shortcut key control function is offered: “space” for playing or pausing; “left” for executing previous line and “right” for next line, and “down” for repeating the present line.

(4) Parameter area: in this area, those speech synthesis parameters including volume, speech speed, and intonation can be adjusted at any time.

4.2 Dynamic visualization of speech

Hereby the “dynamic visualization of speech” means the simultaneous highlighting of the text line and geometric figure elements with the automatic speech synthesis in order to build a speech-visualized environment of cognition and teaching. On the basis of invoking text-to-speech kernel developed by Beijing Yuyintianxia Technology Co., Ltd (emTTS), the paper further studies the technologies related to TTS in geometry exercises.

The emTTS kernel is of the universal type that realizes text-to-speech and basic adjustment of volume, speed, tone and speaker. It is mainly used in the automatic reading of news, articles and vehicle navigation. When the kernel is not optimized specifically for the purpose of reading the geometry question and solution text, it will encounter incorrect synthesis as the common TTS kernel (such as the identification of △⊙⊥//∠∴∵≌Rt). In order to realize dynamic visualization of speech, we develop and enhance the emTTS for a second time and focus on finding solutions to following three problems:

(1) Semantic recognition of geometric and mathematic languages

To semantically understand the text of geometric questions and enable the software to correctly synthesize the speech of geometric signs and read them out as we do. There are three mathematic signs involved in the geometric questions, namely graphs, text and symbols. We need to conduct normalization processing with text properties identification, because to enable the students to master the correct pronunciation of the signs is an important part in geometry teaching. This lecturing tool pre-processes the text of geometric question first (such as removing the space), and then normalizes the signs on the basis of dictionary and context matching by converting “Rt”, “∽”, “≥”, “⊙”, “⊥”, “∵”, and “△” to be “直角”, “相似于”, “大于等于”, “圆”, “垂直”, “因为”, “三角形” *(English word-for-word translation: “right angle”, “resembles”, “no less than”, “circle”, “is vertical to”, “because”, and “triangle”)*. The semantic comprehension of those geometric symbols is superior to that in emTTS kernel.

(2) Control of synchronic figure highlighting

When synthesizing the speech, the lecturing tool extracts alphabetic sequence as keywords (e.g. “AB” and “O” from “straight line AB is the tangent line of ⊙*O*”), and uses the current line number and keywords as the basis for figure highlighting positioning (see Tetrad in Section 4.3), and passes them to the highlighting module of JS which finds out the predicate as per the received line number and keywords and executes corresponding highlighting function (see Dynamic highlighting plan in Section 4.4) to realize dynamic highlighting. This helps to achieve the effect of synchronizing speech synthesis of the keywords with dynamic highlighting through highlighting function invoking. For example, as for “直线 *AB* 是⊙*O* 的切线”*(English word-for-word translation: “straight line AB is the tangent line of ⊙O”)*, the reading of AB is synchronized with highlighting of the straight line AB in the figure and O with circle O.

(3) Special control of synchronic highlighting of geometric relationship

The tool also identifies some special relationships in the geometric figures, such as “CD⊥AB”, “线段 CD 与 AB 垂直”*(English word-for-word translation: “line segment CD is vertical to AB”)*, “OD∥AC”, “OD 平行于 AC”*(English word-for-word translation : “OD is parallel to AC”)* , “AC=BC”, “*∠ADP=∠AEP”*, and “△*ABC*≌△*DEF*”. According to the relationship between two figure elements such as “⊥”, “∥”, “=” and “≌” extracted in the way as described in Section 4.3, the paper adopts different strategies in highlighting: highlighting in blue for the isolated figure elements; and highlighting of the first figure element in red and that of second in blue and end of highlighting for both of them at the same time when they are two figure elements related to each other, so that the geometric relations can be presented more directly to the user. For those geometry questions containing many related angles and segments, special synchronic highlighting is helpful for the users in quickly matching the text with the figure and shortening the time needed for comprehension.

4.3 Smart identification of geometric language

The natural language of middle school geometry is a mathematic symbolic language with strong regularity and diversity of ways of describing. It should be converted into formalized language which the computer can understand and use in realizing automatic plotting. For example, "*AB*⊥*CD*" expresses the same meaning as "线段 AB 与 CD 垂直" *(English word-for-word translation: "line segment AB is vertical to CD")*. The study employs following three steps to extract the geometric elements from geometry exercise text for automatic plotting.

(1) Use the Extended Backus Naur Form (EBNF) that can be easily expressed to present the formalized language of the geometry exercise pattern matching. The results are explained in Fig. 6, including geometric signs (such as "⊙", "∠", "//" and "="), and specific prefixes (e.g. "射线", "延长") *(English word-for-word translation : "ray", "extend")* and suffixes (e.g. "共圆", "共线") *(English word-for-word translation: "concyclic", "collinear")*. For the purpose of figure highlighting, specific prefixes indicating figure type are ignored. For example, polygon prefixes such as "△", "平行四边形" and "梯形"*(English word-for-word translation: "parallelogram" and "trapezoid")* are being ignored in the identification. In such case, the identification of the vertex sequence is enough in building the geometric information for highlighting, so the grammatical complexity can be reduced.

```
Rules ::= Point | Line | Extend | Circle | Angle | Polygon | TwoLines | Collinear
Point ::= '点'? Capital
Line ::= ('射线'|'线段'|'直线')? Capital2 | ('射线'|'线段'|'直线') Lower | '线段'? Capital2 '的' ('延长线'|'中垂线')
Extend ::= '延长'   '线段'?   Capital2   ('到' (Point|Line))?
Circle ::= ('圆'|'⊙') (Capital Capital2? | Lower)
Angle ::= ('角'|'∠') (Capital Capital2? | Greek | Digit)
Polygon ::= Capital Capital Capital+
TwoLines ::= Line   (Relation1   Line | ('和' | '与')   Line   Relation2)
Relation1 ::=   '//'  |  '⊥'  |  '='  |  '＝' | ('平行' | '垂直') '于'?  |  '等于'  |  '交'
Relation2 ::= '平行'  |  '相等'  |  '等长'  |  '垂直'  |  '相交'
Collinear ::= Capital (Connector? Capital)+ (('在'|'位于') '同一个' ('圆'|'直线') '上' | '共线'|'共圆')
Connector ::= ','  |  '，'  |  '、'  |  '和'  |  '与'  |  '及'
Capital2 ::= Capital Capital
Capital ::= [A-Z]    Lower ::= [a-z]    Digit ::= [0-9]    Greek ::= [\u0370-\u03FF]
```

Fig. 6(a) EBNF description of Chinese geometry exercise highlighting identification

Examples of parallel relationship between lines of English geometry exercise		
AB//CD	AB and CD is parallel	the lines AB and CD are parallel
AB is parallel to CD	AB & CD is parallel	AB is a line parallel to CD
AB parallel CD	line AB is parallel to line CD	two parallel lines AB and CD
AB parallel to CD	line AB parallel to line CD	

Note: All the word "line" can be replaced by "segment" or "radial".

```
Parallel ::= Capital2 '//' Capital2
        | Capital2 ('is' ('a' ('line'|'segment'|'radial'))?)?
              'parallel' 'to'? ('line'|'segment'|'radial')? Capital2
        | Capital2 ('and'|'&') Capital2 ('is'|'are') 'parallel'
        | 'parallel' ('lines'|'segments'|'radials') Capital2 'and' Capital2
Capital2 ::= [A-Z][A-Z]
```

Fig. 6(b) EBNF description of English geometry exercise highlighting identification (partial)

Note: the figure provides EBNF description of only parallel relationship between lines.

(2) Extend EBNF by removing the non-terminals, and use syntax diagram structuring tool (http://bottlecaps.de/rr) to carry out sequencing by merging as shown in Fig.7. This method utilizes the mature grammatical analysis tool and can avoid the complicated regular analysis of grammar. The syntax diagram reveals that when combined in different ways or having different contexts, the letters can express different meanings such as point, segment, line，ray, triangle, quadrangle, polygon and extension line. With the help of the syntax diagram, the conflicts among different matching rules can be identified and avoided.

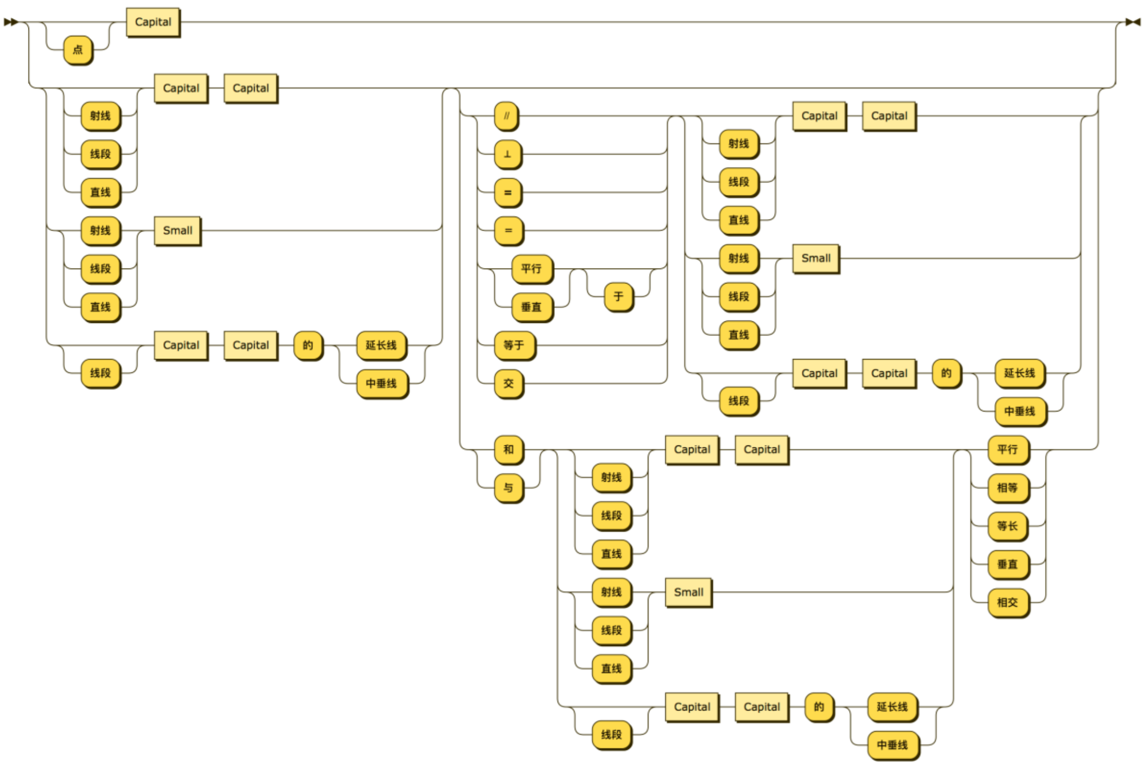


Fig. 7(a) Syntax diagram of Chinese geometry exercise highlighting identification (partial)

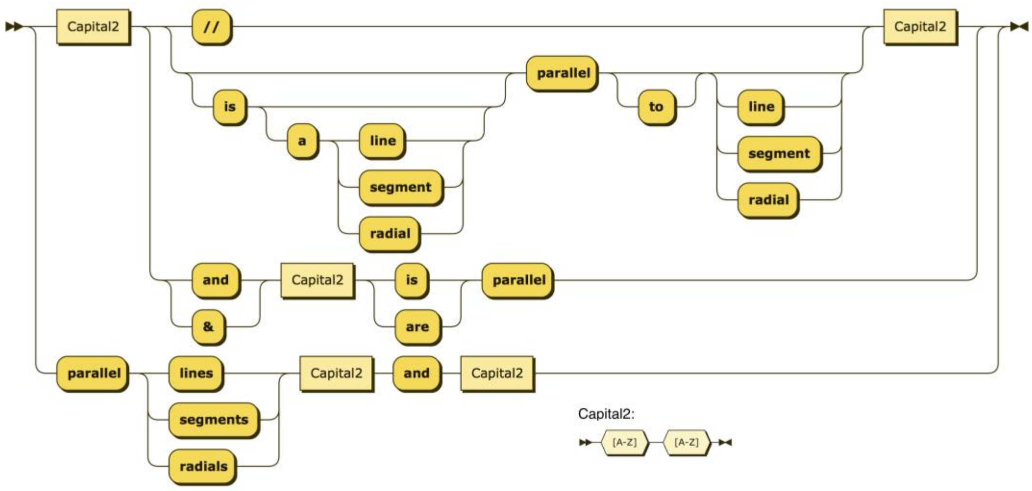


Fig. 7(b) Syntax diagram of English geometry exercise highlighting identification (partial)

Note: the figure only provides the syntax graph of parallel relationship between lines.

Table 3(a) Production rule of Chinese geometry exercise for automatic plotting (partial)

Condition: RegExp	**Input Text**	**Conclusion: First-Order Predicate**
[A-Z]{2}(平行(于)?|//)[A-Z]{2}	*AB* // *CD*	segment(AB， //)、segment(CD)
[A-Z]{2}(和|与|、)[A-Z]{2}平行	*AB* 和 *CE* 平行	segment(AB， //)、segment(CE)
[A-Z]{2}(和|与|、)[A-Z]{2}垂直	*AC* 与 *EF* 垂直	segment(AC， ⊥)、segment(EF)
[A-Z]{2}(=|＝|等于)[A-Z]{2}	*AB*=*CD*	segment(AB， ＝)、segment(CD)
[A-Z]{4}共圆	四点 *ABEP* 共圆	circle3p(ABEP)
(圆|⊙)[A-Z]{3}	圆 *ABC*	circle3p(ABC)
(圆|⊙)[A-Z]	⊙*O*	circle(O)
(角|∠)[A-Z]{3}	∠*PAB*	angle3p(PAB)
(角|∠)([A-Z]|[0-9]|[\u0370-\u03ff])	∠*β*	angle(β)
[A-Z]{3,8}	平行四边形 *ABCD*	polygon(ABCD)
[A-Z]{2}的延长线|延长[A-Z]{2}	*CD* 的延长线	extension(CD)
射线([A-Z]{2}|[a-z])	射线 *AB*	ray(AB)
(线段)?([A-Z]{2}|[a-z])	*AB*	segment(AB)
[A-Z]	点 *P*	point(P)

(3) Regular expression is built on the basis of the sequenced and optimized syntax diagram and used as the precondition in production rule of automatic plotting. The conclusion acts as the first-order predicate logic description of automatic plotting. The conflict-solving strategy adopts rule sequencing: after the preprocess of blank characters removal, a line of text undergoes rule matching in turn, and the rule that matched first and having the foremost position will be the best matched rule which will be then used to extract the predicate, keywords and relational symbols (as for algorithm, see Fig. 8. Table 3 provides some examples of production rule, the predicates in the conclusion part represent the type names of automatic plotting and highlighting. As for binary geometric relationships such as “*AB* // *CD*”,

the figure highlighting information is described and expressed with two first-order predicates instead of second-order ones such as “perpendicular (segment (CD), segment (EF))”, because it is unnecessary to reserve the relationship type for present geometry lecturing tool for the time being.

Table 3(b) Production rule of English geometry exercise for automatic plotting (partial)

<table>
<tr><th>Condition: RegExp</th><th>Input Text</th><th>Conclusion</th></tr>
<tr><td>parallel(lines|segments|radials)[A-Z]{2}
and[A-Z]{2}</td><td>two parallel lines AB and CD</td><td>segment(AB, ∥),
segment(CD)</td></tr>
<tr><td>[A-Z]{2}\/\/[A-Z]{2}|[A-Z]{2}
(and|&)[A-Z]{2}(is|are)parallel</td><td>AB//CD
AB and CD is parallel</td><td>segment(AB, ∥),
segment(CD)</td></tr>
<tr><td>[A-Z]{2}(is(a(line|segment|radial))?)?
parallel(to)?(line|segment|radial)?[A-Z]{2}</td><td>AB is parallel to CD
AB parallel CD</td><td>segment(AB, ∥),
segment(CD)</td></tr>
</table>

Note: in the table, only production rule of automatic plotting for parallel relationship between lines in geometry exercises in English is generated. The profound analysis of automatic plotting of all geometric elements hasn’t been carried out yet.

Table 4 Analysis example of geometry exercise text

<table>
<tr><th>Input text line Array</th><th>Analysis results：Tetrad (line number, keywords, predicate, relation symbol)</th></tr>
<tr><td>['AB∥CD，AC 与 EF 垂直',
'四点 ABEP 共圆',
'∠PAB=∠β']
['AB∥CD，AC is vertical to EF',
' The four points ABEP are concyclic ',
'∠PAB=∠β']</td><td>[{"line":1,"name":"segment","key":"AB","rel":"∥"},
{"line":1,"name":"segment","key":"CD"},
{"line":1,"name":"segment","key":"AC","rel":"⊥"},
{"line":1,"name":"segment","key":"EF"},
{"line":2,"name":"circle3p","key":"ABEP"},
{"line":3,"name":"angle3p", "key":"PAB"},
{"line":3,"name":"angle", "key":"β"}]</td></tr>
<tr><td>['直线 EF 与⊙O 相切于点 M']
[' The line EF is tangent to ⊙O at point M ']</td><td>[{"line":1,"name":"line", "key":"EF"},
{"line":1,"name":"circle","key":"O"},
{"line":1,"name":"point", "key":"M"}]</td></tr>
</table>

Note: the description of “Input text line Array” column in English is for effect only, and it hasn’t been fully developed yet.

For the text of geometry exercise, each line undergoes rule matching, and the result is expressed with tetrad (line number, keywords, predicate, relation symbol). The JSON form and analytic example are listed in Table 4. For instance, the result of “AC 与 EF 垂直” *(English word-for-word translation: “AC is vertical to EF”)* in 6th line is (6, AC, 'segment' , '⊥') and (6, 'EF', 'segment' , null) in which “relation symbol” are used for delaying the highlighting duration. “Keywords” are the alphabetic sequence in the matched contents (incompleteness of all matched contents is to reduce the data complexity at synchronic highlighting interface and enhance the substitutability of the text-to-speech engine) . For example, the keywords in “∠PBA” and “四点 ABEP 共圆” are “PBA” and “ABEP” respectively.

```
function pickKey(t) {return t.replace(/[^A-Za-z0-9\u0370-\u03ff]/g, '');} //Extract letters and figures
function parse2Segments(text) {
  var a = text.replace(/[^A-Z].+$/, '');       //Extract the first alphabetic sequence
  return [['segment', a], ['segment', pickKey(text.substring(a.length))]];
}
var rules = [                    //The matching rule set
  { re: /[A-Z]{2}(平行(于)?|∥)[A-Z]{2}|[A-Z]{2}(和|与|、)[A-Z]{2}平行/,
    type: '∥', parse: parse2Segments },
  //...
  { type: 'circle3p', re: /[A-Z]{4}共圆|(圆|⊙)[A-Z]{3}/ },
  //...
  { type: 'extension', re: /[A-Z]{2}的延长线|延长[A-Z]{2}/ },
  { type: 'segment', re: /(线段)?[A-Z]{2}/ },
  { type: 'circle', re: /(⊙|圆)[A-Z]/ },
  { type: 'point', re: /[A-Z]/ }
];
function applyRule(queue, line, rule, text) {
  var r = rule.parse ? rule.parse(text) : [rule.type, pickKey(text)];
  if (r[0] instanceof Array) { //Add the results of two figures in case of graphic relationship result
    queue.push({ line: line, name: r[0][0], key: r[0][1], rel: rule.type
               }, { line: line, name: r[1][0], key: r[1][1] });
  } else {                     //Direct add the result in case of a simple one
    queue.push({ line: line, name: r[0], key: r[1] });
  }
}
function parseLines(lines) {            //Analyze the text lines of geometric exercise
  var queue = [];
  lines.forEach(function(text, line) { //Analyze each line
    text = text.replace(/\s+/g, '');    //Select the text of one line, eliminate the blank characters
    while (text) {                      //Start text matching until there is no item for matching or the line end
      pos = 9999; rule = null;          //Initialize the matching location to be the max value
      rules.forEach(function(r) {       //Try each rule in turn
        t = text.search(r.re);          //Memorize the matching location of the rule
        //Select the item having its matching location in front of others as the best match candidate
        if (t >= 0 && pos > t) {
          pos = t; rule = r;
        }
      });
      if (rule) {                               //Find the best match
        t = text.match(rule.re)[0];             //Find the matched text
        applyRule(queue, line + 1, rule, t);    //Extract the keyword and predicate
        text = text.substring(pos + t.length);  //Continue with subsequent matching in the line
      } else { break; }                         //End the line when there is no matching item left
    }
  });
  return queue;
}
```

Fig. 8 Matching algorithm based on rule sequencing

4.4 Automatic plotting and highlighting display

The study about using dynamic geometry software to realize automatic plotting of geometric knowledge concentrates on full-automatic smart plotting [2,5]. However, affected by the complexity of natural language and unpredictable figure position, there are problems such as non-aesthetic figure and over-complex system. The paper adopts the plan that combines artificial intervention with automatic plotting: plotting initial figure of the geometry exercise, auxiliary figures in the solution and the figures that can be hardly drawn through command interface (such as angle sign and equivalence sign), saving those preset figures in a GGB file, uploading those preset figures on the webpage during lecturing (with C++ converting GGB file to be BASE64 coding string and passing the string to GGB interface for uploading [1]), and dynamically creating highlighted figures. This can solve the difficulty in figure positioning, and make full use of the usability of GGB to reduce the complexity of the geometry lecturing tool.

The middle school geometry exercise usually includes question and several solutions and resulting figures, as the auxiliary lines are different in the different solutions. As for the dynamic switching between questions and figures corresponding to different solutions, the paper adopts the algorithm in Fig. 9 to dynamically switch the display conditions for preset figures. As for the figures specially designed for the solutions, their display condition (on the "Advanced" page in figure attribute dialog box) are set to be "group == 1" (meaning it can't be displayed unless in the first solution) or "line>=12" (meaning it can't be displayed until from the 12th line of the geometry exercise). By comparison, the figures matching the exercise contents are not provided with display condition but displayed all the time. The variables "group" and "line" can be initialized in the command column of GGB software (e.g. inputting "group =1"). During the lecturing, their values are dynamically set through evalCommand function invoked at GGB interface so as to achieve dynamic display.

```
var Hi = window.Hi = window.Hi || { defaultTimeout: 1500 };
Hi.init = function(text) {          //Initialize the environmental parameters of GGB highlighting
  Hi.lines = text.replace(/\n\r|\r\n/g, '\n').split('\n'); //Decompose the text line
  Hi.queue = parseLines(Hi.lines).map(function(r) {
    var timeout = r.rel ? 'Hi.defaultTimeout * 2' : 'Hi.defaultTimeout';
    var s = 'Hi.' + r.name + "('" + r.key + "', {timeout: " + timeout + '})';
    r.show = new Function(s); //Construct highlighting function as per the predicate
    return r;
  });
};
Hi.seek = function(line, key) { //Highlight specific figure according to the line number and keyword
  for (var i = 0; i < Hi.queue.length; i++) {
    var item = Hi.queue[i];
    if (item.line === line && item.key === key) {
      ggbApplet.evalCommand ('line=' + line);
      ggbApplet.evalCommand ('step=' + (i + 1));
      ggbApplet.evalCommand ('order=' + (item.line * 10 + item.col + 1));
      item.show();    //Invoke highlighting functions, such as Hi.segment ('AB', {timeout:1000})
      break;
    }
  }
};
```

Fig. 9 Plotting command mapping and highlighting positioning algorithm

The dynamic highlighting plan in the paper is to map the predicate logic description resulting from semantic comprehension in Section 4.3 to GGB plotting command (see "Hi.init" dynamic creation of display function part in algorithm listed in Fig. 9), compile JS highlighting function of the same name for each predicate and invoke GGB interface function therein (such as highlighting function named as "Hi.angle3p" for predicate "angle3p" as shown in Fig.10), set highlighting color and attributes like overstriking of the temporarily created figures, and achieve highlighting display effect through delayed figure elimination. For instance, for the predicate logic description in Table 3, the plotting command in Table 5 is used for automatic plotting. The predicate logic of binary geometric relation such as "AB // CD" is mapped to several plotting commands and different time of delayed deletion is set to present overlapped highlighting effect. For example, segment AB is highlighted until the highlighting of CD is completed.

```
function addHighLight(name, attr) {             //Set the display properties of highlighted figure
   ggbApplet.setLabelVisible(name, false);   //Highlight only the figure instead of its label
   ggbApplet.setColor(name, 0, 0, 255);       //Set the default highlighting color to be blue
   ggbApplet.setFixed(name, true);              //Set the highlighted figure to be immovable
   if (attr && attr.timeout) {                   //Delay the deletion of highlighted figure
      setTimeout(function(){ggbApplet.deleteObject(name);}, attr.timeout);
   }
   return name;
}
//Segment highlighting function, such as invoking Hi.segment('AB', {timeout: 1000})
Hi.segment = function(ab, attr) {
   //Create segment according to the end points,request construction of end points in GGB file in advance
   var hi = ab + '_{his}';
   ggbApplet.evalCommand(hi + '=Segment[' + ab[0] + ',' + ab[1] + ']');
   ggbApplet.setLineThickness(hi, 7);       //Overstrike the highlighted part
   return addHighLight(hi, attr);
};
//Highlighting the function of angle in form of ∠ABC, such as invoking Hi.angle3p('PAB',{timeout: 1000})
Hi.angle3p = function(abc, attr) {
   Hi.segment(abc[0] + abc[1], attr);       //Highlight the first side
   Hi.segment(abc[1] + abc[2], attr);       //Highlight the second side
};
```

Fig. 10 Example of highlighting function

Table 5 Automatic plotting command mapping example

Input Text	**Conclusion:** **first-order predicate**	**GGB Plotting command** **or function**
AB // CD	segment(AB， //)、 segment(CD)	Segment[A,B]、Segment[C,D]
AB=CD	segment(AB， =)、 segment(CD)	
四点 ABEP 共圆 *The four points ABEP are concyclic*	circle3p(ABEP)	Circle[A,B,E]
圆 ABC *Circle ABC*	circle3p(ABC)	Circle[A,B,C]
⊙O	circle(O)	Circle[O, 查的同心圆名称]
∠PAB	angle3p(PAB)	Angle[P,A,B]
平行四边形 ABCD *Parallelogram ABCD*	polygon(ABCD)	Polygon[A,B,C,D]
CD 的延长线 *The extension line of CD*	extension(CD)	Ray[D, Reflect[C,D]]
射线 AB *Ray AB*	ray(AB)	Ray[A,B]
AB	segment(AB)	Segment[A,B]

Note: the description of "Input Text" column in English is for effect only, and it hasn't been fully developed yet.

5 Innovations points

The major innovations of the study are as follows:

(1) simultaneous interaction of “text+ speech+ figure” in the exercise: speech synthetic reading progress is coordinated with text line highlighting and figure element highlighting in order to display the dynamic evolution process of geometric auxiliary lines and marks, and construct visual-audio cognitive environment;

(2) free control of lecturing progress and skipping to any parts of the lecture in order to change the lecturing procedure: the teacher can control the lecturing progress by clicking “previous line”, “next line” and “repeat” or double-clicking any line to skip to that part and change the lecturing process;

(3) adoption of automatic text-to-speech: this mode needs no artificial lecturing, it can reduce repetitive work in teaching and avoid the problem of poor pronunciation; the teachers can repeatedly revise the answer (including Problem-solving ideas, solution and remark) according to teaching feedback so as to create excellent examples, it can promote internet circulation and teaching inheritance;

(4) semantic comprehension and smart extraction of geometric elements on the basis of JavaScript regular expression. It can be applied to common geometry questions and answer text. With synchronically highlighted automatic plotting of geometry exercise based on GGB, it is unnecessary to find out the corresponding figure artificially.

6 Outlook

The lecturing tool in discussion needs further accumulation and improvement through actual application. Further study may be performed from following aspects:

(1) Collect and test more samples of plane geometry exercises in Chinese in order to further raise the accuracy of plane geometry exercises lecturing. Further research on how to support the analytic geometry and solid geometry may be performed;

(2) The study adopts the plan combining artificial intervention with automatic plotting. It means the initial figures of the exercises should be prepared by the user with GGB plotting software in advance, which makes the tool difficult to be promoted. In later stage, it is necessary to study the full-auto smart plotting technology. Besides the present automatic plotting and highlighting, it is required to gradually realize full-auto plotting including initial figures so as to bring down the preparation and promoting difficulty.

(3) Fully support the lecturing of geometry exercises in English and realize multi-lingual geometry lecturing. English differs greatly from Chinese in linguistic type, writing system,

phonetic system, morphology and syntax. To enable the tool to fully support geometry lecturing in English, it is necessary to develop following work:

- Collect plenty of geometry exercise samples in English (as my native language is Chinese, I have encountered and accumulated a lot of geometric exercise samples in Chinese in my past study, but only a few in English. To better analyze and semantically comprehend the geometry exercises in English needs analysis of a great amount of geometry exercises in English);
- Find English TTS kernel support, or an open source English TTS kernel for the lecturing tool. An English TTS should be developed and integrated into the C++ main program;
- The C++ main program should perform specific pre-analysis on the geometry exercises in English. Common English TTS kernel can only support common text, but does not do well in those with strong specialty. For the latter, special pre-processing is needed in order to ensure correct speech and extraction of alphabetic sequence as keyword;
- Achieve semantic comprehension of geometry exercises text in English at JS terminal. On the basis of a profound study of geometry exercises in English, use EBNF to express the pattern matching of such exercises, then perform merging, sequencing and optimization to form syntax graph. Next exerts the iteration of the syntax graph to assist thinking, avoid conflict concerning rules matching and take corresponding strategy, and finally construct the regular expressions for the geometry exercises in English as the preconditions of the production rule set in automatic plotting. The conclusion of production rules is the first-order predicate logic which can be identical with that of geometry exercises in Chinese.

7 Conclusion

In view of the limitations of several common geometry lecturing modes, the paper proposes an automatic geometry lecturing mode based on the semantic comprehension of geometry exercise, GGB automatic plotting and speech synthesis kernel (TTS), and develops the speech-visualized geometry lecturing tool with HTML5 and Hybrid architecture as the basis. The new tool features synchronic speech with figure highlighting, easy revision, and random display.

The promotion of computer in teaching lays a foundation for promotion of this lecturing mode, the question text and figure GGB file demand little space which is convenient for transmission. The geometry lecturing mode in discussion is adapted to internet teaching and innovative teaching in AI age.

References

[1] Zhang Yungui, Study on the construction of education resources cloud platform based on GeoGebra [J]. *China Modern Educational Equipment*, 2017 (1): 19-21

[2] Chen Xiaoyu. Design and implementation of e-geometric textbook system [D]. Beijing University of Aeronautics and Astronautics, 2011

[3] Hongyu, Sealing webkit kernel to be duilib browser control [EB/OL]. [2014-07-26]. http://blog.csdn.net/zhuhongshu/article/details/38145651.

[4] Huanghuan, Liu Qingtang, and Chenmao. Merging algorithm research oriented to understanding of geometry proposition [J]. *Computer Science*, 2013, 40 (8): 196-199.

[5] Guo Haiyan. A study of dynamic geometry natural language plotting method in restricted domain [D]. Central China Normal University, 2012.

[6] Ma Chuangxin, The research of knowledge representation [J]. *Modern Information*, 2014, 34(3):21-24

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The only participant of the research is He Wanrong. The research report is compiled by He Wanrong under the guidance of the tutors.

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I like sport and singing, and I am rather good at ping-pong. I am the financial supervisor of my class. In addition, I also like to participate in all sorts of activities. In senior high school, I served as a voluntary guide in Beijing Paleozoology Museum. I also took part in the 12-day 2017 Winter Princeton Teenager Leader Summit Conference, and I am a lecturer of Juvenile Finance of the 6th Students Autonomous Classroom of Tsinghua High School.

I have an interest in a lot of things and am very creative. As I always have a lot of innovative ideas, I am fond of science and invention, especially that in the CS field. In Senior 1, I led my team to successfully complete research and development of an voice-controlled e-book page-turner for which a "patent for invention" and "utility model patent" are applying.

I study really hard and obtain outstanding achievements in integrated academic performance (ranking 7th-22th in my grade). I won the following important awards in the Senior high school：

[Comprehensive]: 2017 Tsinghua High School Enlightenment Award, Principal Award

(Only one student won this award in 11th grade)

[Mathematics]:	2017 Beijing Senior High School Math Final Contest,	2nd Prize in Beijing
	2017 Beijing Senior 1 Math Final Contest,	1st Prize in Beijing
[Physics]:	The 30th Beijing Senior 1 Dynamics Final Contest,	1st Prize in Beijing;
[English]:	The 16th National Creative English Final Competition,	1st Prize in National
[Chinese]:	The 12th National Creative Writing Final Competition,	2nd Prize in National

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It is declared hereby that the paper submitted by the team is the research achievement obtained under the guidance of the tutors. As far as the team knows, unless specially indicated in the paper and listed in the acknowledgement, no published or composed research achievement of others is contained in the paper. I shall take all the legal consequences of any false statement.

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论文题目：基于 GeoGebra 自动作图的几何讲题工具研究

基于 GeoGebra 自动作图的几何讲题工具研究

何婉榕

摘要：针对常见的中文几何题解答存在图文缺乏互动、讲解和图形难以同步定位等问题，设计了集成 HTML5 动态几何软件的 Win32 C++语音讲题工具，实现自由控制讲题进度、双击跳转讲题、图形和文字同步互动亮显及自动同步朗读。基于 JavaScript 正则式实现几何题文本的语义理解并设计了自动作图的产生式规则集。在按行合成语音中对数学语言进行归一化处理和图形亮显的信息提取。本研究可为 GeoGebra 集成开发和基于语义理解的自动作图应用提供参考，可用于几何的创新性教学和知识库构建领域。

关键词：几何教学，语义识别，自动作图，语音合成，GeoGebra

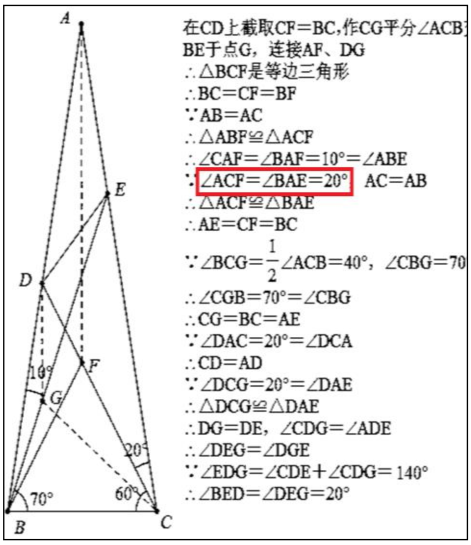
目录

声　明

本论文是针对中文的几何题进行研究的。英汉语言分属不同的语系，各方面差异较大。因时间关系，还未深入研究英文的几何题。论文翻译成英文后，会遇到以下问题，论文中有些章节中不可避免的会出现中文汉字（例如：中文几何题文本，EBNF 模式匹配描述，优化后的语法图，构造的正则表达式规则）。对于英文论文中出现汉字部分的内容，会采取以下的处理方式：（1）配上斜体字书写的英文逐字翻译，见 4.2 章和 4.3 章；（2）专门研究了英文几何题中平行关系的语义理解，增加了图 6(b)，图 7(b)，表 3(b)；（3）对于尚未研究的部分，给出了预期的英文效果，见表 4 和表 5。要全面支持英文几何题的讲题，还有很多工作需要做，在进一步展望中会谈到需开展的工作。以后会全面深入研究英文几何题，并形成新的论文。

1 引言

中学几何题通常包含题目文本、解答文本和几何图形等内容，涉及的几何元素较多，需要在讲解时显示相应的几何图形。



书本和习题册中的“静态图文讲题方式”是最原始简单的方式，但文字讲解和图形缺少互动难以同步定位，图形的动态演变过程也难以展示。如右图所示，看几何题解答时，需要在文本解答和几何图形上,来回切换找对应的图形几何元素。例如：读到“∠ACF=∠BAE=20°”时，我们需找到“∠ACF”和“∠BAE”，并分别做好 20°的标记，接下来再读下一行，再找再标记，一直到读完，找的过程非常辛苦。

传统课堂上的“板书画图老师讲解方式”作图不标准，不利于分享和动态变化。几何画板等动态几何软件是理想的动态交互式几何教学软件，使用动态几何软件制作课件的新型讲题方式的研究较多，但制作成本高，讲解和图形难以同步定位和随意跳转定位。另外，这几种几何题的讲解方式要么没有语音讲读，要么需要老师配音，存在发言不标准的情况，需要兼顾效果和成本。

中文几何题的考试在中国中考、高考和数学竞赛中都占据了非常重要的地位。如何能更高效直观的进行几何题教学是一个值得探讨的问题。研究可自动文本朗读、自动作图和图文同步亮显的几何讲题工具将有助于提高教学效率和增强教学效果，也适应人工智能时代互联网教学的趋势。

具备自动作图和数形结合功能的动态几何软件可满足几何讲题工具的图形需求。其中，GeoGebra（简称 GGB）以其功能强大（包含几何、代数、符号计算、表格等功能）、开源和跨平台等特点成为理想选择（对比分析见表 1）。

表 1　常见的动态几何软件的对比

	几何画板	超级画板	网络画板	GeoGebra	JSXGraph
版权国别	美国	中国	中国	奥地利/IGI	德国
功能规模	大	较大	中	大	小
使用平台	Windows	Windows	浏览器	跨平台/浏览器	浏览器
开源	闭源	闭源	闭源	开源	开源
二次开发	积件库	软件内定制	未知	Java/JavaScript	JavaScript
收费	收费	部分收费	免费	免费	免费

GGB 的 JavaScript（简称 JS）移植版（Math Apps Bundle）适合二次开发，国内已有教学网站和几何知识库的应用案例[1-2]。较多传统教学软件是 Windows 桌面应用软件，通过集成 WebKit 内核（有 WKE 和 CEF 等开源实现版本）可在现有软件中使用 HTML5 功能和避开 IE 控件的问题，让 C++与 JS 代码的相互调用变得容易[3]。因此可用 C++和 JS 基于 GGB 快速开发几何教学软件。

自动作图和同步亮显需要将几何题的自然语言转化为形式化语言以便实现计算机深层语义理解。语义理解的文本处理方法主要有中文分词、规则匹配、本体匹配和深度学习等[4-5]。其结果可采用有限状态机、产生式、谓词逻辑、语义网和本体等多种方法表示和管理[6]，其中产生式和谓词逻辑法以其易于理解和实现的优点得到广泛应用。中学几何题及其解析内容具有较规范的数学符号语言，适合采用规则匹配、产生式和谓词逻辑的理解和表示方法[2,4-5]。将语义理解的结果进一步映射到 GGB 的作图指令就可实现几何语言的自动作图[2]。

基于以上研究成果的总结，本文将研究从中文几何题自然语言到 GGB 自动作图的语义理解方法，并将 HTML5 的 GGB 软件集成到用 VC++/MFC 开发的语音朗读软件中，实现语音与文字图形亮显同步的几何讲题工具。

调研后发现，大量研究都集中在如何做好几何动态教学课件和图形自动作图上，目前还没有类似本论文的课题研究。

2 应用效果

本几何讲题工具需要用户先编辑好每道几何例题的以下两个文件：

（1）题目和解答的文字描述，存储成文本文本（例如“例题 1.txt”）。

（2）用 GGB 软件绘制题目和解答的图形，在同一目录下存成同名文件（例如“例题 1.ggb”）。其中，对解答部分的图形设置显示条件为相应的起始行号。

然后点击讲题工具的“打开几何题目”按钮，选择几何题的文本文件，将展示 GGB 几何图形和题目文本，如图 1 所示，进入自由控制讲题状态。

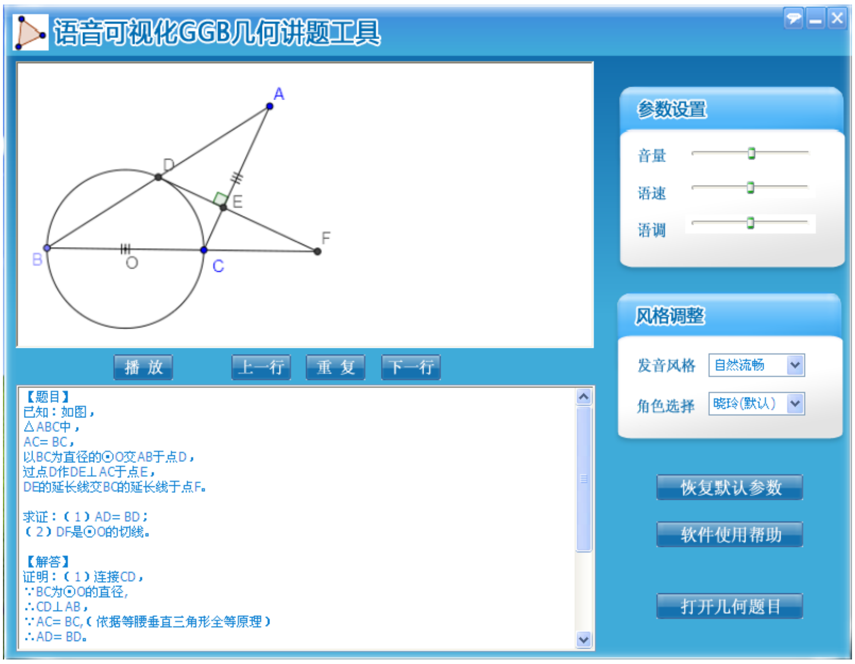


图 1　语音可视化讲题就绪界面

语音合成朗读的语音进度与图形亮显同步进行。文本区的朗读进度亮显和图形区的同步亮显效果如图 2 和图 3。图 2 中展示了题目 1 的系列亮显：三角形的亮显，垂直关系的特殊亮显，平行关系的特殊亮显。图 3 中展示了题目 2 的系列亮显：角度相等的特殊亮显，圆的亮显。

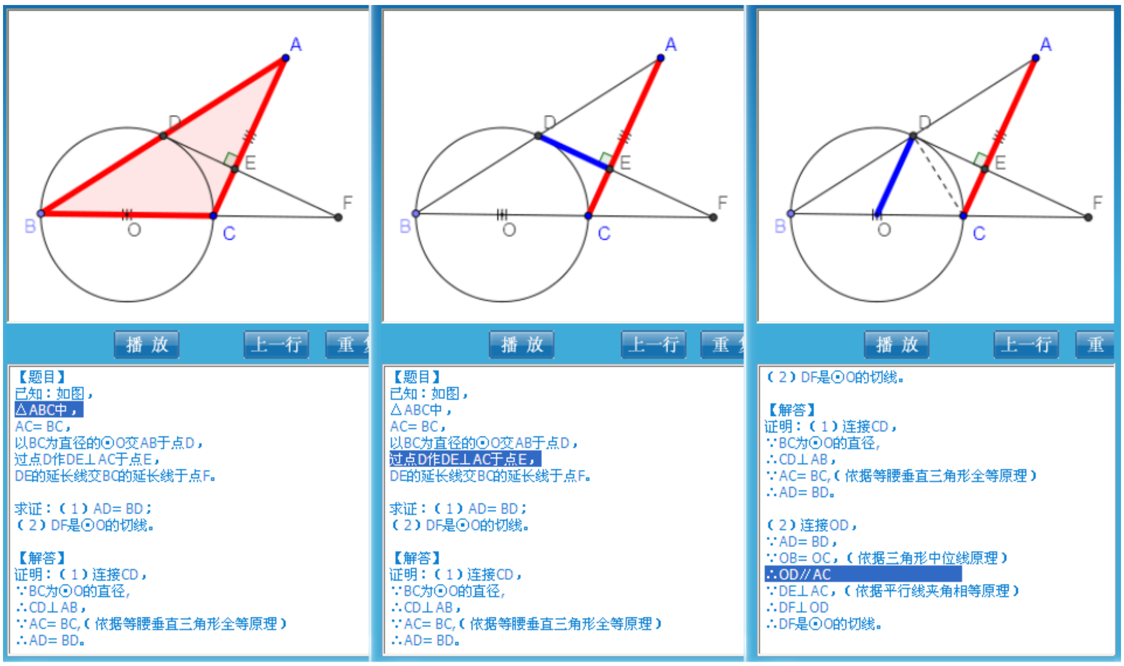


图 2　文本朗读与图形亮显同步的界面（题目 1）

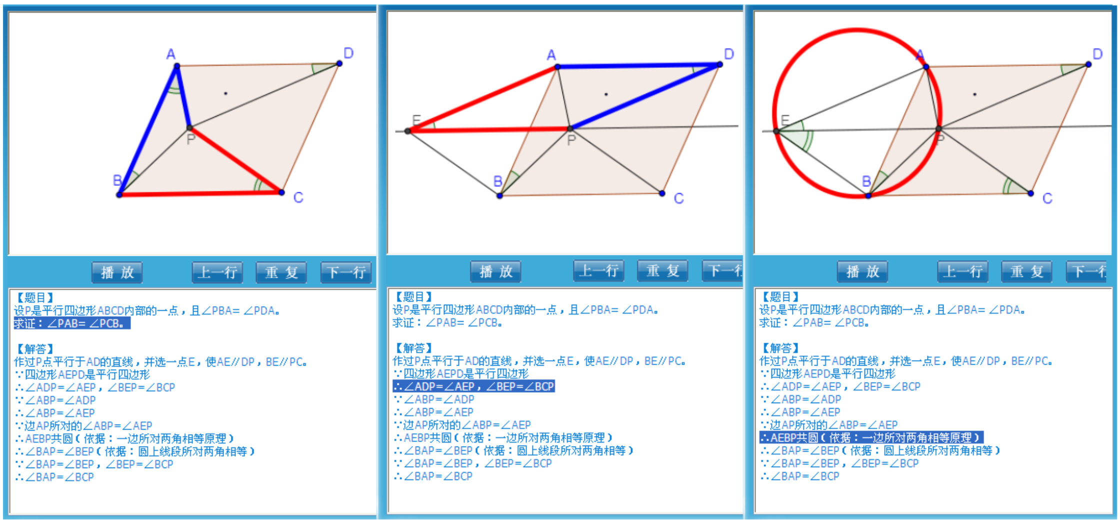


图 3　文本朗读与图形亮显同步的界面（题目 2）

本讲题方式（第 1 种）和常见的讲题方式的对比情况如表 2 所示，结果如下：

（1）第 4 种讲题方式是最原始和简单的方式，也是目前书本例题习题的主要应用方式，没有任何互动性；

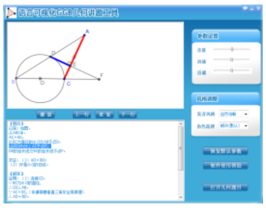
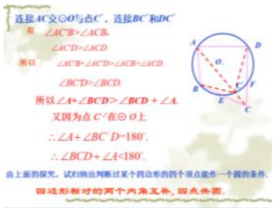
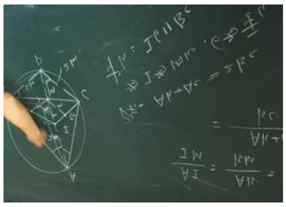
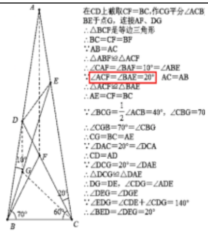
（2）第 3 种讲题方式是目前课堂教学大量使用的方式，属于一次性讲题，非常生动形象，被普遍接受，但需人工讲题，可重复性和可传播性差，还存在作图不标准，带口音讲题的情况。老师在教学过程中进行了大量重复劳动，难以适应人工智能时代的要求；

（3）第 2 种讲题方式是是动态课件讲题，实现了一定的动态性，但仍需人工讲题，且不能自由控制讲题进度。

（4）第 1 种是本论文的讲题方式，解决了以上的不足，可以与第 3 种黑板板书老师讲题互为补充。这种方式“图形+文字+语音”同步互动，非常生动，尤其适合这些场合下学生自己看几何题解答：在家学习时，在外旅游时，用手机搜题 APP 查题看题时。

（5）常用的搜题 APP 有：学霸君，作业帮，小猿搜题。搜题 APP 已经积累了大量习题解答，解答中除了常规的解题过程，很多还有解题思路解析和考点的点评，这些都可以成为本讲题方式的基本素材。搜题 APP 在部分题目中已有视频讲题。本讲题方式可以给搜题 APP 带来创造性的改变。

表 2　四种几何讲题方式的优缺点对比

讲题方式	创新型		传统型	
	第 1 种 GGB 图文同步语音讲题	第 2 种 动态课件老师讲题	第 3 种 黑板板书老师讲题	第 4 种 书本习题答案
图例				
图形标准性	标准	标准	不标准	标准
生动形象性	非常好	比较好	非常好	很差
语音合成功能	有	无，老师讲题	无，老师讲题	无
可重复看否	可以	可以	不可以	可以
跳转讲题	双击直接跳转	不可以，仅上下步	不可以	可以
修改方便性	非常方便	方便	不方便	方便
动态几何功能	有	有	没有	没有
互联网传播	好传播	好传播	不便传播	好传播

注：当前流行的几何动态课件主要有三种方式：PPT 课件（常与视频结合），几何画板课件，GGB 几何课件。

3 系统架构

本几何讲题工具采用如图 4 所示的 Hybrid 混合应用架构设计，在 C++桌面应用中内嵌 HTML5 的动态几何软件和几何语义识别模块。

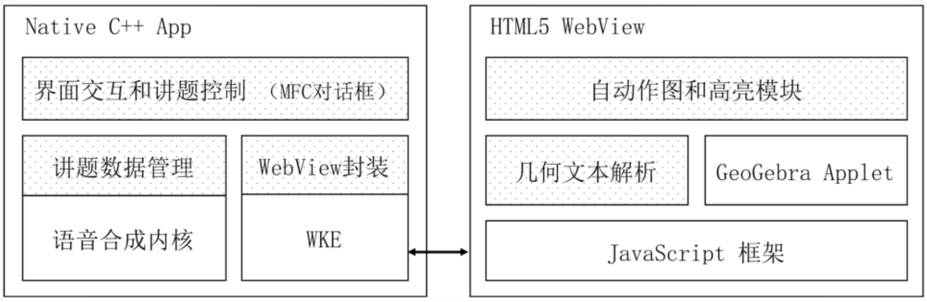


图 4　讲题工具的 Hybrid 架构

主程序基于 VC++ 2008 和 MFC 框架开发，集成了语音合成内核（TTS）模块和 WKE 浏览器内核模块。HTML5 应用基于 JS/HTML 语言和 GGB 小应用接口开发，包含了 GGB 离线运行文件（Math Apps Bundle）[1]。HTML5 应用作为 Web 视图嵌入到对话框主界面中，由“Web View 封装”类实现窗口生命周期、窗口区域、界面显示和交互消息的传递。本地应用调用 WKE 接口的“runJS(expression)”函数执行 JS 语句。几何文本解析模块和亮显自动作图模块，是基于 JS 开发的核心模块。

对于 HTML5 浏览器内核的选型，WKE 内核（基于 WebKit 的精简版分支开源项目）体积小（动态库约 11MB）且使用简单，CEF 和 Blink 内核体积大且编译困难，经试验后本讲题工具选择了满足 GGB 集成需求的 WKE 内核。

4 几个关键技术的实现

4.1 讲题交互控制

本讲题工具除了逐行顺序朗读和暂停恢复朗读的基本控制外，还实现了跳步讲题，如图 5 所示，允许用户自由控制讲题进度。可通过点击或连续点击“上一行”和“下一行”按钮进行跳步，点击“重复”按钮重复朗读和亮显当前行，还可通过双击文本区的任意一行改变讲题流程，双击文本解答区的任何一行，即能看到图形区的同步亮显，也能听到本行的语音合成朗读。其原理是以行为基本单位进行语音合成，朗读过程中动态提取和传递关键字实现语音朗读和图形动态同步亮显。另外用户还可以不用鼠标直接用键盘快捷键控制：“空格键”执行播放暂停，“左键”执行上一行，“右键”执行下一行，“下键”执行重复本行，更加方便用户的自由控制讲题。

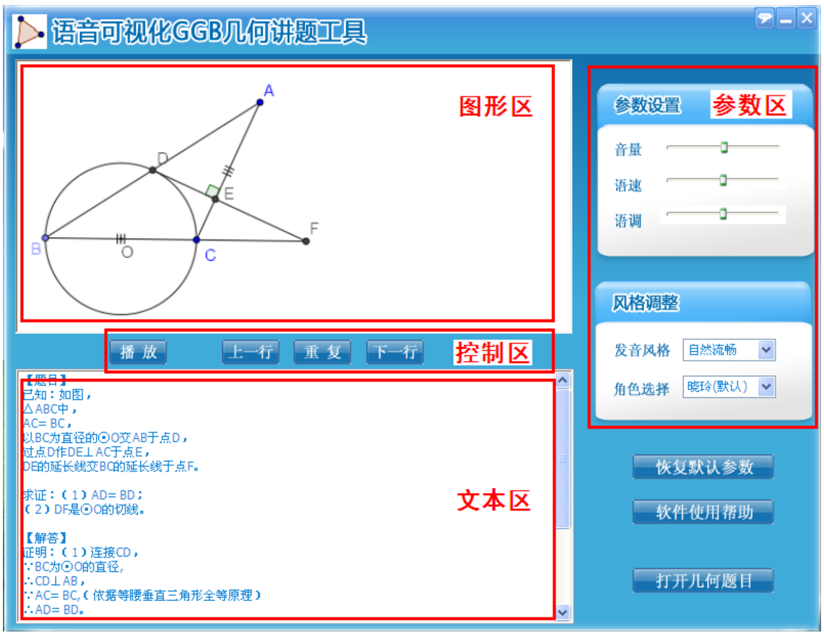


图 5　几何讲题工具的交互控制界面

在图 5 中，讲题界面有下列四个主要的交互区域。

（1）图形区：显示 GGB 图形，随语音朗读的进度同步亮显相应图形元素。

（2）文本区：即题目和解答文本区，会同步亮显当前正在合成朗读的行。

（3）控制区：实现了播放、暂停、上一行、下一行和重复本行等朗读进度控制，双击文本区的某行会直接朗读该行。同时提供键盘快捷控制功能：“空格

键”播放或暂停，“左键”执行上一行，“右键”执行下一行，“下键”重复本行。

（4）参数区：可随时调整音量、语速和语调等语音合成参数。

4.2 语音动态可视化

本文的“语音动态可视化”是指几何题语音合成自动朗读时，同步亮显正在合成的几何题文本行，并同步亮显正在朗读的动态几何图形元素，构建视听双通道的认知和教学环境。本文在调用集成北京宇音天下科技有限公司的语音合成内核（emTTS 内核）的基础上，进一步研究几何题目语音合成的相关技术。

emTTS 内核是通用语音合成内核，实现了文字转语音和音量语速语调发音人等朗读参数的控制等基本的语音合成功能，主要用于新闻、文章和车载导航等的自动朗读。但对几何题目的文本朗读没有进行专门的优化，和常见的语音合成内核一样都会出现有些合成不正确的问题（例如“△⊙⊥∥∠∴∵≌Rt”识别的正确性问题）。为了实现“语音动态可视化”，我们对 emTTS 内核进行二次开发和增强，主要解决以下三个问题：

（1）几何数学语言的语义理解。

对几何题目文本进行几何语义的理解，让软件能够按人的朗读方式把几何符号进行正确的语音合成朗读。几何题有图形、文字和符号等三种数学语言，我们需进行词性识别符号识别后进行归一化处理，让学生掌握其正确的读法是几何教学的重要内容。本讲题工具先将几何题目文本进行预处理（例如去除空格），然后采用基于字典和上下文匹配的符号替换进行归一化处理，例如将“Rt”“∽”“≥”“⊙”“⊥”“∵”“△”分别转换为“直角”“相似于”“大于等于”“圆”“垂直”“因为”“三角形”，这些几何符号的语义理解的优先级高于 emTTS 内核里的语义理解。

（2）同步亮显图形的控制。

本讲题工具在按行合成语音的过程中，将字母序列提取为关键字（例如对“直线 *AB* 是⊙*O* 的切线”将提取到关键字“AB”和“O”），将当前行号和关键字作为图形亮显的上下文定位信息（见 4.3 节的“四元组”描述）传递给 JS 的亮显模块。在 JS 的亮显模块中根据行号和关键字找到谓词，执行相应的亮显函数（见 4.4 节的“动态亮显方案”描述）实现动态亮显。这样就实现了在恰好语音合成到这些关键字时，同步调用亮显函数实现动态亮显。例如：对“直线 *AB* 是⊙*O* 的切线”， 朗读到“AB”时直线 AB 会亮显；朗读到“O”时圆 O 会亮显。

（3）图形关系的同步亮显特殊控制。

本工具对几何图形的中一些特殊关系进行识别，例如：“CD⊥AB”，“线段

CD 与 *AB* 垂直”，“OD∥AC”，“*OD* 平行于 *AC*”，“*AC*＝*BC*”“∠*ADP*=∠*AEP*”“△*ABC*≌△*DEF*”。本文按 4.3 节所述提取“⊥”“∥”“=”“≌”等两个几何元素之间的关系，在亮显时采取不同的策略：对于孤立的几何元素，默认用蓝色进行亮显，对于有关系的几何元素，本文对关系中的第一个几何元素用红色亮显，关系中的第二个元素用蓝色亮显，且前一个元素的亮显时间一直会延迟到后一个元素亮显结束，以便能更直观地向用户呈现几何关系。对于有较多关系角度和关系线段的几何题，同步特殊亮显能让用户快速图文对照定位，缩短理解时间。

4.3 几何语言的智能识别

中学几何题的自然语言是描述多样化和规律性较强的数学符号语言，需要转换为形式化语言以便计算机能理解并实现自动作图。例如“*AB*⊥*CD*”与“线段 *AB* 和 *CD* 垂直”是相同含义的表述。本研究采用下列三个步骤从几何题文本提取需要自动作图的几何元素。

（1）采用易于表达的扩展巴科斯范式（EBNF）作为表述几何题模式匹配的形式化语言，结果如图 6 所示，包括几何符号（例如“⊙”“∠”“∥”“＝”）、特定前缀（例如：“射线”，“延长”）和后缀（例如：“共圆”，“共线”）。针对图形亮显用途对表示图形类型的特定前缀进行裁减，不特别识别“△”“平行四边形”“梯形”等多边形前缀，识别其顶点序列即可构建亮显的几何信息，以此降低语法复杂性。

```
亮显规则 ::= 点 | 线 | 延长到 | 圆 | 角 | 多边形 | 线对 | 共线
点 ::='点'? 大写字母
线 ::=('射线'|'线段'|'直线')? (双字母|小写字母) | '线段'? 双字母 '的延长线'
延长到 ::='延长' '线段'? 双字母 ('到' (点|线))?
圆 ::=('圆'|'⊙') 大写字母 双字母?
角 ::=('角'|'∠') (大写字母 双字母?| 希腊字母 | 数字)
多边形 ::= 大写字母 大写字母 大写字母+
线对 ::= 线 ((关系符号 | 关系名1) 线 |('和'|'与') 线 关系名2)
关系符号 ::='∥' | '⊥' | '=' | '＝'
关系名1 ::=('平行' | '垂直')'于'? | '等于' | '交'
关系名2 ::='平行' | '相等' | '等长' | '垂直' | '相交'
共线 ::= 大写字母 (点连接符? 大写字母)+ ('共线'|'共圆')
点连接符 ::=',' | '，' | '、' | '和' | '与' | '及'
双字母 ::= 大写字母 大写字母
```

图 6(a) 中文几何题亮显识别的 EBNF 描述

英文几何题线平行关系的文本示例		
AB//CD	AB and CD is parallel	the lines AB and CD are parallel
AB is parallel to CD	AB & CD is parallel	AB is a line parallel to CD
AB parallel CD	line AB is parallel to line CD	two parallel lines AB and CD
AB parallel to CD	line AB parallel to line CD	

注：所有的单词“line”都可以被“segment”和“radial”替换

```
Parallel ::= Capital2 '//' Capital2
        | Capital2 ('is' ('a' ('line'|'segment'|'radial'))?)?
                'parallel' 'to'? ('line'|'segment'|'radial')? Capital2
        | Capital2 ('and'|'&') Capital2 ('is'|'are') 'parallel'
        | 'parallel' ('lines'|'segments'|'radials') Capital2 'and' Capital2
Capital2 ::= [A-Z][A-Z]
```

图 6(b) 英文几何题线平行关系的亮显识别的 EBNF 描述

（2）对 EBNF 描述进行去除非终结符的展开处理，并采用语法图构建工具（http://bottlecaps.de/rr）进行合并排序处理，结果如图 7 所示。此方法可避免复杂的正规文法分析工作，复用较成熟的语法分析工具的能力。从语法图易知字母的不同组合和不同的上下文分别有点、直线、线段、射线、三角形、四边形、多边形和延长线等语义，从而识别多个匹配规则的冲突并采用相应的策略。

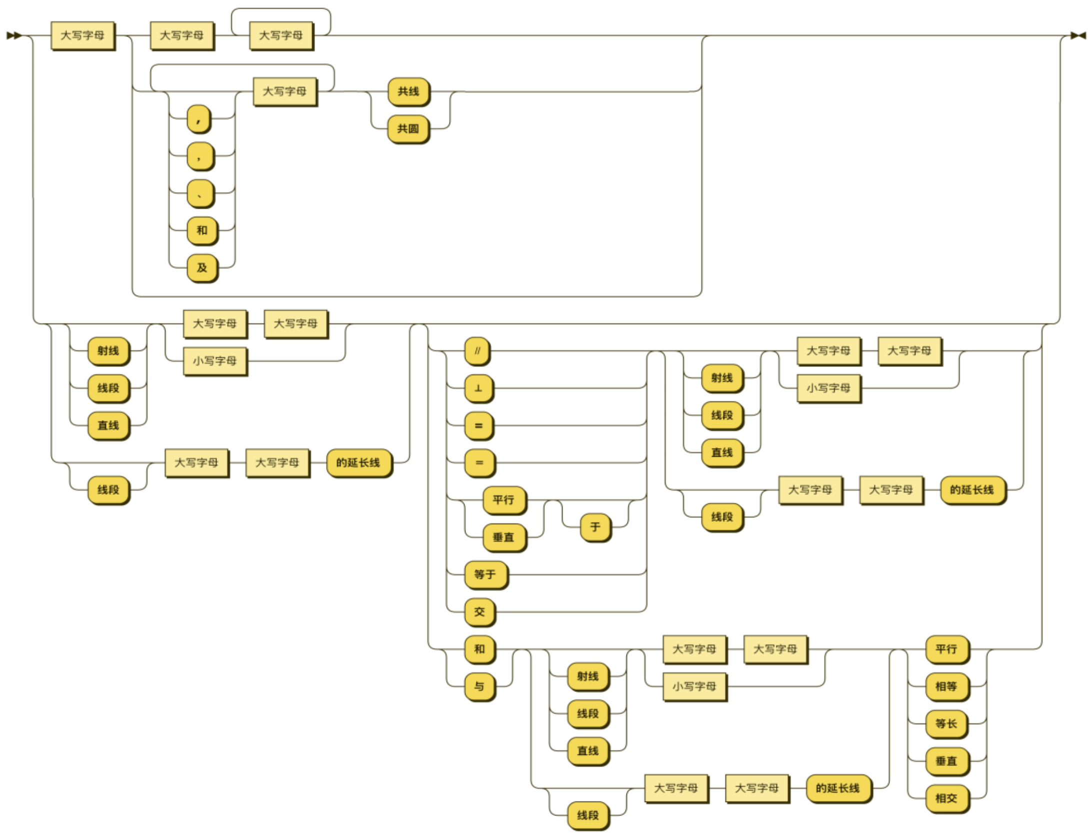


图 7(a) 中文几何题亮显识别的语法图（局部）

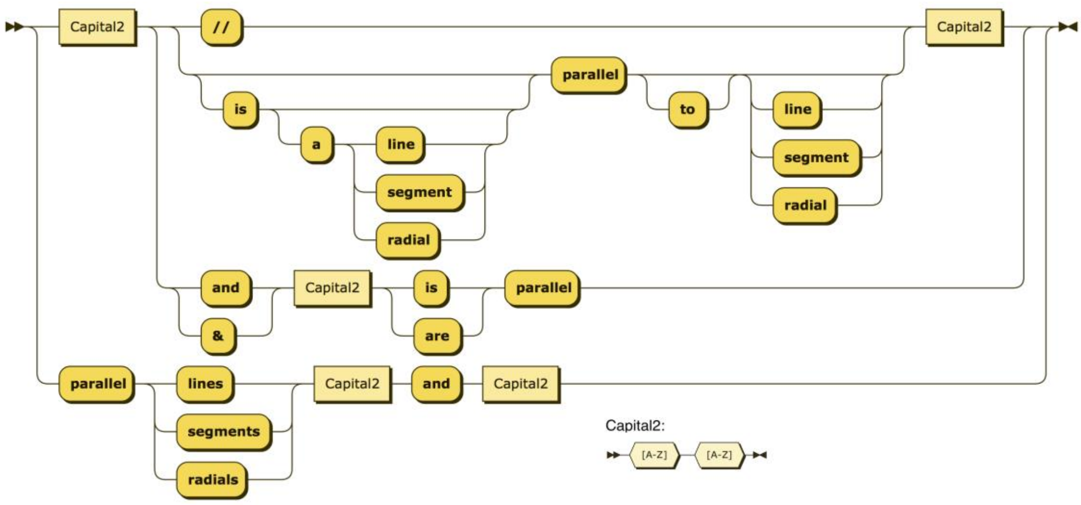


图 7(b) 英文几何题线平行关系的亮显识别的语法图

（3）从已排序优化的语法图构造正则表达式，将其作为自动作图产生式规则的条件部分。而结论部分则为自动作图的一阶谓词逻辑描述。采用规则排序的冲突解决策略，一行文本经去除空白字符的预处理后依次应用每个规则进行规则匹配，最先匹配成功且位置最靠前的规则为最佳匹配规则，然后应用此规则提取谓词、关键字和关系符号，算法见图 8 描述。表 3 列举了产生式规则的部分示例，其中结论部分的谓词表示自动作图和亮显的类型名。对于“*AB*∥*CD*”等的二元图形关系采用两个一阶谓词逻辑描述表达图形亮显信息，不使用类似于“perpendicular(segment(CD), segment(EF))”的二阶谓词逻辑描述图形关系是因为本几何讲读工具暂时还没有保留关系类型的必要。

表 3 (a) 中文几何题自动作图的产生式规则（局部）

条件：正则表达式	输入文本	结论：一阶谓词逻辑
[A-Z]{2}(平行(于)?|∥)[A-Z]{2}	*AB*∥*CD*	segment(AB，∥)、segment(CD)
[A-Z]{2}(和|与|、)[A-Z]{2}平行	*AB* 和 *CE* 平行	segment(AB，∥)、segment(CE)
[A-Z]{2}(和|与|、)[A-Z]{2}垂直	*AC* 与 *EF* 垂直	segment(AC，⊥)、segment(EF)
[A-Z]{2}(=|＝|等于)[A-Z]{2}	*AB*=*CD*	segment(AB，＝)、segment(CD)
[A-Z]{4}共圆	四点 *ABEP* 共圆	circle3p(ABEP)
(圆|⊙)[A-Z]{3}	圆 *ABC*	circle3p(ABC)
(圆|⊙)[A-Z]	⊙*O*	circle(O)
(角|∠)[A-Z]{3}	∠*PAB*	angle3p(PAB)
(角|∠)([A-Z]|[0-9]|[\u0370-\u03ff])	∠*β*	angle(β)
[A-Z]{3,8}	平行四边形 *ABCD*	polygon(ABCD)
[A-Z]{2}的延长线|延长[A-Z]{2}	*CD* 的延长线	extension(CD)
射线([A-Z]{2}|[a-z])	射线 *AB*	ray(AB)
(线段)?([A-Z]{2}|[a-z])	*AB*	segment(AB)
[A-Z]	点 *P*	point(P)

表 3 (b) 英文几何题自动作图的产生式规则（局部）

条件：正则表达式	输入文本	结论：一阶谓词逻辑
parallel(lines|segments|radials)[A-Z]{2} and[A-Z]{2}	two parallel lines *AB* and *CD*	segment(AB, //), segment(CD)
[A-Z]{2}\/\/[A-Z]{2}|[A-Z]{2} (and|&)[A-Z]{2}(is|are)parallel	*AB*//*CD* *AB* and *CD* is parallel	segment(AB, //), segment(CD)
[A-Z]{2}(is(a(line|segment|radial))?)? parallel(to)?(line|segment|radial)?[A-Z]{2}	*AB* is parallel to *CD* *AB* parallel *CD*	segment(AB, //), segment(CD)

注：表 3(b) 仅生成了英文几何题的线平行关系的自动作图的产生式规则，还没有深入分析所有几何元素的自动作图映射。

对几何题的每行文本依次进行规则匹配，结果用四元组（行号，关键字，谓词，关系符号）表达，其 JSON 格式和解析示例如表 4 所示。例如第六行的“AC 与 EF 垂直”的结果是（6，AC，'segment'，'⊥'）、（6，'EF'，'segment'，null）。其中“关系符号”用于延长高亮显示时间。“关键字”是匹配内容中的字母序列（不包含完整的匹配内容是为了降低同步亮显接口的数据复杂性，增强文字转语音引擎的可替代性）。例如原文“∠PBA”和“四点 ABEP 共圆”的关键字分别是“PBA”和“ABEP”。

表 4　几何题文本解析示例

输入文本行数组	解析结果：四元组 （行号，谓词，关键字，关系符号）
['AB // CD，AC 与 EF 垂直', '四点 ABEP 共圆', '∠PAB=∠β'] *['AB // CD，AC is vertical to EF',* *' The four points ABEP are concyclic ',* *'∠PAB= ∠β']*	[{"line":1,"name":"segment","key":"AB","rel":" // "}, {"line":1,"name":"segment","key":"CD"}, {"line":1,"name":"segment","key":"AC","rel":"⊥"}, {"line":1,"name":"segment","key":"EF"}, {"line":2,"name":"circle3p","key":"ABEP"}, {"line":3,"name":"angle3p", "key":"PAB"}, {"line":3,"name":"angle", "key":"β "}]
['直线 EF 与⊙O 相切于点 M'] *[' The line EF is tangent to ⊙O at point M ']*	[{"line":1,"name":"line", "key":"EF"}, {"line":1,"name":"circle","key":"O"}, {"line":1,"name":"point", "key":"M"}]

注：此列“输入文本行数组”的英文描述仅为效果图，并没实际研发出来。

```
function pickKey(t) {return t.replace(/[^A-Za-z0-9\u0370-\u03ff]/g, '');} //提取字母和数字
function parse2Segments(text) {
  var a = text.replace(/[^A-Z].+$/, '');     // 提取第一个字母序列
  return [['segment', a], ['segment', pickKey(text.substring(a.length))]];
}
var rules = [          // 匹配规则集，简单规则省略 parse 函数
  { re: /[A-Z]{2}(平行(于)?|∥)[A-Z]{2}|[A-Z]{2}(和|与|、)[A-Z]{2}平行/,
    type: '∥', parse: parse2Segments },
  //...
  { type: 'circle3p', re: /[A-Z]{4}共圆|(圆|⊙)[A-Z]{3}/ },
  //...
  { type: 'extension', re: /[A-Z]{2}的延长线|延长[A-Z]{2}/ },
  { type: 'segment', re: /(线段)?[A-Z]{2}/ },
  { type: 'circle', re: /(⊙|圆)[A-Z]/ },
  { type: 'point', re: /[A-Z]/ }
];
function applyRule(queue, line, rule, text) {
  var r = rule.parse ? rule.parse(text) : [rule.type, pickKey(text)];
  if (r[0] instanceof Array) {  // 如果是图形关系结果，则分别添加两个图形的结果
    queue.push({ line: line, name: r[0][0], key: r[0][1], rel: rule.type
           }, { line: line, name: r[1][0], key: r[1][1] });
  } else {                       // 是简单结果，则直接添加结果
    queue.push({ line: line, name: r[0], key: r[1] });
  }
}
function parseLines(lines) {                  // 解析几何题的文本行
  var queue = [];
  lines.forEach(function(text, line) {       // 解析每一行
    text = text.replace(/\s+/g, '');          // 取一行文本，去除空白字符
    while (text) {                            // 开始匹配文本，直到没有匹配项或已到行尾
      pos = 9999; rule = null;                // 匹配位置初始化为最大值
      rules.forEach(function(r) {             // 顺序尝试每个规则
        t = text.search(r.re);                // 记下该规则的匹配位置
        if (t >= 0 && pos > t) {              // 如果匹配位置更靠前，则为最佳匹配候选项
          pos = t; rule = r;
        }
      });
      if (rule) {                                   // 找到最佳匹配项
        t = text.match(rule.re)[0];                 // 匹配到的文本
        applyRule(queue, line + 1, rule, t);        // 提取关键字和谓词
        text = text.substring(pos + t.length);      // 继续该行的后续匹配
      } else { break; }                             // 没有匹配项就结束本行
    }
  });
  return queue;
}
```

图 8　基于规则排序的匹配算法

4.4 自动作图和高亮显示

基于动态几何软件实现几何知识自动作图的研究集中在实现全自动智能作图[2,5]。但受自然语言复杂性和图形位置难以预测的影响，存在图形不够美观和系统过于复杂的问题。本文采用人工干预和自动作图结合的方案：提前绘制几何题的初始图形、解答内容的辅助图形和通过指令接口无法准确绘制的图形（例如角度符号和相等符号），将这些预置图形保存在一个 GGB 文件中，在讲读运行时在网页中加载这些预置图形（由 C++应用将 GGB 文件转换为 BASE64 编码串，传递给 GGB 接口实现加载[1]）并动态创建亮显效果的图形。这样就解决了图形定位问题，充分利用 GGB 软件的易用性降低几何讲读工具的复杂性。

中学几何题通常有题目和数量不等的解答部分，对应的图形通常会有差异，

例如解答部分有辅助线。对于动态切换题目和不同解答部分对应的图形，本文采用如图 9 所列算法动态切换预置图形的显示条件来实现：对解答部分特有的图形，设置其显示条件（在图形属性对话框的"高级"页面中）为"group==1"（表示仅在第一个解答部分才显示）或"line>=12"（表示从几何题的第 12 行起才显示），而对题目内容对应的图形则不设置显示条件，让其总是显示。"group"和"line"变量可以在 GGB 软件的指令栏中初始化（例如输入"group=1"），在讲读运行时调用 GGB 接口的 evalCommand 函数动态设置值，从而实现动态显示。

```
var Hi = window.Hi = window.Hi || { defaultTimeout: 1500 };
Hi.init = function(text) {     // 根据几何题文本, 初始化 GGB 亮显的环境参数
  Hi.lines = text.replace(/\n\r|\r\n/g, '\n').split('\n'); //拆分文本行
  Hi.queue = parseLines(Hi.lines).map(function(r) {
    var timeout = r.rel ? 'Hi.defaultTimeout * 2' : 'Hi.defaultTimeout';
    var s = 'Hi.' + r.name + "('" + r.key + "', {timeout: " + timeout + '})';
    r.show = new Function(s); // 根据谓词构造亮显函数
    return r;
  });
};
Hi.seek = function(line, key) { // 根据行号和关键字亮显特定图形
  for (var i = 0; i < Hi.queue.length; i++) {
    var item = Hi.queue[i];
    if (item.line === line && item.key === key) {
      ggbApplet.evalCommand ('line=' + line);
      ggbApplet.evalCommand ('step=' + (i + 1));
      ggbApplet.evalCommand ('order=' + (item.line * 10 + item.col + 1));
      item.show();  // 调用亮显函数, 例如 Hi.segment ('AB', {timeout:1000})
      break;
    }
  }
};
```

图 9　作图指令映射和亮显定位的算法

本文的动态亮显方案是将 4.3 节的语义理解结果的谓词逻辑描述映射到 GGB 的作图指令（见图 9 所列算法的"Hi.init"动态创建显示函数部分），然后为每个谓词编写同名的 JS 亮显函数，在其中调用 GGB 的接口函数（例如谓词"angle3p"对应的亮显函数名为"Hi.angle3p"，实现见图 10 所示），并为临时创建的图形设置亮显颜色和加粗等属性，通过延时删除图形实现高亮显示效果。例如对表 3 中的谓词逻辑描述采用表 5 所示的作图指令自动作图。其中"$AB // CD$"等二元图形关系的谓词逻辑将映射到多个作图指令，通过设置不同的延时删除时间达到重叠亮显效果，例如让线段 AB 亮显持续到线段 CD 亮显完成后才消失。

```
function addHighLight(name, attr) {          // 设置亮显图形的显示属性
  ggbApplet.setLabelVisible(name, false);   // 不显示标签，仅亮显图形
  ggbApplet.setColor(name, 0, 0, 255);       // 默认蓝色亮显
  ggbApplet.setFixed(name, true);            // 亮显图形不可拖动
  if (attr && attr.timeout) {                // 延时删除亮显图形
    setTimeout(function(){ggbApplet.deleteObject(name);}, attr.timeout);
  }
  return name;
}
// 线段亮显函数，例如调用 Hi.segment('AB', {timeout: 1000})
Hi.segment = function(ab, attr) {
  var hi = ab + '_{his}';  // 下面根据端点创建线段，要求提前在 GGB 文件中画出端点
  ggbApplet.evalCommand(hi + '=Segment[' + ab[0] + ',' + ab[1] + ']');
  ggbApplet.setLineThickness(hi, 7);      // 加粗亮显
  return addHighLight(hi, attr);
};
// ∠ABC 形式的角度亮显函数，例如调用 Hi.angle3p('PAB', {timeout: 1000})
Hi.angle3p = function(abc, attr) {
  Hi.segment(abc[0] + abc[1], attr);     // 亮显第一条边
  Hi.segment(abc[1] + abc[2], attr);     // 亮显第二条边
};
```

图 10　亮显函数示例

表 5　自动作图的指令映射示例

输入文本	结论：一阶谓词逻辑	GGB 作图指令或函数
AB // CD	segment(AB，//)、segment(CD)	Segment[A,B]、Segment[C,D]
AB=CD	segment(AB，＝)、segment(CD)	
四点 ABEP 共圆 *The four points ABEP are concyclic*	circle3p(ABEP)	Circle[A,B,E]
圆 ABC *Circle ABC*	circle3p(ABC)	Circle[A,B,C]
⊙O	circle(O)	Circle[O，查的同心圆名称]
∠PAB	angle3p(PAB)	Angle[P,A,B]
平行四边形 ABCD *Parallelogram ABCD*	polygon(ABCD)	Polygon[A,B,C,D]
CD 的延长线 *The extension line of CD*	extension(CD)	Ray[D, Reflect[C,D]]
射线 AB *Ray AB*	ray(AB)	Ray[A,B]
AB	segment(AB)	Segment[A,B]

注：此列“输入文本”的英文描述仅为效果图，并没完全实际研发出来。

5 创新点

本研究的创新点主要有以下几点：

（1）讲题中“文字+语音+图形”同步互动。语音合成朗读的进度与文本行亮显及图形元素亮显同步进行，形象地展示解题过程中几何辅助线和几何标记的动态演变过程，构建视听双通道的认知环境；

（2）可以自由控制讲题进度，随意跳转改变讲题流程。既可以通过“上一行”“下一行”“重复”控制讲题进度，也可以双击任意一行跳转改变讲题流程；

（3）采用了文字自动语音合成的朗读方式，无需人工讲题。这样减少了教学过程中的重复劳动，也避免了发音不正的问题。老师可根据教学反馈多次修改解答内容（包括解题思路、解答过程和点评等）形成精品例题，利于互联网传播和传承教学。

（4）基于 JavaScript 正则式实现几何题的语义理解和几何元素智能提取，适合常见的几何题目和解答文本。基于 GGB 实现了几何题同步亮显的自动作图，无需人工找对应图形。

6 未来展望

本讲题工具还需在实际应用中不断积累和完善，可以从以下几个方面做进一步的研究：

（1）采集和测试更多的中文平面几何题的真题样本，进一步提高平面几何真题讲题的正确率。还可进一步研究支持解析几何和立体几何。

（2）目前本研究采用人工干预和自动作图相结合的方案，题目的初始化图形需用户自己利用 GGB 作图软件先做做好，给推广带来了一定的难度。后期需要深入研究全自动智能作图技术，除了现有的亮显自动作图外，逐步实现初始图形的全自动作图,以便降低制作和推广难度。

（3）全面支持英文几何题的讲题，实现多语种的几何讲题。英汉语言在语言类型、文字系统、语音系统以及词法和句法上的差异很大。要全面支持英文几何题的讲题，还需要从以下方面开展工作：

- 寻找大量英文几何题的样本。（中文是我的母语，在之前的学习经历中，已经见过和积累了大量中文几何题的样本。英文几何题见的很少，只有在大量的英文几何题真题分析上，才能更好的分析和进行语义理解），
- 需要为讲题工具找英文语音合成内核的研发友情支持，或者找一个开源的英文语音合成内核。C++主程序端需开发集成英文语音合成；

- C++主程序端，需要对英文几何题文本的进行专门的预分析处理分析，一般英文语音合成内核只对常见的文本支持的好，对于领域性比较强的英文几何题一般合成都不太好，需专门进行预处理，才能保证正确合成朗读和正确提取字母串关键字。
- JS 程序端英文几何题文本的语义理解。在研究大量英文几何题真题的基础上，用扩展的巴科斯范式（EBNF）来表述英文几何题模式匹配，再合并排序优化成语法图，用语法图不断迭代来辅助思考，避免匹配规则的冲突并采用相应的策略，最终构造出英文几何题的正则表达式，作为自动作图的产生式规则的条件部分。产生式规则的结论部分，即一阶谓词逻辑，是可以和中文几何题的完全一致的。

7 结束语

本文针对常见的几何讲题方式的局限性，提出了基于几何语义理解、GGB 自动作图和语音合成内核（TTS）的几何自动讲题方式，并基于 HTML5 和 Hybrid 混合架构开发了语音可视化几何讲题工具，具有语音朗读与图形亮显同步、易于修改和随机回放的特点。

教学中电脑的普及给这种讲题方式奠定了一定的基础，并且几何题文本和图形 GGB 文件占用空间很小，非常便于传播。本几何讲题方式适合人工智能时代的互联网教学和创新性教学。

参考文献

[1] 张云贵．GeoGebra 教育资源云平台的构建研究[J]. 中国现代教育装备, 2017(1):19-21.

[2] 陈肖宇. 电子几何教科书系统的设计与实施[D]. 北京航空航天大学，2011.

[3] 洪宇. 将 webkit 内核封装为 duilib 的浏览器控件[EB/OL]. [2014-07-26]. http://blog.csdn.net/zhuhongshu/article/details/38145651.

[4] 黄焕, 刘清堂, 陈矛. 面向平面几何命题理解的融合算法研究[J]. 计算机科学, 2013, 40(8):196-199.

[5] 郭海燕. 受限域下动态几何自然语言作图方法研究[D]. 华中师范大学, 2012.

[6] 马创新. 论知识表示[J]. 现代情报, 2014, 34(3):21-24.

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本人喜欢运动，唱歌等，乒乓球打得不错，担任班上的财务委员。喜欢参加各项课内外活动，高中阶段，在北京古动物馆做过志愿者讲解员；参加了 12 天的 2017 冬季普林斯顿青少年领袖峰会；成为“清华附中第六届自主课堂”讲师，给大家讲《青年理财》等。

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【数学】：2017 年全国高中数学联赛　　北京市二等奖

【数学】：2017 年北京中学生高一数学竞赛　　北京市一等奖

【物理】：第 30 届北京市高一物理力学竞赛　　北京市一等奖

【英语】：第 16 届全国英语创新大赛　　全国一等奖

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