Comment on "Cosmological Topological Massive Gravitons and Photons"

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Abstract

In a recent paper [1] it was shown that all global energy eigenstates of asymptotically AdS_3 chiral gravity have nonnegative energy at the linearized level. This result was questioned [2] by Carlip, Deser, Waldron and Wise (CDWW), who work on the Poincare patch. They exhibit a linearized solution of chiral gravity and claim that it has negative energy and is smooth at the boundary. We show that the solution of CDWW is smooth only on that part of the boundary of AdS_3 included in the Poincare patch. Extended to global AdS_3, it is divergent at the boundary point not included in the Poincare patch. Hence it is consistent with the results of [1].

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We consider the theory of TMG (topologically massive gravity) with a negative cosmological constant. This is described by [3][4][5][2]

$$I = \frac{1}{16\pi G}\int d^3x\sqrt{-g}(R+\frac{2}{\ell^2}) + \frac{1}{16\pi G\mu}I_{CS} \tag{1}$$

where I_{cs} is the Chern-Simons term

$$I_{cs} = -\frac{1}{2}\int d^3x\sqrt{-g}\epsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}[\partial_\mu\Gamma^{\sigma}_{\rho\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}]. \tag{2}$$

TMG has an AdS_3 vacuum solution:

$$ds^2 = \ell^2(-\cosh^2\rho d\tau^2 + \sinh^2\rho d\phi^2 + d\rho^2) \tag{3}$$

For generic signs and values of the parameters TMG will have solutions which are asymptotically AdS_3 (by which we always mean obeying the precise boundary conditions given in Brown and Henneaux [6] and discussed in the context of TMG recently in [7]) but have negative energies. This limits the physical interest of the generic TMG theory. However in [1] it was pointed out that for the case $\mu\ell = 1$ of so-called chiral gravity all known linearized and nonlinear asymptotically AdS_3 solutions have nonnegative energy. It was further conjectured that both the classical and quantum theories of chiral gravity are consistent and stable. The terminology chiral gravity was adopted because the central charge of the dual CFT, if it exists, must be [6][8][9] $(c_L, c_R) = \frac{3\ell}{2G}(1-\frac{1}{\mu\ell}, 1+\frac{1}{\mu\ell})$ and hence is purely chiral at $\mu\ell = 1$.

Solutions of the linearized equations which are energy eigenstates (where energy is defined as in [10][11]using the global time τ) fall into $SL(2,R)_L \times SL(2,R)_R$ representations which can be characterized by the $(L_0, \bar{L}_0)$ eigenvalues (h_L, h_R) of the highest weight states. In [1] it was shown that there are three representations labelled by the highest weights (h_L, h_R)

$$(2,0), \quad (0,2), \tag{4}$$

corresponding to massless left and right moving boundary gravitons and

$$(\frac{3}{2}+\frac{\mu\ell}{2}, -\frac{1}{2}+\frac{\mu\ell}{2}), \tag{5}$$

[2] We have chosen here the standard sign in front of the Einstein-Hilbert action so that BTZ black holes have positive energy for large μ.This contrasts with most of the literature which chooses the opposite sign in order that massive gravitons have positive energy (for large μ).

corresponding to the so-called massive gravitons. For $\mu\ell > 1$, despite the positivity of the weights, the massive gravitons have negative energy because the kinetic term has an overall negative prefactor of $(\frac{1}{\mu\ell} - 1)$.

It was pointed out [1] that for chiral gravity, the Compton wavelength of the massive graviton becomes of order the AdS_3 radius, the weights and the wavefunctions of the massless left-moving and massive gravitons degenerate and the energies of both becomes zero. Hence all known asymptotically AdS_3 solutions have nonnegative energy in this special case.

CDWW claim there are asymptotically AdS_3 solutions of TMG with negative energy even at the chiral point.[3] They work in Poincare coordinates for which the line element is (CDWW equation (35))

$$ds^2 = \frac{2dx^+dx^- + dz^2}{z^2}. \tag{6}$$

They find a continuum (rather than a discrete set) of solutions of the linearized wave equations, work out the linearized Einstein tensor $\mathcal{H}_{\rho\sigma}$ (CDWW equation (44)). They find that e.g. $\mathcal{H}_{--}$ is given by (CDWW equation (48) and page 12)

$$\mathcal{H}_{--} \sim zJ_3(z) \to z^4, \quad z \to 0 \tag{7}$$

where J is a Bessel function. This indeed vanishes at the boundary $z = 0$ of the Poincare patch. However this does not cover all of the boundary points of global AdS_3. The missing boundary point can be approached by fixing Poincare time $x^+ - x^- = constant$ and taking $z \to \infty$. From Bessel function asymptotics it is then easily seen that the curvature diverges as

$$\mathcal{H}_{--} \sim \sqrt{z}\cos z, \quad , \quad z \to \infty \tag{8}$$

In a local orthonormal frame the divergence is faster by a factor of z^2. A simple analogy is the function $f(z) = z$ in the upper half complex z plane. It vanishes everywhere on the boundary $Im\ z = 0$. However the boundary of the Poincare disc includes the point $Im\ z = \infty$ where f diverges.

In conclusion, the curvature tensor of the CDWW negative-energy massive graviton solutions of chiral gravity diverges on the boundary of global

[3] Actually to be more precise CDWW use a negative Newton's constant so the energy is positive, but with our sign choice it is negative.

AdS_3. This is fully consistent with the observation of [1] that all asymptotically AdS_3 $(L_0, \bar{L}_0)$ eigensolutions of linearized chiral gravity have nonnegative energy.

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