Weak gravity conjecture in the asymptotical dS and AdS background

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(Dated: December 22, 2013)

The cosmological observations provide a strong evidence that there is a positive cosmological constant in our universe and thus the spacetime is asymptotical de Sitter space. The conjecture of gravity as the weakest force in the asymptotical dS space leads to a lower bound on the U(1) gauge coupling g, or equivalently, the positive cosmological constant gets an upper bound $\rho_V \leq g^2 M_p^4$ in order that the U(1) gauge theory can survive in four dimensions. This result has a simple explanation in string theory, i.e. the string scale $\sqrt{\alpha'}$ should not be greater than the size of the cosmic horizon. Our proposal in string theory can be generalized to U(N) gauge theory and gives a guideline to the microscopic explanation of the de Sitter entropy. The similar results are also obtained in the asymptotical anti-de Sitter space.

PACS numbers:

Particle physicists have told us the low energy physics is perfectly described by the standard model. Recent cosmological observations provide a strong evidence that there is a positive cosmological constant in our universe. As the only well formulated candidate for quantum gravity, string theory shall be connected with the phenomena in our universe. A central topic along the line in string theory is to answer why the theory for the low energy phenomena is the standard model, why there is a cosmological constant, and why it is the value we observed.

String theory is only consistent in ten dimensions in order to cancel the conformal anomaly on the string world sheet. To understand four dimensional physics in our real world, we must compactify string theory on some manifolds. Recent developments for the flux compactifications [1], however, suggest that a huge number of at least semi-classically consistent string vacua emerge in string theory, named string landscape [2]. It may or may not provide an opportunity for us to explore the specific low energy phenomena in the experiments from the viewpoint of string theory. Given the numerous "vacua" in the string landscape, the most urgent problem is to find a reliable vacuum selecting principle.

In [3], Vafa proposed that self-consistence of a quantum theory of gravity offers a way to pick out which effective field theories can arise. Many vacua in the string landscape, although consistent semi-classically, are actually inconsistent on the quantum level, called swampland in [3]. Self-consistent landscape is surrounded by the swampland. Then the problem is translated into finding the criteria to pick out really self-consistent landscape from the swampland. More criteria for self-consistent effective field theory have been proposed in [4, 5]. Eventually we expect that more and more consistent conditions will be found and the range for otherwise free parameters will be narrowed down and the predictions of string theory can be checked in the experiments.

The authors in [4, 5] proposed some criteria for the self-consistent effective field theory in four-dimensional asymptotical flat spacetime. However, there are a huge number of string vacua with positive or negative cosmological constants in string landscape. What is more, our universe is asymptotical de sitter space which is favored by recent cosmological observations. So we are motivated to explore the criterion for self-consistent effective field theory in asymptotical de Sitter and Anti-de Sitter background. On the other hand, the well-defined string theory and quantum field theory in asymptotical dS space are still unknown. But we try to explore them heuristically in this note.

In [4], gravity is conjectured as the weakest force. It is the claim that for a $U(1)$ theory there exists a charged particle whose mass is smaller than its charge in some appropriate unit. The conjecture is supported by string theory and some evidence involving black holes and symmetries. The conjecture leads to an intrinsic bound on the UV cutoff Λ for a consistent U(1) gauge field theory in asymptotical flat four dimensional spacetime which takes the form

$$\Lambda \leq g M_p \sim g/\sqrt{G}, \tag{1}$$

where g is the U(1) gauge coupling. An intrinsic radius of curvature appears in the asymptotical de Sitter and Anti-de Sitter background. An infrared cut-off of the effective field theory should be greater than, if not equal to, the radius of the background. Moreover, the UV cutoff of the effective field theory should be smaller than the curvature radius, otherwise the full quantum gravity or string theory must be invoked to describe the situation. We shall show that the latter condition combined with the conjecture of gravity as the weakest force yields a lower bound on the U(1) gauge theory; or equivalently, an upper bound on the cosmological constant. We also propose a heuristic explanation in string theory for this bound of the coupling constant.

Let us start with the metric of Schwarzschild-de Sitter solution in four dimensional spacetime

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_2^2, \tag{2}$$

with

$$f(r) = 1 - \frac{2Gm}{r} - \frac{r^2}{L^2}, \tag{3}$$

where G is the Newton constant and $L = \sqrt{3/(8\pi G\rho_V)}$ is the size of the pure de Sitter space with a positive cosmological constant ρ_V. For a U(1) gauge theory, the mass scale of the minimally charged monopole is Λ/g^2 and its size is of order $1/\Lambda$, where Λ is the cutoff of the field theory. The size of the black hole horizon r_- and cosmic horizon r_+ satisfies

$$r_\pm^3 - L^2 r_\pm + 2G\frac{\Lambda}{g^2}L^2 = 0. \tag{4}$$

Requiring that this monopole is not black and smaller than the cosmic horizon, namely $r_- \le \frac{1}{\Lambda} \le r_+$, yields

$$\frac{1}{\Lambda^3} - \frac{L^2}{\Lambda} + 2G\frac{\Lambda}{g^2}L^2 \le 0 \tag{5}$$

or,

$$\Lambda^4 - \frac{g^2}{2G}\Lambda^2 + \frac{g^2}{2GL^2} \le 0. \tag{6}$$

Demanding that there is a solution for the inequality (6) yields

$$g \ge \frac{\sqrt{8G}}{L}, \tag{7}$$

or equivalently,

$$\rho_V \le g^2/G^2 \sim g^2 M_p^4. \tag{8}$$

If there is a very weak U(1) gauge theory with gauge coupling $g \sim 10^{-60}$, the cosmological constant is roughly the same as that we observed. Solving the inequality (6), we find a bound on the cutoff for U(1) gauge theory which takes the form

$$\frac{g}{2\sqrt{G}}\sqrt{1-\sqrt{1-\frac{8G}{g^2L^2}}} \le \Lambda \le \frac{g}{2\sqrt{G}}\sqrt{1+\sqrt{1-\frac{8G}{g^2L^2}}}. \tag{9}$$

For a fixed gauge coupling, in the limit with $\rho_V \to 0$ or $L \to \infty$, eq. (9) is just the same as eq. (1). When gauge coupling goes to its lower bound, the UV cutoff for this U(1) gauge field theory is $\Lambda \sim g/\sqrt{G} \sim 1/L$.

Surprisingly, eq.(7) shows that a positive cosmological constant induces a lower bound on the U(1) gauge coupling. Or equivalently, the positive cosmological constant can not be arbitrarily large in order that a consistent U(1) gauge theory can survive. The most important input at this point is the requirement that the size of the minimally charged monopole is not larger than the cosmic horizon in the asymptotical de Sitter space. This is also the condition for us to trust the above estimates about the mass scale and the size of the monopole.

There is a simple physical explanation of eq.(7). $1/\Lambda$ is roughly the shortest physical length for the U(1) gauge field theory. It is natural to demand $1/\Lambda$ be no larger than the size of cosmic horizon, namely $\Lambda > 1/L$. Together with $\Lambda \le gM_p$, this directly leads to eq.(7) and (8).

Another heuristic consideration leading to eq.(7) is the following. In order that there is no naked singularity in the space-time, the mass of the minimally charged monopole is not greater than the mass parameter of the Nariai Black hole L/G, namely

$$\frac{\Lambda}{g^2} \le \frac{L}{G}, \quad \text{or} \quad \Lambda \le \frac{g^2 L}{G}. \tag{10}$$

On the other hand, the size of the monopole $1/\Lambda$ should not be larger than the size scale of the cosmic horizon L; or equivalently, $\Lambda \ge 1/L$. Substituting this relationship into eq. (10), we obtain eq. (7) and (8) again.

In the more formal derivation using the metric (2), we did not introduce in $f(r)$ the contribution of the magnetic charge which is roughly $Gg^{-2}r^{-2}$, this term is smaller than GMr^{-1} if the horizon size is larger than $1/\Lambda$. Or it is larger than GMr^{-1} if the horizon size is smaller than $1/\Lambda$, in this case we obtain the condition $\Lambda \le gM_p$. Thus, if we include the term $Gg^{-2}r^{-2}$ in the above formal discussion, we will end up with inequalities similar to (9).

In [4], the authors argued that the absence of global symmetries in quantum gravity requires that the field theory description should break down in the limit $g \to 0$, since the symmetry can be identified as a global symmetry. The way to avoid this problem in [4] is the UV cutoff also goes to zero when $g \to 0$. In a asymptotical de Sitter space, an intrinsic lower bound on the gauge coupling is induced by the size of the cosmic horizon (7), which shows that the effective gauge field theory already breaks down before taking the limit $g \to 0$. The gauge coupling characterizes the strength of the local interaction. Eq. (7) implies that the size of the system can affect the local interaction in quantum theory.

We pause to discuss the most important premise in our discussion, namely a minimally charged monopole should not be a black hole. The following reasoning not only applies to an asymptotic Minkowski space, it also applies to an asymptotic de Sitter space. Imagining that a minimal charged monopole is indeed a black hole, thus it will Hawking radiate (the horizon size is greater than the field theory UV cut-off which in turn is greater than the Planck scale). During the radiation process, neutral particles as well as charged particles can be radiated. If only neutral particles are radiated, the black hole's mass becomes smaller while its magnetic charges remains the same, this implies that there exists monopoles with smaller mass. If charges are radiated, these charges must be smaller than that of the original monopole, this contradicts our assumption. Of course the above argument does not apply to a general black hole with magnetic charge, since it may be formed of many minimally charged monopoles and other matter, thus is neither minimally charged nor with smaller mass. [4] derives the weak gravity inequality using the absence remnants, which is valid also in an asymptotic de Sitter space.

We now switch to string theory. Consider the brane world scenario in Type IIB string theory. The tension of the D3-brane $T_3 \sim M_s^4/g_s$ is taken as the effective cosmological constant on the brane and the U(1) gauge coupling is related to the string coupling g_s by $g \sim g_s^{1/2}$. According to eq. (8), we obtain a constraint on the string scale and the string coupling, namely

$$M_s^2 \leq g_s M_p^2. \tag{11}$$

Note that the string theory in four dimensions is reduced from ten dimensions. For toroidal compactification, if the average size of the extra dimension is R, the Planck scale in four dimensions is

$$M_p^2 \sim R^6 M_s^8/g_s^2 = (RM_s)^6 M_s^2/g_s^2. \tag{12}$$

Requirement (11) implies

$$g_s \leq (RM_s)^6. \tag{13}$$

In general we assume $RM_s > 1$; otherwise, we switch to a T-dual description. For weakly coupled string theory $g_s \leq 1$, this condition is always satisfied. The constraint on the string coupling is quite loose.

In string theory, we can find a simple explanation about the lower bound on the gauge coupling or string coupling in the asymptotical de Sitter space. Only when the length of string $\sqrt{\alpha'}$ is shorter than the size of the cosmic horizon, the stringy effects can be ignored and the description of the effective field theory is reliable. This is just the condition that the Hawking temperature is lower than the string Hagedorn temperature. Thus we require

$$\sqrt{\alpha'} \leq L, \quad \text{or} \quad \rho_V \leq \frac{1}{G\alpha'}. \tag{14}$$

In four dimensions Newton's constant is related to the string coupling and string length square $\alpha' \sim 1/M_s^2$ by

$$G \sim g_s^2 \alpha' \tag{15}$$

up to a coefficient which depends on the compactification. Substituting eq. (15) into eq. (14), we obtain

$$\rho_V \leq g_s^2 M_p^4. \tag{16}$$

In an asymptotical de Sitter space string coupling can not be arbitrarily weak. For a given string coupling, an upper bound on the cosmological constant appears; Above the bound, the effective gauge field theory on the brane breaks down.

Even though a well-defined string theory in asymptotical de Sitter is still unknown, the above discussions provide a useful constraint on possible realizations of de Sitter space. We now re-investigate the brane world scenario in Type II B string theory more carefully. Assume the string theory in four dimensions be reduced from ten dimensions by toroidal compactification. Eq. (15) is modified to

$$G \sim g_s^2 \alpha' (RM_s)^{-6} \tag{17}$$

Thus eq. (16) takes the form

$$\rho_V \leq g_s^2 M_p^4 (RM_s)^{-6}. \tag{18}$$

Identifying ρ_V with $T_3 \sim M_s^4/g_s$ and using eq. (17) we find $g_s \leq (RM_s)^6$ which is exactly the same as eq. (13). This result obtained in string theory exactly matches the result obtained in the effective field theory.

We can go one step further in string theory. The field theoretical argument can not be generalized to the Non-Abelian gauge field theory, while the string theory argument can. Consider a stack of N D3-branes. The fields of the open string theory are in the adjoint representation of SU(N). For a stack of D3-brane, the effective cosmological constant becomes $NT_3 \sim NM_s^4/g_s$. In this case we simply obtain the constraint on the string coupling as

$$g_s N \leq (RM_s)^6. \tag{19}$$

The combination of the string coupling and N is nothing but t' Hooft coupling. This is to be expected. For fixed string coupling and size of the extra dimensions, an upper bound on the rank of the gauge group is obtained. Eq.(19) can be obtained from eq.(8) provided g^2 in eq.(8) is replaced by $g^2 N$.

The above discussions in strng theory can be generalized to diverse dimensions. For simplicity, we investigate a stack of N D9-brane. The Hubble parameter H on the brane takes the form

$$H \sim \sqrt{G_{10} T_9 N} \sim \frac{1}{l_s} \sqrt{g_s N}, \tag{20}$$

where the ten-dimensional Newton constant is given by $G_{10} \sim g_s^2 l_s^8$ and $T_9 \sim 1/(g_s l_s^{10})$. Requiring $\sqrt{\alpha'} \leq H^{-1}$ yields

$$g_s N \leq 1. \tag{21}$$

The 't Hooft coupling must be not greater than 1 in order that the gauge field theory on the brane is effective. The entropy of de Sitter space on the brane is

$$S \sim \frac{1}{H^8 G_{10}} \sim \frac{1}{(g_s N)^6} N^2. \tag{22}$$

For $g_s N \leq 1$, $S \geq N^2$. This is a reasonable result since the number of adjoint fields is no less than N^2. Recall the argument about de Sitter entropy in [6]. The authors considered a system of N unstable D9-brane in Type II A string theory. The basic requirement for eternal inflation is that the Hubble time H^{-1} should be larger than the time scale for the open string tachyon to fall off the top of its potential l_s, which yields $g_s N \geq 1$ by using eq. (20). The gauge field theory on the brane breaks down for eternal inflation unless $g_s N \sim 1$. For fixed t' Hooft coupling $g_s N \sim 1$, we can take $N \to \infty$ and $g_s \to 0$ without any need for closed string quantum corrections. Now the de Sitter entropy is just the square of the number of branes N^2. This provides a tentative calculation for the de Sitter entropy in string theory. We hope we can work out the details on this argument in the future. This is different with the case AdS/CFT where we require t' Hooft coupling is much greater than one in order that we can trust the geometry [7].

In an asymptotical Anti-de Sitter space-time, the similar results can be obtained. The metric of Schwarzschild Anti-de Sitter solution in four dimensional spacetime takes the form

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_2^2, \tag{23}$$

with

$$f(r) = 1 - \frac{2Gm}{r} + \frac{r^2}{L^2}, \tag{24}$$

where $L = \sqrt{-3/(8\pi G \rho_V)}$ is the size of the anti-de Sitter space with a negative cosmological constant ρ_V. The radius of the black hole r_{bh} satisfies

$$r_{bh}^3 + L^2 r_{bh} - 2GmL^2 = 0. \tag{25}$$

Requiring that the minimally charged monopole should not be black yields

$$\frac{1}{\Lambda^3} + \frac{L^2}{\Lambda} - 2G\frac{\Lambda}{g^2}L^2 \geq 0, \tag{26}$$

or equivalently,

$$\Lambda^4 - \frac{g^2}{2G}\Lambda^2 - \frac{g^2}{2GL^2} \leq 0. \tag{27}$$

Solving this inequality, we obtain the bound on the intrinsic UV cutoff for the U(1) gauge field theory, namely

$$\Lambda \leq \frac{g}{2\sqrt{G}} \left(1 + \sqrt{1 + \frac{8G}{g^2 L^2}} \right)^{1/2}. \tag{28}$$

On the other hand, we also require that the minimal physical length $1/\Lambda$ should be shorter than the radius of anti-de Sitter background; otherwise, the gauge field theory breaks down. Thus $g \geq \sqrt{G}/L$ or $|\rho_V| \leq g^2 M_p^4$. With the viewpoint of string theory, the similar results are also obtained.

In [8], the authors generalized the arguments in four dimensions in [4] to lower dimensions. Our previous discussions can also be used to investigate the cases in lower dimensions and the similar results are obtained.

To summarize, we have investigated the constraints on the effective gauge field theory in an asymptotical de Sitter and an anti-de Sitter background. But string theory still survives when the constraints are violated. A lower bound on the gauge coupling results from the requirement that the shortest length for the effective gauge field theory should be shorter than the radius of the background curvature. This result has a simple explanation in string theory. The discusisons in string theory can be generalized to diverse dimensions and the non-Abelian gauge field theory.

We also want to stress that we don't provide any concrete example to show how certain theories in certain de Sitter space cannot arise in string theory. We only say that there is no local field theory description for length scales shorter than the de Sitter radius if the latter itself is shorter than the string scale.

Acknowledgement. The work of QGH is supported by a grant from NSFC, a grant from China Postdoctoral Science Foundation and and a grant from K. C. Wang Postdoctoral Foundation. The work of ML and WS is supported by a grant from CAS and a grant from NSFC.

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