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Hilbert Space Operators in Quantum Physics

Second Edition

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To our wives and daughters

Preface to the second edition

Almost fifteen years later, and there is little change in our motivation. Mathematical physics of quantum systems remains a lively subject of intrinsic interest with numerous applications, both actual and potential.

In the preface to the first edition we have described the origin of this book rooted at the beginning in a course of lectures. With this fact in mind, we were naturally pleased to learn that the volume was used as a course text in many points of the world and we gladly accepted the offer of *Springer Verlag* which inherited the rights from our original publisher, to consider preparation of a second edition.

It was our ambition to bring the reader close to the places where real life dwells, and therefore this edition had to be more than a corrected printing. The field is developing rapidly and since the first edition various new subjects have appeared; as a couple of examples let us mention quantum computing or the major progress in the investigation of random Schrödinger operators. There are, however, good sources in the literature where the reader can learn about these and other new developments.

We decided instead to amend the book with results about new topics which are less well covered, and the same time, closer to the research interests of one of us. The main change here are two new chapters devoted to quantum waveguides and quantum graphs. Following the spirit of this book we have not aspired to full coverage — each of these subjects would deserve a separate monograph — but we have given a detailed enough exposition to allow the interested reader to follow (and enjoy) fresh research results in this area. In connection with this we have updated the list of references, not only in the added chapters but also in other parts of the text in the second part of the book where we found it appropriate.

Naturally we have corrected misprints and minor inconsistencies spotted in the first edition. We thank the colleagues who brought them to our attention, in particular to Jana Stará, who indicated numerous improvements. As with the first edition, we have asked a native speaker to try to remove the foreign "accent" from our writing; we are grateful to Mark Harmer for accepting this role.

Prague, December 2007

Pavel Exner
Miloslav Havlíček

Preface

Relations between mathematics and physics have a long and entangled tradition. In spite of repeated clashes resulting from the different aims and methods of the two disciplines, both sides have always benefitted. The place where contacts are most intensive is usually called mathematical physics, or if you prefer, physical mathematics. These terms express the fact that mathematical methods are needed here more to understand the essence of problems than as a computational tool, and conversely, the investigated properties of physical systems are also inspiring from the mathematical point of view.

In fact, this field does not need any advocacy. When A. Wightman remarked a few years ago that it had become "socially acceptable", it was a pleasant understatement; by all accounts, mathematical physics is flourishing. It has long left the adolescent stage when it cherished only oscillating strings and membranes; nowadays it has built synapses to almost every part of physics. Evidence that the discipline is developing actively is provided by the fruitful oscillation between the investigation of particular systems and synthetizing generalizations, as well as by discoveries of new connections between different branches.

The drawback of this rapid development is that it has become virtually impossible to write a textbook on mathematical physics as a single topic. There are, of course, books which cover a wide range of problems, some of them indeed monumental, but even they are like cities which govern the territory while watching the frontier slowly moving towards the gray distance. This is simply the price we have to pay for the flood of ideas, concepts, tools, and results that our science is producing.

It was not our aim to write a poor man's version of some of the big textbooks. What we want is to give students basic information about the field, by which we mean an amount of knowledge that could constitute the basis of an intensive one–year course for those who already have the necessary training in algebra and analysis, as well as in classical and quantum mechanics. If our exposition should kindle interest in the subject, the student will be able, after taking such a course, to read specialized monographs and research papers, and to discover a research topic to his or her taste. We have mentioned that the span of the contemporary mathematical physics is vast; nevertheless the cornerstone remains where it was laid by J. von Neumann, H. Weyl, and the other founding fathers, namely in regions connected with quantum theory. Apart from its importance for fundamental problems such as the constitution of matter, this claim is supported by the fact that quantum theory is gradually becoming a basis for most branches of *applied* physics, and has in this way entered our everyday life.

The mathematical backbone of quantum physics is provided by the theory of linear operators on Hilbert spaces, which we discuss in the first half of this book. Here we follow a well–trodden path; this is why references in this part aim mostly at standard book sources, even for the few problems which maybe go beyond the standard curriculum. To make the exposition self–contained without burdening the main text, we have collected the necessary information about measure theory, integration, and some algebraic notions in the appendices.

The physical chapters in the second half are not intended to provide a self–contained exposition of quantum theory. As we have remarked, we suppose that the reader has background knowledge up to the level of a standard quantum mechanics course; the present text should rather provide new insights and help to reach a deeper understanding. However, we attempt to describe the mathematical foundations of quantum theory in a sufficiently complete way, so that a student coming from mathematics can start his or her way into this part of physics through our book.

In connection with the intended purpose of the text, the character of referencing changes in the second part. Though the material discussed here is with a few exceptions again standard, we try in the notes to each chapter to explain extensions of the discussed results and their relations to other problems; occasionally we have set traps for the reader's curiosity. The notes are accompanied by a selective but quite broad list of references, which map ways to the areas where real life dwells.

Each chapter is accompanied by a list of problems. Solving at least some of them in full detail is the safest way for the reader to check that he or she has indeed mastered the topic. The problem level ranges from elementary exercises to fairly complicated proofs and computations. We have refrained from marking the more difficult ones with asterisks because such a classification is always subjective, and after all, in real life you also often do not know in advance whether it will take you an hour or half a year to deal with a given problem.

Let us add a few words about the history of the book. It originates from courses of lectures we have given in different forms during the past two decades at Charles University and the Czech Technical University in Prague. In the 1970s we prepared several volumes of lecture notes; ten years later we returned to them and rewrote the material into a textbook, again in Czech. It was prepared for publication in 1989, but the economic turmoil which inevitably accompanied the welcome changes delayed its publication, so that it appeared only recently.

In the meantime we suffered a heavy blow. Our friend and coauthor, Jiří Blank, died in February 1990 at the age of 50. His departure reminded us of the bitter truth that we usually are able to appreciate the real value of our relationships with fellow humans only after we have lost them. He was always a stabilizing element of our triumvirate of authors, and his spirit as a devoted and precise teacher is felt throughout this book; we hope that in this indirect way his classes will continue.

Preparing the English edition was therefore left to the remaining two authors. It has been modified in many places. First of all, we have included two chapters and some other material which was prepared for the Czech version but then left out due to editorial restrictions. Though the aim of the book is not to report on the present state of research, as we have already remarked, the original manuscript was finished four years ago and we felt it was necessary to update the text and references in some places. On the other hand, since the audience addressed by the English text is different — and is equipped with different libraries — we decided to rewrite certain parts from the first half of the book in a more condensed form.

One consequence of these alterations was that we chose to do the translation ourselves. This decision contained an obvious danger. If you write in a language which you did not master during your childhood, the result will necessarily contain some unwanted comical twists reminiscent of the famous character of Leo Rosten. We are indebted to P. Moylan and, in particular, to R. Healey, who have read the text and counteracted our numerous petty attacks against the English language; those clumsy expressions that remain are, of course, our own.

There are many more people who deserve our thanks: coauthors of our research papers, colleagues with whom we have had the pleasure of exchanging ideas, and simply friends who have supported us during difficult times. We should not forget about students in our courses who have helped just by asking questions; some of them have now become our colleagues. In view of the book complex history, the list should be very long. We prefer to thank all of them anonymously. However, since every rule should have an exception, let us name J. Dittrich, who read the manuscript and corrected numerous mistakes. Last but not least we want to thank our wives, whose patience and understanding made the writing of this book possible.

Prague, July 1993

Pavel Exner
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