A NON-RELATIVISTIC MODEL OF TWO-PARTICLE DECAY

II. REDUCED RESOLVENT*)

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We continue discussion of the Lee-type decay model described in the first part of this paper. After separation of the centre-of-mass motion, we deduce meromorphic structure of the reduced resolvent for small values of the coupling constant.

1. INTRODUCTION

In the first part of this paper [1], hereafter referred to as I, we have formulated a non-relativistic model of two-particle decay and studied its Galilean invariance. Here we are going to study the model further. First we separate the centre-of-mass motion. Then we turn to discussion of the reduced resolvent which contains the essential dynamical information. We show that under mild assumptions about the interaction Hamiltonian, it has a meromorphic structure. The unperturbed Hamiltonian has a simple eigenvalue embedded in the continuous spectrum; the corresponding pole shifts under influence of the perturbation to the second sheet of the analytically continued reduced resolvent. Further properties of the model will be discussed in the subsequent parts of the paper.

The second part is a direct continuation of I and leans fully on its results. The theorems, formulae and references given in I are not repeated but referred to by their numbers following the digit I. In the forthcoming parts of the paper we shall proceed similarly.

2. SEPARATION OF THE CENTRE-OF-MASS MOTION

The state Hilbert space (I.2.1) decomposes naturally into the tensor product of spaces referring to relative and centre-of-mass motion, $\mathscr{H} = \mathscr{H}^{\mathrm{cm}} \otimes \mathscr{H}^{\mathrm{rel}}$. With the usual licence, we write

$$\mathscr{H} = L^2(\mathbb{R}^3) \otimes (\mathbb{C} \oplus L^2(\mathbb{R}^3)), \tag{2.1}$$

where the bilinear mapping $\otimes : L^2(\mathbb{R}^3) \times [\mathbb{C} \oplus L^2(\mathbb{R}^3)] \to \mathscr{H}$ is defined by

$$\left(\psi \otimes \begin{pmatrix} \alpha \\ \varphi \end{pmatrix}\right)(X, x) := \begin{pmatrix} \alpha\psi(X) \\ \psi(X)\,\varphi(x) \end{pmatrix};$$

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one can check easily that it has the required properties (cf. [I.29], Chapt. 6 or [2]). The Hamiltonian (I.2.5) can be then expressed as

$$H_g = \overline{H_0^{\mathrm{cm}} \otimes I + I \otimes H_g^{\mathrm{rel}}}\,, \tag{2.2}$$

where $H_0^{\mathrm{cm}} = -(1/2M)\,\Delta_X$ and $H_g^{\mathrm{rel}} = H_0^{\mathrm{rel}} + gV$ with

$$H_0^{\mathrm{rel}} = \begin{pmatrix} E & 0 \\ 0 & -\dfrac{1}{2m}\,\Delta_x \end{pmatrix}, \tag{2.3a}$$

$$V = \begin{pmatrix} 0 & (v,\cdot) \\ v & 0 \end{pmatrix}; \tag{2.3b}$$

we omit here the superscript "rel" for convenience. The operators H_0^{cm} and H_g^{rel} are self-adjoint and the relation (2.2) implies ([I. 29], Theorem 6.10)

$$\mathrm{e}^{-\mathrm{i}H_g t} = \mathrm{e}^{-\mathrm{i}H_0{}^{\mathrm{cm}}t} \otimes \mathrm{e}^{-\mathrm{i}H_g{}^{\mathrm{rel}}t} \tag{2.4}$$

for all $t \in \mathbb{R}$. Hence the total propagator decomposes naturally and its centre-of-mass part represents a free motion. For our purposes, only the relative part is important. We are interested in the situation when the initial state of the system represents an undecayed heavy particle,

$$\Psi = \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} = \psi_u \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.5}$$

at $t = 0$. Then the state vector factorizes at each $t > 0$,

$$\Psi_t \equiv \mathrm{e}^{-\mathrm{i}H_g t}\,\Psi = \psi_{t,u}^{\mathrm{cm}} \otimes \begin{pmatrix} \psi_{t,u}^{\mathrm{rel}} \\ \psi_{t,d}^{\mathrm{rel}} \end{pmatrix}, \tag{2.6a}$$

where

$$\begin{pmatrix} \psi_{t,u}^{\mathrm{rel}} \\ \psi_{t,d}^{\mathrm{rel}} \end{pmatrix} := \mathrm{e}^{-\mathrm{i}H_g{}^{\mathrm{rel}}t} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.6b}$$

and

$$\psi_{t,u}^{\mathrm{cm}}(X) = \left(\frac{M}{2\pi\,\mathrm{i}t}\right)^{3/2} \operatorname*{l.i.m.}_{\alpha\to\infty} \int_{|Y|\le\alpha} \exp\left(\mathrm{i}M|X-Y|^2/2t\right) \psi_u(Y)\,\mathrm{d}Y \tag{2.6c}$$

(cf. Ref. [I.27], Sect IX.7). The same can be expressed in other words. The decay is described fully by the reduced propagator

$$U_t := \mathrm{pr}_{\mathscr{H}_u}\,\mathrm{e}^{-\mathrm{i}H_g t} \equiv E_u\,\mathrm{e}^{-\mathrm{i}H_g t} \restriction \mathscr{H}_u\,, \tag{2.7a}$$

where E_u denotes the projection onto $\mathscr{H}_u$ (Ref. [I.3], Chapt. 1). However, the latter equals $I \otimes E_u^{\mathrm{rel}}$, where E_u^{rel} projects onto the one-dimensional subspace $\mathscr{H}_u^{\mathrm{rel}} \sim \mathbb{C}$ in $\mathscr{H}^{\mathrm{rel}}$, and therefore the relation (2.4) implies

$$U_t = \mathrm{e}^{-\mathrm{i}H_0{}^{\mathrm{cm}}t} \otimes \mathrm{pr}_{\mathscr{H}_u{}^{\mathrm{rel}}}\,\mathrm{e}^{-\mathrm{i}H_g{}^{\mathrm{rel}}t}\,. \tag{2.7b}$$

Before proceeding further, let us mention how this decomposition looks like in the p-representation. The operator (I.2.10) may be expressed as

$$F = F_3 \otimes \begin{pmatrix} 1 & 0 \\ 0 & F_3 \end{pmatrix} \tag{2.8}$$

and transforms H_g into $\overline{\hat{H}_0^{\mathrm{cm}} \otimes I + I \otimes \hat{H}_g^{\mathrm{rel}}}$, where $\hat{H}_0^{\mathrm{cm}}$ is a multiplication operator, $(\hat{H}_0^{\mathrm{cm}}\hat{\psi})(\boldsymbol{P}) = (\boldsymbol{P}^2/2M)\,\hat{\psi}(\boldsymbol{P})$ and

$$\hat{H}_g^{\mathrm{rel}} = \begin{pmatrix} E & g(\hat{v},\cdot) \\ g\hat{v} & \dfrac{\boldsymbol{p}^2}{2m} \end{pmatrix} \tag{2.9}$$

3. THE REDUCED RESOLVENT

In what follows, we shall be concerned mostly with the relative motion, and therefore we omit the superscripts "rel". Let us first recall that the reduced propagator (2.7) is determined by the reduced resolvent

$$R_u(z, H_g) := \mathrm{pr}_{\mathscr{H}_u} (H_g - z)^{-1} \tag{3.1}$$

as

$$U_t \Psi = \int_R \mathrm{e}^{-\mathrm{i}\lambda t}\, \mathrm{d}F_\lambda\, \Psi\,, \tag{3.2a}$$

where the vector-valued measure is given by

$$\tfrac{1}{2}\{F([\lambda, \mu]) + F((\lambda, \mu))\}\, \Psi = \tag{3.2b}$$

$$= \frac{1}{2\pi\mathrm{i}} \lim_{\eta \to 0^+} \int_\lambda^\mu [R_u(\xi + \mathrm{i}\eta, H_g) - R_u(\xi - \mathrm{i}\eta, H_g)]\, \Psi\, \mathrm{d}\xi$$

(Ref. [I.3], Sect. 3.1); if $R_u(\cdot, H_g)$ has a pole near the real axis, then the reduced propagator is dominated by the corresponding exponential term. In our case, the (relative) subspace $\mathscr{H}_u$ is one-dimensional, so $R_u(z, H_g)$ and U_t act simply as multiplication by numbers $r_u(z, H_g)$ and $u(t)$, respectively.

Since $E > 0$ by assumption, the unperturbed Hamiltonian has the simple eigenvalue E embedded in the non-simple continuous spectrum $\sigma_c(H_0) = \mathbb{R}^+$. Spectrum of the operator H_g can be found easily. In particular, $\sigma_{\mathrm{ess}}(H_g) = \sigma_{\mathrm{ess}}(H_0) = \mathbb{R}^+$, because the operator V is of rank two, and therefore relatively compact with respect to H_0 (cf. [I.27], Sect XIII.4); it means that $\sigma(H_g) \supset \sigma(H_0)$.

The perturbation problem for the embedded eigenvalue E can be solved explicitly, because the interaction Hamiltonian fulfils the Friedrichs condition

$$E_d V E_d = 0\,. \tag{3.3}$$

A simple algebraic argument (cf. [I.3], Proposition 3.2.1) gives

Proposition 3.1: The reduced resolvent (3.1) acts as multiplication by

$$r_u(z, H_g) = [-z + E + g^2 G(z)]^{-1}, \tag{3.4}$$

where

$$G(z) := \int_{R^3} \frac{|\hat{v}(\boldsymbol{p})|^2}{z - (\boldsymbol{p}^2/2m)} \, d\boldsymbol{p}, \tag{3.5a}$$

for $z \in \varrho(H_g)$, in particular, for each non-real $z \in \mathbb{C}$.

Notice that proving this assertion, one can work in the p-representation, where $\hat{H}_0$ and $(\hat{H}_0 - z)^{-1}$ act as multiplication operators; the relation (2.8) shows that $r_u(z, \hat{H}_g) = r_u(z, H_g)$. The crucial observation is now that $r_u(\cdot, H_g)$ may be continued analytically across $\mathbb{R}^+$, even if the full resolvent is not having a cut there. We shall prove that the perturbation shifts the pole corresponding to the unperturbed eigenvalue from the real axis to the second sheet of the analytically continued $r_u(\cdot, H_g)$. Let us first collect the hypotheses concerning the function $\boldsymbol{v}$:

Assumptions 3.2: (a) v is rotationally invariant. In that case, the same is true for $\hat{v}$, and we shall write $\hat{v}(\boldsymbol{p}) = \hat{v}_1(p)$, having in mind that $\hat{v}_1$ is not Fourier image of v_1 from I,

$$\hat{v}_1(p) = \left(\frac{2}{\pi}\right)^{1/2} \underset{\alpha \to \infty}{\text{l.i.m.}} \int_0^\alpha \frac{r \sin pr}{p} v_1(r) \, dr. \tag{3.6}$$

The relation (3.5a) can be now rewritten as

$$G(z) = 4\pi \int_0^\infty \frac{|\hat{v}_1(p)|^2 p^2}{z - (p^2/2m)} \, dp. \tag{3.5b}$$

(b) The function $\lambda \mapsto |\hat{v}_1[\sqrt{(2m\lambda)}]|^2 \sqrt{(2m\lambda)}$ can be continued analytically to an open set $\Omega \subset \mathbb{C}$ containing the point E and such that $\Omega \cap \mathbb{R} \subset \mathbb{R}^+$, i.e., there is a holomorphic function $f: \Omega \to \mathbb{C}$ such that $f(\lambda) = |\hat{v}_1[\sqrt{(2m\lambda)}]|^2 \sqrt{(2m\lambda)}$ for $\lambda \in \Omega \cap \mathbb{R}$. For convenience, we write $f(z) = |\hat{v}_1[\sqrt{(2mz)}]|^2 \sqrt{(2mz)}$ for non-real z too.

(c)
$$\hat{v}_1[\sqrt{(2mE)}] \neq 0$$

Now we shall prove two auxiliary assertions.

Lemma 3.3: Let the function $\lambda \mapsto |\hat{v}_1[\sqrt{(2m\lambda)}]|^2 \sqrt{(2m\lambda)}$ has a bounded derivative in an open set $J \subset \mathbb{R}^+$. Define

$$I(\lambda, v) := \mathscr{P} \int_0^\infty \frac{|\hat{v}_1(p)|^2 p^2}{\lambda - (p^2/2m)} \, dp, \tag{3.7}$$

where $\mathscr{P}$ denotes principal value, then the function $I(\cdot, v)$ is finite and continuous in J.

Proof: Choose an arbitrary $\lambda_0 \in J$. Due to the assumption, there are positive C, δ such that

$$(3.8) \qquad \left| |\hat{v}_1(p)|^2 p - |\hat{v}_1[\sqrt{(2m\lambda)}]|^2 \sqrt{(2m\lambda)} \right| \leqq C|p - \sqrt{(2m\lambda)}|$$

holds for all p, $\sqrt{(2m\lambda)} \in (\sqrt{(2m\lambda_0)} - \delta, \sqrt{(2m\lambda_0)} + \delta)$. The integral (3.7) can be then written as

$$I(\lambda, v) = \sum_{k=1}^{3} I_k(\lambda, v) = \left(\int_0^{\sqrt{[2m(\lambda-\varrho)]}} + \mathscr{P} \int_{\sqrt{[2m(\lambda-\varrho)]}}^{\sqrt{[2m(\lambda+\varrho)]}} + \int_{\sqrt{[2m(\lambda+\varrho)]}}^{\infty} \right) \frac{|\hat{v}_1(p)|^2 p^2}{\lambda - (p^2/2m)} \, \mathrm{d}p$$

for some $\varrho \in (0, \lambda)$. We can choose $\varrho \in (0, \lambda_0)$ and $\delta_1 \in (0, \frac{1}{2}\delta)$ in such a way that $|\sqrt{[2m(\lambda \pm \varrho)]} - \sqrt{(2m\lambda)}| < \frac{1}{2}\delta$ hods for

$$(3.9a) \qquad \sqrt{(2m\lambda)} \in (\sqrt{(2m\lambda_0)} - \delta_1, \sqrt{(2m\lambda_0)} + \delta_1)$$

so that

$$(3.9b) \qquad |\sqrt{[2m(\lambda \pm \varrho)]} - \sqrt{(2m\lambda_0)}| < \delta \,.$$

In what follows, we shall consider only those λ which fulfil the condition (3.9a). The integrals $I_k(\lambda, v)$, $k = 1, 3$, are finite and $I_k(\cdot, v)$ are continuous at λ_0 due to the dominated-convergence theorem. It is sufficient therefore to consider the second integral. A simple intergration yields

$$\mathscr{P} \int_{\sqrt{[2m(\lambda-\varrho)]}}^{\sqrt{[2m(\lambda+\varrho)]}} \frac{p \, \mathrm{d}p}{\lambda - (p^2/2m)} = \lim_{\eta \to 0^+} \left(\int_{\sqrt{[2m(\lambda-\varrho)]}}^{\sqrt{(2m\lambda)} - \eta} + \int_{\sqrt{(2m\lambda)}+\eta}^{\sqrt{[2m(\lambda+\varrho)]}} \right) \frac{p \, \mathrm{d}p}{\lambda - (p^2/2m)} =$$

$$= m \lim_{\eta \to 0^+} \ln \frac{\eta \sqrt{\left(\frac{2\lambda}{m}\right)} + \frac{\eta^2}{2m}}{\eta \sqrt{\left(\frac{2\lambda}{m}\right)} - \frac{\eta^2}{2m}} = 0$$

so we have

$$(3.10) \qquad I_2(\lambda, v) = \mathscr{P} \int_{\sqrt{[2m(\lambda-\varrho)]}}^{\sqrt{[2m(\lambda+\varrho)]}} \frac{|\hat{v}_1(p)|^2 p - |\hat{v}_1[\sqrt{(2m\lambda)}]|^2 \sqrt{(2m\lambda)}}{\lambda - (p^2/2m)} \, p \, \mathrm{d}p \,.$$

According to the conditions (3.8) and (3.9), the following inequality holds

$$\left| \frac{|\hat{v}_1(p)|^2 p - |\hat{v}_1[\sqrt{(2m\lambda)}]|^2 \sqrt{(2m\lambda)}}{\lambda - (p^2/2m)} \, p \right| \leqq \frac{2mCp}{p + \sqrt{(2m\lambda)}} \leqq 2mC \,.$$

Thus the rhs of (3.10) makes sense as a Lebesgue integral and $I_2(\cdot, v)$ is continuous at λ_0 by the dominated-convergence theorem. □

Lemma 3.4: Adopt the assumptions (a) and (b). The function G_Ω defined by

$$(3.11) \qquad G_\Omega(z) = \begin{cases} G(z) & \ldots \operatorname{Im} z > 0 \\ 4\pi I(z, v) - 4\pi^2 \, im |\hat{v}_1[\sqrt{(2mz)}]|^2 \sqrt{(2mz)} & \ldots z \in \mathbb{R} \cap \Omega \\ G(z) - 8\pi^2 \, im |\hat{v}_1[\sqrt{(2mz)}]|^2 \sqrt{(2mz)} & \ldots z \in \Omega, \operatorname{Im} z < 0 \end{cases}$$

is holomorphic in $\{z \in \mathbb{C}: \operatorname{Im} z > 0\} \cup \Omega$.

Proof: One has to check that within Ω, the relation

$$\lim_{\substack{\xi \to \lambda \\ \eta \to 0^+}} G(\xi \pm i\eta) = 4\pi I(\lambda, v) \mp 4\pi^2 \, im |\hat{v}_1[\sqrt{(2m\lambda)}]|^2 \sqrt{(2m\lambda)} \tag{3.12}$$

holds. We notice first that $I(\lambda, v)$ makes sense due to (b), because the assumption of the preceding lemma is fulfilled in that case. Hence one can choose a sufficiently small ϱ and express $G(z)$ as a sum of three integrals in analogy with the above proof; the dominated-convergence theorem implies $G_k(\xi \pm i\eta) \to 4\pi I_k(\lambda, v)$ for $k = 1, 3$. Further we have

$$\lim_{\substack{\xi \to \lambda \\ \eta \to 0^+}} G_2(\xi \pm i\eta) = \tag{3.13}$$

$$= 4\pi \lim_{\substack{\xi \to \lambda \\ \eta \to 0^+}} \int_{\sqrt{[2m(\lambda-\varrho)]}}^{\sqrt{[2m(\lambda+\varrho)]}} \frac{|\hat{v}_1(p)|^2 \, p - |\hat{v}_1[\sqrt{(2m\xi)}]|^2 \sqrt{(2m\xi)}}{\xi \pm i\eta - (p^2/2m)} \, p \, dp \, +$$

$$+ \, 4\pi \lim_{\substack{\xi \to \lambda \\ \eta \to 0^+}} |\hat{v}_1[\sqrt{(2m\xi)}]|^2 \sqrt{(2m\xi)} \int_{\sqrt{[2m(\lambda-\varrho)]}}^{\sqrt{[2m(\lambda+\varrho)]}} \frac{p \, dp}{\xi \pm i\eta - (p^2/2m)} \, .$$

The first limit equals $4\pi I_2(\lambda, v)$ according to (3.10) and the dominated-convergence theorem. One obtains easily

$$\lim_{\substack{\xi \to \lambda \\ \eta \to 0^+}} \int_{\sqrt{[2m(\lambda-\varrho)]}}^{\sqrt{[2m(\lambda+\varrho)]}} \frac{p \, dp}{\xi \pm i\eta - (p^2/2m)} = m \lim_{\substack{\xi \to \lambda \\ \eta \to 0^+}} \ln \frac{\xi - \lambda \pm i\eta + \varrho}{\xi - \lambda \pm i\eta - \varrho} = \mp \, i\pi m \tag{3.14}$$

and we arrive at the relation (3.12). Combining it with the preceding lemma, we see that the function (3.11) is continuous in $\{z \in \mathbb{C}: \operatorname{Im} z > 0\} \cup \Omega$.[1]) Then it is uniformly continuous in any compact subset of it, and therefore the limit in (3.12) is uniform with respect to λ in any compact subinterval of $\mathbb{R} \cap \Omega$. Since $G_\Omega(\cdot)$ is easily seen to be holomorphic in the upper and lower halfplanes (within Ω in the last case), the assertion follows from the edge-of-wedge theorem [4]. □

Remark 3.5: One has to check the uniform convergence, because the remark following Theorem 2–13 of Ref. [4] is not correct: a counterexample is represented by $F(z) = z \, e^{i/z^2}$.

Now we are in position to prove the main result of this section.

Theorem 3.6: Assume (a)–(c). Then there is a connected complex neighbourhood $\Omega_1 \subset \Omega$ of the point E and a positive ε such that for each $g \in (-\varepsilon, \varepsilon)$,

$$r_u^\Omega(z, H_g) := [-z + E + g^2 \, G_\Omega(z)]^{-1} \tag{3.15}$$

represents analytic continuation of (3.4) to $\{z \in \mathbb{C}: \operatorname{Im} z > 0\} \cup \Omega$. The function

[1]) In fact, we need Lemma 3.3 only to ensure finiteness of $I(\lambda, v)$ since the continuity follows from the existence of a finite limit on the real axis (cf. Ref. [3], Theorem 146).

$r_u^{\Omega}(\cdot, H_g)$ has just one singularity in Ω_1, a simple pole at $z = z_p(g)$, where the function $z_p = \lambda_p - i\delta_p$ belongs to $C^\infty[-\varepsilon, \varepsilon]$ and its real and imaginary parts fulfil

$$\lambda_p(g) = E + 4\pi g^2 \mathscr{P} \int_0^\infty \frac{|\hat{v}_1(p)|^2 p^2}{E - (p^2/2m)} \, dp + O(g^4), \tag{3.16a}$$

$$\delta_p(g) = 4\pi^2 m g^2 |\hat{v}_1[\sqrt{(2mE)}]|^2 \sqrt{(2mE)} + O(g^4). \tag{3.16b}$$

Proof: The assertion concerning analytic continuation follows from Lemma 3.4. Only possible singularities of (3.15) are zeros of the function $f(g, z) := z - E - g^2 G_\Omega(z)$ defined for $g \in \mathbb{R}$ and z from the analycity domain of G_Ω. For small enough g, one can use the implicit-function theorem (cf. Ref. [3], Theorem 210 or Ref. [5], Theorems III.28, III.31). The function f is infinitely differentiable with respect to both g and z, further we have $f(0, E) = 0$ and $(\partial f/\partial z)(0, E) = 1 \neq 0$. Then there is a neighbourhood $(-\varepsilon', \varepsilon')$ of the point $g = 0$ and a unique function $z_p \in C^\infty[-\varepsilon', \varepsilon']$ such that $f(g, z_p(g)) = 0$ for $|g| < \varepsilon'$, i.e., $z_p(g) = E + g^2 G_\Omega(z_p(g))$. Continuity of the partial derivatives of f implies particularly that $(\partial f/\partial z)(\cdot, z_p(\cdot))$ is continuous in $(-\varepsilon', \varepsilon')$, and therefore there is a positive $\varepsilon \leqq \varepsilon'$ such that $(\partial f/\partial z) . (g, z_p(g)) \neq 0$ for $g \in (-\varepsilon, \varepsilon)$. Consequently, $r_u^{\Omega}(\cdot, H_g)$ has a simple pole at $z_p(g)$. The first few terms of the Taylor expansion of z_p can be easily calculated: we obtain

$$\left.\frac{dz_p}{dg}\right|_{g=0} = \left.\frac{d^3 z_p}{dg^3}\right|_{g=0} = 0 \quad \text{and} \quad \left.\frac{d^2 z_p}{dg^2}\right|_{g=0} = 2G_\Omega(E)$$

which imply (3.16). □

Remark 3.7: In fact, we have proved the theorem using the assumptions (a) and (b) only. The assumption (c) is important, however, since it determines the leading order in the formula (3.16b) which yields the decay width. We shall return to this problem in a sequel to this paper.

Remark 3.8: The proved theorem represents a particular case of much more general results obtained by Howland and Baumgärtel – cf. ref. [I.3], sect. 3.3. The deduction is, however, more illustrative in comparison with the general case since the Friedrichs condition (3.3) makes it possible to avoid use of the factorization technique.

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References

[1] Dittrich J., Exner P.: Czech. J. Phys. B *37* (1987) 503.
[2] Blank J., Exner P.: Acta Univ. Carolinae, Math. et Phys. *17* (1977) 75; *18* (1978) 3.
[3] Jarník V.: Differential Calculus II. Academia, Prague, 1956 (in Czech).
[4] Streater R. F., Wightman A. S.: PCT, Spin, Statistics and All That. Benjamin, New York, 1964.
[5] Schwartz L.: Analyse mathématique I. Hermann, Paris, 1967.