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# **Mathematical Results in Quantum Mechanics**

QMath7 Conference, Prague, June 22–26, 1998

Jaroslav Dittrich  
Pavel Exner  
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**Springer Basel AG**

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# Preface

At the age of almost three quarters of a century, quantum mechanics is by all accounts a mature theory. There were times when it seemed that it had borne its best fruit already and would give way to investigation of deeper levels of matter. Today this sounds like rash thinking. Modern experimental techniques have led to discoveries of numerous new quantum effects in solid state, optics and elsewhere. Quantum mechanics is thus gradually becoming a basis for many branches of applied physics, in this way entering our everyday life.

While the dynamic laws of quantum mechanics are well known, a proper theoretical understanding requires methods which would allow us to derive the abundance of observed quantum effects from the first principles. In many cases the rich structure hidden in the Schrödinger equation can be revealed only using sophisticated tools. This constitutes a motivation to investigate rigorous methods which yield mathematically well-founded properties of quantum systems.

Among the traditional meetings where such problems are discussed there was a conference series launched in 1987 at Dubna, Russia. The present volume collects some of the contributions presented at the seventh gathering of this series, so far the largest, held on June 22–26, 1998, at the Czech Technical University in Prague. The coverage is by far not exhaustive because of the conference size, the natural volume restriction, the scope of the series *Operator Theory: Advances and Applications*, and the decision of some participants not to write again about results they have published recently. These and other reasons have led to the fact that this book is just a selection representing less than one-third of the talks. It is clear that many interesting results had to be left out. To give the reader a better idea, we include at the end the titles of the other contributions together with the list of participants; it is easy in this wired world to ask anybody for a copy of a paper.



We want to thank all the participants who made the QMath7 conference interesting and full of inspiration indeed. We are no less grateful to the institutions which provided financial support: the Ministry of Education of the Czech Republic, the Physics Section of the Union of Czech Mathematicians and Physicists, and the Foundation for Support of Theoretical Physics at Slemeno. Last but not least we thank the referees who reviewed the contributions rapidly but attentively; their remarks and suggestions helped to improve the book.

Prague, November 1998

*The editors*