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Two-component model of a spin-polarized transport

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Effect of the spin-involved interaction of electrons with impurity atoms or defects to the transport properties of a two-dimensional electron gas is described by using a simplifying two-component model. Components representing spin-up and spin-down states are supposed to be coupled at a discrete set of points within a conducting channel. The used limit of the short-range interaction allows to solve the relevant scattering problem exactly. By varying the model parameters different transport regimes of two-terminal devices with ferromagnetic contacts can be described. In a quasi-ballistic regime the resulting difference between conductances for the parallel and antiparallel orientation of the contact magnetization changes its sign as a function of the length of the conducting channel if appropriate model parameters are chosen. This effect is in agreement with recent experimental observation.

Spin-polarized transport in two-dimensional electron systems has been a field of growing interest during the last years. Typically, the experiments are performed using a two-terminal device with ferromagnetic metal contacts. A spin-polarization of the injected current is expected from the different densities of states for spin-up and spin-down electrons in the ferromagnetic source. This leads to a spin dependent interface-resistance, which also exists at the interface of the second ferromagnetic contact, the drain. Together with spin-involved scattering processes in the studied electron system this should result in a conductance which depends on the relative magnetization of the two contacts [1].

The quantum mechanical nature of spin places it out of reach of many of the forces in a solid and the orientation of a carrier's spin can be very long-lived. The conductance $G^{\uparrow\uparrow}$ of a two-terminal device with parallel orientation of magnetic moments of the contacts is thus expected to be higher than the conductance $G^{\uparrow\downarrow}$ for the case of antiparallel moment orientation [1,2]. However, just the opposite results have been reported recently [3] for a two-dimensional electron gas confined in an InAs channel with the permalloy source and drain. It has been found that an ensemble average of the conductance difference $G^{\uparrow\uparrow} - G^{\uparrow\downarrow}$ decreases as function of the channel length reaching negative values in a quasi-ballistic regime when the electron mean free path l_e becomes comparable with the channel length.

In the absence of magnetic impurities the natural candidate for spin dephasing and precession effects is spin-orbit coupling to impurity atoms or defects. General theoretical approach to its description is to use the contribution to the Hamiltonian which stems directly from the quadratic in v/c expansion of the Dirac equation [4]

$$\hat{H}_{SO} = -\frac{\hbar}{4m^2c^2}\nabla V(\vec{r}) \cdot (\hat{\sigma} \times \vec{p}) \, , \qquad (1)$$

where $\hat{\sigma} \equiv \{\sigma_x, \sigma_y, \sigma_z\}$ denotes Pauli matrices, $V(\vec{r})$ is a background potential, ∇ stands for the spatial gradient and m is the electron mass.

Influence of the spin-orbit interaction on the electron transport properties of two-dimensional mesoscopic systems has been studied since the early 1980's. At that time it was found that it is responsible for so-called antilocalization effect [7]. Later the attention has been turned to the effects caused by a Rashba term [5,6] in two-dimensional [8–10] and quasi-one-dimensional systems [11–15]. Realistic transport theory for fully quantum coherent systems including the spin-orbit coupling to the impurities or defects has not yet been reported. The problem becomes complicated even if electron motion is restricted to the two-dimensional space. In general, the spin-orbit term, Eq.(1), turns the problem back to three dimensions.

The goal of this paper is to reveal those features of the transport properties which can be caused by the spin-orbit interaction induced by a scattering potential. In the interesting case of a quasi-ballistic regime, which shows chaotic features, it is very difficult to estimate deviation from the exact solution caused by any used approximation. For this reason we have employed simplifying two-component model with point interaction for which exact solution, including fully the quantum coherence, can be found. Non-zero spin-orbit coupling is assumed to be associated with short-range scattering potentials only. Although the treatment is far from a realistic transport theory, it might be useful to understand some mysteries of the recent experimental observation.

Free electron system is a typical two-component system if the electron spin is taken into account. If there are no spin-involved forces, electron states are represented by plane waves $\exp(i\vec{k}\vec{r})$ with $\vec{k}$ being a wave vector. Orientation of the electron spin is given by the quantum number $s_z = \pm\frac{1}{2}$ and the electron system can be splitted into two independent subsystems, each of them composed of electrons having the same spin orientation. However, any perturbation of the background potential can cause a coupling between subsystems due to non-zero spin-orbit term defined by Eq.(1).

Let us first consider a single scattering potential acting on a two-dimensional electron gas within a finite region of a radius r_0. In accord with standard scattering theory an incoming wave belonging to one particular subsystem, say of the spin up states, can be scattered into states belonging to both subsystems. In the short-range limit, $kr_0 << 1$, only s-part of the incoming wave gives non-zero contribution to the scattering process. For given energy $E = \hbar^2 k^2/2m$, $(k = |\vec{k}|)$, the corresponding solution of the radial Schrödinger equation has two components, $\Psi_\uparrow(r)$ and $\Psi_\downarrow(r)$. Outside the scattering region they can be written as follows [16,17]

$$\Psi_\uparrow(r) \;=\; J_0(kr) \;+\; a(k) H_0^{(1)}(kr) \qquad (2)$$

$$\Psi_\downarrow(r) \;=\; b(k) H_0^{(1)}(kr), \qquad (3)$$

where $J_0(z)$ denotes the cylindrical Bessel function and Hankel functions $H_0^{(1)}(z)$ represent scattered outgoing waves that for large arguments have the following asymptotic form

$$H_0^{(1)}(kr \to \infty) \;\sim\; \frac{1-i}{\sqrt{\pi k}} \, \frac{e^{ikr}}{\sqrt{r}} \; . \qquad (4)$$

Taking into account the time reversal symmetry and assuming that the system is invariant with respect of the subsystem interchange, the amplitudes $a(k)$ and $b(k)$ in the short-range limit have to be of the following general form [17]

$$a(k) \;=\; \frac{1 + \frac{2i}{\pi}\left(\gamma + \ln\frac{k}{2} - A\right)}{\left[1 + \frac{2i}{\pi}\left(\gamma + \ln\frac{k}{2} - A\right)\right]^2 \;+\; \frac{4}{\pi^2}|C|^2} \; , \qquad (5)$$

$$b(k) \;=\; \frac{2i}{\pi} C \left[1 + \frac{2i}{\pi}\left(\gamma + \ln\frac{k}{2} - A\right)\right]^{-1} a(k) \; , \qquad (6)$$

where A and C are real model parameters. If C is chosen to be zero, the parameter A represents a strength of the scattering process within one particular subsystem and for a potential well of the radius r_0 it takes the value $A = \ln r_0$. Non-zero values of the parameter C give rise to a spin-flip process.

There are two relevant physical quantities characterizing scattering event: the total scattering cross-section

$$\sigma_0 \;=\; a(k)^2 + b(k)^2 \qquad (7)$$

and the spin-flip probability $t^{\uparrow\downarrow}$

$$t^{\uparrow\downarrow} \;=\; \frac{b(k)^2}{a(k)^2 + b(k)^2} \; . \qquad (8)$$

Note, that the assumption of the system invariance with respect of the subsystem interchange leads to the independence of σ_0 and $t^{\uparrow\downarrow}$ on the spin orientation of the incoming electron. This assumption has been used for the sake of simplicity despite of the fact that it need not to be satisfied in real systems, e.g. due to a Rashba term.

The scattering problem for a two-dimensional strip with a finite number of short-range scatterers, as sketched in Fig. 1, can be solved exactly. The detail analysis in the case of a one-component system has already been reported [18] and generalization to a two-component system is straightforward. For simplicity we have assumed that all scatterers are identical, i.e. they give the same scattering cross-section σ_0 and spin-flip probability $t^{\uparrow\downarrow}$ if they would be placed alone within the two-dimensional space. Scattering matrix has been obtained numerically for a given configuration of point scatterers randomly distributed within a strip region of the length L.

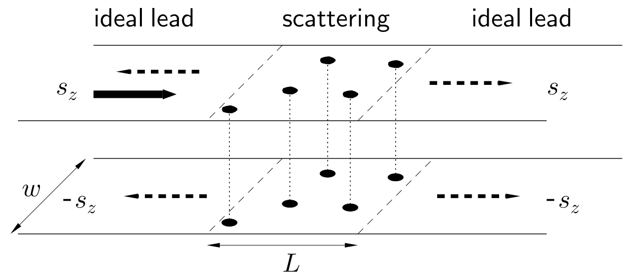

FIG.1. Scheme of the scattering process in a two-component system. Upper and lower strips of the width w represent the spin-subsystems. Scatterers, black points, serve also as connection points between subsystems giving rise to a spin-flip processes. Thick full and dashed lines represent an incoming and outgoing waves, respectively.

Spin-dependent transport properties are determined by partial transmission coefficients representing transition between left and right subsystems of asymptotic spin-up or spin-down states. They are defined as the sum of transmission probabilities over all relevant modes of asymptotic states. To simplify the description by excluding the quantum fluctuations from our consideration we have used configurationally averaged values of the partial scattering coefficients to define 2×2 transmission and reflection matrices $\mathbf{T}$ and $\mathbf{R}$, respectively:

$$\mathbf{T} \equiv \begin{pmatrix} T^{\uparrow\uparrow} & T^{\uparrow\downarrow} \\ T^{\downarrow\uparrow} & T^{\downarrow\downarrow} \end{pmatrix} \; , \; \mathbf{R} \equiv \begin{pmatrix} R^{\uparrow\uparrow} & R^{\uparrow\downarrow} \\ R^{\downarrow\uparrow} & R^{\downarrow\downarrow} \end{pmatrix} \; . \qquad (9)$$

For the considered symmetrical system $T^{\uparrow\uparrow} \equiv T^{\downarrow\downarrow}$, $T^{\uparrow\downarrow} \equiv T^{\downarrow\uparrow}$, $R^{\uparrow\uparrow} \equiv R^{\downarrow\downarrow}$ and $R^{\uparrow\downarrow} \equiv R^{\downarrow\uparrow}$.

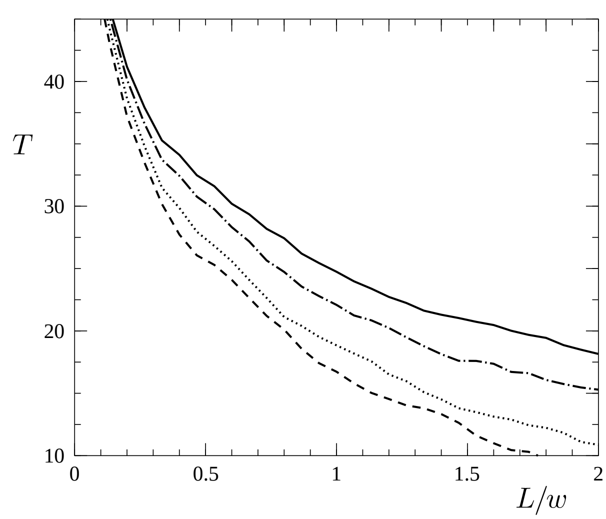

FIG.2. The total transmission coefficient T as function of the scattering region length L for several values of the spin-flip probability: $t^{\uparrow\downarrow} = 8.1 \times 10^{-3}$ (full line), $t^{\uparrow\downarrow} = 4.2 \times 10^{-3}$ (dashed-dotted line), $t^{\uparrow\downarrow} = 1.1 \times 10^{-3}$ (dotted line), $t^{\uparrow\downarrow} = 0.0005 \times 10^{-3}$ (dashed line) .

In Fig. 2 and Fig. 3 the dependence of transmission coefficients on the length L of the scattering region is shown for different spin-flip probabilities $t^{\uparrow\downarrow}$. The used energy corresponds to 31 occupied subbands. Concentration of scatterers $(750/w^2)$ and the scattering cross-section $\sigma_0 = 0.1217$ were held fixed. Probability of an injected electron to cross scattering region increases with increasing spin-flip probability as shown in Fig. 2 where the total transmission coefficients $T = 2(T^{\uparrow\uparrow} + T^{\uparrow\downarrow})$ are plotted. This tendency is in agreement with expected antilocalization effect [7].

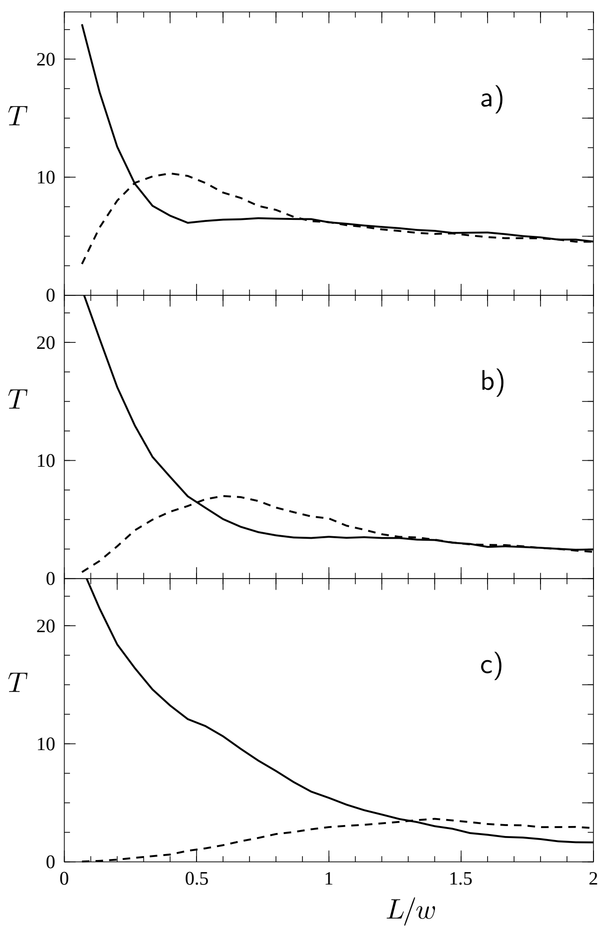

FIG.3. The partial transmission coefficients $T^{\uparrow\uparrow}$ (full line) and $T^{\uparrow\downarrow}$ (dashed line) as function of the scattering region length L for several values of the spin-flip probability: a) $t^{\uparrow\downarrow} = 8.1\times10^{-3}$; b) $t^{\uparrow\downarrow} = 0.2\times10^{-3}$; c) $t^{\uparrow\downarrow} = 0.0005\times10^{-3}$.

The more interesting is the dependence of partial transmission coefficients. For some values of the spin-flip probability and lengths L, $T^{\uparrow\uparrow}$ becomes less than $T^{\uparrow\downarrow}$, as can be seen in Fig. 3. It means that the polarization of the transmitted current has opposite orientation than the polarization of the injected current. This surprising result we ascribe to the non-trivial quantum coherence in two-component systems. Wave interference leading to the weak localization is one-component effect. It take place within one particular subsystem only. In the considered case of a weak spin-flip process ($t^{\uparrow\downarrow} << 1$) it becomes dominant within the subsystem with incoming waves while the localization effect within second subsystem do not need to be for given length L still well developed.

The above described effect disappears if the scattering cross-section is substantially enlarged. The localization becomes dominant effect of the scattering process in the both subsystems and inequality $T^{\uparrow\uparrow} > T^{\uparrow\downarrow}$ remains valid for all lengths of the scattering region.

The device conductance of a two-component quantum system is determined by a matrix of partial transmission coefficients, $\mathbf{T}_{\rm dev}$,

$$G = \frac{e^2}{h}(1\,,1)\,\mathbf{T}_{\rm dev}\begin{pmatrix}1\\1\end{pmatrix}\;;\;\mathbf{T}_{dev}\equiv\begin{pmatrix}T_{\rm dev}^{\uparrow\uparrow} & T_{\rm dev}^{\uparrow\downarrow}\\ T_{\rm dev}^{\downarrow\uparrow} & T_{\rm dev}^{\downarrow\downarrow}\end{pmatrix}\,, \quad (10)$$

which depends on the properties of ferromagnetic contacts and their interfaces with the two-dimensional electron gas. To estimate their effect we have used the idea of polarization filters [14]. The source and the drain are considered to be standard reservoirs and all spin-dependent effects are modeled by filters placed within the asymptotic region of ideal leads. If the coherence is supposed to be completely destroyed at the filter boundaries the conductance can be expressed as a function of the already defined coefficients $T^{\uparrow\uparrow}$, $T^{\uparrow\downarrow}$, $R^{\uparrow\uparrow}$ and $R^{\uparrow\downarrow}$ describing scattering process of the same device without filters.

Experiments on spin-injection into a two-dimensional systems usually show a large interface resistance. For this reason non-zero probabilities, α and β, of spin-up and spin-down electrons to be reflected by the filter will be considered. For the sake of the simplicity we assume that reflected electrons will be equally distributed between available quantum channels without any change of their spin orientation. In this case the filtering effect can be described by 2×2 diagonal matrix

$$\mathbf{F}_{s,d}\equiv\begin{pmatrix}\alpha_{s,d} & 0\\ 0 & \beta_{s,d}\end{pmatrix}\,, \quad (11)$$

where indices s and d represent the source-filter and the drain-filter, respectively.

For the case of N available quantum channels (subbands) within each subsystem, the above described model leads to the following expression for the transmission matrix $\mathbf{T}_{\rm dev}$ entering the Eq.(10)

$$\mathbf{T}_{dev} = (\mathbf{1}-\mathbf{F}_d)\,\mathbf{N}\mathbf{M}_d\,\mathbf{T}\,\mathbf{K}_{d,s}\,\mathbf{N}\mathbf{M}_s\,(\mathbf{1}-\mathbf{F}_s) \quad (12)$$

where $\mathbf{N}$ stands for the product of N and unit matrix $\mathbf{1}$. The effect of multiple reflections between filters and the scattering region is represented by matrices $\mathbf{M}_s$ and $\mathbf{M}_d$

$$\mathbf{M}_s\;=\;(\mathbf{N}-\mathbf{F}_s\mathbf{R})^{-1}\;;\;\mathbf{M}_d\;=\;(\mathbf{N}-\mathbf{R}\mathbf{F}_d)^{-1}\,, \quad (13)$$

and

$$\mathbf{K}_{d,s}\;=\;[\mathbf{1}-\mathbf{M}_s\mathbf{F}_s\mathbf{T}\mathbf{F}_d\mathbf{M}_d\mathbf{T}]^{-1}\,. \quad (14)$$

Device conductance, Eq.(10) depends on reflection probabilities $\alpha_{s,d}$ and $\beta_{s,d}$ modelling the effect of ferromagnetic contacts. For the case of the parallel orientation of the contact magnetization the conductance $G^{\uparrow\uparrow}$ can be obtained by setting $\alpha_s \equiv \alpha_d$ and $\beta_s \equiv \beta_d$. To get $G^{\uparrow\downarrow}$ for the antiparallel contact magnetization $\alpha_s \equiv \beta_d$ and $\beta_s \equiv \alpha_d$ have to be used. While the conductance strongly depends on the used values of reflection probabilities, the sign of the conductance difference $\Delta G \equiv G^{\uparrow\uparrow} - G^{\uparrow\downarrow}$, is not affected.

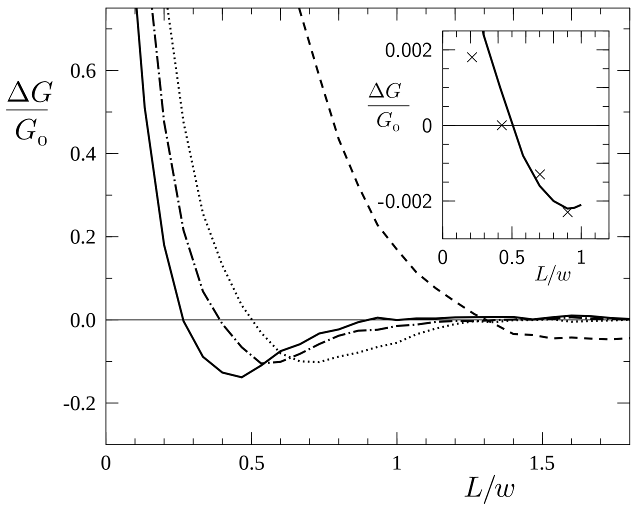

FIG.4. Relative conductance change $\Delta G/G_0$ as function of the scattering region length L for several values of the spin-flip probability: $t^{\uparrow\downarrow} = 8.1 \times 10^{-3}$ (full line), $t^{\uparrow\downarrow} = 1.1 \times 10^{-3}$ (dashed-dotted line), $t^{\uparrow\downarrow} = 0.2 \times 10^{-3}$ (dotted line), $t^{\uparrow\downarrow} = 0.0005 \times 10^{-3}$ (dashed line). In the inset crosses represent experimental data obtained by Hu et al. and full line is the result of the model calculation for the following parameters: $N = 173$, $\alpha_s = 0.01$, $\beta_s = 0$, scatterer concentration $1500/w^2$, $\sigma_0 = 0.1341$ and $t^{\uparrow\downarrow} = 4.8 \times 10^{-3}$.

In Fig. 4 the relative conductance change

$$\frac{\Delta G}{G_0} \equiv 2\frac{G^{\uparrow\uparrow} - G^{\uparrow\downarrow}}{G^{\uparrow\uparrow} + G^{\uparrow\downarrow}} \qquad (15)$$

as a function of the scattering-region length L is shown for the case of ideal filters, $\alpha_s = 0$ and $\beta_s = 1$. It corresponds to injection of fully polarized current and vanishing interface resistance. All other used model parameters are the same as that for transmission coefficients presented in Fig. 2 and Fig. 3. The model parameters of the scatterers, σ_0 and $t^{\uparrow\downarrow}$, and their concentration have been chosen to give pronounced minimum in the dependence $\Delta G/G_0$ on L.

To model the device studied by C. M. Hu et al. [3] with hundreds occupied subbands it is necessary to perform calculation for much larger energy and take into account boundary resistance between ferromagnetic contacts and two-dimensional electron gas by using non-zero value for α_s. Note that $\alpha_s \sim 0.01$ lowers the values of the relative conductance change approximately hundred times and nearly quantitative agreement with the measured data can be reached as shown in the inset of the Fig. 4.

The presented two-component model does not allow to consider asymmetry of scattering processes which leads to the spin-Hall effect [14]. However, if the polarization vectors of the contact magnetization lies within the plane of the electron gas and they are perpendicular to the applied current, as in the case studied by C. M. Hu et al. [3], this effect is suppressed.

The main result of the described model is that in mesoscopic disordered systems the quantum coherence affected by spin-flip processes can lead to the higher conductance of two-terminal devices with antiparallel contact magnetization than that for parallel configuration as observed in recent experiments by by C. M. Hu et al. [3]. Similar effects can also be expected for other two-component systems. For example in double-layer systems the defects of the barrier separating two-dimensional electron gases can act as their connection points. Instead of a spin-flip process there will be a real electron transition between electron layers and in particular cases the conductance measured between contacts at different layers could be larger than that between contacts of the same electron layer.

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