Constrained energy minimization based upscaling for coupled flow and mechanics

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Abstract

In this paper, our aim is to present (1) an embedded fracture model (EFM) for coupled flow and mechanics problem based on the dual continuum approach on the fine grid and (2) an upscaled model for the resulting fine grid equations. The mathematical model is described by the coupled system of equation for displacement, fracture and matrix pressures. For a fine grid approximation, we use the finite volume method for flow problem and finite element method for mechanics. Due to the complexity of fractures, solutions have a variety of scales, and fine grid approximation results in a large discrete system. Our second focus in the construction of the upscaled coarse grid poroelasticity model for fractured media. Our upscaled approach is based on the nonlocal multicontinuum (NLMC) upscaling for coupled flow and mechanics problem, which involves computations of local basis functions via an energy minimization principle. This concept allows a systematic upscaling for processes in the fractured porous media, and provides an effective coarse scale model whose degrees of freedoms have physical meaning. We obtain a fast and accurate solver for the poroelasticity problem on a coarse grid and, at the same time, derive a novel upscaled model. We present numerical results for the two dimensional model problem.

Introduction

In the reservoir simulation, mathematical modeling of the fluid flow and geomechanics in the fractured porous media plays an important role. A coupled poroelastic models can help for better understanding of the processes in the fractured reservoirs. In this work, we consider an embedded fracture model (EFM) for coupled flow and mechanics problems based on the dual continuum approach. The mathematical model is

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described by the coupled system of equations for displacement and fracture/matrix pressures [35]. Coupling of the fracture and matrix equations is derived from the mass exchange between the two continua (transfer term) and based on the embedded fracture model. For the geomechanical effect, we consider deformation of the porous matrix due to pressure change, where pressure plays a role of specific source term for deformation [28, 27, 26, 29, 5, 6]. Fundamentally, the system of equations is coupled between flow and geomechanics, where displacement equation includes the volume force, which is proportional to the pressure gradient, and the pressure equations include the term, which describes the compressibility of the medium.

Fracture networks commonly have complex geometries with multiple scales, and usually have very small thickness compared to typical reservoir sizes. Due to high permeability, fractures have a significant impact on the flow processes. A common approach to the fracture modeling is to model them as lower dimensional problems [33, 15, 20, 13]. The result is a coupled mixed dimensional flow models, where we consider flow in the two domains (matrix and fracture) with mass transfer between them. In this work, the fractures are not resolved by grid but included as an overlaying continuum with an exchange term between fracture and matrix that appears as an additional source (Embedded Fracture Model (EFM)) [23, 37, 36]. This approach is related to the class of multicontinuum model [3, 39, 14]. Instead of the dualcontinuum approach, we represent fractures directly using lower dimensional flow model embedded in a porous matrix domain. In EFM, we have two independent grids for fracture networks and matrix, where simple structured meshes can be used for the matrix.

For geomechanics, we derive an embedded fracture model, where each fracture provides an additional source term for the displacement equation. This approach is based on the mechanics with dual porosity model [42, 45]. In this model, we suppose displacement continuity on the fracture interface. For the discrete fracture model, a specific enrichment of the finite element space can be used for accurate solution of the elasticity problem with displacement discontinuity [1]. In this paper, we focus on the fully coupled poroelastic model for embedded fracture model and construct an upscaled model for fast coarse grid simulations. For the fine grid approximation, we use the finite volume method (FVM) for flow problem and the finite element method (FEM) for geomechanics. FVM is widely used as discretization for the simulation of flow problems [4, 38]. We use a cell centered finite volume approximation with two point flux approximation (TPFA) for pressure. FEM is typically used for approximating the solid deformation problem. We use a continuous Galerkin method with linear basis functions with accurate approximation of the coupling term.

Fine grid simulation of the processes in fractured porous media leads to very expensive simulations due to the extremely large degrees of freedoms. To reduce the cost of simulations, multiscale methods or upscaling techniques are used, for example, in [24, 18, 41, 32, 25]. In our previous works, we presented multiscale model reduction techniques based on the Generalized multiscale finite element method (GMsFEM) for flow in fractured porous media [2, 9, 19]. In GMsFEM approach, we solve a local spectral problem for the multiscale basis construction [16, 17, 8, 7]. This gives us a systematic way to construct the missing degrees of freedom via multiscale basis functions. In this work, we construct an upscaled coarse grid poroelasticity model with embedded fracture model. Our approach uses the general concept of nonlocal multicontinua (NLMC) upscaling for flow [10, 11] and significantly generalized it to the coupled flow and mechanics problems. The local problems for the upscaling involves computations of local basis functions via an energy minimization principle and the degrees of freedom are chosen such that they represent physical parameters related to the coupled flow and mechanics problem. We summarize below the main goals of our work:

- a new fine grid embedded fracture model for poroelastic media (coupled system),
- a new accurate and computationally effective fully coarse grid model for coupled multiphysics problem using NLMC whose degrees of freedoms have physical meaning on the coarse grid.

Nonlocal multicontinua (NLMC) upscaling for processes in the fractured porous media provides an effective coarse scale model with physical meaning, and leads to a fast and accurate solver for coupled poroelasticity problem. To capture fine scale processes at the coarse grid model, local multiscale basis functions are presented. Constructing the basis functions based on the constrained energy minimization problem in the oversampled local domain is subject to the constraint that the local solution vanishes in other continua except the one for which it is formulated. Multiscale basis functions have spatial decay property in local domains and separate background medium and fractures. The proposed upscaled model has only one coarse degree of freedom (DOF) for each fracture network. Numerical results show that our NLMC method for fractured porous media provides an accurate and efficient upscaled model on the coarse grid.

The paper is organized as follows. In Section 1, we construct an embedded fracture model for poroelastic media. Next, we construct fine grid approximation using FVM for flow problem and FEM for mechanics in Section 2. In Section 3, we construct an upscaled coupled coarse grid poroelasticity model using NLMC method and present numerical results in Section 4.

1 Embedded fracture model for poroelastic medium

The proposed mathematical model of a coupled flow and mechanics in fractured poroelastic medium contains an interacting model for fluid flow in the porous matrix, flow in fracture network and mechanical deformation. The matrix is assumed to be linear elastic and isotropic with now gravity effects. The mechanical and flow models are coupled through hydraulic loading on the fracture walls and using the effective stress concept [42, 35]. For fluid flow, we consider a mixed dimensional formulation, where we have a coupled problem for fluid flow in the porous matrix in $\Omega \in \mathcal{R}^d$ (d = 2,3), and flow in the fracture network on $\gamma \in \mathcal{R}^{d-1}$ (see Figure 1 for d = 2).

Porous matrix flow model. Using the mass conservation and Darcy law in the domain Ω :

$$\frac{\partial m}{\partial t} + \operatorname{div}(\rho q_m) = \rho f_m, \quad q_m = -\frac{k_m}{\nu_f} \operatorname{grad} p_m, \quad x \in \Omega,$$
(1)

where m is the fluid mass, p_m is the matrix pressure, q_m is Darcy velocity, ν_f is the viscosity, ρ is the fluid density, and f_m is the source term.

Due to the motion of the solid skeleton and Biot's theory, we have the following relationships [12, 28, 27, 26]

$$m - m_0 = \rho \left(\frac{1}{M} (p_m - p_0) + \alpha \varepsilon^v \right), \tag{2}$$



Figure 1: Schematic illustration of the problem with embedded fracture model.

where subscript 0 means reference state, α is Biot coefficient, M is Biot's modulus, ε^{v} is the volumetric strain (the trace of the strain tensor, $\varepsilon^{v} = \text{tr } \varepsilon$) and

$$\frac{1}{M} = \Phi c_f + \frac{1}{N}, \quad \frac{1}{N} = \frac{\alpha - \Phi_0}{K_s}, \quad c_f = \frac{1}{\rho} \frac{d\rho}{dp_m}.$$

Here K_s is the solid grain stiffness, c_f is the fluid compressibility and Φ is the Lagrange's porosity (also known as reservoir porosity).

From equation (2), we can express the reservoir porosity change induced by mechanical deformation as

$$\Phi - \Phi_0 = \alpha \epsilon^v + \frac{1}{N} (p_m - p_0), \tag{3}$$

The permeability of the matrix is updated using the current porosity by the power-law relationship

$$k_m = k_0 \left(\frac{\Phi}{\Phi_0}\right)^n. \tag{4}$$

where the cubic law with n = 3 usually used [40, 44].

Therefore by assuming slightly compressible fluids, for the fluid flow in the porous matrix, we have the following parabolic equation

$$\frac{1}{M}\frac{\partial p_m}{\partial t} + \alpha \frac{\partial \varepsilon^v}{\partial t} - \operatorname{div}\left(\frac{k_m}{\nu_f}\operatorname{grad} p_m\right) = f_m,$$

defined in the domain Ω .

For the case of fractures porous medium, we should add mass trasfer term between matrix and fracture

$$\frac{1}{M}\frac{\partial p_m}{\partial t} + \alpha \frac{\partial \varepsilon^{\nu}}{\partial t} - \operatorname{div}\left(\frac{k_m}{\nu_f}\operatorname{grad} p_m\right) + L_{mf} = f_m,\tag{5}$$

where for the mass exchange between matrix and fracture, we assume a linear relationship

$$L_{mf} = \beta_{mf}(p_m - p_f).$$

This mass exchange term occurs only on the fracture boundary.

Fracture flow model. For the highly permeable fractures, we use the following reduced dimension model for the fluid flow on $\gamma \in \mathcal{R}^{(d-1)}$ [35, 21]:

$$\frac{\partial(\rho b)}{\partial t} + \operatorname{div}(\rho q_f) - \rho L_{mf} = \rho f_f, \quad q_f = -b \frac{k_f}{\nu_f} \operatorname{grad} p_f, \quad x \in \gamma,$$
(6)

where b is the fracture aperture, p_f is the fracture pressure, q_f is the average velocity of fluid along the fracture plane that can be calculated using the cubic low $(k_f = b^2)$. For the calculation of the fracture aperture b, we can use following relation $b(t) = zp_f(t)$, where $z = \frac{2(1-\nu^2)}{E}$ and deformation proportional to the fracture pressure p_f , where η is the Poisson's ratio, E is the elastic modulus [22, 34].

Since

$$\frac{\partial(\rho b)}{\partial t} = \rho \frac{\partial b}{\partial t} + b \frac{\partial \rho}{\partial t} = \rho \left(\frac{\partial b}{\partial t} + bc_f \frac{\partial p_f}{\partial t} \right),\tag{7}$$

and by assuming slightly compressible fluids [21]

$$\rho\left(\frac{\partial b}{\partial t} + bc_f \frac{\partial p_f}{\partial t}\right) \approx \rho_0 \left(\frac{\partial b}{\partial t} + bc_f \frac{\partial p_f}{\partial t}\right),$$
$$\operatorname{div}(\rho q_f) \approx \rho_0 \operatorname{div} q_f, \quad \rho L_{mf} \approx \rho_0 L_{mf}, \quad \rho f_f \approx \rho_0 f_f.$$

Therefore, we have the following equation on fracture γ

$$\frac{\partial b}{\partial t} + bc_f \frac{\partial p_f}{\partial t} - \operatorname{div} \left(b \frac{k_f}{\nu_f} \operatorname{grad} p_f \right) + L_{fm} = f_f, \quad x \in \gamma,$$
(8)

where, for the mass exchange between matrix and fracture, we assume a linear relationship between the flux and pressure difference, namely,

$$L_{fm} = \beta_{fm} (p_f - p_m).$$

Let $\beta_{fm} = \eta_f \beta$ and $\beta_{mf} = \eta_m \beta$, where β is the transfer term proportional to the matrix permeability, η_f and η_m are the geometric factors, that will be described in next section. Then we have

$$a_m \frac{\partial p_m}{\partial t} + \alpha \frac{\partial \varepsilon^v}{\partial t} - \operatorname{div}(b_m \operatorname{grad} p_m) + \eta_m \beta(p_m - p_f) = f_m, \quad x \in \Omega,$$

$$a_f \frac{\partial p_f}{\partial t} + \frac{\partial b}{\partial t} - \operatorname{div}(b_f \operatorname{grad} p_f) + \eta_f \beta(p_f - p_m) = f_f. \quad x \in \gamma,$$
(9)

where $a_m = 1/M$, $a_f = bc_f$, $b_m = k_m/\nu_f$, $b_f = bk_f/\nu_f$. β is the transfer term proportional to the matrix and fracture probabilities, η_f and η_m are geometric factors that will be defined in next section. Pressure coupling term expresses the conservation of the flow rate (the fluid that is lost in the fractures goes into the porous matrix). Here we assume that the fractures have constant aperture, in general case, we can use $b(t) = z p_f(t)$ as a relationship between fracture width and pressure.

Mechanical deformation model. The balance of a linear momentum in the porous matrix is given by

$$-\operatorname{div} \sigma_T = 0, \quad \sigma_T = \sigma - \alpha p_m \mathcal{I}, \quad x \in \Omega,$$
(10)

where p_m is the matrix pressure, σ_T is the total stress tensor, σ is the effective stress [35]. Relation between the stress σ and strain ε tensors is given as

$$\sigma = \lambda \varepsilon^{v} \mathcal{I} + 2\mu \varepsilon(u), \quad \varepsilon(u) = 0.5 (\nabla u + (\nabla u)^{T}),$$

where u is the displacement vector in the porous matrix, and λ and μ are the Lame's coefficients.

For incorporating of the fracture pressure into the model, we assume negligible shear traction on the fracture walls and consider normal tractions on the fractures [35] with $\tau_f = -p_f n_f$, where n_f is the normal vector to the fracture surface. After some manipulation, we obtain following equation in domain Ω

$$-\operatorname{div}\left(\sigma - \alpha p_m \mathcal{I}\right) + r_f p_f = 0, \quad x \in \Omega, \tag{11}$$

where r_f comes from the integration over fracture surface $(\int_{\gamma} p_f n_f ds)$ and contains direction of the fracture pressure influence. In presented model, we follow the classic dual porosity model and add fracture pressure effects as additional source (reaction) term. In more general case, fractures are modeled by an interface condition, where displacements have discontinuity across a fracture but stress is continuous [1].

2 Fine grid approximation of the coupled system

Let $\mathcal{T}_h = \bigcup_i \varsigma_i$ be a fine scale finite element partition of the domain Ω and $\mathcal{E}_{\gamma} = \bigcup_l \iota_l$ is the fracture mesh (see Figure 1). The implementation is based on the open-source library FEniCS [30, 31]. We use geometry objects for construction of the discrete system for coupled problem. For approximation of the flow part of the system, we use cell centered finite volume approximation with two point flux approximation. For displacement, we use Galerkin method with linear basis functions [43].

In this work, we use the two dimensional problem for illustration of the robustness of our method. In particular, we consider the following coupled system of equations for displacements (two displacements, u_x and u_y) and fluid pressures (fracture and matrix, p_f and p_m)

$$a_{m}\frac{\partial p_{m}}{\partial t} + \alpha \frac{\partial \varepsilon^{v}}{\partial t} - \operatorname{div}(b_{m}\operatorname{grad} p_{m}) + \eta_{m}\beta(p_{m} - p_{f}) = f_{m}, \quad x \in \Omega,$$

$$a_{f}\frac{\partial p_{f}}{\partial t} + \frac{\partial b}{\partial t} - \operatorname{div}(b_{f}\operatorname{grad} p_{f}) + \eta_{f}\beta(p_{f} - p_{m}) = f_{f}, \quad x \in \gamma,$$

$$-\operatorname{div}\left(\sigma(u) - \alpha p_{m}\mathcal{I}\right) + r_{f}p_{f} = 0, \quad x \in \Omega.$$
(12)

Using implicit scheme for approximation of time, a finite volume approximation for pressures and standard Galerkin method for displacements, we have following approximation

$$\int_{\Omega} a_m \frac{p_m - \check{p}_m}{\tau} d\Omega + \int_{\Omega} \alpha \frac{\varepsilon^v - \check{\varepsilon}^v}{\tau} d\Omega - \int_{\Omega} \operatorname{div}(b_m \operatorname{grad} p_m) d\Omega + \int_{\Omega} \eta_m \beta(p_m - p_f) d\Omega = \int_{\Omega} f_m d\Omega,$$

$$\int_{\gamma} a_f \frac{p_f - \check{p}_f}{\tau} d\gamma + \int_{\gamma} \frac{b - \check{b}}{\tau} d\gamma - \int_{\gamma} \operatorname{div}(b_f \operatorname{grad} p_f) d\gamma - \int_{\gamma} \eta_f \beta(p_m - p_f) d\gamma = \int_{\gamma} f_f d\gamma,$$

$$\int_{\Omega} (\sigma(u), \varepsilon(v)) d\Omega - \int_{\Omega} (\alpha p_m \mathcal{I}, \varepsilon(v)) d\Omega + \int_{\Omega} (r_f p_f, v) d\Omega = 0,$$
(13)

where $(\check{p}_m, \check{p}_f, \check{u})$ are solutions from the previous times step and τ is the given time step.

Using the two point flux approximation for pressure equations, we obtain

$$a_{m} \frac{p_{m,i} - \check{p}_{m,i}}{\tau} |\varsigma_{i}| + \alpha \frac{\varepsilon_{i}^{v} - \varepsilon_{i}^{v}}{\tau} |\varsigma_{i}| + \sum_{j} T_{ij}(p_{m,i} - p_{m,j}) + \beta_{il}(p_{m,i} - p_{f,l}) = f_{m}|\varsigma_{i}|, \quad \forall i = 1, N_{f}^{m}$$

$$a_{f} \frac{p_{f,l} - \check{p}_{f,l}}{\tau} |\iota_{l}| + \frac{b_{l} - \check{b}_{l}}{\tau} |\iota_{l}| + \sum_{n} W_{ln}(p_{f,l} - p_{f,n}) - \beta_{il}(p_{m,i} - p_{f,l}) = f_{f}|\iota_{l}|, \quad \forall l = 1, N_{f}^{f}$$

$$(14)$$

where $T_{ij} = b_m |E_{ij}| / \Delta_{ij}$ ($|E_{ij}|$ is the length of interface between cells ς_i and ς_j , Δ_{ij} is the distance between mid point of cells ς_i and ς_j), $W_{ln} = b_f / \Delta_{ln}$ (Δ_{ln} is the distance between points l and n), $|\varsigma_i|$ and $|\iota_l|$ is the volume of the cells cells ς_i and ι_l . N_f^m is the number of cells in \mathcal{T}_h , N_f^f is the number of cell for fracture mesh \mathcal{E}_{γ} . Here, we use $\eta_m = 1/|\varsigma_i|$ and $\eta_f = 1/|\iota_l|$. Also, $\beta_{il} = \beta$ if $\mathcal{E}_{\gamma} \cap \partial \varsigma_i = \iota_l$ and equals zero otherwise.

Matrix form. Combining the above schemes, we have following discrete system of equations for $y = (p_m, p_f, u_x, u_y)$ in the matrix form

$$\left(\frac{1}{\tau}M + A\right)y = F,\tag{15}$$

where

Here D is the elasticity stiffness matrix

$$\begin{split} D_x &= [d_{ij}^x] = \int_{\Omega} \sigma_x(\psi_i) : \varepsilon_x(\psi_j) \, d\Omega, \quad D_y = [d_{ij}^y] = \int_{\Omega} \sigma_y(\psi_i) : \varepsilon_y(\psi_j) \, d\Omega, \quad D_{xy} = [d_{ij}^{xy}] = \int_{\Omega} \sigma_x(\psi_i) : \varepsilon_y(\psi_j) \, d\Omega, \\ B_{m,x} &= [b_{ij}^{m,x}] = \int_{\Omega} (\alpha p_{m,i}, \varepsilon_x(\psi_j)) d\Omega, \quad B_{m,y} = [b_{ij}^{m,y}] = \int_{\Omega} (\alpha p_{m,i}, \varepsilon_y(\psi_j)) d\Omega, \\ B_{f,x} &= [b_{lj}^{f,x}] = -\int_{\Omega} (r_f p_{f,l}, \psi_j) d\Omega, \quad B_{f,y} = [b_{lj}^{f,y}] = -\int_{\Omega} (r_f p_{f,l}, \psi_j) d\Omega, \\ \text{with linear basis functions } \psi_i \text{ and } \sigma = \begin{pmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{yx} & \sigma_y \end{pmatrix} \text{ and } \varepsilon = \begin{pmatrix} \varepsilon_x & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_y \end{pmatrix} \\ \text{We remark that the dimension of fine grid problem is given by} \end{split}$$

$$N_f = N_f^m + N_f^f + 2N_f^v$$

where N_f^v is the number of vertices on the fine grid.

3 Coarse grid upscaled model for coupled problem

Consider a coarse grid partition $\mathcal{T}_H = \{K_i\}$ of the domain, where K_i is the *i*-th coarse cell. Let K_i^+ be the oversampled region for the coarse cell K_i obtained by enlarging K_i by a few coarse grid blocks. For our coarse grid approximation, we will construct multiscale basis functions using the nonlocal multicontinua method (NLMC)[11]. In general, the construction of the multiscale basis functions starts with an auxiliary space, which is constructed by solving local spectral problems [10], and then we take eigenvectors that correspond to small (contrast dependent) eigenvalues as basis functions. These spectral basis functions represent the channels (high contrast features). Using the auxiliary space, the target multiscale space is obtained by solutions of constraint energy minimization problems in oversampling domain K_i^+ . subject to a set of orthogonality conditions related to the auxiliary space. More precisely, for each auxiliary basis function, we will find a corresponding multiscale basis function such that it is orthogonal to all other auxiliary basis functions with respect to a weight inner product. Our basis functions have a nice decay property away from the target coarse element. In this paper, we use the NLMC method. In the NLMC method, we use a simplified construction that separate continua in each local domain K_i (coarse cell). Instead of using an auxiliary space, we obtain the required basis functions by minimizing an energy over an oversampling domain K_i^+ subject to the conditions that the minimizer has mean value zero on all fractures and matrix except the fracture or matrix that the basis function is formulated for. The resulting multiscale basis functions have a spatial decay property in local domains and separate background medium and fractures.



Figure 2: Multiscale basis functions on mesh 20×20 for local domain K^4 for pressures and displacements.

For the fractures, we write $\gamma = \bigcup_{l=1}^{L} \gamma^{(l)}$, where $\gamma^{(l)}$ is the *l*-th fracture network and *L* is the total number of fracture networks. We also write $\gamma_j = \bigcup_{l=1}^{L_j} \gamma_j^{(l)}$, where $\gamma_j^{(l)} = K_j \cap \gamma^{(l)}$ is the fracture inside the coarse

cell K_j and L_j is the number of fractures in K_j . Again, for construction of multiscale basis functions, we solve constrained energy minimization problem in the oversampled local domains subject to the constraint that the local solution has zero mean on other continua except the one for which it is formulated.

For construction for coarse grid approximation for the coupled problem, we construct multiscale basis function for (p_m, p_f, u_x, u_y) . For simplicity, we ignore the coupling term between pressure and displacements and find multiscale basis functions for pressure and displacements separately. In general coupled poroelastic basis functions can be constructed using the coupled poroelastic system and the constrained energy minimization principle.

Multiscale basis function for matrix and fracture pressures. To define our multiscale basis functions, we will minimize an energy subject to some constraints. In the following, we will define the constraints. We remark that we will find a set of multiscale basis functions for each coarse cell K_i , and these basis functions have support in K_i^+ . Thus the following constraints are needed for each K_i , and they are defined within K_i^+ . For each coarse cell $K_j \in K_i^+$:

(1) background medium (ψ_l^0) :

$$\int_{K_j} \psi_0^i \, dx = \delta_{i,j}, \quad \int_{\gamma_j^{(l)}} \psi_0^i \, ds = 0, \quad l = \overline{1, L_j},$$

We note that these constraints are defined for the matrix part in K_i , and they require the resulting basis function to have mean value on each continuum in K_i^+ except the continuum corresponding to the matrix part in K_i .

(2) *l*-th fracture network in K_i (ψ_l^i):

$$\int_{K_j} \psi_l^i \, dx = 0, \quad \int_{\gamma_j^{(l)}} \psi_l^i \, ds = \delta_{i,j} \delta_{m,l}, \quad l = \overline{1, L_j},$$

where L_i is the number of fracture networks in K_i . We note that these constraints are defined for a fracture network in K_i , and they require the resulting basis function to have mean value on each continuum in K_i^+ except the continuum corresponding to a specific fracture network in K_i

For the construction of the multiscale basis functions, we solve the following local problems in K_i^+ using an operator restricted in K_i^+ . This results in solving the following local problems in K_i^+ :

$$\begin{pmatrix} A_m^{K_i^+} + Q^{K_i^+} & -Q^{K_i^+} & C_m^T & 0\\ -Q^{K_i^+} & A_f^{K_i^+} + Q^{K_i^+} & 0 & C_f^T\\ C_m & 0 & 0 & 0\\ 0 & C_f & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_m \\ \psi_f \\ \mu_m \\ \mu_f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F_m \\ F_f \end{pmatrix}$$
(16)

with zero Dirichlet boundary conditions on ∂K_i^+ for ψ_m and ψ_f . Here we used Lagrange multipliers μ_m and μ_f to impose the constraints for multiscale basis construction. We remark that we have used the notations $\psi_m, \psi_f, \mu_m, \mu_f$ to denote the vector representations of the corresponding functions in terms of fine scale basis. For example ψ_m is the vector of coefficients of the matrix pressure expanded in terms of fine scale basis. We set $F_m = \delta_{i,j}$ and $F_f = 0$ for construction of multiscale basis function for porous matrix $\psi^0 = (\psi_m^0, \psi_f^0)$. For multiscale basis function for fracture network, we set $F_m = 0$ and $F_f = \delta_{i,j} \delta_{m,l}$. In Figure 2, we depict multiscale basis functions for oversampled region $K_i^+ = K_i^4$ (four oversampling coarse cell layers) on coarse mesh 20×20 .

Multiscale basis function for displacements. The construction is similar to that of pressure. More precisely, we construct a set of basis functions $\psi^{X,i} := (\psi_x^{X,i}, \psi_y^{X,i})$ and $\psi^{Y,i} := (\psi_x^{Y,i}, \psi_y^{Y,i})$, which minimize the energy for elasticity problem operator restricted in the region K_i^+ and satisfy the constraints described below for all $K_j \subset K_i^+$:

(1) X-component, $\psi^{X,i}$:

$$\int_{K_j} \psi_x^{X,i} \, dx = \delta_{i,j}, \quad \int_{K_j} \psi_y^{X,i} \, dx = 0$$

(2) Y-component, $\psi^{Y,i}$:

$$\int_{K_j} \psi_x^{Y,i} \, dx = 0, \quad \int_{K_j} \psi_y^{Y,i} \, dx = \delta_{i,j}.$$

For further error reduction, we can add additional basis function for heterogeneous source term.

This results in solving the following local problems in K_i^+ :

$$\begin{pmatrix} D_x^{K_i^+} & D_{xy}^{K_i^+} & S_x^T & 0\\ D_{xy}^{K_i^+} & D_{yy}^{K_i^+} & 0 & S_y^T\\ S_x & 0 & 0 & 0\\ 0 & S_y & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_x\\ \psi_y\\ \mu_x\\ \mu_y \end{pmatrix} = \begin{pmatrix} 0\\ 0\\ F_x\\ F_y \end{pmatrix}$$
(17)

with zero Dirichlet boundary conditions on ∂K_i^+ for ψ_x and ψ_y . We set $(F_x, F_y) = (\delta_{i,j}, 0)$ and $(0, \delta_{i,j})$ for construction of multiscale basis function for X and Y-components. In Figure 2, we depict multiscale basis functions for displacements in oversampled domain $K_i^+ = K_i^4$.

In general, the permeability or elastic coefficients can be heterogeneous, where for high-construct cases more basis should be used and constrained energy minimization (CEM) GMsFEM can identify important modes [10].

We note that, the fracture contributions are divided in each coarse cell and then coupled. Each local fracture network introduce an additional degree of freedom for current coarse cell. In general, CEM-GMsFEM can be applied, where local spectral problem automatically identify important modes [10].

Coarse scale coupled system. We first define a projection matrix using the multiscale basis functions

$$R = \begin{pmatrix} R_{mm} & R_{mf} & 0 & 0 \\ R_{fm} & R_{ff} & 0 & 0 \\ 0 & 0 & R_{xx} & R_{xy} \\ 0 & 0 & R_{yx} & R_{yy} \end{pmatrix}.$$

where

$$\begin{split} R_{mm}^{T} &= \left[\psi_{m}^{0,0}, \psi_{m}^{1,0} \dots \psi_{m}^{N_{c},0}\right], \quad R_{ff}^{T} = \left[\psi_{f}^{0,1} \dots \psi_{f}^{0,L_{0}}, \psi_{f}^{1,1} \dots \psi_{f}^{1,L_{1}}, \dots, \psi_{f}^{N_{c},1} \dots \psi_{f}^{N_{c},L_{N_{c}}}\right], \\ R_{mf}^{T} &= \left[\psi_{f}^{0,0}, \psi_{f}^{1,0} \dots \psi_{f}^{N_{c},0}\right], \quad R_{fm}^{T} = \left[\psi_{m}^{0,1} \dots \psi_{m}^{0,L_{0}}, \psi_{m}^{1,1} \dots \psi_{m}^{1,L_{1}}, \dots, \psi_{m}^{N_{c},1} \dots \psi_{m}^{N_{c},L_{N_{c}}}\right], \\ R_{xx}^{T} &= \left[\psi_{x}^{X,0}, \dots \psi_{x}^{X,N_{c}}\right] \quad R_{xy}^{T} = \left[\psi_{y}^{X,0}, \dots \psi_{y}^{X,N_{c}}\right] \quad R_{yx}^{T} = \left[\psi_{x}^{Y,0}, \dots \psi_{x}^{Y,N_{c}}\right] \quad R_{yy}^{T} = \left[\psi_{y}^{Y,0}, \dots \psi_{y}^{Y,N_{c}}\right]. \end{split}$$

In the above definition, $\psi_m^{i,l}$ is the basis function for matrix pressure corresponding to the coarse block K_i and the continuum l. The definition for $\psi_f^{i,l}$ is the basis function for fracture pressure corresponding to the coarse block K_i and the continuum l. The notation $\psi_m^{i,l}$ stands for both the function and its vector representation in fine grid basis. We note that we construct only decoupled multiscale basis functions for flow and mechanics. Coupled construction of the multiscale basis functions can provide better results and will be considered and investigated in the future works.

Finally, we obtain following upscaled coarse grid model

$$\left(\frac{1}{\tau}\bar{M} + \bar{A}\right)\bar{y} = \bar{F},\tag{18}$$

where $\bar{A} = RAR^T$, $\bar{F} = RF$, $\bar{y} = (\bar{p}_m, \bar{p}_f, \bar{u}_x, \bar{u}_y)$. Here $\bar{p}_m, \bar{p}_f, \bar{u}_x, \bar{u}_y$ are the average solution on coarse grid cell for matrix, fracture, displacement X and Y components, respectively. For mass matrix, we can use a property of the constructed multiscale basis functions, and obtain diagonal mass matrix by direct calculation on the coarse grid

$$\bar{M} = \begin{pmatrix} \bar{M}_m & 0 & 0 & 0\\ 0 & \bar{M}_f & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where $\bar{M}_m = \text{diag}\{a_m|K_i|\}, \bar{M}_f = \text{diag}\{a_f|\gamma_i|\}$. The coarse grid upscaled model has only one coarse degree of freedom (DOF) for each fracture network and provides an effective coarse scale model with physical meaning, and leads to a fast and accurate solver for the coupled poroelasticity problem.

4 Numerical results

We present numerical results for poroelastic model in Ω with length of 1 meter in both directions. We consider two test cases: (1) domain with 30 fractures and (2) domain with 60 fractures. In Figures 3 and 4, we show computational coarse and fine grids, where the fractures are depicted with red color and fine mesh with blue color. For fracture network, we constructed separate mesh and for domain Ω , we use structured fine mesh. We consider two coarse grids with 400 cells and with 1600 cells. The coarse grids are uniform.

For coupled poroelastic model, we use following parameters:

- Elastic parameters: $\mu = \frac{E}{2(1+\nu)}$ and $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$, where $E = 10 \times 10^9$, $\nu = 0.3$ and $\alpha = 0.1$,
- Flow parameters $a_m = 10^{-6}$, $a_f = 10^{-7}$, $b_m = 10^{-11}$, $b_f = 10^{-6}$ and $\beta = 10^{-10}$.

Boundary condition for the displacement: $u_x = 0.0$ on the left and right boundaries, $u_y = 0.0$ on the bottom and top. We set a point source at the two coarse cells with q = 0.01 and set initial pressure $p_0 = 10^7$. We simulate $t_{max} = 10$ years with 50 time steps for multiscale and fine scale solvers.

We use DOF_c to denote problem size of the coarse-grid upscaled model and DOF_f for the fine grid system size. To compare the results, we use the relative L^2 error between coarse cell average of the



Figure 3: Computational grids with 30 fracture lines. First: Coarse grid 20×20 with 400 cells. Second: Coarse grid 40×40 with 1600 cells. Third: Fine grid for matrix domain Ω with 14641 vertices and 28800 cells (blue). Fine gird for fracture domain γ with 1042 cells (red and white)



Figure 4: Computational grids with 60 fracture lines. First: Coarse grid 20×20 with 400 cells. Second: Fine grid for matrix domain Ω with 14641 vertices and 28800 cells (blue). Fine gird for fracture domain γ with 1312 cells (red and white)

<i>K</i> ^s	0	0	0	K^s	e_p	e_{u_x}	e_{u_y}
	$\begin{array}{c c} c_p & c_{u_x} & c_{u_y} \\ \hline \\ $			Coarse grid 40×40			
1	Coarse		20	1	1.986	96.667	95.454
	4.740	86.865	82.598	2	0.191	78.718	74.957
2	0.723	43.721	37.034	3	0.174	30.550	25.220
3	0.369	6.716	4.668	4	0.158	4 1302	3 321
4	0.359	2.718	2.854	6	0.157	1 1 27	1 933
			,	0	0.157	1.127	1.233

Table 1: Numerical results of relative errors (%) at the final simulation time. $DOF_f = 59124$ and $DOF_c = 1393$. Test case with 30 fractures.



Figure 5: Fine scale solution for pressure $(p^* = (p - p_0)/p_0)$ and displacements (from left to right) for the different time layers t_5 , t_{15} and t_{50} (from top to bottom). Test case with 30 fractures.

fine-scale solution $\bar{p}_m^{fine}, \bar{u}_x^{fine}, \bar{u}_y^{fine}$ and upscaled coarse grid solutions $\bar{p}_m, \bar{u}_x, \bar{u}_y$

$$e_{p} = ||\bar{p}_{m}^{fine} - \bar{p}_{m}||_{L^{2}}, \quad e_{u_{x}} = ||\bar{u}_{x}^{fine} - \bar{u}_{x}||_{L^{2}}, \quad e_{u_{y}} = ||\bar{u}_{y}^{fine} - \bar{u}_{y}||_{L^{2}}, \\ ||\bar{v}_{f} - \bar{v}||_{L^{2}}^{2} = \frac{\sum_{K} (\bar{v}_{f}^{K} - \bar{v}^{K})^{2}}{\sum_{K} (\bar{v}_{f}^{K})^{2}}, \quad \bar{v}_{f}^{K} = \frac{1}{|K|} \int_{K} v_{f} \, dx, \quad v = p, u_{x}, u_{y},$$

$$(19)$$

for matrix pressure and displacements.

Fine grid solution for computational domain with 30 fractures is presented in Figure 5 for the different time instants t_5 , t_{15} and t_{50} , where $t_n = n\tau$. On the first column of the figure, we depict pressure $p^* = (p-p_0)/p^0$, on the second and third row columns – displacements u_x and u_y . Comparison of the fine grid and coarse grid upscaled solutions are presented in Figure 6 at final time. We perform computations on the coarse grid with 400 cells with 4 oversampling layers in the construction of basis functions ($K^+ = K^4$). In the first column, we depict fine grid pressure solution; in the second column – reconstructed fine scale solution from upscaled



Figure 6: Numerical results for pressure $(p^* = (p - p_0)/p_0)$ and displacements at final time. Test case with 30 fractures. First row: pressure, p_m . Second row: displacement, u_x . Third row: displacement, u_y . First column: fine scale solution with $DOF_f = 59124$. Second column: reconstructed fine scale solution from upscaled coarse grid solution with $DOF_c = 1393$. Third column: coarse cell average for fine scale solution. Fourth column: coarse cell average for upscaled coarse grid. Coarse grid 20×20 ($K^+ = K^4$).

coarse grid solution, in the third column – coarse cell average for fine scale solution and in the fourth column – coarse cell average for upscaled coarse grid. Fine grid system has size $DOF_f = 59124$. By performing NLMC method, we reduce size of system to $DOF_c = 1393$. At final time, we have less than one percent of error for pressure and near 2.5% for displacement.

In Table 1, we present relative errors at final time for two coarse grids and for different numbers of oversampling layers for the oversample region K^s with s = 1, 2, 3, 4 and 6, where K^s is obtained by extending K by s coarse grid layers. From the numerical results, we observe a good convergence behavior, when we take



Figure 7: Fine scale solution for pressure $(p^* = (p - p_0)/p_0)$ and displacements (from left to right) for the different time layers t_5 , t_{15} and t_{50} (from top to bottom). Test case with 60 fractures.

sufficient number of oversampled layers. For the coarse mesh with 400 cells, when we take 4 oversampling layers, we have 0.359% relative error for pressure, for displacement – 2.718% (u_x) and 2.854% (u_y). For the coarse mesh with 1600 cells with 6 oversampling layers, relative error is 0.157% for pressure, for displacement – 1.127% (u_x) and 1.233% (u_y). We note that, on the 20 × 20 coarse mesh, the size of upscaled system is $DOF_c = 1393$ and for the 40 × 40 coarse mesh, we have $DOF_c = 5165$. From the Table 1, we observe that we can use smaller number of oversampling layers for pressure than for displacements. For pressure is enough to take 2 oversampling layers for obtaining errors smaller than one percent for both coarse grids, on the other hand for displacements we should take 4 or 6 oversampling layers in coarse grids 20 × 20 and 40 × 40, respectively. Note tat, in general the presented algorithm can work with different numbers of oversampling layers for pressure and displacement due to coupled construction of the coarse grid system.

In Figure 9, we present relative errors for pressure and displacements vs time with different number of



Figure 8: Numerical results for pressure $(p^* = (p - p_0)/p_0)$ and displacements at final time. Test case with 60 fractures. First row: pressure, p_m . Second row: displacement, u_x . Third row: displacement, u_y . First column: fine scale solution with $DOF_f = 59394$. Second column: reconstructed fine scale solution from upscaled coarse grid solution with $DOF_c = 1484$. Third column: coarse cell average for fine scale solution. Fourth column: coarse cell average for upscaled coarse grid. Coarse grid 20×20 ($K^+ = K^4$)

oversampling layers K^s , s = 1, 2, 3, 4, 6. All results show good accuracy of the proposed method for coupled poroelasticity problems in fractured media.

Next, we consider test case with 60 fractures. In Figure 7, we shown solution of the problem for the different time layers t_5 , t_{15} and t_{50} . Comparison of the fine grid and coarse grid upscaled solutions are presented in Figure 8 at final time for coarse grid with 400 cells and 4 oversampling layers ($K^+ = K^4$). In the first column, we depict fine grid pressure solution; in the second column – reconstructed fine scale solution from upscaled coarse grid solution, in the third column – coarse cell average for fine scale solution and in



Figure 9: Relative errors by time for coarse mesh 20×20 with K^s . Test case with 30 fractures.



Figure 10: Relative errors by time for coarse mesh 20×20 with K^4 . Test cases with 30 and 60 fractures.

the fourth column – coarse cell average for upscaled coarse grid. Fine grid system has size $DOF_f = 59394$. By performing NLMC method, we reduce size of system to $DOF_c = 1484$. At final time, we have 0.4217% of error for pressure and for displacement – 2.739% (u_x) and 3.124% (u_y) . In Figure 10, we depict relative errors vs time for K^4 on coarse mesh 20 × 20 for test case with 30 and 60 fractures. We obtain similar results with good accuracy for both test cases.

Next, we discuss the computational advantages of our approach. The computatonal time is divided into offline and online stages. In offline stage (preprocessing), we generate local domains, calculate multiscale basis functions and generate coarse grid system. In online stage, we solve coarse grid problem, with different imput parameters (source term, boundary conditions, time steps, etc.). Let DOF_f is the size of fine scale solution $y = (p_m, p_f, u)$, then the dimension of the fine grid coupled problem is $DOF_f \times DOF_f$. The coarse grid system size is DOF_c for coupled poroelasticity problem, that depends on the coarse grid size and the number of local multiscale basis functions. In each local domain (coarse grid cell), we have degree of freedom

for displacement X and Y components, vof matrix pressure and additional degree of freedom for each fracture network in current coarse cell. We note that, the number of degree of freedom is similar to classic embedded fracture model (EFM). For two dimensional problems ,we have $M = 3 + M_i$ ($(p_m, p_f^1 ... p_f^{M_i}, u_x, u_y)$) degrees of freedoms in local domain, where M_i is the number of the fracture networks in coarse cell K_i . Therefore, the size of coarse grid system is $DOF_c = \sum_{K_i} (3 + M_i)$.

Let N_f is the number of cells for fracture network mesh, N_c and N_v are the number of cells and vertices on fine grid for domain Ω . Then for two dimensional problems with finite element approximation for displacements equation and finite volume approximation for flow problem, we have $DOF_f = N_f + N_c + 2N_v$. Then, we can compare a computational cost of solving coarse and fine grid problems. For example in test case with 30 fractures, a coarse solution has $DOF_c = 1393$ in the coarse grid with 400 cells, where we have 400 and 800 degrees of freedom for matrix pressure and displacement X and Y components, and 193 degrees of freedom for fractures. For the fine scale system $DOF_f = 59124$ on fine grid with $N_v = 14641$ vertices and $N_c = 28800$ cells. Then, we can obtain accurate solution for multiscale solver using only 2.3% from DOF_f . We note that, the number of M_i in K_i and therefore size of coarse grid system is independent on fine grid size and a few basis functions can approximate the fine scale solution accurately no matter how fine is the fine grid. When we use classic direct solver, the solution time of the time dependent coupled fine grid problem is 81.17 seconds and 5.74 seconds for coarse grid. We have computational gain in the simulations, because in each time step, the proposed method solves a small coarse grid system compared to the fine-grid system.

5 Conclusion

In this paper, our goal is to develop an upscaled model for a poroelastic system in fractured media. There are several contributions. First, we construct an embedded fracture model for a coupled flow and mechanics system. Secondly, based on this system, we develop a nonlocal upscaled model for efficient numerical simulations. The construction of the upscaled model is motivated by the NLMC method. The main idea is to construct basis functions for each continuum within a local coarse region such that the resulting coarse degrees of freedom have physical meanings. Moreover, these basis functions have decay property thanks to an energy minimization principle, which can guarantee an accurate approximation of the solution. We have presented several numerical tests to show that our upscaled model can give accurate solutions with a small computational cost.

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