

Strong semismoothness of Fischer-Burmeister complementarity function associated with symmetric cones

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Abstract. We provide an affirmative answer to an question that the Fischer-Burmeister complementarity function associated with symmetric cones, named the FB SC complementarity function, is globally Lipschitz continuous and strongly semismooth everywhere for \mathbb{H}^n and \mathbb{Q}^n . This is achieved with the help of embedding \mathbb{H}^n and \mathbb{Q}^n into certain \mathbb{S}^m .

Key words. Fischer-Burmeister function, symmetric cones, strong semismoothness.

1 Introduction

Let $\mathbb{A} = (\mathbb{V}, \langle \cdot, \cdot \rangle, \circ)$ be an n -dimensional Euclidean Jordan algebra (see Section 2) and \mathcal{K} be the symmetric cone in \mathbb{V} . We call $\phi : \mathbb{V} \times \mathbb{V} \rightarrow \mathbb{V}$ a complementarity function

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associated with symmetric cone (SC complementarity function for short) if

$$\phi(x, y) = 0 \iff x \in \mathcal{K}, \quad y \in \mathcal{K}, \quad x \circ y = 0. \quad (1)$$

This class of functions and the merit functions induced play an important role in the development of algorithms for the symmetric cone complementarity problem (SCCP) and the symmetric cone programming (SCP), and attract much attention in current optimization field; see [8, 9, 10, 13, 14, 18].

A popular choice for ϕ is the natural residual (NR) SC complementarity function [7]

$$\phi_{\text{NR}}^{\text{sc}}(x, y) := x - (x - y)_+ \quad \forall x, y \in \mathbb{V}, \quad (2)$$

where $(z)_+$ denotes the minimum metric projection of z onto \mathcal{K} . This function is shown to be strongly semismooth in [18]. Recently, the nonsingularity of Clarke's generalized Jacobian of the KKT nonsmooth system based on $\phi_{\text{NR}}^{\text{sc}}$ for linear SCP is studied in [11, 19]. These works lay the foundations for the corresponding nonsmooth Newton methods and smoothing Newton methods of the SCCPs and the SCPs.

Another popular choice of ϕ is the Fischer-Burmeister(FB) SC complementarity function [7] defined as

$$\phi_{\text{FB}}^{\text{sc}}(x, y) := (x^2 + y^2)^{1/2} - (x + y) \quad \forall x, y \in \mathbb{V}, \quad (3)$$

where $x^2 = x \circ x$, and $x^{1/2}$ denotes the unique square root of $x \in \mathcal{K}$, i.e., $x^{1/2} \circ x^{1/2} = x$. Compared with the function $\phi_{\text{NR}}^{\text{sc}}$, this function has a remarkable advantage, namely, its squared norm induces a continuously differentiable merit function, and furthermore, the merit function has a globally Lipschitz continuous gradient; see [12, 14] for details. This will greatly facilitate the globalization of nonsmooth Newton methods based on $\phi_{\text{FB}}^{\text{sc}}$.

It is known that when \mathbb{V} is the space of all $n \times n$ symmetric matrices with a specific Jordan product, \mathcal{K} corresponds to positive semidefinite cone, whereas when \mathbb{V} is the \mathbb{R}^n space with a specific Jordan product, \mathcal{K} corresponds to the Lorentz cone (also known as second-order cone), see [3]. Moreover, it was shown in [17] that $\phi_{\text{FB}}^{\text{sc}}$ is strongly semismooth under the aforementioned two cases. Whether such property holds for general Euclidean Jordan algebra has been an open question thereafter. In this paper, we provide an almost-complete answer for it and explain why the incomplete part occurs.

2 Preliminaries

This section recalls some results on Euclidean Jordan algebras that will be used in subsequent analysis. More detailed expositions of Euclidean Jordan algebras can be found

in Koecher's lecture notes [5] and the monograph by Faraut and Korányi [3].

Let \mathbb{V} be an n -dimensional vector space over the real field \mathbb{R} , endowed with a bilinear mapping $(x, y) \mapsto x \circ y$ from $\mathbb{V} \times \mathbb{V}$ into \mathbb{V} . The pair (\mathbb{V}, \circ) is called a *Jordan algebra* if

- (i) $x \circ y = y \circ x$ for all $x, y \in \mathbb{V}$,
- (ii) $x \circ (x^2 \circ y) = x^2 \circ (x \circ y)$ for all $x, y \in \mathbb{V}$.

Note that a Jordan algebra is not necessarily associative, i.e., $x \circ (y \circ z) = (x \circ y) \circ z$ may not hold for all $x, y, z \in \mathbb{V}$. We call an element $e \in \mathbb{V}$ the *identity* element if $x \circ e = e \circ x = x$ for all $x \in \mathbb{V}$. A Jordan algebra (\mathbb{V}, \circ) with an identity element e is called a *Euclidean Jordan algebra* if there is an inner product, $\langle \cdot, \cdot \rangle_{\mathbb{V}}$, such that

- (iii) $\langle x \circ y, z \rangle_{\mathbb{V}} = \langle y, x \circ z \rangle_{\mathbb{V}}$ for all $x, y, z \in \mathbb{V}$.

Given a Euclidean Jordan algebra $\mathbb{A} = (\mathbb{V}, \circ, \langle \cdot, \cdot \rangle_{\mathbb{V}})$, we denote the set of squares as

$$\mathcal{K} := \{x^2 \mid x \in \mathbb{V}\}.$$

By [3, Theorem III.2.1], \mathcal{K} is a symmetric cone. This means that \mathcal{K} is a self-dual closed convex cone with nonempty interior and for any two elements $x, y \in \text{int}(\mathcal{K})$, there exists an invertible linear transformation $\mathcal{T} : \mathbb{V} \rightarrow \mathbb{V}$ such that $\mathcal{T}(\mathcal{K}) = \mathcal{K}$ and $\mathcal{T}(x) = y$.

For any given $x \in \mathbb{A}$, let $\zeta(x)$ be the degree of the minimal polynomial of x , i.e.,

$$\zeta(x) := \min \{k : \{e, x, x^2, \dots, x^k\} \text{ are linearly dependent}\}.$$

Then the *rank* of \mathbb{A} is defined as $\max\{\zeta(x) : x \in \mathbb{V}\}$. In this paper, we use r to denote the rank of the underlying Euclidean Jordan algebra. Recall that an element $c \in \mathbb{V}$ is *idempotent* if $c^2 = c$. Two idempotents c_i and c_j are said to be *orthogonal* if $c_i \circ c_j = 0$. One says that $\{c_1, c_2, \dots, c_k\}$ is a *complete system of orthogonal idempotents* if

$$c_j^2 = c_j, \quad c_j \circ c_i = 0 \text{ if } j \neq i \text{ for all } j, i = 1, 2, \dots, k, \quad \text{and } \sum_{j=1}^k c_j = e.$$

An idempotent is *primitive* if it is nonzero and cannot be written as the sum of two other nonzero idempotents. We call a complete system of orthogonal primitive idempotents a *Jordan frame*. Now we state the second version of the spectral decomposition theorem.

Theorem 2.1 [3, Theorem III.1.2] *Suppose that \mathbb{A} is a Euclidean Jordan algebra with the rank r . Then for any $x \in \mathbb{V}$, there exists a Jordan frame $\{c_1, \dots, c_r\}$ and real numbers $\lambda_1(x), \dots, \lambda_r(x)$, arranged in the decreasing order $\lambda_1(x) \geq \lambda_2(x) \geq \dots \geq \lambda_r(x)$, such that*

$$x = \lambda_1(x)c_1 + \lambda_2(x)c_2 + \dots + \lambda_r(x)c_r.$$

The numbers $\lambda_j(x)$ (counting multiplicities), which are uniquely determined by x , are called the eigenvalues and $\text{tr}(x) = \sum_{j=1}^r \lambda_j(x)$ the trace of x .

Since, by [3, Prop. III.1.5], a Jordan algebra (\mathbb{V}, \circ) with an identity element $e \in \mathbb{V}$ is Euclidean if and only if the symmetric bilinear form $\text{tr}(x \circ y)$ is positive definite, we may define another inner product on \mathbb{V} by $\langle x, y \rangle := \text{tr}(x \circ y)$ for any $x, y \in \mathbb{V}$. The inner product $\langle \cdot, \cdot \rangle$ is associative by [3, Prop. II. 4.3], i.e., $\langle x, y \circ z \rangle = \langle y, x \circ z \rangle$ for any $x, y, z \in \mathbb{V}$. Every Euclidean Jordan algebra can be written as a direct sum of so-called *simple* ones, which are not themselves direct sums in a nontrivial way. In finite dimensions, the simple Euclidean Jordan algebras come from the following five basic structures.

Theorem 2.2 [3, Chapter V.3.7] *Every simple Euclidean Jordan algebra is isomorphic to one of the following.*

- (i) *The Jordan spin algebra \mathbb{L}^n .*
- (ii) *The algebra \mathbb{S}^n of $n \times n$ real symmetric matrices.*
- (iii) *The algebra \mathbb{H}^n of all $n \times n$ complex Hermitian matrices.*
- (iv) *The algebra \mathbb{Q}^n of all $n \times n$ quaternion Hermitian matrices.*
- (v) *The algebra \mathbb{O}^3 of all 3×3 octonion Hermitian matrices.*

3 Main results

As mentioned earlier, the Fischer-Burmeister SC complementarity function $\phi_{\text{FB}}^{\text{sc}}$ defined as in (3) was shown to be strongly semismooth in [17] for classes (i) and (ii) of Theorem 2.2. Thus, we only need to check the remainder classes (iii)-(v). Let us start with class (iii).

Class(iii): The algebra \mathbb{H}^n of $n \times n$ complex Hermitian matrices.

A square matrix A of complex entries is said to be *Hermitian* if $A^* := \bar{A}^T = A$, where ‘bar’ denotes the complex conjugate, and the superscript ‘T’ means the transpose. Let \mathbb{H}^n be the set of all $n \times n$ complex Hermitian matrices. On \mathbb{H}^n , we define the Jordan product and inner product by $X \circ Y := \frac{1}{2}(XY + YX)$ and $\langle X, Y \rangle := \text{trace}(XY)$. Then \mathbb{H}^n is a Euclidean Jordan algebra of rank n with e being the $n \times n$ identity matrix I .

For example, \mathbb{H}^2 is the set which contains all

$$\begin{bmatrix} \alpha_1 & \beta \\ \bar{\beta} & \alpha_2 \end{bmatrix}, \quad \alpha_1, \alpha_2 \in \mathbb{R} \text{ and } \beta \in \mathbb{C}.$$

We also know that each complex number $a + bi$ can be represented as a 2×2 real matrix:

$$a \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

where $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ satisfies $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}^2 = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Hence, we can embed $\begin{bmatrix} \alpha_1 & \beta \\ \bar{\beta} & \alpha_2 \end{bmatrix}$ into an element in \mathbb{S}^4 :

$$\mathbb{H}^2 \ni \begin{bmatrix} \alpha_1 & \beta \\ \bar{\beta} & \alpha_2 \end{bmatrix} \mapsto \begin{bmatrix} \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_1 \end{bmatrix} & \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \\ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} & \begin{bmatrix} \alpha_2 & 0 \\ 0 & \alpha_2 \end{bmatrix} \end{bmatrix} \in \mathbb{S}^4$$

where $\beta = a + ib$.

It is easy to check that this embedding is one-to-one and onto, and also preserves the Jordan algebra structures on the both sides by matrix block multiplication. Therefore, we can view \mathbb{H}^2 as a Jordan sub-algebra of \mathbb{S}^4 . For general n it is also true that \mathbb{H}^n is a Jordan sub-algebra of \mathbb{S}^{2n} . In fact, the general embedding map is given by

$$\mathbb{H}^n \ni \begin{bmatrix} \alpha_1 & \beta & \cdots & \gamma \\ \bar{\beta} & \alpha_2 & \cdots & \delta \\ \vdots & \vdots & \ddots & \vdots \\ \bar{\gamma} & \bar{\delta} & \cdots & \alpha_n \end{bmatrix} \mapsto \begin{bmatrix} \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_1 \end{bmatrix} & \begin{bmatrix} a & b \\ -b & a \end{bmatrix} & \cdots & \begin{bmatrix} c & d \\ -d & c \end{bmatrix} \\ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} & \begin{bmatrix} \alpha_2 & 0 \\ 0 & \alpha_2 \end{bmatrix} & \cdots & \begin{bmatrix} e & f \\ -f & e \end{bmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{bmatrix} c & -d \\ d & c \end{bmatrix} & \begin{bmatrix} e & -f \\ f & e \end{bmatrix} & \cdots & \begin{bmatrix} \alpha_n & 0 \\ 0 & \alpha_n \end{bmatrix} \end{bmatrix} \in \mathbb{S}^{2n}$$

where $\beta = a + ib$, $\gamma = c + id$, $\delta = e + if$.

Class(iv): The algebra \mathbb{Q}^n of $n \times n$ quaternion Hermitian matrices.

The linear space of quaternions over \mathbb{R} , denoted by \mathbb{Q} , is 4-dimensional vector space [20] with a basis $\{1, i, j, k\}$. This space becomes an associated algebra via the following multiplication table.

	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

For any $x = x_01 + x_1i + x_2j + x_3k \in \mathbb{Q}$, we define its *real part* by $\mathbb{R}(x) := x_0$, its *conjugate* by $\bar{x} := x_01 - x_1i - x_2j - x_3k$, and its norm by $|x| = \sqrt{x\bar{x}}$. A square matrix A with quaternion entries is called *Hermitian* if A coincides with its conjugate transpose. Let \mathbb{Q}^n be the set of all $n \times n$ quaternion Hermitian matrices. For any $X, Y \in \mathbb{Q}^n$, we define

$$X \circ Y := \frac{1}{2}(XY + YX) \quad \text{and} \quad \langle X, Y \rangle := \mathbb{R}(\text{trace}(XY)).$$

Then \mathbb{Q}^n is a Euclidean Jordan algebra of rank n with e being the $n \times n$ identity matrix I . Analogous to complex number, each quaternion $x = a1 + bi + cj + dk \in \mathbb{Q}$ can be

represented as a 4×4 real matrix $\begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{bmatrix}$ which is also equivalent to

$$a \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + b \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} + d \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}.$$

Following the same lines for \mathbb{H}^n , we can embed \mathbb{Q}^n into \mathbb{S}^{4n} such that \mathbb{Q}^n can be viewed as a Jordan sub-algebra of \mathbb{S}^{4n} . Again, the embedding map under the case for \mathbb{Q}^2 is

$$\mathbb{Q}^2 \ni \begin{bmatrix} \alpha_1 & x \\ \bar{x} & \alpha_2 \end{bmatrix} \mapsto \begin{bmatrix} \begin{bmatrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 \\ 0 & 0 & 0 & \alpha_1 \end{bmatrix} & \begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{bmatrix} \\ \begin{bmatrix} a & -b & -c & -d \\ b & a & d & -c \\ c & -d & a & b \\ d & c & -b & a \end{bmatrix} & \begin{bmatrix} \alpha_2 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 \\ 0 & 0 & 0 & \alpha_2 \end{bmatrix} \end{bmatrix} \in \mathbb{S}^8$$

where $x = a1 + bi + cj + dk$.

Moreover, the general embedding map under this case is given by

$$\mathbb{Q}^n \ni \begin{bmatrix} \alpha_1 & x & \cdots & y \\ \bar{x} & \alpha_2 & \cdots & z \\ \vdots & \vdots & \ddots & \vdots \\ \bar{y} & \bar{z} & \cdots & \alpha_n \end{bmatrix} \mapsto$$

$$\left[\begin{array}{c} \begin{bmatrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & \alpha_1 & 0 \\ 0 & 0 & 0 & \alpha_1 \end{bmatrix} \\ \begin{bmatrix} a & -b & -c & -d \\ b & a & d & -c \\ c & -d & a & b \\ d & c & -b & a \end{bmatrix} \\ \vdots \\ \begin{bmatrix} e & -f & -g & -h \\ f & e & h & -g \\ g & -h & e & f \\ h & g & -f & e \end{bmatrix} \end{array} \right] \left[\begin{array}{c} \begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ -c & d & a & -b \\ -d & -c & b & a \end{bmatrix} \\ \begin{bmatrix} \alpha_2 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 \\ 0 & 0 & \alpha_2 & 0 \\ 0 & 0 & 0 & \alpha_2 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} p & -q & -r & -s \\ q & p & s & -r \\ r & -s & p & q \\ s & r & -q & p \end{bmatrix} \end{array} \right] \cdots \left[\begin{array}{c} \begin{bmatrix} e & f & g & h \\ -f & e & -h & g \\ -g & h & e & -f \\ -h & -g & f & e \end{bmatrix} \\ \begin{bmatrix} p & q & r & s \\ -q & p & -s & r \\ -r & s & p & -q \\ -s & -r & q & p \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \alpha_n & 0 & 0 & 0 \\ 0 & \alpha_n & 0 & 0 \\ 0 & 0 & \alpha_n & 0 \\ 0 & 0 & 0 & \alpha_n \end{bmatrix} \end{array} \right] \in \mathbb{S}^{4n}$$

where $x = a1 + bi + cj + dk$, $y = e1 + fi + gj + hk$ and $z = p1 + qi + rj + sk$.

In summary, \mathbb{H}^n and \mathbb{Q}^n are Jordan sub-algebras of \mathbb{S}^m for certain m . Hence the strong semismoothness of $\phi_{\text{FB}}^{\text{sc}}$ for classes (iii) and (iv) follows from the result of [17]. Thus, we conclude the following theorem.

Theorem 3.1 *The Fischer-Burmeister SC complementarity function $\phi_{\text{FB}}^{\text{sc}}$ defined as in (3) is strongly semismooth for each one of the following.*

- (i) *The Jordan spin algebra \mathbb{L}^n .*
- (ii) *The algebra \mathbb{S}^n of $n \times n$ real symmetric matrices.*
- (iii) *The algebra \mathbb{H}^n of all $n \times n$ complex Hermitian matrices.*
- (iv) *The algebra \mathbb{Q}^n of all $n \times n$ quaternion Hermitian matrices.*

Suppose that \mathbb{A} is a Euclidean Jordan algebra which is a direct sum of ones taken only from classes (i)-(iv) of Theorem 3.1. Theorem 3.1 says that the Fischer-Burmeister SC complementarity function $\phi_{\text{FB}}^{\text{sc}}$ defined on such \mathbb{A} is strongly semismooth. The exceptional case where we cannot draw a conclusion is \mathbb{O}^3 which is also called Albert algebra, a 27-dimensional Jordan algebra. Since \mathbb{O} is not an associative algebra, there is no way (to our best knowledge) to represent an element in \mathbb{O} as a real matrix. Hence we can not embed \mathbb{O}^3 into \mathbb{S}^m as what we do for classes (iii)-(iv). This is the big hurdle which causes the uncertainty of the function $\phi_{\text{FB}}^{\text{sc}}$ being strongly semismooth under this case of \mathbb{O}^3 .

In fact, the aforementioned result could be obtained by using analysis associated with Euclidean Jordan algebra. However, in that approach there comes up similar barrier during some analysis procedure. Moreover, the arguments by employing direct analysis associated with Jordan algebra are harder to follow. Therefore, we decide to use the current way to present this result. Even though the outcome is not perfect because there is one case not concluded, we still think the update result should be known in public so that subsequent research can be continued. We leave this unsolved case for future study. For readers who are interested in knowing more details about the structure of \mathbb{O} (so that they can understand why it is a difficult case), please refer to [6].

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