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Overview

Financial Signal Processing and Machine Learning

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1.1 Introduction

In the last decade, we have seen dramatic growth in applications for signal-processing and machine-learning techniques in many enterprise and industrial settings. Advertising, real estate, healthcare, e-commerce, and many other industries have been radically transformed by new processes and practices relying on collecting and analyzing data about operations, customers, competitors, new opportunities, and other aspects of business. The financial industry has been one of the early adopters, with a long history of applying sophisticated methods and models to analyze relevant data and make intelligent decisions – ranging from the quadratic programming formulation in Markowitz portfolio selection (Markowitz, 1952), factor analysis for equity modeling (Fama and French, 1993), stochastic differential equations for option pricing (Black and Scholes, 1973), stochastic volatility models in risk management (Engle, 1982; Hull and White, 1987), reinforcement learning for optimal trade execution (Bertsimas and Lo, 1998), and many other examples. While there is a great deal of overlap among techniques in machine learning, signal processing and financial econometrics, historically, there has been rather limited awareness and slow permeation of new ideas among these areas of research. For example, the ideas of stochastic volatility and copula modeling, which are quite central in financial econometrics, are less known in the signal-processing literature, and the concepts of sparse modeling and optimization that have had a transformative impact on signal processing and statistics have only started to propagate slowly into financial

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applications. The aim of this book is to raise awareness of possible synergies and interactions among these disciplines, present some recent developments in signal processing and machine learning with applications in finance, and also facilitate interested experts in signal processing to learn more about applications and tools that have been developed and widely used by the financial community.

We start this chapter with a brief summary of basic concepts in finance and risk management that appear throughout the rest of the book. We present the underlying technical themes, including sparse learning, convex optimization, and non-Gaussian modeling, followed by brief overviews of the chapters in the book. Finally, we mention a number of highly relevant topics that have not been included in the volume due to lack of space.

1.2 A Bird's-Eye View of Finance

The financial ecosystem and markets have been transformed with the advent of new technologies where almost any financial product can be traded in the globally interconnected cyberspace of financial exchanges by anyone, anywhere, and anytime. This systemic change has placed real-time data acquisition and handling, low-latency communications technologies and services, and high-performance processing and automated decision making at the core of such complex systems. The industry has already coined the term *big data finance*, and it is interesting to see that technology is leading the financial industry as it has been in other sectors like e-commerce, internet multimedia, and wireless communications. In contrast, the knowledge base and exposure of the engineering community to the financial sector and its relevant activity have been quite limited. Recently, there have been an increasing number of publications by the engineering community in the finance literature, including *A Primer for Financial Engineering* (Akansu and Torun, 2015) and research contributions like Akansu *et al.*, (2012) and Pollak *et al.*, (2011). This volume facilitates that trend, and it is composed of chapter contributions on selected topics written by prominent researchers in quantitative finance and financial engineering.

We start by sketching a very broad-stroke view of the field of finance, its objectives, and its participants to put the chapters into context for readers with engineering expertise. Finance broadly deals with all aspects of money management, including borrowing and lending, transfer of money across continents, investment and price discovery, and asset and liability management by governments, corporations, and individuals. We focus specifically on trading where the main participants may be roughly classified into hedgers, investors, speculators, and market makers (and other intermediaries). Despite their different goals, all participants try to balance the two basic objectives in trading: to maximize future expected rewards (returns) and to minimize the risk of potential losses.

Naturally, one desires to buy a product cheap and sell it at a higher price in order to achieve the ultimate goal of profiting from this trading activity. Therefore, the expected return of an investment over any holding time (horizon) is one of the two fundamental performance metrics of a trade. The complementary metric is its variation, often measured as the standard deviation over a time window, and called investment risk or market risk.[1] Return and risk are two typically conflicting but interwoven measures, and risk-normalized return (Sharpe ratio)

[1] There are other types of risk, including credit risk, liquidity risk, model risk, and systemic risk, that may also need to be considered by market participants.

finds its common use in many areas of finance. Portfolio optimization involves balancing risk and reward to achieve investment objectives by optimally combining multiple financial instruments into a portfolio. The critical ingredient in forming portfolios is to characterize the statistical dependence between prices of various financial instruments in the portfolio. The celebrated Markowitz portfolio formulation (Markowitz, 1952) was the first principled mathematical framework to balance risk and reward based on the covariance matrix (also known as the variance-covariance or VCV matrix in finance) of returns (or log-returns) of financial instruments as a measure of statistical dependence. Portfolio management is a rich and active field, and many other formulations have been proposed, including risk parity portfolios (Roncalli, 2013), Black–Litterman portfolios (Black and Litterman, 1992), log-optimal portfolios (Cover and Ordentlich, 1996), and conditional value at risk (cVaR) and coherent risk measures for portfolios (Rockafellar and Uryasev, 2000) that address various aspects ranging from the difficulty of estimating the risk and return for large portfolios to the non-Gaussian nature of financial time series, and to more complex utility functions of investors.

The recognition of a price inefficiency is one of the crucial pieces of information to trade that product. If the price is deemed to be low based on some analysis (e.g. fundamental or statistical), an investor would like to buy it with the expectation that the price will go up in time. Similarly, one would shortsell it (borrow the product from a lender with some fee and sell it at the current market price) when its price is forecast to be higher than what it should be. Then, the investor would later buy to cover it (buy from the market and return the borrowed product back to the lender) when the price goes down. This set of transactions is the building block of any sophisticated financial trading activity. The main challenge is to identify price inefficiencies, also called *alpha* of a product, and swiftly act upon it for the purpose of making a profit from the trade. The efficient market hypothesis (EMH) stipulates that the market instantaneously aggregates and reflects all of the relevant information to price various securities; hence, it is impossible to beat the market. However, violations of the EMH assumptions abound: unequal availability of information, access to high-speed infrastructure, and various frictions and regulations in the market have fostered a vast and thriving trading industry.

Fundamental investors find alpha (i.e., predict the expected return) based on their knowledge of enterprise strategy, competitive advantage, aptitude of its leadership, economic and political developments, and future outlook. Traders often find inefficiencies that arise due to the complexity of market operations. Inefficiencies come from various sources such as market regulations, complexity of exchange operations, varying latency, private sources of information, and complex statistical considerations. An *arbitrage* is a typically short-lived market anomaly where the same financial instrument can be bought at one venue (exchange) for a lower price than it can be simultaneously sold at another venue. Relative value strategies recognize that similar instruments can exhibit significant (unjustified) price differences. Statistical trading strategies, including statistical arbitrage, find patterns and correlations in historical trading data using machine-learning methods and tools like factor models, and attempt to exploit them hoping that these relations will persist in the future. Some market inefficiencies arise due to unequal access to information, or the speed of dissemination of this information. The various sources of market inefficiencies give rise to trading strategies at different frequencies, from high-frequency traders who hold their positions on the order of milliseconds, to midfrequency trading that ranges from intraday (holding no overnight position) to a span of a few days, and to long-term trading ranging from a few weeks to years. High-frequency trading requires state-of-the-art computing, network communications, and trading infrastructure: a large number of trades are made where each position is held for a very short time period and typically produces a small return with very little risk. Longer term strategies are less dependent on latency and sophisticated technology, but individual positions are typically held for a longer time horizon and can pose substantial risk.

1.2.1 Trading and Exchanges

There is a vast array of financial instruments ranging from stocks and bonds to a variety of more sophisticated products like futures, exchange-traded funds (ETFs), swaps, collateralized debt obligations (CDOs), and exotic options (Hull, 2011). Each product is structured to serve certain needs of the investment community. Portfolio managers create investment portfolios for their clients based on the risk appetite and desired return. Since prices, expected returns, and even correlations of products in financial markets naturally fluctuate, it is the portfolio manager's task to measure the performance of a portfolio and maintain (rebalance) it in order to deliver the expected return.

The market for a security is formed by its buyers (bidding) and sellers (asking) with defined price and order types that describe the conditions for trades to happen. Such markets for various financial instruments are created and maintained by exchanges (e.g., the New York Stock Exchange, NASDAQ, London Stock Exchange, and Chicago Mercantile Exchange), and they must be compliant with existing trading rules and regulations. Other venues where trading occurs include dark pools, and over-the-counter or interbank trading. An order book is like a look-up table populated by the desired price and quantity (volume) information of traders willing to trade a financial instrument. It is created and maintained by an exchange. Certain securities may be simultaneously traded at multiple exchanges. It is a common practice that an exchange assigns one or several market makers for each security in order to maintain the robustness of its market.

The health (or liquidity) of an order book for a particular financial product is related to the bid–ask spread, which is defined as the difference between the lowest price of sell orders and the highest price of buy orders. A robust order book has a low bid–ask spread supported with large quantities at many price levels on both sides of the book. This implies that there are many buyers and sellers with high aggregated volumes on both sides of the book for that product. Buying and selling such an instrument at any time are easy, and it is classified as a high-liquidity (liquid) product in the market. Trades for a security happen whenever a buyer–seller match happens and their orders are filled by the exchange(s). Trades of a product create synchronous price and volume signals and are viewed as discrete time with irregular sampling intervals due to the random arrival times of orders at the market. Exchanges charge traders commissions (a transaction cost) for their matching and fulfillment services. Market-makers are offered some privileges in exchange for their market-making responsibilities to always maintain a two-sided order book.

The intricacies of exchange operations, order books, and microscale price formation is the study of market microstructure (Harris, 2002; O'Hara, 1995). Even defining the price for a security becomes rather complicated, with irregular time intervals characterized by the random arrivals of limit and market orders, multiple definitions of prices (highest bid price, lowest ask price, midmarket price, quantity-weighted prices, etc.), and the price movements occurring at discrete price levels (ticks). This kind of fine granularity is required for designing high-frequency trading strategies. Lower frequency strategies may view prices as regular discrete-time time series (daily or hourly) with a definition of price that abstracts away the details of market microstructure and instead considers some notion of aggregate transaction costs. Portfolio allocation strategies usually operate at this low-frequency granularity with prices viewed as real-valued stochastic processes.

1.2.2 Technical Themes in the Book

Although the scope of financial signal processing and machine learning is very wide, in this book, we have chosen to focus on a well-selected set of topics revolving around the concepts of high-dimensional covariance estimation, applications of sparse learning in risk management and statistical arbitrage, and non-Gaussian and heavy-tailed measures of dependence.[2]

A unifying challenge for many applications of signal processing and machine learning is the high-dimensional nature of the data, and the need to exploit the inherent structure in those data. The field of finance is, of course, no exception; there, thousands of domestic equities and tens of thousands of international equities, tens of thousands of bonds, and even more options contracts with various strikes and expirations provide a very rich source of data. Modeling the dependence among these instruments is especially challenging, as the number of pairwise relationships (e.g., correlations) is quadratic in the number of instruments. Simple traditional tools like the sample covariance estimate are not applicable in high-dimensional settings where the number of data points is small or comparable to the dimension of the space (El Karoui, 2013). A variety of approaches have been devised to tackle this challenge – ranging from simple dimensionality reduction techniques like principal component analysis and factor analysis, to Markov random fields (or sparse covariance selection models), and several others. They rely on exploiting additional structure in the data (sparsity or low-rank, or Markov structure) in order to reduce the sheer number of parameters in covariance estimation. Chapter 1.3.5 provides a comprehensive overview of high-dimensional covariance estimation. Chapter 1.3.4 derives an explicit eigen-analysis for the covariance matrices of AR processes, and investigates their sparsity.

The sparse modeling paradigm that has been highly influential in signal processing is based on the premise that in many settings with a large number of variables, only a small subset of these variables are active or important. The dimensionality of the problem can thus be reduced by focusing on these variables. The challenge is, of course, that the identity of these key variables may not be known, and the crux of the problem involves identifying this subset. The discovery of efficient approaches based on convex relaxations and greedy methods with theoretical guarantees has opened an explosive interest in theory and applications of these methods in various disciplines spanning from compressed sensing to computational biology (Chen *et al.*, 1998; Mallat and Zhang, 1993; Tibshirani, 1996). We explore a few exciting applications of sparse modeling in finance. Chapter 1.3.1 presents sparse Markowitz portfolios where, in addition to balancing risk and expected returns, a new objective is imposed requiring the portfolio to be sparse. The sparse Markowitz framework has a number of benefits, including better statistical out-of-sample performance, better control of transaction costs, and allowing portfolio managers and traders to focus on a small subset of financial instruments. Chapter 1.3.2 introduces a formulation to find sparse eigenvectors (and generalized eigenvectors) that can be used to design sparse mean-reverting portfolios, with applications

[2] We refer the readers to a number of other important topics at the end of this chapter that we could not fit into the book.

to statistical arbitrage strategies. In Chapter 1.3.3, another variation of sparsity, the so-called group sparsity, is used in the context of causal modeling of high-dimensional time series. In group sparsity, the variables belong to a number of groups, where only a small number of groups is selected to be active, while the variables within the groups need not be sparse. In the context of temporal causal modeling, the lagged variables at different lags are used as a group to discover influences among the time series.

Another dominating theme in the book is the focus on non-Gaussian, non-stationary and heavy-tailed distributions, which are critical for realistic modeling of financial data. The measure of risk based on variance (or standard deviation) that relies on the covariance matrix among the financial instruments has been widely used in finance due to its theoretical elegance and computational tractability. There is a significant interest in developing computational and modeling approaches for more flexible risk measures. A very potent alternative is the cVaR, which measures the expected loss below a certain quantile of the loss distribution (Rockafellar and Uryasev, 2000). It provides a very practical alternative to the value at risk (VaR) measure, which is simply the quantile of the loss distribution. VaR has a number of problems such as lack of coherence, and it is very difficult to optimize in portfolio settings. Both of these shortcomings are addressed by the cVaR formulation. cVaR is indeed coherent, and can be optimized by convex optimization (namely, linear programming). Chapter 1.3.9 describes the very intriguing close connections between the cVaR measure of risk and support vector regression in machine learning, which allows the authors to establish out-of-sample results for cVaR portfolio selection based on statistical learning theory. Chapter 1.3.9 provides an overview of a number of regression formulations with applications in finance that rely on different loss functions, including quantile regression and the cVaR metric as a loss measure.

The issue of characterizing statistical dependence and the inadequacy of jointly Gaussian models has been of central interest in finance. A number of approaches based on elliptical distributions, robust measures of correlation and tail dependence, and the copula-modeling framework have been introduced in the financial econometrics literature as potential solutions (McNeil *et al.*, 2015). Chapter 1.3.7 provides a thorough overview of these ideas. Modeling correlated events (e.g., defaults or jumps) requires an entirely different set of tools. An approach based on correlated Poisson processes is presented in Chapter 1.3.8. Another critical aspect of modeling financial data is the handling of non-stationarity. Chapter 1.3.6 describes the problem of modeling the non-stationarity in volatility (i.e. stochastic volatility). An alternative framework based on autoregressive conditional heteroskedasticity models (ARCH and GARCH) is described in Chapter 1.3.7.

1.3 Overview of the Chapters

1.3.1 Chapter 2: "Sparse Markowitz Portfolios" by Christine De Mol

Sparse Markowitz portfolios impose an additional requirement of sparsity to the objectives of risk and expected return in traditional Markowitz portfolios. The chapter starts with an overview of the Markowitz portfolio formulation and describes its fragility in high-dimensional settings. The author argues that sparsity of the portfolio can alleviate many of the shortcomings, and presents an optimization formulation based on convex relaxations. Other related problems, including sparse portfolio rebalancing and combining multiple forecasts, are also introduced in the chapter.

1.3.2 Chapter 3: "Mean-Reverting Portfolios: Tradeoffs between Sparsity and Volatility" by Marco Cuturi and Alexandre d'Aspremont

Statistical arbitrage strategies attempt to find portfolios that exhibit mean reversion. A common econometric tool to find mean reverting portfolios is based on co-integration. The authors argue that sparsity and high volatility are other crucial considerations for statistical arbitrage, and describe a formulation to balance these objectives using semidefinite programming (SDP) relaxations.

1.3.3 Chapter 4: "Temporal Causal Modeling" by Prabhanjan Kambadur, Aurélie C. Lozano, and Ronny Luss

This chapter revisits the old maxim that correlation is not causation, and extends the definition of Granger causality to high-dimensional multivariate time series by defining graphical Granger causality as a tool for temporal causal modeling (TCM). After discussing computational and statistical issues, the authors extend TCM to robust quantile loss functions and consider regime changes using a Markov switching framework.

1.3.4 Chapter 5: "Explicit Kernel and Sparsity of Eigen Subspace for the AR(1) Process" by Mustafa U. Torun, Onur Yilmaz and Ali N. Akansu

The closed-form kernel expressions for the eigenvectors and eigenvalues of the AR(1) discrete process are derived in this chapter. The sparsity of its eigen subspace is investigated. Then, a new method based on rate-distortion theory to find a sparse subspace is introduced. Its superior performance over a few well-known sparsity methods is shown for the AR(1) source as well as for the empirical correlation matrix of stock returns in the NASDAQ-100 index.

1.3.5 Chapter 6: "Approaches to High-Dimensional Covariance and Precision Matrix Estimation" by Jianqing Fan, Yuan Liao, and Han Liu

Covariance estimation presents significant challenges in high-dimensional settings. The authors provide an overview of a variety of powerful approaches for covariance estimation based on approximate factor models, sparse covariance, and sparse precision matrix models. Applications to large-scale portfolio management and testing mean-variance efficiency are considered.

1.3.6 Chapter 7: "Stochastic Volatility: Modeling and Asymptotic Approaches to Option Pricing and Portfolio Selection" by Matthew Lorig and Ronnie Sircar

The dynamic and uncertain nature of market volatility is one of the important incarnations of nonstationarity in financial time series. This chapter starts by reviewing the Black–Scholes formulation and the notion of implied volatility, and discusses local and stochastic models of volatility and their asymptotic analysis. The authors discuss implications of stochastic volatility models for option pricing and investment strategies.

1.3.7 Chapter 8: "Statistical Measures of Dependence for Financial Data" by David S. Matteson, Nicholas A. James, and William B. Nicholson

Idealized models such as jointly Gaussian distributions are rarely appropriate for real financial time series. This chapter describes a variety of more realistic statistical models to capture cross-sectional and temporal dependence in financial time series. Starting with robust measures of correlation and autocorrelation, the authors move on to describe scalar and vector models for serial correlation and heteroscedasticity, and then introduce copula models, tail dependence, and multivariate copula models based on vines.

1.3.8 Chapter 9: "Correlated Poisson Processes and Their Applications in Financial Modeling" by Alexander Kreinin

Jump-diffusion processes have been popular among practitioners as models for equity derivatives and other financial instruments. Modeling the dependence of jump-diffusion processes is considerably more challenging than that of jointly Gaussian diffusion models where the positive-definiteness of the covariance matrix is the only requirement. This chapter introduces a framework for modeling correlated Poisson processes that relies on extreme joint distributions and backward simulation, and discusses its application to financial risk management.

1.3.9 Chapter 10: "CVaR Minimizations in Support Vector Machines" by Junya Gotoh and Akiko Takeda

This chapter establishes intriguing connections between the literature on cVaR optimization in finance, and the support vector machine formulation for regularized empirical risk minimization from the machine-learning literature. Among other insights, this connection allows the establishment of out-of-sample bounds on cVaR risk forecasts. The authors further discuss robust extensions of the cVaR formulation.

1.3.10 Chapter 11: "Regression Models in Risk Management" by Stan Uryasev

Regression models are one of the most widely used tools in quantitative finance. This chapter presents a general framework for linear regression based on minimizing a rich class of error measures for regression residuals subject to constraints on regression coefficients. The discussion starts with least squares linear regression, and includes many important variants such as median regression, quantile regression, mixed quantile regression, and robust regression as special cases. A number of applications are considered such as financial index tracking, sparse signal reconstruction, mutual fund return-based style classification, and mortgage pipeline hedging, among others.

1.4 Other Topics in Financial Signal Processing and Machine Learning

We have left out a number of very interesting topics that all could fit very well within the scope of this book. Here, we briefly provide the reader some pointers for further study.

In practice, the expected returns and the covariance matrices used in portfolio strategies are typically estimated based on recent windows of historical data and, hence, pose significant uncertainty. It behooves a careful portfolio manager to be cognizant of the sensitivity of portfolio allocation strategies to these estimation errors. The field of robust portfolio optimization attempts to characterize this sensitivity and propose strategies that are more stable with respect to modeling errors (Goldfarb and Iyengar, 2003).

The study of market microstructure and the development of high-frequency trading strategies and aggressive directional and market-making strategies rely on short-term predictions of prices and market activity. A recent overview in Kearns and Nevmyvaka (2013) describes many of the issues involved.

Managers of large portfolios such as pension funds and mutual funds often need to execute very large trades that cannot be traded instantaneously in the market without causing a dramatic market impact. The field of optimal order execution studies how to split a large order into a sequence of carefully timed small orders in order to minimize the market impact but still execute the order in a timely manner (Almgren and Chriss, 2001; Bertsimas and Lo, 1998). The solutions for such a problem involve ideas from stochastic optimal control.

Various financial instruments exhibit specific structures that require dedicated mathematical models. For example, fixed income instruments depend on the movements of various interest-rate curves at different ratings (Brigo and Mercurio, 2007), options prices depend on volatility surfaces (Gatheral, 2011), and foreign exchange rates are traded via a graph of currency pairs. Stocks do not have such a rich mathematical structure, but they can be modeled by their industry, style, and other common characteristics. This gives rise to fundamental or statistical factor models (Darolles *et al.*, 2013).

A critical driver for market activity is the release of news, reflecting developments in the industry, economic, and political sectors that affect the price of a security. Traditionally, traders act upon this information after reading an article and evaluating its significance and impact on their portfolio. With the availability of large amounts of information online, the advent of natural language processing, and the need for rapid decision making, many financial institutions have already started to explore automated decision-making and trading strategies based on computer interpretation of relevant news (Bollen *et al.*, 2011; Luss and d'Aspremont, 2008) ranging from simple sentiment analysis to deeper semantic analysis and entity extraction.

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