

Asset Allocation with Gross Exposure Constraints for Vast Portfolios with High Frequency Data

Ke Yu

Princeton University

A joint work with Professor Jianqing Fan and Yingying Li

March 27, 2009

Outline

Motivation

Problem Setting

Risk Characteristics and Asymptotics

Empirical Studies

Simulation

Motivation

Markowitz portfolio allocation problem:

$$\begin{aligned} \min \quad & \mathbf{w}^T \Sigma \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T \boldsymbol{\mu} \geq \mu_b \\ & \mathbf{w}^T \mathbf{1} = 1 \end{aligned} \tag{1}$$

where $\Sigma = \text{var}(\mathbf{R})$.

Motivation

It is a simple quadratic programming problem with linear constraint. However, the solution produced by the typical low frequency approach has many problems. For example, it tends to produce extreme long and short positions which makes the portfolio unstable.

Motivation

Fan, Zhang and Yu (2008) showed that, using the daily closing price data, the desired portfolio features can be achieved by adding the $L - 1$ norm constraint to the original problem.

$$\begin{aligned} \min \quad & \mathbf{w}^T \Sigma \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T \mathbf{1} = 1 \\ & \|\mathbf{w}\|_1 \leq c \end{aligned} \tag{2}$$

Motivation

We would like to explore the use of high frequency data to further improve the portfolio allocation.

In the previous literature, high frequency data has only been studied on a financial econometrics level, but never on a financial engineering level, which means that it is rarely used to make portfolio allocation decisions.

Motivation

Our goal is to see if, by using high frequency data, we can shorten the scale of the time window we need to estimate the covariation structure and improve the asset allocation.

Outline

Motivation

Problem Setting

Risk Characteristics and Asymptotics

Empirical Studies

Simulation

Problem Setting

For asset price processes \mathbf{S}_t , the log prices $\mathbf{X}_t = \ln \mathbf{S}_t$ follow the diffusion processes $d\mathbf{X}_t = \boldsymbol{\mu}_t dt + \boldsymbol{\sigma}_t d\mathbf{B}_t$.

We would like to minimize

$$\begin{aligned} \text{var}_t\left(\int_t^{T+t} \mathbf{w}^T d\mathbf{X}_u\right) &= \mathbb{E}_t\left(\int_t^{T+t} \mathbf{w}^T \boldsymbol{\sigma}_u \boldsymbol{\sigma}'_u \mathbf{w} du\right) \\ &= \mathbf{w}^T \mathbb{E}_t\left(\int_t^{T+t} \boldsymbol{\Sigma}_u du\right) \mathbf{w} \end{aligned} \quad (3)$$

where $\boldsymbol{\Sigma}_u = \boldsymbol{\sigma}_u \boldsymbol{\sigma}'_u$.

Problem Setting

The realized volatility $\int_{t-h}^t \boldsymbol{\sigma}_u \boldsymbol{\sigma}'_u du$ is used to approximate the conditional expectation of the future realized volatility $E_t(\int_t^{T+t} \boldsymbol{\Sigma}_u du)$.

We are facing two major challenges, non-synchronous trading and microstructure noise in high frequency data.

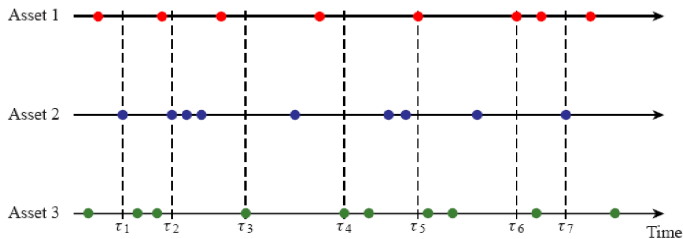
Problem Setting

In reality, microstructure noise cannot be neglected. The log asset price processes \mathbf{X}_t are actually driven by underlying processes \mathbf{Y}_t , which follow the diffusion processes $d\mathbf{Y}_t = \boldsymbol{\mu}_t dt + \boldsymbol{\sigma}_t d\mathbf{B}_t$.

Zhang (2006) suggested the TSRC (Two Time-Scale Realized Covariation) approach to deal with the issue. Barndorff-Nielsen, Hansen, Lunde and Shephard (2008) and others suggested alternative approaches. We applied the former one. To fix ideas, TSRC can be viewed as a modified version of the realized covariance of \mathbf{Y} , which is $\sum_{j=1}^n [\mathbf{Y}_{t_j} - \mathbf{Y}_{t_{j-1}}][\mathbf{Y}_{t_j} - \mathbf{Y}_{t_{j-1}}]'$.

Problem Setting

To deal with non-synchronous trading, we use the concept of "refresh time" introduced by Barndorff-Nielsen, Hansen, Lunde and Shephard (2008).



Problem Setting

In terms of volatility estimation, we proposed the pairwise-refresh-time estimator, and compared it with the all-refresh-time estimator.

For all-refresh-time estimator, as the number of assets increases, the frequency of the refresh times is decreasing and a large amount of data is likely to be thrown away. It will not be a problem for pairwise-refresh-time estimator, which improves the precision of estimation.

Outline

Motivation

Problem Setting

Risk Characteristics and Asymptotics

Empirical Studies

Simulation

Risk Characteristics and Asymptotics

Let us briefly revisit the portfolio optimization problem with the L_1 norm constraint:

$$\begin{aligned} \min \quad & \mathbf{w}^T \Sigma \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^T \mathbf{1} = 1 \\ & \|\mathbf{w}\|_1 \leq c \end{aligned}$$

Risk Characteristics and Asymptotics

Let

$$R(\mathbf{w}) = \mathbf{w}^T \Sigma \mathbf{w}, \quad R_n(\mathbf{w}) = \mathbf{w}^T \hat{\Sigma} \mathbf{w}$$

be respectively the theoretical and empirical portfolio risks.

And let

$$\mathbf{w}_{opt} = \underset{\mathbf{w}^T \mathbf{1}=1, \|\mathbf{w}\|_1 \leq c}{\operatorname{argmin}} R(\mathbf{w}), \quad \hat{\mathbf{w}}_{opt} = \underset{\mathbf{w}^T \mathbf{1}=1, \|\mathbf{w}\|_1 \leq c}{\operatorname{argmin}} R_n(\mathbf{w})$$

be respectively the theoretical optimal allocation vector we want and empirical optimal allocation vector we get.

Risk Characteristics and Asymptotics

We are interested in the behaviors and asymptotics of $|R(\hat{\mathbf{w}}_{opt}) - R(\mathbf{w}_{opt})|$, $|R(\hat{\mathbf{w}}_{opt}) - R_n(\hat{\mathbf{w}}_{opt})|$ and $|R(\mathbf{w}_{opt}) - R_n(\hat{\mathbf{w}}_{opt})|$.

The theorems are under derivation.

Outline

Motivation

Problem Setting

Risk Characteristics and Asymptotics

Empirical Studies

Simulation

Empirical Studies

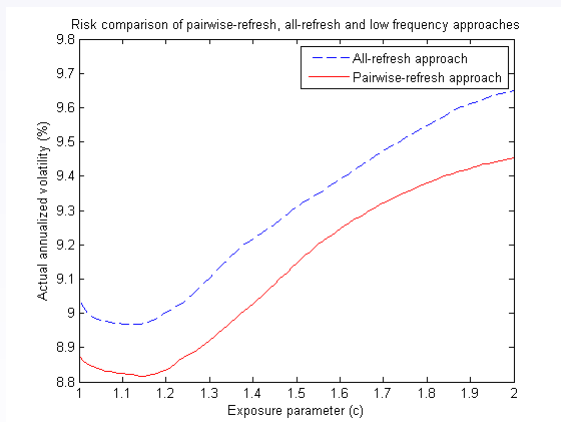


Figure: Comparison between all-refresh and pairwise-refresh approaches

Empirical Studies

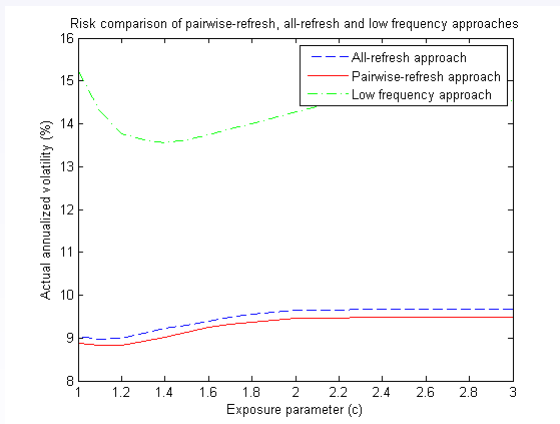


Figure: Comparison between the high frequency approaches and low frequency approach

Outline

Motivation

Problem Setting

Risk Characteristics and Asymptotics

Empirical Studies

Simulation

Simulation

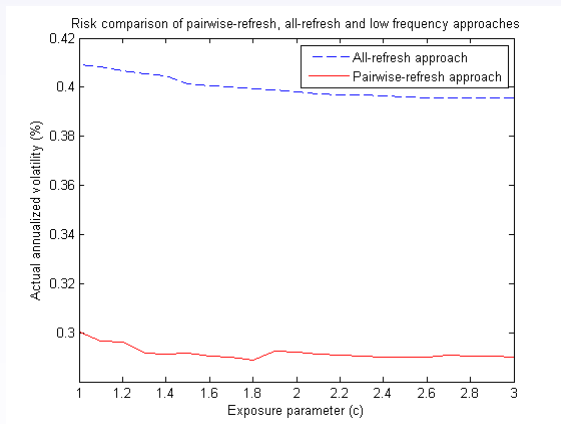


Figure: Comparison between all-refresh and pairwise-refresh approaches

Simulation

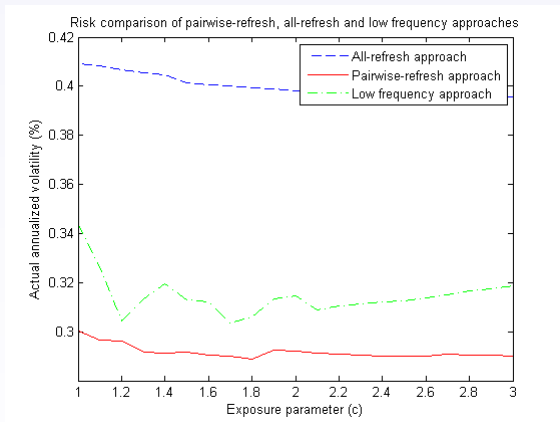


Figure: Comparison between the high frequency approaches and low frequency approach