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SOME RECENT RESULTS ON COMPRESSIBLE FLOW WITH VACUUM

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Dedicated to Professor Fon-Che Liu on his sixtieth birthday

Abstract. In this paper, we will survey some recent results on the study of the viscous and invisid compressible flow with vacuum. It is well-known that the study on vacuum has significance in the investigation on some important physical phenomena. However, most of the important questions about vacuum are still open due to the singularities caused by vacuum which need new mathematical tools and techniques to handle.

1. Introduction

In this paper, we will consider the viscous and invisid compressible flow with vacuum. The study of the vacuum phenomena for the compressible flow is of great importance both theoretically and practically. In the following, we are interested in the time evolution of the vacuum boundary which separates the vacuum and the gas, and the singular behavior of the solutions near this interface. For illustration, we will mainly consider two mathematical models: the Euler equations and the Navier-Stokes equations for isentropic compressible flow. Some recent results on these models will be summarized and some open problems will be raised. Similar problems have been and can be studied for the models for non-isentropic flow and those in muti-dimensional spaces. Furthermore, similar vacuum problems can be raised for the systems of Euler-Possion equations governing the evolution of the gaseous star, and the Maxwell equations for electro-magnetic fields where vacuum region is nontrivial.

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The remaining of the paper will be organized as follows. In Section 2, we will consider the Euler equations for isentropic flow with vacuum. Since the system fails to be strictly hyperbolic at the vacuum states, the classical analysis of strictly hyperbolic equations is not applicable to the present situation. Most of the main open problems on this system are still open, such as what the canonical behavior of the solutions near the vacuum boundary is and what the large time behavior of the solutions is. It can be seen that this kind of study will help us to understand the singularity, evolution of vacuum boundary and other complicated phenomena caused by vacuum. It would also help the design of the numerical scheme for the understanding of the evolution of the vacuum boundary.

In Section 3, we will consider the Navier-Stokes equations for isentropic flow. It has been shown that the classical Navier-Stokes equations with constant viscosity coefficient is not stable with vacuum [11]. By deriving the Navier-Stokes equations from Boltzmann equations using Chapman-Enskog expansion up to the second order, the viscosity coefficient depends on the temperature. For isentropic gas, this dependence is reduced to the dependence on the density. Therefore, even though the system with constant viscosity coefficient is well understood at the vacuum state, it is interesting and important to study the system when the viscosity coefficient depends on the density. For the latter system, most of the questions on vacuum boundary are still unanswered.

2. Euler Equations with Vacuum

Consider the compressible Euler equations in R^3 for isentropic flow with damping in Eulerian coordinates:

$$\begin{aligned} &\rho_t + \nabla \cdot (\rho\vec{u}) = 0, \\ &\rho\vec{u}_t + \rho\vec{u} \cdot \nabla\vec{u} + \nabla p(\rho) = -k\rho\vec{u}, \end{aligned} \tag{2.1}$$

where ρ, u and $p(\rho)$ are density, velocity and pressure, respectively, and $k > 0$ is the linear frictional coefficient. When the initial density function has compact support, the vacuum boundary Γ is defined as

$$\Gamma = \overline{\{(\vec{x}, t) \mid \rho(\vec{x}, t) > 0\}} \cap \overline{\{(\vec{x}, t) \mid \rho(\vec{x}, t) = 0\}}.$$

The main difficulty caused by vacuum is that the system becomes degenerate, that is, characteristics coincide and become zero in the Lagrangian coordinates. Therefore, even though the system is symmetrizable, in general the coefficients do not satisfy the usual local existence theories. For Euler equations without damping, the author in [36] gave a sufficient condition for blowup of C^1 solutions when $\inf \rho_0(\vec{x}) > 0$, where $\rho_0(\vec{x})$ is the initial density. The nonexistence of C^1 solutions in [36] is related to the shock formation. However, in the following discussion, we will concentrate on the singularity of the solutions at the vacuum states when the solutions contain no shocks. Hence, the time when the regular solutions blow up in our discussion is before the time when shock forms.

The one-dimensional Euler equations with damping can be written in the Lagrangian coordinates as follows:

$$\begin{aligned} &v_t - u_x = 0, \\ &u_t + p(v)_x = -ku, \end{aligned} \tag{2.2}$$

where $v = 1/\rho$ is the specific volume. Most of the following results are on this one-dimensional system.

When there is no frictional damping, it was shown that the shock waves vanish at the vacuum and the singular, non-shock behavior at the vacuum is similar to that of the centered rarefaction waves [21]. Under some special condition on the initial velocity, the authors in [9] give the existence of global smooth solutions for Euler equations for an isentropic perfect gas in the d-dimensional Euclidean space. Precisely, the conditions in [21] are: (i) the initial velocity belongs to the Sobolev space of order m with $m > 1 + d/2$, and for any x the spectrum of its Jacobian at x is nonnegative; (ii) the initial density has compact support and its $(\gamma - 1)/2$ power is small enough in the Sobolev space of order m.

When there is linear frictional damping and the solution is away from vacuum, the dissipative effect of damping prevents shock forming. Since no other singularity emerges from the flow, it was shown that the system is time-asymptotically equivalent to the porous media equation [13]. When the system contains both linear damping and vacuum, so far this kind of equivalence was only proved for a class of special solutions [18].

The main open problems on the system (2.2) with vacuum are still unsolved. One of them is to study the singular evolution of the interface, that is, to give a detailed description of how the solution near the interface changes and how it becomes the canonical kind of behavior after finite time. Another would be the study of the nonlinear stability of the traveling wave solutions and the class of special solutions corresponding to the Barenblatt's solutions for the porous media equation obtained in [18]. It will also be interesting and important to give the time asymptotic equivalence between the Euler equations with damping and the porous media equation when vacuum occurs. In the following, we will summarize some of the recent results which partially answer the above questions.

In [18], the author constructs a class os spherical symmetric solutions which converge to the self-similar solutions of the porous media equation

$$\begin{aligned} k\rho_t &= p(\rho)_{xx}, \\ p(\rho)_x &= -k\rho u. \end{aligned}$$

It is also conjectured in [18] that the general behavior of the solution near Γ satisfies

$$0 < |\frac{dc^2}{dx}| < \infty \tag{2.3}$$

in the Eulerian coordinates after finite time, where $c = \sqrt{dp(\rho)/d\rho}$ is the sound speed. Notice that this singular behavior implies that the pressure has a bounded nonzero effect on the evolution of the vacuum boundary. This also implies that if initial data $(\rho, u)(x, 0)$ are smooth, then there is a waiting time. After that time, the boundary will move due to the effect of the pressure. In [18], a class of traveling wave solutions with infinite total mass is also constructed. To our knowledge, these two kinds of solutions are the only nontrivial global solutions to the Euler equations with vacuum and damping where the vacuum boundary is clearly presented.

Along this direction, the authors in [20] show that the regular solutions cannot be global if the density function has compact support. And the local existence of solutions when they have the property that dc^α/dx, $0 < \alpha \le 1$, is bounded away from zero across Γ is also proved. It is noticed that this kind of phenomenon exists and α remains the same locally in time. When the initial data connect to vacuum states discontinuously, local existence for Euler equations without damping was proved in [19] by a polygonal method introduced in [6] for scalar equations and generalized in [16] for p-systems.

According to the argument in [18], the general behavior of the solutions should be the one corresponding to $\alpha = 2$. Very recently, the authors in [22] obtained the local existence when $\alpha = 2$ by introducing a coordinate transformation. For illustration, let $x = 0$ be the vacuum boundary in the Lagrangian coordinates and the pressure function satisfy the γ-law, i.e., $p(v) = \sigma^2 v^{-\gamma}$, $\gamma > 1$. By the transformation $x = y^{(2\gamma/(\gamma-1))}$, the system (2.2) can be rewritten as

$$\begin{aligned} &\phi(v)_t + \bar{\mu} u_y = 0, \\ &u_t + \bar{\mu}\phi(v)_y = -ku, \end{aligned} \tag{2.4}$$

where $\phi(v) = \frac{2\sqrt{\gamma}\sigma}{\gamma-1} v^{-\frac{\gamma-1}{2}}$, and $\bar{\mu} = \frac{(\gamma-1)\sigma}{\sqrt{\gamma}}(vy^{\frac{2}{\gamma-1}})^{-\frac{\gamma+1}{2}}$. Notice that at the vacuum boundary, both $\phi(v)_y$ and $\bar{\mu}$ are bounded away from zero if (2.3) holds. Thus the local existence of solutions with the property (2.3) can be proved by the fixed-point theorem. However, this kind of transformation works only for $\alpha = 2$. Up to now, the local existence of solutions for $1 < \alpha < 2$ remains open.

The above analysis can also be applied to the system of Euler-Poisson equations for gaseous star to obtain local existence which includes the stationary solutions. The Euler-Poisson equations can be written as

$$\begin{aligned} &\rho_t + \nabla \cdot (\rho \vec{u}) = 0, \\ &\rho \vec{u}_t + \rho \vec{u} \cdot \nabla \vec{u} + \nabla P(\rho) + \rho \nabla \Phi = 0, \\ &\Delta \Phi = 4\pi \rho, \end{aligned} \tag{2.5}$$

where Φ is the gravitational potential. When $6/5 < \gamma < 2$, there exist stationary solutions with spherical symmetry (cf. [26]),

$$\rho = \left(\frac{\sigma^2 A^2 \gamma}{4\pi(\gamma - 1)} \right)^{\frac{1}{2-\gamma}} \theta(A|x|)^{\frac{1}{\gamma-1}}, \qquad u = 0,$$

where A is an arbitrary positive constant and $\theta(r)$ is the "Lane-Emden function" of index $1/(\gamma - 1)$ satisfying (cf. [5]),

$$\frac{d^2}{dr^2}\theta + \frac{2}{r}\frac{d}{dr}\theta + \theta^{\frac{1}{\gamma-1}} = 0, \quad \theta(0) = 1, \quad \frac{d}{dr}\theta(0) = 0.$$

Furthermore, there exists $R = R_\gamma$ such that $\theta(r) > 0$ when $0 \le r < R$ and $\theta(R) = 0$. And the solutions behave like

$$\rho(r) = C(R - r)^{\frac{1}{\gamma-1}} [1 + P(R - r, (R - r)^{\frac{\gamma}{\gamma-1}})]$$

when $r \to R - 0$. Here C is a positive constant and P is a double power series with positive radii of convergence. Notice that this behavior is precisely the one stated in (2.3). The local existence result in [26] does not include this family of stationary solutions. However, by using the above coordinate transformation the local existence in [22] includes this family of stationary solutions.

When $\inf \rho_0(\vec{x}) = 0$, the author in [27] proved the nonglobal existence of regular solutions by assuming that the initial data $(\rho_0(\vec{x}), \vec{u}_0(\vec{x}))$ have compact support, where $\vec{u}_0(\vec{x})$ is the initial velocity. For the Euler-Poisson equations governing gaseous stars, the authors in [25] proved the nonglobal existence of tame solutions under the condition of spherical symmetry. This nonglobal existence result can be generalized to the boundary condition $dc^2/dx = 0$ instead of $dc/dx = 0$ at vacuum. Local existence of tame or regular solutions for these two systems was proved in [24, 26], by using the symmetrization and the fixed-point theorem. Solutions thus obtained correspond to those of $0 < \alpha < \frac{2}{3}$ or $\alpha = 1$ if the solutions are in the space H^2. Using the same symmetrization, the author in [7] uses the paradifferential calculus of

J.M. Bony, and the author in [4] uses the Gagliardo-Nirenberg inequality and Littlewood-Paley theory to obtain local existence of solutions to (2.5) without assuming the support of the initial density being compact.

Notice that the regular solution defined in [20] is different from that in [26], where $\rho^{\gamma-1} \in C^1([0,T) \times R^3)$ instead of $\rho^{((\gamma-1)/2)} \in C^1([0,T) \times R^3)$ is required. Since $c = \sigma\sqrt{\gamma}\rho^{((\gamma-1)/2)}$ is a continuous function, the regular solution defined in [20] is more general than the one in [26], and thus the nonexistence theorem in [20] generalizes that of [26]. So far the proofs for the nonexistence of global regular solutions either for invisid models given above or for the viscous models (cf. [20, 24, 15, 27, 38] etc.) are based on the analysis of the support of the density. As shown by the special class of global solutions obtained in [18], the support of the density there is infinite. Therefore, deeper understanding on the development of the singularity at the vacuum is desirable.

3. Navier-Stokes Equations with Vacuum

The one-dimensional compressible Navier-Stokes equations for isentropic gas flow in Eulerian coordinates can be written as

$$\begin{aligned} &\rho_\tau + (\rho u)_\xi = 0, \\ &(\rho u)_\tau + (\rho u^2 + p(\rho))_\xi = (\mu u_\xi)_\xi, \end{aligned} \tag{3.1}$$

where $\mu \geq 0$ is the viscosity coefficient.

We are interested in the study of the existence and uniqueness of the weak solutions to (3.1) when the density function has compact support. The weak solution in consideration should have enough regularity for the study of the behavior of the vacuum boundary and the behavior of the solution near the boundary or when time tends to infinity. The general global existence of weak solutions to Navier-Stokes equations is foundamental [17], but it is not applicable to the present situation because we need more regularity of the solution to study its behavior. For simplicity, we also consider the polytropic gas, i.e., $p(\rho) = \sigma^2\rho^\gamma$ with constant $\gamma > 1$. Assume that the entire gas initially occupies only a finite interval $[a, b] \subset R^1$ with one side or two sides connecting to vacuum, and the viscosity coefficient μ is a functional of the density ρ; for example, $\mu = c\rho^\theta$, where $c > 0$ and $\theta \geq 0$ are constants. The important feature of this problem is that the interface separating the gas and vacuum propagates with finite speed and has some singular behavior. This system models many interesting physical phenomena, such as the gaseous stars in astrophysics. This kind of study will help us to understand the singularity, evolution of vacuum boundary and other complicated phenomena caused by vacuum for the viscous compressible flow.

When the viscosity coefficient is constant, this problem is well investigated when the density function connects to vacuum either with a jump or continuously. For this case, both the large-time behavior and the interface behavior have been obtained for the initial boundary value problem. In [38], the dispersive behavior of the total pressure is described for smooth solutions in multi-dimensional space, which yields the nonexistence of global smooth solutions with compact supported initial data.

However, the problem becomes more complicated when the viscosity depends on the density because of the higher degeneracy and weaker dissipation. Such a dependence can be seen when the Navier-Stokes equations are derived from the Boltzmann equations through the Chapman-Enskog expansion to the second order, where the dependence of the viscosity coefficient on the temperature is translated to the dependence on the density for the isentropic flow. For this problem, there are recently some results on the global existence of solutions when the density function connects to the vacuum with a jump. But the global existence when the density function connects to vacuum continuously remains open. Furthermore, the exact large-time behavior and interface behavior when the density function connects to the vacuum either with a jump or continuously are still unknown.

In the following, we will summarize some recent results and state some other open problems on this subject. Notice that the situation when the gravity is taken into account can be discussed similarly [31].

When the viscosity coefficient is constant, i.e., $\theta = 0$, the study in [11] shows that the continuous dependence on the initial data of the solutions to the Navier-Stokes equations with vacuum fails. The main reason for the failure at the vacuum comes from the kinematic viscosity coefficient being independent of the density. In [31], the author studies the free boundary value problem of (3.1) with one boundary fixed and the other connected to vacuum. She proved the global existence of weak solutions. Similar results were obtained in [25] for the system of spherically symmetric motion of viscous gases. In fact, the free boundary problem of the one-dimensional viscous gases which expand into the vacuum has been studied by many people; see [31, 32, 38] and the references therein. A further understanding of the regularity and behavior of solutions near the interface between the gas and vacuum is given in [23].

As mentioned above, it is more reasonable to consider the case when the viscosity coefficient depends on the density for the vacuum problem, i.e., $\theta > 0$. For this case, the local existence of weak solutions to Navier-Stokes equations with vacuum was studied in [19], where the initial density was assumed to be connected to vacuum with discontinuities. This property, as shown in [19] maintains as time evolves. Under the assumption that the gas connects to vacuum with a jump, the authors in [33] recently obtained the global existence and uniqueness of weak solutions when $\mu = c\rho^\theta$ and $0 < \theta < 1/3$. As pointed out in [8], for some physical models or when considering the second approximation of the Chapman-Enskog expression to the nonlinear Boltzmann equations for a rarefied gas with hard sphere interaction potential, the viscosity of gas is proportional to the square root of the temperature. Under this hypothesis, the temperature is of the order of $\rho^{\gamma-1}$ for the perfect gas where the pressure is proportional to the product of the density and the temperature. In this consideration, the result in [33] includes the case when $1 < \gamma < 5/3$. Since this result does not include the important case when $\gamma = 5/3$ which corresponds to the monatomic gas, it is necessary to generalize it to larger θ. Under this motivation, the authors in [39] consider this problem and generalize the result in [33] to $0 < \theta < 1/2$, which corresponds to the case when $1 < \gamma < 2$ in the above consideration. Very recently, the authors in [15] further generalize this result to the case of $0 < \theta < 1$ which corresponds to $1 < \gamma < 3$. Note also that the solution defined in [15] is in weaker sense.

It is noticed that the above analysis is based on the uniform positive lower bound estimate of the density with respect to the construction of the approximate solutions. This estimate is crucial because the other estimates for the convergence of a subsequence of the approximate solutions and the uniqueness of the solution thus obtained follow from it by standard techniques as long as the vacuum does not appear in the solutions. The authors in [39] obtain the positive lower bound of the density in more general setting than the one in [33] by minimizing the constant α_1 such that the term $\int_0^1 \rho^{\alpha_1} u^2 dx$ has an upper bound estimate in Lagrangian coordinates. Here we normalize the total mass to 1 and assume the support of the density function to be $[0, 1]$ in Lagrangian coordinates. And this minimizing procedure is carried out by deriving a recurrence relation of α_m, where $\int_0^1 \rho^{\alpha_m} u^{2^m} dx$ is bounded. Further estimation in [15] yields the lower bound for density under more general conditions.

The lower bound of the density is easier to get when the viscosity coefficient is constant because we have the following expression for the density:

$$\rho(x, t) = \rho(0, t) \exp\left\{ -\int_0^t \rho(x, s) u_x(x, s) ds \right\}$$

in Lagrangian coordinates. This gives a simple connection between the density at time t and $t = 0$ along the particle path with a factor in exponential form. By using this expression and the standard energy estimates, the lower bound of the density can be readily obtained. However, when $\theta \neq 0$, the above expression reduces to the difference between the density to some power at time t and $t = 0$. Thus, it is not easy to obtain the lower bound for the density. Nevertheless, the lower bound at the boundary can be obtained by using the conservation of momentum in the whole interval when the density connects to vacuum with a jump. That is, we have

$$\rho(t) = \rho(0)\left(\frac{\gamma-\theta}{\gamma-\theta+\rho(0)^{\gamma-\theta}t}\right)^{\frac{1}{\gamma-\theta}},$$

for the density at one side of the vacuum boundary. Notice that when $\theta = 0$, the above decay rate for density is valid also in the gas region. Whether this decay rate holds in the gas region for $\theta \neq 0$ is unknown. All the analyses in [15, 33, 39] depend on the above lower bound estimate for density at the boundary. This is the main reason why those analyses cannot be applied to the study of the problem when the gas connects to vacuum continuously. It is because the density is zero at the boundary in this case. The optimal decay rate is necessary for the study of the expansion of the vacuum boundaries and the detailed description of the solution near these interfaces.

When the gas connects to the vacuum continuously, another interesting problem is to study how the regularity property of the initial data near the vacuum boundary affects the behavior of the solution and what is the canonical behavior of the solution near the vacuum boundary.

There has been a lot of investigation on the Navier-Stokes equations when the interface separating the gas and the vacuum is not specified, both for smooth initial data or discontinuous initial data, and one-dimensional or multidimensional problems. For these results, please refer to [12, 35] and the references therein.

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