CITY UNIVERSITY OF HONG KONG
香港城市大學

An Improved Convergence Rate of Glimm Scheme for General Systems of Hyperbolic Conservation Laws

一般雙曲守恆律系統的 Glimm 格式收斂率

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by

Hua Jiale

華嘉樂

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Abstract

The study of Cauchy problem of systems of hyperbolic conservation laws is very important in both theory and application. Many fundamental equations in physics are in the form of conservation laws. One of the well known examples of such systems is the Euler equations of gas dynamics for an inviscid, non-heat conducting gas in Eulerian coordinates. To study such problems, Glimm introduced a scheme to construct the solution [14]. Later on, T.P.Liu introduced a deterministic version of Glimm scheme [22], which can be used in real computation and yield some convergence rate. In real application, the convergence rate is quite important to justify the computation and control the error in approximation.

We study the convergence rate of Glimm scheme for general systems of hyperbolic conservation laws. It was shown in the previous work that for systems of hyperbolic conservation laws with each characteristic field being genuinely nonlinear or linearly degenerate, the convergence rate of Glimm scheme is $o(1)s^{\frac{1}{2}}|\ln s|$, cf. [11]. Here s is the mesh size. This is the case for Euler equations. But there are also systems with some characteristic fields being neither genuinely non-linear nor linearly degenerate. As for general systems without the assumption that each characteristic field is either genuinely nonlinear or linearly degenerate, the convergence rate was only proved to be $o(1)s^{\frac{1}{4}}|\ln s|$ in [29]. In this thesis, we improve this result to be $o(1)s^{\frac{1}{3}}|\ln s|^{1+\alpha}$, where α is any positive constant. This is achieved by yielding a sharper estimate of the error arising from the wave tracing argument. In order to get such a result, a careful analysis of the interaction

between small waves is conducted. This convergence rate is sharper compared to the one in [29]. However, it is still slower than $o(1)s^{\frac{1}{2}}|\ln s|$ given in [11] for systems with each characteristic field being genuinely nonlinear or linearly degenerate.

In the thesis, a brief review of the basic ideas used in the proof is given, including Glimm scheme and the deterministic version of Glimm scheme, the wave tracing argument, equidistributed sequence, Glimm type functional and so on. Then a theorem of the refinement of wave tracing is proved. The proof of the convergence rate follows.

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