alg-geom/9603020

Calabi-Yau Varieties and Pencils of K3 Surfaces

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Abstract: In this note, we give a list of Calabi-Yau hypersurfaces in weighted projective 4-spaces with the property that a hypersurface contains naturally a pencil of K3 variety. For completeness we also obtain a similar list in the case K3 hypersurfaces in weighted projective 3-spaces. The first list significantly enlarges the list of K3-fibrations of [1] which has been obtained on some assumptions on the weights. Our lists are expected to correspond to examples of the so-called heterotic-type II duality [2][3].

3/2/96

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1. Problems

Let $w_1,..,w_{n+1}$ be positive integers, and put $d := \sum w_i$. We call the weight vector $\hat{w} = (w_1,..,w_{n+1})$ admissible if the generic weighted degree d hypersurface in $\mathbf{C}^{n+1}$ is smooth away from the origin. This means that the weighted projectivized hypersurface in $\mathbf{P}[w]$ is transversal, ie. it only acquires singularities from the ambient space $\mathbf{P}[\hat{w}]$. For $n = 3$, there is a list of admissible weights of Reid-Yonemura (see [4]). For $n = 4$, there is a list admissible weights obtained by Klemm-Schimmrigk [5] and Kreuzer-Skarke [6].

Given an admissible weight $\hat{w} = (w_1,..,w_{n+1})$ we can consider in $\mathbf{P}[\hat{w}]$ the generic Calabi-Yau variety given by

$$\hat{X}_a = \{z | \sum_{\hat{w}\cdot\nu=d} a_\nu z^\nu = 0\}. \tag{1.1}$$

Suppose we intersect this variety with the coordinate hyperplane $z_{n+1} = 0$.

Problem 1.1. *When is $X_a := \hat{X}_a \cap \{z_{n+1} = 0\}$ isomorphic to a transversal Calabi-Yau variety?*

Note that by permuting the weights, this includes the cases $\hat{X}_a \cap \{z_i = 0\}$ for any i. More generally,

Problem 1.2. *When is there a 1-parameter family of hypersurfaces Z_λ such that $X_{a,\lambda} := \hat{X}_a \cap Z_\lambda$ is isomorphic to a transversal Calabi-Yau variety?*

For $n = 4$ and with some assumptions on the weights, a short list of such cases has been tabulated in [1]. We say that ν is compatible with the weight $\hat{w}$ if $\hat{w}\cdot\nu = d$.

Let $w = (w_1,..,w_n)$ and $\bar{w}$ its normalization, ie. $\bar{w} := (w_1/\delta_1,..,w_n/\delta_n)$ where $\delta_i := lcm(\rho_1,..,\hat{\rho}_i,..,\rho_n)$ and $\rho_i := gcd(w_1,..,\hat{w}_i,..,w_n)$. It is well known that $\phi : \mathbf{P}[w] \to \mathbf{P}[\bar{w}]$ is an isomorphism under the normalization map $(z_1,..,z_n) \mapsto (z_1^{\rho_1},..,z_n^{\rho_n})$. It is easy to show that $\delta_1\rho_1 = \cdots = \delta_n\rho_n$; we call this integer k.

We require that the image $\bar{X}_a = \phi X_a$ is a transversal Calabi-Yau variety in $\mathbf{P}[\bar{w}]$. If $x_1,..,x_n$ are the quasi-homogeneous coordinates of $\mathbf{P}[\bar{w}]$, then a Calabi-Yau variety can be written as

$$\bar{X}_b = \{x | \sum_{\bar{w}\cdot\mu=\bar{d}} b_\mu x^\mu = 0\}, \tag{1.2}$$

where $\bar{d} := \sum_{i=1}^n \bar{w}_i = \sum w_i/\delta_i$. Pulling this back by the normalization map, we see that

$$\phi^{-1}\bar{X}_b = \{z| \sum_{\bar{w}\cdot\mu=\bar{d}} b_\mu \prod z_i^{\rho_i\mu_i} = 0\} \subset \mathbf{P}[w]. \qquad (1.3)$$

If we require that $\phi^{-1}\bar{X}_b = X_a$ for some a, then we conclude that

(a) every $\mu \in \mathbf{Z}_+^n$ compatible with $\bar{w}$ has $\sum w_i\rho_i\mu_i = d$, and $\nu = (\rho_1\mu_1, .., \rho_n\mu_n, 0)$ is an exponent compatible with $\hat{w}$.

(b) every exponent $\nu \in \mathbf{Z}_+^{n+1}$ compatible with $\hat{w}$ having $\nu_{n+1} = 0$ is of the form $\nu = (\rho_1\mu_1, .., \rho_n\mu_n, 0)$ for some exponent μ compatible with $\bar{w}$.

We claim that $d = k\bar{d}$. This follows from (a) and the fact that $k = \rho_i\delta_i$. Since $\mu = (1, .., 1)$ is compatible with $\bar{w}$, it follows from (a) that $w \cdot \rho = d$. Thus our task is to search through the list of normalized admissible weights $\hat{w}$ ($n = 4$) satisfying

$$\begin{aligned} &(i) \quad \hat{w}\cdot\nu = d, \quad \nu_5 = 0 \ \Rightarrow \rho_i|\nu_i \ \ \forall i \\ &(ii) \quad w\cdot\rho = d \\ &(iii) \quad \bar{w} \ \ admissible \end{aligned} \qquad (1.4)$$

On the last condition, we will check that $\bar{w}$ be in the Reid-Yonemura list. It is also clear that (i)–(iii) implies that $\hat{X}_a \cap \{z_{n+1}\}$ is isomorphic to $\bar{X}_a$ in the admissible $\mathbf{P}[\bar{w}]$. Our computer search shows that there are 628 admissible weights $\hat{w}$ of length 5 satisfying (i)–(iii).

Example: Take $\hat{w} = (42, 27, 8, 4, 3)$, $d = 84$. We consider the intersection $X := \hat{X} \cap \{z_3 = 0\}$. Then $\rho = (1, 1, 3, 1)$, and so condition (ii) holds. The normalized weight of $w = (42, 27, 4, 3)$ becomes $\bar{w} = (14, 9, 4, 1)$, which is an admissible weight of length 4 (see [4]), and so condition (iii) holds. The equations for X in $\mathbf{P}[\hat{w}]$ is $z_3 = 0$ plus that of $\hat{X}$. The latter is given by the generic sum of the monomials with admissible exponent ν with $\nu_5 = 0$. There are exactly 24 such exponents:

$$\begin{aligned} &z_5^{28},\ z_5^{24}z_4^3,\ z_5^{20}z_4^6,\ z_5^{16}z_4^9,\ z_5^{12}z_4^{12},\ z_5^8z_4^{15},\ z_5^4z_4^{18},\ z_4^{21},\ z_5^{19}z_2, \\ &z_5^{15}z_4^3z_2,\ z_5^{11}z_4^6z_2,\ z_5^7z_4^9z_2,\ z_5^3z_4^{12}z_2,\ z_5^{10}z_2^2,\ z_5^6z_4^3z_2^2, \\ &z_5^2z_4^6z_2^2,\ z_5z_2^3,\ z_5^{14}z_1,\ z_5^{10}z_4^3z_1,\ z_5^6z_4^6z_1,\ z_5^2z_4^9z_1,\ z_5^5z_2z_1,\ z_5z_4^3z_2z_1,\ z_1^2. \end{aligned} \qquad (1.5)$$

Condition (i) holds because the exponent ν_4 of z_4 is always a multiple of $\rho_3 = 3$. The equation for the isomorphic image $\bar{X}$ of X in $\mathbf{P}[14, 9, 4, 1]$ is the generic sum of the above monomials with the replacement, $z_1 \mapsto x_1$, $z_2 \mapsto x_2$, $z_4 \mapsto x_3^3$, $z_5 \mapsto x_4$.

We note that given an admissible weight $\hat{w}$, the Calabi-Yau varieties in $\mathbf{P}[\hat{w}]$ can give two distinct transversal Calabi-Yau varieties when intersect with two different coordinate hyperplanes $z_i = 0$.

1.1. the second problem

We consider our second problem under the following assumption. We assume that Z_λ is of the form $\lambda_1 z_{n+1} = \lambda_2 p(z)$ where $\lambda = [\lambda_1, \lambda_2]$ is regarded as a point in $\mathbf{P}^1$, and $p(z)$ a fixed nonzero quasi-homogeneous polynomial independent of z_{n+1} and has degree w_{n+1}. When $\lambda_2 = 0$ this reduces to the case in the first problem. This generalization turns out to require just some minor modification. Specifically, in addition to conditions (i)–(iii), we must require that the weight component
(iv) w_{n+1} can be partitioned by the components $w_1, .., w_n$.

This is true iff p exists. Note that as λ varies the intersections $\hat{X}_a \cap Z_\lambda$ form a pencil of codimension one subvarieties in $\hat{X}_a$. In the case of $n = 4$ we require that they are transversal K3 varieties when $\lambda_1 \neq 0$. In our list of 628 cases above, we find that all of them admit this description hence enlarging the list of [1].

The table given in the appendix is the list of the 628 cases. The number denoted i between 1 and 5 in the table indicates Z_λ is of the form $\lambda_1 z_i = \lambda_2 p(z)$ as in the case $i = n + 1$ discussed above. Some of the examples in this list have been studied in great details in the context of mirror symmetry (see for example [7][8][9]), and in connection with string duality in [2][10] and others.

We note that the conditions we impose in our method for enumerating K3 pencils are only sufficient but not necessary. There is in fact a criterion given in [11] for K3 pencils using the intersection ring of the Calabi-Yau variety. In fact in [9] (see the conclusion section there) we have already used this criterion to give a few examples of K3 pencils in which we have computed the intersection ring. For example, the Calabi-Yau hypersurfaces in $\mathbf{P}[8, 3, 3, 1, 1]$ was found to have a K3 pencil according to the criterion of [11], but this example fails to satisfy conditions (i)-(iii) above. In [9], we have also given an algorithm for computing the intersection ring of Calabi-Yau hypersurfaces in weighted projective spaces. This algorithm can in principle be carried out for all of the list [5], and be used to check the criterion above. But the actual computation can be enormous.

For completeness, we also do the case of $n = 3$. Thus we search through the list of transversal K3 hypersurfaces in [4] which admits a pencil of elliptic curves in one of the following transversal weighted projective spaces $\mathbf{P}[1, 1, 1], \mathbf{P}[2, 1, 1], \mathbf{P}[3, 2, 1]$. The $n = 3$ analogues of conditions (i)-(iii) are satisfied by 18 admissible weights, and all of them satisfy condition (iv).

Acknowledgement: We thank S. Kachru and C. Vafa for helpful discussions, and C. Doran for pointing out an error in the early version of this paper.

$\hat{w} = (w_1, .., w_4)$	i	$\bar{w}$	$\hat{w} = (w_1, .., w_4)$	i	$\bar{w}$
$(4, 3, 3, 2)$	1	$(2, 1, 1)$	$(4, 3, 3, 2)$	2	$(3, 2, 1)$
$(2, 2, 1, 1)$	3	$(1, 1, 1)$	$(4, 4, 3, 1)$	3	$(1, 1, 1)$
$(4, 2, 1, 1)$	3	$(2, 1, 1)$	$(6, 3, 2, 1)$	3	$(2, 1, 1)$
$(10, 5, 4, 1)$	3	$(2, 1, 1)$	$(6, 4, 1, 1)$	3	$(3, 2, 1)$
$(9, 6, 2, 1)$	3	$(3, 2, 1)$	$(12, 8, 3, 1)$	3	$(3, 2, 1)$
$(21, 14, 6, 1)$	3	$(3, 2, 1)$	$(3, 3, 2, 1)$	3	$(1, 1, 1)$
$(9, 4, 3, 2)$	2	$(3, 2, 1)$	$(8, 4, 3, 1)$	3	$(2, 1, 1)$
$(12, 7, 3, 2)$	2	$(2, 1, 1)$	$(18, 11, 4, 3)$	2	$(3, 2, 1)$
$(15, 10, 4, 1)$	3	$(3, 2, 1)$	$(18, 12, 5, 1)$	3	$(3, 2, 1)$

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2. Appendix

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
480	287	(882, 588, 251, 36, 7)	3	(21, 14, 6, 1)
376	201	(280, 140, 109, 16, 15)	3	(14, 7, 4, 3)
324	212	(630, 420, 179, 24, 7)	3	(15, 10, 4, 1)
256	147	(200, 100, 77, 15, 8)	3	(10, 5, 3, 2)
240	173	(504, 336, 143, 18, 7)	3	(12, 8, 3, 1)
216	141	(200, 100, 79, 16, 5)	3	(10, 5, 4, 1)
192	110	(60, 60, 43, 9, 8)	3	(5, 5, 3, 2)
180	114	(143, 110, 44, 30, 3)	4	(13, 10, 4, 3)
180	114	(130, 100, 40, 27, 3)	4	(13, 10, 4, 3)
168	95	(144, 67, 48, 20, 9)	2	(12, 5, 4, 3)
160	115	(160, 80, 61, 15, 4)	3	(8, 4, 3, 1)
160	115	(160, 80, 63, 12, 5)	3	(8, 4, 3, 1)
156	86	(77, 56, 42, 30, 5)	4	(11, 8, 6, 5)
156	86	(66, 48, 36, 25, 5)	4	(11, 8, 6, 5)
144	131	(378, 252, 107, 12, 7)	3	(9, 6, 2, 1)
144	98	(162, 99, 32, 27, 4)	3	(18, 11, 4, 3)
144	91	(80, 56, 32, 21, 3)	4	(10, 7, 4, 3)
120	86	(48, 48, 35, 9, 4)	3	(4, 4, 3, 1)
120	69	(100, 35, 32, 25, 8)	3	(20, 8, 7, 5)
120	69	(54, 42, 25, 24, 5)	3	(9, 7, 5, 4)
120	65	(60, 40, 36, 35, 9)	3	(12, 9, 8, 7)
120	65	(48, 32, 28, 27, 9)	4	(12, 9, 8, 7)
112	76	(98, 49, 24, 21, 4)	3	(14, 7, 4, 3)
112	63	(55, 30, 28, 20, 7)	3	(11, 7, 6, 4)
112	63	(44, 24, 21, 16, 7)	3	(11, 7, 6, 4)
108	60	(50, 30, 25, 24, 21)	1	(25, 10, 8, 7)
108	60	(25, 25, 20, 16, 14)	1	(25, 10, 8, 7)
96	167	(225, 200, 150, 24, 1)	4	(9, 8, 6, 1)
96	167	(216, 192, 144, 23, 1)	4	(9, 8, 6, 1)
96	87	(120, 60, 47, 8, 5)	3	(6, 3, 2, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
96	79	(88, 64, 21, 16, 3)	3	(11, 8, 3, 2)
96	65	(63, 42, 35, 24, 4)	4	(9, 6, 5, 4)
96	59	(99, 44, 22, 18, 15)	2	(33, 22, 6, 5)
96	59	(44, 33, 33, 12, 10)	2	(33, 22, 6, 5)
96	59	(44, 39, 22, 15, 12)	1	(22, 13, 5, 4)
96	59	(56, 33, 20, 12, 11)	2	(14, 11, 5, 3)
96	59	(42, 22, 15, 11, 9)	2	(14, 11, 5, 3)
96	57	(44, 32, 24, 15, 5)	4	(11, 8, 6, 5)
96	55	(38, 24, 19, 18, 15)	1	(19, 8, 6, 5)
96	55	(19, 19, 16, 12, 10)	1	(19, 8, 6, 5)
84	104	(294, 196, 56, 39, 3)	4	(21, 14, 4, 3)
84	54	(36, 27, 27, 10, 8)	2	(27, 18, 5, 4)
84	54	(38, 33, 19, 15, 9)	1	(19, 11, 5, 3)
84	54	(22, 19, 19, 10, 6)	2	(19, 11, 5, 3)
84	50	(36, 31, 18, 15, 8)	2	(6, 5, 4, 3)
84	50	(34, 21, 18, 17, 12)	1	(17, 7, 6, 4)
84	50	(17, 17, 14, 12, 8)	1	(17, 7, 6, 4)
80	68	(112, 56, 32, 21, 3)	4	(14, 7, 4, 3)
80	51	(32, 16, 15, 12, 5)	3	(8, 5, 4, 3)
72	68	(108, 49, 36, 20, 3)	2	(9, 5, 3, 1)
72	68	(108, 53, 36, 15, 4)	2	(9, 5, 3, 1)
72	65	(36, 36, 25, 8, 3)	3	(3, 3, 2, 1)
72	59	(56, 35, 18, 14, 3)	3	(8, 5, 3, 2)
72	57	(50, 35, 20, 12, 3)	4	(10, 7, 4, 3)
72	50	(72, 32, 16, 15, 9)	2	(24, 16, 5, 3)
72	50	(44, 27, 20, 9, 8)	2	(11, 9, 5, 2)
72	49	(34, 30, 17, 12, 9)	1	(17, 10, 4, 3)
72	49	(28, 24, 15, 8, 5)	3	(7, 6, 5, 2)
72	49	(28, 11, 11, 10, 6)	2	(14, 11, 5, 3)
72	49	(20, 17, 17, 8, 6)	2	(17, 10, 4, 3)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
72	48	$(33, 24, 18, 10, 5)$	4	$(11, 8, 6, 5)$
72	47	$(60, 21, 16, 15, 8)$	3	$(20, 8, 7, 5)$
72	47	$(27, 21, 12, 10, 5)$	4	$(9, 7, 5, 4)$
72	46	$(33, 18, 14, 12, 7)$	3	$(11, 7, 6, 4)$
72	44	$(24, 16, 14, 9, 9)$	4	$(12, 9, 8, 7)$
72	44	$(24, 21, 20, 12, 7)$	2	$(7, 6, 5, 3)$
72	44	$(18, 15, 14, 9, 7)$	3	$(7, 6, 5, 3)$
72	44	$(21, 18, 16, 9, 8)$	3	$(8, 7, 6, 3)$
64	47	$(36, 16, 15, 8, 5)$	3	$(9, 5, 4, 2)$
64	43	$(22, 12, 8, 7, 7)$	4	$(11, 7, 6, 4)$
60	194	$(465, 248, 186, 30, 1)$	4	$(15, 8, 6, 1)$
60	194	$(450, 240, 180, 29, 1)$	4	$(15, 8, 6, 1)$
60	59	$(90, 55, 16, 15, 4)$	3	$(18, 11, 4, 3)$
60	49	$(25, 25, 12, 10, 3)$	3	$(5, 5, 3, 2)$
60	44	$(22, 16, 12, 5, 5)$	4	$(11, 8, 6, 5)$
54	56	$(35, 35, 21, 12, 2)$	4	$(5, 5, 3, 2)$
50	44	$(35, 25, 20, 12, 3)$	4	$(7, 5, 4, 3)$
48	83	$(156, 91, 39, 24, 2)$	4	$(12, 7, 3, 2)$
48	59	$(96, 40, 32, 21, 3)$	4	$(12, 5, 4, 3)$
48	53	$(52, 40, 16, 9, 3)$	4	$(13, 10, 4, 3)$
48	43	$(28, 21, 21, 10, 4)$	2	$(21, 14, 5, 2)$
48	43	$(32, 27, 16, 15, 6)$	1	$(16, 9, 5, 2)$
48	41	$(36, 21, 12, 8, 7)$	2	$(9, 7, 3, 2)$
48	41	$(27, 14, 9, 7, 6)$	2	$(9, 7, 3, 2)$
48	41	$(22, 10, 9, 9, 4)$	3	$(11, 9, 5, 2)$
48	39	$(18, 14, 8, 5, 5)$	4	$(9, 7, 5, 4)$
48	39	$(26, 18, 15, 13, 6)$	1	$(13, 6, 5, 2)$
48	39	$(40, 24, 21, 20, 15)$	1	$(20, 8, 7, 5)$
48	39	$(32, 21, 16, 15, 12)$	1	$(16, 7, 5, 4)$
48	39	$(15, 15, 14, 12, 4)$	1	$(15, 7, 6, 2)$

Euler #	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
48	39	(13, 13, 12, 10, 4)	1	(13, 6, 5, 2)
48	38	(40, 21, 16, 12, 7)	2	(10, 7, 4, 3)
48	37	(24, 12, 10, 9, 5)	3	(8, 5, 4, 3)
48	36	(36, 24, 16, 15, 5)	4	(9, 6, 5, 4)
48	35	(16, 15, 12, 12, 5)	2	(5, 4, 3, 3)
48	35	(12, 10, 7, 7, 6)	3	(7, 6, 5, 3)
48	35	(12, 10, 9, 9, 5)	2	(5, 4, 3, 3)
44	51	(70, 35, 18, 14, 3)	3	(10, 5, 3, 2)
42	55	(49, 35, 21, 12, 2)	4	(7, 5, 3, 2)
40	69	(110, 55, 33, 20, 2)	4	(10, 5, 3, 2)
40	41	(40, 25, 20, 12, 3)	4	(8, 5, 4, 3)
36	116	(133, 114, 76, 18, 1)	4	(7, 6, 4, 1)
36	116	(126, 108, 72, 17, 1)	4	(7, 6, 4, 1)
36	38	(28, 21, 21, 8, 6)	2	(21, 14, 4, 3)
36	38	(26, 24, 13, 9, 6)	1	(13, 8, 3, 2)
36	38	(16, 13, 13, 6, 4)	2	(13, 8, 3, 2)
36	35	(21, 18, 10, 6, 5)	3	(7, 6, 5, 2)
36	34	(66, 31, 15, 12, 8)	2	(11, 5, 4, 2)
36	34	(22, 15, 12, 11, 6)	1	(11, 5, 4, 2)
36	34	(11, 11, 10, 8, 4)	1	(11, 5, 4, 2)
32	103	(102, 85, 68, 16, 1)	4	(6, 5, 4, 1)
32	103	(96, 80, 64, 15, 1)	4	(6, 5, 4, 1)
32	46	(70, 35, 20, 12, 3)	4	(14, 7, 4, 3)
32	33	(16, 8, 6, 5, 5)	4	(8, 5, 4, 3)
24	80	(216, 144, 43, 27, 2)	3	(12, 8, 3, 1)
24	49	(100, 40, 33, 25, 2)	3	(10, 5, 4, 1)
24	48	(55, 40, 12, 10, 3)	3	(11, 8, 3, 2)
24	47	(72, 35, 24, 9, 4)	2	(6, 3, 2, 1)
24	41	(105, 42, 30, 28, 5)	3	(15, 6, 5, 4)
24	41	(90, 36, 25, 24, 5)	3	(15, 6, 5, 4)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
24	38	(42, 21, 9, 8, 4)	4	(14, 7, 4, 3)
24	38	(63, 28, 15, 14, 6)	2	(21, 14, 5, 2)
24	38	(32, 20, 9, 8, 3)	3	(8, 5, 3, 2)
24	38	(27, 18, 16, 8, 3)	3	(9, 8, 6, 1)
24	38	(32, 21, 20, 7, 4)	2	(8, 7, 5, 1)
24	38	(24, 15, 14, 7, 3)	3	(8, 7, 5, 1)
24	38	(33, 16, 9, 8, 6)	2	(11, 8, 3, 2)
24	36	(35, 20, 12, 10, 3)	3	(7, 4, 3, 2)
24	34	(27, 18, 15, 8, 4)	4	(9, 6, 5, 4)
24	33	(18, 7, 7, 6, 4)	2	(9, 7, 3, 2)
24	33	(25, 12, 10, 10, 3)	2	(5, 3, 2, 2)
24	33	(27, 12, 10, 6, 5)	3	(9, 5, 4, 2)
24	33	(20, 9, 8, 8, 3)	2	(5, 3, 2, 2)
24	32	(14, 12, 5, 5, 4)	3	(7, 6, 5, 2)
24	32	(32, 15, 12, 8, 5)	2	(8, 5, 3, 2)
24	32	(30, 17, 12, 9, 4)	2	(5, 3, 2, 2)
24	31	(28, 15, 12, 8, 5)	2	(7, 5, 3, 2)
24	30	(54, 25, 12, 9, 8)	2	(9, 4, 3, 2)
24	29	(15, 15, 12, 10, 8)	1	(15, 6, 5, 4)
24	28	(24, 18, 17, 9, 4)	3	(4, 3, 3, 2)
24	27	(27, 18, 12, 10, 5)	4	(9, 6, 5, 4)
24	27	(30, 14, 12, 9, 7)	2	(10, 7, 4, 3)
24	27	(8, 6, 6, 5, 5)	4	(5, 4, 3, 3)
20	50	(91, 56, 18, 14, 3)	3	(13, 8, 3, 2)
18	53	(63, 49, 21, 12, 2)	4	(9, 7, 3, 2)
16	31	(18, 8, 5, 5, 4)	3	(9, 5, 4, 2)
16	29	(20, 16, 9, 8, 3)	3	(5, 4, 3, 2)
12	41	(54, 33, 9, 8, 4)	4	(18, 11, 4, 3)
12	36	(26, 21, 15, 13, 3)	1	(13, 7, 5, 1)
12	36	(16, 15, 15, 12, 2)	2	(15, 8, 6, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
12	36	(14, 13, 13, 10, 2)	2	(13, 7, 5, 1)
8	29	(28, 16, 9, 8, 3)	3	(7, 4, 3, 2)
6	23	(21, 10, 9, 6, 5)	2	(7, 5, 3, 2)
0	251	(903, 602, 258, 42, 1)	4	(21, 14, 6, 1)
0	251	(882, 588, 252, 41, 1)	4	(21, 14, 6, 1)
0	131	(253, 138, 92, 22, 1)	4	(11, 6, 4, 1)
0	131	(242, 132, 88, 21, 1)	4	(11, 6, 4, 1)
0	121	(153, 136, 102, 16, 1)	4	(9, 8, 6, 1)
0	119	(210, 105, 84, 20, 1)	4	(10, 5, 4, 1)
0	119	(200, 100, 80, 19, 1)	4	(10, 5, 4, 1)
0	89	(225, 150, 45, 28, 2)	4	(15, 10, 3, 2)
0	89	(96, 80, 48, 15, 1)	4	(6, 5, 3, 1)
0	89	(90, 75, 45, 14, 1)	4	(6, 5, 3, 1)
0	83	(252, 168, 71, 7, 6)	3	(6, 4, 1, 1)
0	77	(70, 56, 42, 13, 1)	4	(5, 4, 3, 1)
0	77	(65, 52, 39, 12, 1)	4	(5, 4, 3, 1)
0	71	(52, 52, 39, 12, 1)	4	(4, 4, 3, 1)
0	71	(48, 48, 36, 11, 1)	4	(4, 4, 3, 1)
0	65	(168, 112, 32, 21, 3)	4	(21, 14, 4, 3)
0	59	(165, 110, 30, 22, 3)	3	(15, 10, 3, 2)
0	59	(150, 100, 27, 20, 3)	3	(15, 10, 3, 2)
0	55	(80, 40, 31, 5, 4)	3	(4, 2, 1, 1)
0	55	(147, 98, 36, 7, 6)	3	(21, 14, 6, 1)
0	55	(98, 63, 24, 7, 4)	3	(14, 9, 4, 1)
0	55	(80, 56, 21, 8, 3)	3	(10, 7, 3, 1)
0	55	(70, 49, 18, 7, 3)	3	(10, 7, 3, 1)
0	41	(24, 24, 17, 4, 3)	3	(2, 2, 1, 1)
0	39	(50, 25, 16, 5, 4)	3	(10, 5, 4, 1)
0	39	(75, 35, 24, 10, 6)	3	(15, 7, 6, 2)
0	39	(35, 20, 12, 5, 3)	3	(7, 4, 3, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
0	39	(45, 25, 16, 10, 4)	3	(9, 5, 4, 2)
0	38	(60, 25, 20, 12, 3)	4	(12, 5, 4, 3)
0	35	(63, 28, 18, 14, 3)	2	(21, 14, 6, 1)
0	35	(63, 28, 18, 14, 3)	3	(9, 4, 3, 2)
0	35	(28, 21, 21, 12, 2)	2	(21, 14, 6, 1)
0	35	(28, 21, 21, 12, 2)	4	(4, 3, 3, 2)
0	35	(28, 27, 14, 12, 3)	1	(14, 9, 4, 1)
0	35	(40, 21, 12, 7, 4)	2	(10, 7, 3, 1)
0	35	(30, 14, 9, 7, 3)	2	(10, 7, 3, 1)
0	34	(20, 14, 8, 3, 3)	4	(10, 7, 4, 3)
0	31	(60, 24, 16, 15, 5)	4	(15, 6, 5, 4)
0	31	(20, 20, 12, 5, 3)	3	(4, 4, 3, 1)
0	31	(16, 16, 9, 4, 3)	3	(4, 4, 3, 1)
0	31	(22, 18, 12, 11, 3)	1	(11, 6, 4, 1)
0	31	(24, 19, 15, 12, 2)	2	(5, 4, 2, 1)
0	31	(24, 23, 12, 10, 3)	2	(5, 4, 2, 1)
0	31	(40, 35, 24, 15, 6)	3	(8, 7, 6, 3)
0	31	(35, 16, 15, 10, 4)	2	(7, 4, 3, 2)
0	31	(16, 10, 7, 7, 2)	3	(8, 7, 5, 1)
0	31	(12, 11, 11, 8, 2)	2	(11, 6, 4, 1)
0	29	(45, 20, 10, 9, 6)	2	(15, 10, 3, 2)
0	29	(24, 15, 12, 5, 4)	2	(6, 5, 3, 1)
0	29	(18, 10, 9, 5, 3)	2	(6, 5, 3, 1)
0	29	(13, 12, 12, 9, 2)	1	(3, 2, 2, 1)
0	27	(20, 15, 15, 6, 4)	2	(15, 10, 3, 2)
0	23	(28, 21, 14, 12, 9)	1	(14, 7, 4, 3)
0	23	(14, 9, 7, 6, 6)	1	(7, 3, 2, 2)
0	23	(9, 9, 8, 6, 4)	1	(9, 4, 3, 2)
0	23	(9, 9, 8, 6, 4)	3	(4, 3, 3, 2)
0	23	(7, 7, 6, 4, 4)	1	(7, 3, 2, 2)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
0	22	(24, 10, 9, 6, 5)	2	(8, 5, 3, 2)
0	18	(18, 12, 8, 5, 5)	4	(9, 6, 5, 4)
0	18	(20, 8, 7, 7, 6)	3	(10, 7, 4, 3)
−4	26	(35, 12, 10, 10, 3)	2	(7, 3, 2, 2)
−8	29	(40, 20, 9, 8, 3)	3	(10, 5, 3, 2)
−8	25	(24, 16, 15, 5, 4)	3	(6, 5, 4, 1)
−12	38	(26, 20, 8, 3, 3)	4	(13, 10, 4, 3)
−12	30	(20, 15, 15, 8, 2)	2	(15, 10, 4, 1)
−12	30	(20, 15, 15, 8, 2)	4	(4, 3, 3, 2)
−12	30	(22, 21, 11, 9, 3)	1	(11, 7, 3, 1)
−12	30	(14, 11, 11, 6, 2)	2	(11, 7, 3, 1)
−12	25	(10, 10, 4, 3, 3)	4	(5, 5, 3, 2)
−12	24	(18, 13, 12, 9, 2)	2	(3, 3, 2, 1)
−12	16	(10, 9, 6, 6, 5)	1	(5, 3, 2, 2)
−16	23	(70, 28, 20, 15, 7)	2	(14, 7, 4, 3)
−16	23	(56, 21, 16, 12, 7)	2	(14, 7, 4, 3)
−20	15	(14, 6, 5, 5, 4)	3	(7, 5, 3, 2)
−24	110	(144, 128, 96, 15, 1)	4	(9, 8, 6, 1)
−24	77	(72, 60, 48, 11, 1)	4	(6, 5, 4, 1)
−24	60	(40, 40, 30, 9, 1)	4	(4, 4, 3, 1)
−24	51	(84, 49, 21, 12, 2)	4	(12, 7, 3, 2)
−24	29	(20, 7, 7, 6, 2)	2	(10, 7, 3, 1)
−24	27	(16, 10, 4, 3, 3)	4	(8, 5, 3, 2)
−24	26	(36, 16, 9, 8, 3)	2	(12, 8, 3, 1)
−24	26	(36, 16, 9, 8, 3)	3	(9, 4, 3, 2)
−24	26	(28, 15, 8, 5, 4)	2	(7, 5, 2, 1)
−24	26	(21, 10, 6, 5, 3)	2	(7, 5, 2, 1)
−24	25	(48, 23, 15, 6, 4)	2	(8, 5, 2, 1)
−24	23	(12, 6, 5, 5, 2)	3	(6, 5, 3, 1)
−24	22	(36, 17, 9, 6, 4)	2	(6, 3, 2, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−24	22	(21, 9, 8, 6, 4)	3	(7, 4, 3, 2)
−24	21	(40, 15, 12, 8, 5)	2	(10, 5, 3, 2)
−24	21	(18, 12, 11, 4, 3)	3	(3, 2, 2, 1)
−24	20	(42, 14, 12, 9, 7)	2	(14, 7, 4, 3)
−24	20	(10, 4, 4, 3, 3)	4	(5, 3, 2, 2)
−24	18	(18, 12, 10, 5, 3)	3	(6, 5, 4, 1)
−24	17	(20, 15, 10, 9, 6)	1	(10, 5, 3, 2)
−24	15	(16, 6, 5, 5, 4)	3	(8, 5, 3, 2)
−24	12	(6, 5, 5, 4, 4)	2	(5, 3, 2, 2)
−28	17	(14, 10, 8, 3, 3)	4	(7, 5, 4, 3)
−30	24	(15, 15, 8, 5, 2)	3	(3, 3, 2, 1)
−30	23	(15, 15, 9, 4, 2)	4	(5, 5, 3, 2)
−30	17	(15, 9, 8, 4, 3)	3	(5, 4, 3, 1)
−32	87	(136, 68, 51, 16, 1)	4	(8, 4, 3, 1)
−32	87	(128, 64, 48, 15, 1)	4	(8, 4, 3, 1)
−32	29	(52, 32, 9, 8, 3)	3	(13, 8, 3, 2)
−32	19	(16, 9, 8, 4, 3)	2	(4, 3, 2, 1)
−32	17	(16, 10, 8, 3, 3)	4	(8, 5, 4, 3)
−36	148	(345, 184, 138, 22, 1)	4	(15, 8, 6, 1)
−36	102	(170, 85, 68, 16, 1)	4	(10, 5, 4, 1)
−36	98	(171, 95, 57, 18, 1)	4	(9, 5, 3, 1)
−36	98	(162, 90, 54, 17, 1)	4	(9, 5, 3, 1)
−36	44	(30, 30, 20, 9, 1)	4	(3, 3, 2, 1)
−36	44	(27, 27, 18, 8, 1)	4	(3, 3, 2, 1)
−36	26	(30, 15, 8, 4, 3)	3	(10, 5, 4, 1)
−36	26	(27, 15, 8, 6, 4)	3	(9, 5, 4, 2)
−36	20	(42, 19, 12, 8, 3)	2	(7, 4, 2, 1)
−36	20	(14, 12, 7, 6, 3)	1	(7, 4, 2, 1)
−36	20	(8, 7, 7, 4, 2)	2	(7, 4, 2, 1)
−36	20	(10, 9, 9, 6, 2)	2	(9, 5, 3, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
-36	17	(30, 10, 9, 6, 5)	2	(10, 5, 3, 2)
-36	14	(10, 8, 4, 3, 3)	4	(5, 4, 3, 2)
-40	70	(66, 55, 44, 10, 1)	4	(6, 5, 4, 1)
-40	59	(50, 40, 30, 9, 1)	4	(5, 4, 3, 1)
-40	49	(44, 33, 22, 10, 1)	4	(4, 3, 2, 1)
-40	49	(40, 30, 20, 9, 1)	4	(4, 3, 2, 1)
-40	25	(40, 20, 12, 5, 3)	3	(8, 4, 3, 1)
-40	19	(20, 9, 8, 4, 3)	2	(5, 3, 2, 1)
-42	23	(21, 15, 9, 4, 2)	4	(7, 5, 3, 2)
-42	17	(21, 9, 8, 4, 3)	3	(7, 4, 3, 1)
-48	77	(84, 72, 48, 11, 1)	4	(7, 6, 4, 1)
-48	67	(165, 110, 33, 20, 2)	4	(15, 10, 3, 2)
-48	67	(66, 55, 33, 10, 1)	4	(6, 5, 3, 1)
-48	59	(65, 52, 26, 12, 1)	4	(5, 4, 2, 1)
-48	59	(60, 48, 24, 11, 1)	4	(5, 4, 2, 1)
-48	50	(36, 36, 27, 8, 1)	4	(4, 4, 3, 1)
-48	43	(126, 84, 31, 7, 4)	3	(9, 6, 2, 1)
-48	39	(24, 24, 16, 7, 1)	4	(3, 3, 2, 1)
-48	35	(40, 28, 9, 4, 3)	3	(10, 7, 3, 1)
-48	35	(27, 18, 18, 8, 1)	4	(3, 2, 2, 1)
-48	35	(24, 16, 16, 7, 1)	4	(3, 2, 2, 1)
-48	31	(22, 16, 4, 3, 3)	4	(11, 8, 3, 2)
-48	22	(24, 13, 9, 6, 2)	2	(4, 3, 1, 1)
-48	21	(14, 5, 5, 4, 2)	2	(7, 5, 2, 1)
-48	19	(8, 8, 3, 3, 2)	3	(4, 4, 3, 1)
-48	19	(12, 9, 9, 4, 2)	2	(9, 6, 2, 1)
-48	19	(12, 9, 9, 4, 2)	4	(4, 3, 3, 2)
-48	19	(16, 15, 8, 6, 3)	1	(8, 5, 2, 1)
-48	17	(45, 18, 12, 10, 5)	4	(15, 6, 5, 4)
-48	17	(15, 8, 6, 4, 3)	2	(5, 4, 2, 1)

Euler #	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−48	15	(16, 12, 9, 8, 3)	1	(8, 4, 3, 1)
−48	15	(16, 12, 9, 8, 3)	3	(4, 3, 3, 2)
−48	15	(20, 15, 12, 10, 3)	1	(10, 5, 4, 1)
−48	15	(20, 15, 12, 10, 3)	3	(4, 3, 3, 2)
−48	15	(12, 11, 6, 4, 3)	2	(2, 2, 1, 1)
−48	15	(10, 6, 6, 5, 3)	1	(5, 2, 2, 1)
−48	15	(5, 5, 4, 4, 2)	1	(5, 2, 2, 1)
−48	14	(14, 8, 4, 3, 3)	4	(7, 4, 3, 2)
−48	12	(12, 8, 5, 5, 2)	3	(6, 5, 4, 1)
−48	11	(15, 10, 9, 6, 5)	2	(5, 5, 3, 2)
−48	11	(20, 15, 12, 8, 5)	2	(5, 5, 3, 2)
−50	24	(25, 15, 8, 5, 2)	3	(5, 3, 2, 1)
−54	53	(45, 36, 27, 8, 1)	4	(5, 4, 3, 1)
−56	93	(160, 80, 64, 15, 1)	4	(10, 5, 4, 1)
−56	76	(112, 56, 42, 13, 1)	4	(8, 4, 3, 1)
−60	222	(777, 518, 222, 36, 1)	4	(21, 14, 6, 1)
−60	164	(465, 310, 124, 30, 1)	4	(15, 10, 4, 1)
−60	164	(450, 300, 120, 29, 1)	4	(15, 10, 4, 1)
−60	19	(25, 20, 15, 12, 3)	4	(5, 4, 3, 3)
−60	14	(9, 9, 4, 3, 2)	3	(3, 3, 2, 1)
−64	39	(32, 24, 16, 7, 1)	4	(4, 3, 2, 1)
−64	29	(21, 14, 14, 6, 1)	4	(3, 2, 2, 1)
−64	17	(28, 14, 8, 3, 3)	4	(14, 7, 4, 3)
−64	15	(40, 16, 15, 5, 4)	2	(8, 4, 3, 1)
−64	15	(40, 16, 15, 5, 4)	3	(10, 5, 4, 1)
−64	11	(28, 8, 7, 7, 6)	3	(14, 7, 4, 3)
−64	11	(14, 4, 4, 3, 3)	4	(7, 3, 2, 2)
−64	8	(4, 4, 3, 3, 2)	3	(3, 2, 2, 1)
−66	32	(21, 21, 14, 6, 1)	4	(3, 3, 2, 1)
−66	23	(27, 21, 9, 4, 2)	4	(9, 7, 3, 2)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−72	88	(117, 104, 78, 12, 1)	4	(9, 8, 6, 1)
−72	69	(104, 52, 39, 12, 1)	4	(8, 4, 3, 1)
−72	57	(60, 50, 30, 9, 1)	4	(6, 5, 3, 1)
−72	56	(54, 45, 36, 8, 1)	4	(6, 5, 4, 1)
−72	53	(78, 39, 26, 12, 1)	4	(6, 3, 2, 1)
−72	53	(72, 36, 24, 11, 1)	4	(6, 3, 2, 1)
−72	40	(28, 28, 21, 6, 1)	4	(4, 4, 3, 1)
−72	32	(60, 35, 15, 8, 2)	4	(12, 7, 3, 2)
−72	29	(42, 27, 8, 4, 3)	3	(14, 9, 4, 1)
−72	26	(40, 20, 13, 5, 2)	3	(4, 2, 1, 1)
−72	26	(18, 12, 12, 5, 1)	4	(3, 2, 2, 1)
−72	23	(42, 21, 12, 7, 2)	3	(6, 3, 2, 1)
−72	23	(35, 21, 14, 12, 2)	4	(5, 3, 2, 2)
−72	21	(14, 8, 3, 3, 2)	3	(7, 4, 3, 1)
−72	20	(36, 17, 12, 4, 3)	2	(3, 1, 1, 1)
−72	14	(20, 10, 4, 3, 3)	4	(10, 5, 3, 2)
−72	14	(16, 9, 4, 4, 3)	2	(4, 3, 1, 1)
−72	13	(30, 12, 10, 5, 3)	2	(6, 3, 2, 1)
−72	13	(30, 12, 10, 5, 3)	3	(10, 5, 4, 1)
−72	13	(24, 9, 8, 4, 3)	2	(6, 3, 2, 1)
−72	13	(24, 9, 8, 4, 3)	3	(8, 4, 3, 1)
−72	13	(24, 11, 6, 4, 3)	2	(4, 2, 1, 1)
−72	13	(30, 12, 8, 5, 5)	4	(15, 6, 5, 4)
−72	13	(12, 7, 6, 3, 2)	2	(2, 1, 1, 1)
−72	11	(7, 6, 6, 3, 2)	1	(1, 1, 1, 1)
−72	10	(20, 6, 5, 5, 4)	3	(10, 5, 3, 2)
−72	9	(8, 4, 3, 3, 2)	3	(4, 3, 2, 1)
−72	8	(12, 9, 8, 4, 3)	2	(3, 3, 2, 1)
−72	8	(12, 9, 8, 4, 3)	3	(4, 4, 3, 1)
−72	7	(10, 6, 5, 5, 4)	3	(5, 5, 3, 2)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−80	39	(55, 22, 22, 10, 1)	4	(5, 2, 2, 1)
−80	39	(50, 20, 20, 9, 1)	4	(5, 2, 2, 1)
−80	33	(28, 21, 14, 6, 1)	4	(4, 3, 2, 1)
−80	11	(15, 10, 8, 5, 2)	3	(3, 2, 2, 1)
−84	76	(126, 70, 42, 13, 1)	4	(9, 5, 3, 1)
−84	62	(105, 60, 30, 14, 1)	4	(7, 4, 2, 1)
−84	62	(98, 56, 28, 13, 1)	4	(7, 4, 2, 1)
−84	61	(70, 60, 40, 9, 1)	4	(7, 6, 4, 1)
−84	48	(66, 33, 22, 10, 1)	4	(6, 3, 2, 1)
−84	44	(45, 36, 18, 8, 1)	4	(5, 4, 2, 1)
−84	41	(35, 28, 21, 6, 1)	4	(5, 4, 3, 1)
−84	14	(15, 9, 4, 3, 2)	3	(5, 3, 2, 1)
−84	12	(10, 9, 5, 3, 3)	1	(5, 3, 1, 1)
−84	12	(6, 5, 5, 2, 2)	2	(5, 3, 1, 1)
−84	9	(10, 4, 3, 3, 2)	3	(5, 3, 2, 1)
−84	8	(9, 8, 4, 3, 3)	2	(4, 3, 1, 1)
−88	17	(34, 20, 8, 3, 3)	4	(17, 10, 4, 3)
−90	24	(15, 15, 10, 4, 1)	4	(3, 3, 2, 1)
−90	24	(35, 25, 8, 5, 2)	3	(7, 5, 2, 1)
−90	8	(9, 4, 3, 3, 2)	2	(3, 2, 1, 1)
−92	5	(4, 3, 3, 2, 2)	2	(3, 2, 1, 1)
−96	147	(405, 270, 108, 26, 1)	4	(15, 10, 4, 1)
−96	119	(300, 200, 75, 24, 1)	4	(12, 8, 3, 1)
−96	119	(288, 192, 72, 23, 1)	4	(12, 8, 3, 1)
−96	47	(48, 40, 24, 7, 1)	4	(6, 5, 3, 1)
−96	43	(60, 30, 20, 9, 1)	4	(6, 3, 2, 1)
−96	39	(40, 32, 16, 7, 1)	4	(5, 4, 2, 1)
−96	33	(45, 18, 18, 8, 1)	4	(5, 2, 2, 1)
−96	31	(96, 64, 21, 8, 3)	3	(12, 8, 3, 1)
−96	31	(72, 45, 16, 9, 2)	3	(8, 5, 2, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
-96	27	(24, 18, 12, 5, 1)	4	(4, 3, 2, 1)
-96	23	(60, 40, 9, 8, 3)	3	(15, 10, 3, 2)
-96	19	(30, 15, 9, 4, 2)	4	(10, 5, 3, 2)
-96	19	(15, 10, 10, 4, 1)	4	(3, 2, 2, 1)
-96	17	(12, 12, 6, 5, 1)	4	(2, 2, 1, 1)
-96	17	(14, 14, 7, 6, 1)	4	(2, 2, 1, 1)
-96	14	(24, 10, 8, 3, 3)	4	(12, 5, 4, 3)
-96	11	(18, 8, 4, 3, 3)	2	(6, 4, 1, 1)
-96	11	(18, 8, 4, 3, 3)	4	(9, 4, 3, 2)
-96	11	(8, 3, 3, 2, 2)	2	(4, 3, 1, 1)
-96	9	(20, 9, 4, 4, 3)	2	(5, 3, 1, 1)
-96	7	(8, 6, 4, 3, 3)	1	(4, 2, 1, 1)
-96	7	(8, 6, 4, 3, 3)	4	(4, 3, 3, 2)
-96	7	(9, 6, 4, 3, 2)	3	(3, 2, 2, 1)
-96	5	(6, 4, 3, 3, 2)	2	(2, 2, 1, 1)
-96	5	(6, 4, 3, 3, 2)	3	(3, 3, 2, 1)
-96	5	(4, 3, 3, 3, 2)	1	(2, 1, 1, 1)
-104	17	(12, 8, 8, 3, 1)	4	(3, 2, 2, 1)
-108	14	(26, 16, 4, 3, 3)	4	(13, 8, 3, 2)
-108	13	(10, 10, 5, 4, 1)	4	(2, 2, 1, 1)
-108	6	(3, 3, 2, 2, 2)	1	(3, 1, 1, 1)
-112	20	(49, 21, 14, 12, 2)	4	(7, 3, 2, 2)
-112	20	(21, 14, 7, 6, 1)	4	(3, 2, 1, 1)
-112	20	(24, 16, 8, 7, 1)	4	(3, 2, 1, 1)
-112	10	(16, 8, 3, 3, 2)	3	(8, 4, 3, 1)
-112	7	(20, 8, 5, 5, 2)	2	(4, 2, 1, 1)
-112	7	(20, 8, 5, 5, 2)	3	(10, 5, 4, 1)
-120	109	(240, 128, 96, 15, 1)	4	(15, 8, 6, 1)
-120	108	(264, 176, 66, 21, 1)	4	(12, 8, 3, 1)
-120	76	(132, 72, 48, 11, 1)	4	(11, 6, 4, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
-120	68	$(81, 72, 54, 8, 1)$	4	$(9, 8, 6, 1)$
-120	65	$(110, 55, 44, 10, 1)$	4	$(10, 5, 4, 1)$
-120	49	$(72, 36, 27, 8, 1)$	4	$(8, 4, 3, 1)$
-120	48	$(49, 42, 28, 6, 1)$	4	$(7, 6, 4, 1)$
-120	47	$(77, 44, 22, 10, 1)$	4	$(7, 4, 2, 1)$
-120	39	$(36, 30, 18, 5, 1)$	4	$(6, 5, 3, 1)$
-120	38	$(36, 30, 24, 5, 1)$	4	$(6, 5, 4, 1)$
-120	31	$(35, 28, 14, 6, 1)$	4	$(5, 4, 2, 1)$
-120	29	$(25, 20, 15, 4, 1)$	4	$(5, 4, 3, 1)$
-120	26	$(20, 20, 15, 4, 1)$	4	$(4, 4, 3, 1)$
-120	26	$(16, 16, 12, 3, 1)$	4	$(4, 4, 3, 1)$
-120	25	$(30, 12, 12, 5, 1)$	4	$(5, 2, 2, 1)$
-120	25	$(20, 14, 3, 3, 2)$	3	$(10, 7, 3, 1)$
-120	22	$(60, 40, 12, 5, 3)$	3	$(12, 8, 3, 1)$
-120	22	$(40, 25, 8, 5, 2)$	3	$(8, 5, 2, 1)$
-120	21	$(36, 21, 9, 4, 2)$	4	$(12, 7, 3, 2)$
-120	17	$(12, 12, 8, 3, 1)$	4	$(3, 3, 2, 1)$
-120	17	$(18, 12, 6, 5, 1)$	4	$(3, 2, 1, 1)$
-120	10	$(18, 9, 4, 3, 2)$	3	$(6, 3, 2, 1)$
-120	10	$(8, 8, 4, 3, 1)$	4	$(2, 2, 1, 1)$
-120	10	$(15, 9, 6, 4, 2)$	4	$(5, 3, 2, 2)$
-120	9	$(25, 10, 8, 5, 2)$	3	$(5, 2, 2, 1)$
-120	9	$(10, 5, 5, 4, 1)$	4	$(2, 1, 1, 1)$
-120	9	$(12, 6, 6, 5, 1)$	4	$(2, 1, 1, 1)$
-120	6	$(12, 4, 3, 3, 2)$	2	$(4, 2, 1, 1)$
-120	6	$(12, 4, 3, 3, 2)$	3	$(6, 3, 2, 1)$
-120	5	$(4, 4, 4, 3, 1)$	4	$(1, 1, 1, 1)$
-120	5	$(5, 5, 5, 4, 1)$	4	$(1, 1, 1, 1)$
-128	19	$(16, 12, 8, 3, 1)$	4	$(4, 3, 2, 1)$
-128	7	$(8, 4, 4, 3, 1)$	4	$(2, 1, 1, 1)$

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−130	14	(15, 10, 5, 4, 1)	4	(3, 2, 1, 1)
−132	56	(90, 50, 30, 9, 1)	4	(9, 5, 3, 1)
−132	30	(42, 21, 14, 6, 1)	4	(6, 3, 2, 1)
−132	14	(21, 15, 4, 3, 2)	3	(7, 5, 2, 1)
−132	7	(10, 8, 6, 3, 3)	4	(5, 4, 3, 3)
−132	4	(10, 3, 3, 2, 2)	2	(5, 3, 1, 1)
−132	3	(3, 3, 3, 2, 1)	4	(1, 1, 1, 1)
−136	34	(30, 25, 20, 4, 1)	4	(6, 5, 4, 1)
−138	14	(9, 9, 6, 2, 1)	4	(3, 3, 2, 1)
−144	71	(171, 114, 38, 18, 1)	4	(9, 6, 2, 1)
−144	71	(162, 108, 36, 17, 1)	4	(9, 6, 2, 1)
−144	55	(90, 45, 36, 8, 1)	4	(10, 5, 4, 1)
−144	38	(56, 32, 16, 7, 1)	4	(7, 4, 2, 1)
−144	26	(36, 18, 12, 5, 1)	4	(6, 3, 2, 1)
−144	26	(81, 54, 16, 9, 2)	3	(9, 6, 2, 1)
−144	26	(36, 27, 9, 8, 1)	4	(4, 3, 1, 1)
−144	26	(40, 30, 10, 9, 1)	4	(4, 3, 1, 1)
−144	25	(25, 20, 10, 4, 1)	4	(5, 4, 2, 1)
−144	23	(20, 16, 12, 3, 1)	4	(5, 4, 3, 1)
−144	19	(32, 16, 8, 7, 1)	4	(4, 2, 1, 1)
−144	19	(36, 18, 9, 8, 1)	4	(4, 2, 1, 1)
−144	11	(12, 8, 4, 3, 1)	4	(3, 2, 1, 1)
−144	11	(9, 6, 6, 2, 1)	4	(3, 2, 2, 1)
−144	7	(6, 6, 3, 2, 1)	4	(2, 2, 1, 1)
−144	5	(15, 6, 4, 3, 2)	3	(5, 2, 2, 1)
−144	5	(6, 3, 3, 2, 1)	4	(2, 1, 1, 1)
−150	29	(30, 25, 15, 4, 1)	4	(6, 5, 3, 1)
−152	37	(56, 28, 21, 6, 1)	4	(8, 4, 3, 1)
−152	17	(25, 10, 10, 4, 1)	4	(5, 2, 2, 1)
−152	16	(28, 14, 7, 6, 1)	4	(4, 2, 1, 1)

Euler #	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−156	176	(609, 406, 174, 28, 1)	4	(21, 14, 6, 1)
−156	66	(153, 102, 34, 16, 1)	4	(9, 6, 2, 1)
−156	21	(63, 42, 12, 7, 2)	3	(9, 6, 2, 1)
−156	21	(28, 21, 7, 6, 1)	4	(4, 3, 1, 1)
−156	8	(18, 6, 6, 5, 1)	4	(3, 1, 1, 1)
−156	8	(21, 7, 7, 6, 1)	4	(3, 1, 1, 1)
−160	59	(110, 60, 40, 9, 1)	4	(11, 6, 4, 1)
−160	15	(20, 8, 8, 3, 1)	4	(5, 2, 2, 1)
−168	86	(204, 136, 51, 16, 1)	4	(12, 8, 3, 1)
−168	50	(63, 56, 42, 6, 1)	4	(9, 8, 6, 1)
−168	32	(48, 24, 18, 5, 1)	4	(8, 4, 3, 1)
−168	25	(24, 20, 16, 3, 1)	4	(6, 5, 4, 1)
−168	20	(30, 15, 10, 4, 1)	4	(6, 3, 2, 1)
−168	18	(20, 16, 8, 3, 1)	4	(5, 4, 2, 1)
−168	17	(24, 18, 6, 5, 1)	4	(4, 3, 1, 1)
−168	16	(12, 12, 9, 2, 1)	4	(4, 4, 3, 1)
−168	13	(12, 9, 6, 2, 1)	4	(4, 3, 2, 1)
−168	12	(20, 10, 5, 4, 1)	4	(4, 2, 1, 1)
−168	12	(24, 15, 4, 3, 2)	3	(8, 5, 2, 1)
−168	8	(21, 9, 6, 4, 2)	4	(7, 3, 2, 2)
−168	8	(9, 6, 3, 2, 1)	4	(3, 2, 1, 1)
−168	6	(12, 4, 4, 3, 1)	4	(3, 1, 1, 1)
−168	2	(2, 2, 2, 1, 1)	4	(1, 1, 1, 1)
−172	29	(49, 28, 14, 6, 1)	4	(7, 4, 2, 1)
−172	10	(22, 14, 3, 3, 2)	3	(11, 7, 3, 1)
−180	26	(42, 24, 12, 5, 1)	4	(7, 4, 2, 1)
−180	24	(50, 30, 10, 9, 1)	4	(5, 3, 1, 1)
−180	24	(55, 33, 11, 10, 1)	4	(5, 3, 1, 1)
−180	17	(15, 12, 9, 2, 1)	4	(5, 4, 3, 1)
−180	14	(45, 30, 8, 5, 2)	3	(9, 6, 2, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
-180	14	(20, 15, 5, 4, 1)	4	(4, 3, 1, 1)
-184	9	(16, 8, 4, 3, 1)	4	(4, 2, 1, 1)
-192	77	(180, 96, 72, 11, 1)	4	(15, 8, 6, 1)
-192	51	(117, 78, 26, 12, 1)	4	(9, 6, 2, 1)
-192	47	(88, 48, 32, 7, 1)	4	(11, 6, 4, 1)
-192	41	(54, 48, 36, 5, 1)	4	(9, 8, 6, 1)
-192	35	(63, 35, 21, 6, 1)	4	(9, 5, 3, 1)
-192	27	(28, 24, 16, 3, 1)	4	(7, 6, 4, 1)
-192	21	(24, 20, 12, 3, 1)	4	(6, 5, 3, 1)
-192	19	(40, 24, 8, 7, 1)	4	(5, 3, 1, 1)
-192	16	(24, 12, 8, 3, 1)	4	(6, 3, 2, 1)
-192	13	(16, 12, 4, 3, 1)	4	(4, 3, 1, 1)
-192	11	(30, 20, 4, 3, 3)	4	(15, 10, 3, 2)
-192	11	(36, 24, 7, 3, 2)	3	(6, 4, 1, 1)
-192	8	(6, 4, 4, 1, 1)	4	(3, 2, 2, 1)
-192	5	(4, 4, 2, 1, 1)	4	(2, 2, 1, 1)
-192	3	(9, 3, 3, 2, 1)	4	(3, 1, 1, 1)
-192	3	(4, 2, 2, 1, 1)	4	(2, 1, 1, 1)
-200	25	(40, 20, 15, 4, 1)	4	(8, 4, 3, 1)
-204	16	(30, 18, 6, 5, 1)	4	(5, 3, 1, 1)
-204	14	(42, 28, 8, 3, 3)	4	(21, 14, 4, 3)
-204	9	(6, 6, 4, 1, 1)	4	(3, 3, 2, 1)
-216	92	(240, 160, 64, 15, 1)	4	(15, 10, 4, 1)
-216	68	(165, 88, 66, 10, 1)	4	(15, 8, 6, 1)
-216	66	(156, 104, 39, 12, 1)	4	(12, 8, 3, 1)
-216	42	(90, 60, 20, 9, 1)	4	(9, 6, 2, 1)
-216	36	(45, 40, 30, 4, 1)	4	(9, 8, 6, 1)
-216	33	(60, 30, 24, 5, 1)	4	(10, 5, 4, 1)
-216	33	(50, 25, 20, 4, 1)	4	(10, 5, 4, 1)
-216	21	(32, 16, 12, 3, 1)	4	(8, 4, 3, 1)

Euler #	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−216	18	(18, 15, 12, 2, 1)	4	(6, 5, 4, 1)
−216	17	(45, 30, 9, 4, 2)	4	(15, 10, 3, 2)
−216	17	(18, 15, 9, 2, 1)	4	(6, 5, 3, 1)
−216	13	(15, 12, 6, 2, 1)	4	(5, 4, 2, 1)
−216	9	(15, 6, 6, 2, 1)	4	(5, 2, 2, 1)
−216	6	(12, 6, 3, 2, 1)	4	(4, 2, 1, 1)
−224	17	(28, 16, 8, 3, 1)	4	(7, 4, 2, 1)
−228	12	(18, 9, 6, 2, 1)	4	(6, 3, 2, 1)
−228	8	(27, 18, 4, 3, 2)	3	(9, 6, 2, 1)
−228	8	(12, 9, 3, 2, 1)	4	(4, 3, 1, 1)
−232	9	(8, 6, 4, 1, 1)	4	(4, 3, 2, 1)
−232	5	(6, 4, 2, 1, 1)	4	(3, 2, 1, 1)
−240	137	(462, 308, 132, 21, 1)	4	(21, 14, 6, 1)
−240	34	(81, 54, 18, 8, 1)	4	(9, 6, 2, 1)
−240	23	(72, 48, 12, 11, 1)	4	(6, 4, 1, 1)
−240	23	(78, 52, 13, 12, 1)	4	(6, 4, 1, 1)
−240	11	(8, 8, 6, 1, 1)	4	(4, 4, 3, 1)
−240	9	(20, 12, 4, 3, 1)	4	(5, 3, 1, 1)
−240	7	(24, 16, 3, 3, 2)	3	(12, 8, 3, 1)
−252	76	(210, 140, 56, 13, 1)	4	(15, 10, 4, 1)
−252	18	(54, 36, 9, 8, 1)	4	(6, 4, 1, 1)
−252	18	(21, 18, 12, 2, 1)	4	(7, 6, 4, 1)
−252	2	(6, 2, 2, 1, 1)	4	(3, 1, 1, 1)
−256	23	(40, 20, 16, 3, 1)	4	(10, 5, 4, 1)
−264	48	(108, 72, 27, 8, 1)	4	(12, 8, 3, 1)
−264	28	(63, 42, 14, 6, 1)	4	(9, 6, 2, 1)
−264	26	(36, 32, 24, 3, 1)	4	(9, 8, 6, 1)
−264	20	(36, 20, 12, 3, 1)	4	(9, 5, 3, 1)
−264	15	(24, 12, 9, 2, 1)	4	(8, 4, 3, 1)
−264	15	(42, 28, 7, 6, 1)	4	(6, 4, 1, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
−264	11	(10, 8, 6, 1, 1)	4	(5, 4, 3, 1)
−272	7	(10, 4, 4, 1, 1)	4	(5, 2, 2, 1)
−276	48	(105, 56, 42, 6, 1)	4	(15, 8, 6, 1)
−276	6	(15, 9, 3, 2, 1)	4	(5, 3, 1, 1)
−288	115	(399, 266, 114, 18, 1)	4	(21, 14, 6, 1)
−288	63	(165, 110, 44, 10, 1)	4	(15, 10, 4, 1)
−288	39	(96, 64, 24, 7, 1)	4	(12, 8, 3, 1)
−288	23	(44, 24, 16, 3, 1)	4	(11, 6, 4, 1)
−288	11	(30, 20, 5, 4, 1)	4	(6, 4, 1, 1)
−288	11	(21, 12, 6, 2, 1)	4	(7, 4, 2, 1)
−288	9	(10, 8, 4, 1, 1)	4	(5, 4, 2, 1)
−288	4	(8, 4, 2, 1, 1)	4	(4, 2, 1, 1)
−300	40	(90, 48, 36, 5, 1)	4	(15, 8, 6, 1)
−300	15	(27, 15, 9, 2, 1)	4	(9, 5, 3, 1)
−304	12	(12, 10, 8, 1, 1)	4	(6, 5, 4, 1)
−312	34	(84, 56, 21, 6, 1)	4	(12, 8, 3, 1)
−312	20	(27, 24, 18, 2, 1)	4	(9, 8, 6, 1)
−312	18	(45, 30, 10, 4, 1)	4	(9, 6, 2, 1)
−312	17	(30, 15, 12, 2, 1)	4	(10, 5, 4, 1)
−312	11	(12, 10, 6, 1, 1)	4	(6, 5, 3, 1)
−312	8	(12, 6, 4, 1, 1)	4	(6, 3, 2, 1)
−312	8	(24, 16, 4, 3, 1)	4	(6, 4, 1, 1)
−312	5	(8, 6, 2, 1, 1)	4	(4, 3, 1, 1)
−336	95	(315, 210, 90, 14, 1)	4	(21, 14, 6, 1)
−336	15	(36, 24, 8, 3, 1)	4	(9, 6, 2, 1)
−348	12	(14, 12, 8, 1, 1)	4	(7, 6, 4, 1)
−360	24	(60, 40, 15, 4, 1)	4	(12, 8, 3, 1)
−360	16	(33, 18, 12, 2, 1)	4	(11, 6, 4, 1)
−360	5	(18, 12, 3, 2, 1)	4	(6, 4, 1, 1)
−368	10	(16, 8, 6, 1, 1)	4	(8, 4, 3, 1)

$Euler\ \#$	$h^{1,1}$	$\hat{w} = (w_1, .., w_5)$	i	$\bar{w}$
-372	80	$(273, 182, 78, 12, 1)$	4	$(21, 14, 6, 1)$
-372	36	$(105, 70, 28, 6, 1)$	4	$(15, 10, 4, 1)$
-372	8	$(14, 8, 4, 1, 1)$	4	$(7, 4, 2, 1)$
-372	4	$(10, 6, 2, 1, 1)$	4	$(5, 3, 1, 1)$
-384	25	$(60, 32, 24, 3, 1)$	4	$(15, 8, 6, 1)$
-396	32	$(90, 60, 24, 5, 1)$	4	$(15, 10, 4, 1)$
-408	18	$(48, 32, 12, 3, 1)$	4	$(12, 8, 3, 1)$
-408	10	$(27, 18, 6, 2, 1)$	4	$(9, 6, 2, 1)$
-420	10	$(18, 10, 6, 1, 1)$	4	$(9, 5, 3, 1)$
-432	59	$(210, 140, 60, 9, 1)$	4	$(21, 14, 6, 1)$
-432	13	$(18, 16, 12, 1, 1)$	4	$(9, 8, 6, 1)$
-432	11	$(20, 10, 8, 1, 1)$	4	$(10, 5, 4, 1)$
-456	22	$(60, 40, 16, 3, 1)$	4	$(15, 10, 4, 1)$
-456	18	$(45, 24, 18, 2, 1)$	4	$(15, 8, 6, 1)$
-456	14	$(36, 24, 9, 2, 1)$	4	$(12, 8, 3, 1)$
-480	47	$(168, 112, 48, 7, 1)$	4	$(21, 14, 6, 1)$
-480	47	$(147, 98, 42, 6, 1)$	4	$(21, 14, 6, 1)$
-480	11	$(22, 12, 8, 1, 1)$	4	$(11, 6, 4, 1)$
-480	3	$(12, 8, 2, 1, 1)$	4	$(6, 4, 1, 1)$
-528	7	$(18, 12, 4, 1, 1)$	4	$(9, 6, 2, 1)$
-552	15	$(45, 30, 12, 2, 1)$	4	$(15, 10, 4, 1)$
-564	29	$(105, 70, 30, 4, 1)$	4	$(21, 14, 6, 1)$
-612	12	$(30, 16, 12, 1, 1)$	4	$(15, 8, 6, 1)$
-624	23	$(84, 56, 24, 3, 1)$	4	$(21, 14, 6, 1)$
-624	9	$(24, 16, 6, 1, 1)$	4	$(12, 8, 3, 1)$
-720	17	$(63, 42, 18, 2, 1)$	4	$(21, 14, 6, 1)$
-732	10	$(30, 20, 8, 1, 1)$	4	$(15, 10, 4, 1)$
-960	11	$(42, 28, 12, 1, 1)$	4	$(21, 14, 6, 1)$