The Electromagnetic Christodoulou Memory Effect in Neutron Star Binary Mergers

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Abstract

Gravitational waves are predicted by the general theory of relativity. In [6] D. Christodoulou showed that gravitational waves have a nonlinear memory. We proved in [3] that the electromagnetic field contributes at highest order to the nonlinear memory effect of gravitational waves. In the present paper, we study this electromagnetic Christodoulou memory effect and compute it for binary neutron star mergers. These are typical sources of gravitational radiation. During these processes, not only mass and momenta are radiated away in form of gravitational waves, but also very strong magnetic fields are produced and radiated away. Thus the observed effect on test masses of a laser interferometer gravitational wave detector will be enlarged by the contribution of the electromagnetic field. Therefore, the present results are important for the planned experiments. Looking at the null asymptotics of spacetimes, which are solutions of the Einstein-Maxwell (EM) equations, we derived in [3] the electromagnetic Christodoulou memory effect. Moreover, our results allow to answer astrophysical questions, as the knowledge about the amount of energy radiated away in a neutron star binary merger enables us to gain information about the source of the gravitational waves.

The main goal of this paper is to discuss the electromagnetic Christodoulou memory effect of gravitational waves and to compute this effect for typical sources. In [6] D. Christodoulou showed that gravitational waves have a nonlinear memory. In our paper [3] we proved that for spacetimes solving the Einstein-Maxwell (EM) equations, the electromagnetic field contributes at highest order to the nonlinear memory effect of gravitational waves. In the present paper, we also calculate it for neutron star binary mergers. We find that for typical constellations, very strong magnetic fields enlarge this effect considerably. Fields which are strong enough have so far only been known to be produced during mergers of neutron star binaries. The latter are well known to be frequent events. There is a vast astrophysical literature about this.

Moreover, our results in [3] and in the present paper, are also important from a purely astrophysical point of view. Namely, the knowledge about the amount of energy radiated away in a neutron star binary merger allows to tell in the experiment what type of source the gravitational waves are coming from. Thus, our findings in the gravitational wave experiment will contribute to astrophysical results.

A major goal of general relativity (GR) and astrophysics is to precisely describe and finally observe gravitational radiation, one of the predictions of GR. We know from the work [6] of Christodoulou that also these waves radiate. That is, in a laser interferometer gravitational wave detector, this will show in a permanent displacement of test masses after a wave train passed. The latter is known as the Christodoulou nonlinear memory effect. In

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[6] Christodoulou showed how the nonlinear memory effect can be measured as a permanent displacement of test masses in such a detector. He derived a precise formula for this permanent displacement in the Einstein vacuum (EV) case. The present authors proved in [3] that when electromagnetic fields are present, they will contribute to this nonlinear effect at highest order. In fact, we showed that for the EM equations this permanent displacement exhibits a term coming from the electromagnetic field, which is at the same highest order as the purely gravitational term that governs the EV situation. Moreover, we showed that the instantaneous displacement of the test masses is not changed at leading order by the electromagnetic field. To see this, we investigated spacetimes of solutions of the Einstein-Maxwell (EM) equations at null infinity.

Typical sources for gravitational waves are binary neutron star mergers and binary black hole mergers. As the former are known to be much more frequent, it is likely that gravitational waves as well as the nonlinear memory effect will first be measured from binary neutron star mergers. During such processes mass and momenta are radiated away. Moreover, large magnetic fields are produced and radiated away. The radiation travels at the speed of light. That means, it moves along null hypersurfaces of corresponding spacetimes. Therefore, in order to fully understand all the different situations, one has to investigate spacetimes which are solutions of the Einstein equations. Taking into account the strong magnetic fields which are generated during binary neutron star mergers, we consider spacetimes solving the Einstein-Maxwell equations. As the sources are very far away, we can think of us as doing the experiment at null infinity. Therefore it is very important to understand the geometry of spacetimes especially at null infinity, that is when we let $t \to \infty$ along null hypersurfaces in the corresponding spacetimes.

In this paper, we discuss the electromagnetic Christodoulou memory effect and compute concrete examples for binary neutron star mergers. In [3], we derived this effect in the regime of the EM equations. First, we recall the Bondi mass loss formula obtained in [15] for spacetimes solving the EM equations.

$$\frac{\partial}{\partial u} M\left(u\right) = \frac{1}{8\pi} \int_{S^2} \left(|\Xi|^2 + \frac{1}{2} \left|A_F\right|^2 \right) d\mu_{\overset{\circ}{\gamma}} \tag{1}$$

Compared to the formula obtained in [8] for spacetimes solving the EV equations, we have an additional term, $|A_F|^2$, from the electromagnetic field. (See [3].)

As shown in the work of Christodoulou [6], $\Sigma^+ - \Sigma^-$ is the term which governs the permanent displacement of test particles. Using this fact, Christodoulou shows that the gravitational field has a non-linear "memory" which can be detected by a gravitational-wave experiment in a spacetime solving the EV equations. Here, Σ denotes the asymptotic shear of outgoing null hypersurfaces C_u that are level sets of a foliation by an optical function u, which we will discuss below. Σ^+ and Σ^- are the limits of Σ as u tends to $+\infty$ respectively $-\infty$.

In our paper [3], we study the permanent displacement formula for uncharged test particles of the same gravitational-wave experiment in a spacetime solving the EM equations. We derive $\Sigma^+ - \Sigma^-$ in the EM case, and we find that the electromagnetic field changes the leading order term of the permanent displacement of test particles. Moreover, investigating the experiment for our setting in [3], we prove that the electromagnetic field does not enter the leading order term of the Jacobi equation. As a result, to leading order, it does not change the instantaneous displacement of test particles. But the electromagnetic field does contribute at highest order to the nonlinear effect of the permanent displacement of test masses.

To study the effect of gravitational waves, we follow the method introduced by Christodoulou in [6]. The analysis is based on the asymptotic behavior of the gravitational field obtained at null and spatial infinity. These rigorous asymptotics allow us to study the structure of the spacetimes at null infinity. To foliate the spacetime, we use a time function t and an optical function u. We denote the corresponding lapse functions by ϕ respectively a. Whereas each level set of t, H_t is a maximal spacelike hypersurface, each level set of u, C_u, is an outgoing null hypersurface. Along the null hypersurface C_u, we pick a suitable pair of normal vectors. The flow along these vector fields generates a family of diffeomorphisms ϕ_u of S^2. Using ϕ_u we pull back tensor fields in our spacetime. In this manner, we can study their limit at null infinity along the null hypersurface C_u. Building on these, we then take the limit as u goes to $\pm\infty$, which allows us to investigate the effect of gravitational waves. For a detailed explanation of the structure at null infinity, see [6] by Christodoulou.

Understanding gravitational radiation and therefore null infinity heavily relies on the rigorous understanding of the corresponding spacetimes. The methods introduced in [8], used in [14], [15] and [1], [2], reveal the structure of the null asymptotics of our spacetimes. In these works, stability results were proven. The authors showed that under a smallness condition on asymptotically flat initial data for the EV respectively EM equations, this can be extended uniquely to a smooth, globally hyperbolic and geodesically complete spacetime solving the EV respectively EM equations. The spacetime obtained is globally asymptotically flat. The main achievements are generally two-fold: First, existence and uniqueness theorems were proven. To ensure these, one has to impose smallness conditions. Second, precise descriptions of the asymptotic behavior of the spacetimes were derived. We stress the fact, that the results about null infinity are largely independent of the smallness. An elaborated geometric-analytic procedure led to these results. And many mathematical theorems were proven on the way. However, the outcome exhibits a physical result in point two from which Christodoulou in [6] derived the Christodoulou memory effect of gravitational waves in the EV case and the present authors in [3] in the EM case. In what follows, let us, discuss the new physical results and compute the effects for different binary neutron star constellations.

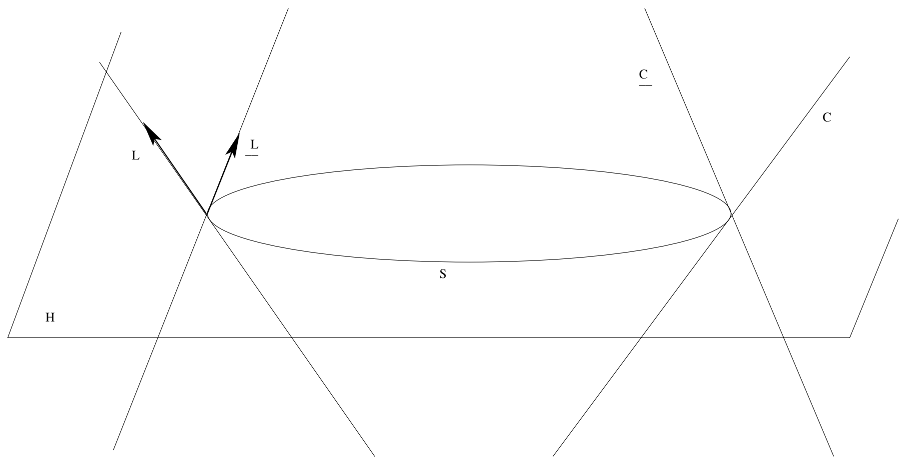
First we recall the Einstein-Maxwell equation. The electromagnetic field is represented by a skew-symmetric 2-tensor $F_{\mu\nu}$. The stress-energy tensor corresponding to $F_{\mu\nu}$ is

$$T_{\mu\nu} = \frac{1}{4\pi}\left(F_{\mu}{}^{\rho}F_{\nu\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)$$

The Einstein-Maxwell equations read:

$$\begin{aligned} R_{\mu\nu} &= 8\pi T_{\mu\nu} \\ D^{\alpha}F_{\alpha\beta} &= 0 \\ D^{\alpha}\, {}^{*}F_{\alpha\beta} &= 0. \end{aligned} \tag{2}$$

Let $S_{t,u}$ be the intersection of the hypersurface H_t and the null cone C_u. Let N be the spacelike unit normal vector of $S_{t,u}$ in H_t and T be the timelike unit normal vector of H_t in the spacetime. Let $\{e_a\}_{a=1,2}$ be an orthonormal frame on $S_{t,u}$. We have the following orthogonal frame (T, N, e_2, e_1). This also gives us a pair of null normal vectors to $S_{t,u}$, namely $L = T+N$ and $\underline{L} = T - N$. Together with $\{e_a\}_{a=1,2}$, they form a null frame. The following is a picture of the null cone C_u together with the null frame $(L, \underline{L}, e_2, e_1)$.



We can decompose the Weyl curvature tensor and the electromagnetic field with respect to the null frame or the orthogonal frame. The asymptotics of these components are studied in [14] and [15]. These asymptotics are important for the understanding of the geometry of null infinity. For simplicity, we will only list the components of the spacetime curvature and electromagnetic field that are used in our discussion. Please see [14] and [15] for more details on the asymptotics.

Let X, Y be arbitrary tangent vectors to S at a point in S. Given the null frame $e_4 = L$, $e_3 = \underline{L}$ and $\{e_a\}_{a=1,2}$, let $\chi(X,Y) = g(\nabla_X L, Y)$ and $\underline{\chi}(X,Y) = g(\nabla_X \underline{L}, Y)$ be the second fundamental forms with respect to L and $\underline{L}$, respectively. Let $\widehat{\chi}$ and $\widehat{\underline{\chi}}$ be their traceless parts. We also need the following null components of the Weyl curvature

$$\underline{\alpha}_W(X,Y) = R(X, \underline{L}, Y, \underline{L})$$

and the electromagnetic field

$$\begin{array}{ll} F_{A3} = \underline{\alpha}(F)_A & F_{A4} = \alpha\,(F)_A \\ F_{34} = 2\rho\,(F) & F_{12} = \sigma\,(F) \end{array} \tag{3}$$

We have the following limit of the above quantities at null infinity

$$\lim_{C_u, t\to\infty} r^2 \widehat{\chi} = \Sigma \ , \quad \lim_{C_u, t\to\infty} r\widehat{\underline{\chi}} = 2\Xi$$

$$\lim_{C_u, t\to\infty} r\underline{\alpha}_W = A_W \ , \quad \lim_{C_u, t\to\infty} r\underline{\alpha}_F = A_F$$

As shown in [6], the permanent displacement of the test masses of a laser interferometer gravitational-wave detector is governed by $\Sigma^+ - \Sigma^-$ where

$$\lim_{u\to\pm\infty} \Sigma = \Sigma^{\pm}$$

Theorem 1 *[14], [15] We have the following equations for Σ, Ξ and A_W*

$$\frac{\partial \Sigma}{\partial u} = -\Xi \quad \textit{and} \quad \frac{\partial \Xi}{\partial u} = -\frac{1}{4} A_W$$

In our paper [3], we prove that in a spacetime solving the Einstein–Maxwell equations, $\Sigma^+ - \Sigma^-$ is governed by the following relation.

Theorem 2 *[3] Let*

$$F(\cdot) = \int_{-\infty}^{\infty} (\mid \Xi(u,\cdot) \mid^2 + \frac{1}{2} \mid A_F(u,\cdot) \mid^2) du \quad . \tag{4}$$

Then $\Sigma^+ - \Sigma^-$ is given by the following equation on S^2:

$$\overset{\circ}{div} \ (\Sigma^+ - \Sigma^-) = \overset{\circ}{\nabla} \Phi \quad . \tag{5}$$

where Φ is the solution with $\bar{\Phi} = 0$ on S^2 of the equation

$$\overset{\circ}{\triangle} \Phi = F - \bar{F} \quad .$$

Comparing this with the EV case studied in the last chapter of [8] and used in [6], where the corresponding formula was $F(\cdot) = \int_{-\infty}^{\infty} \mid \Xi(u,\cdot) \mid^2 du$, we find that new the electromagnetic part $\frac{1}{2} \mid A_F(u,\cdot) \mid^2$ appears in the integral. In fact, in our proof, we derive the limiting formulas and obtain the said electromagnetic contribution in $\Sigma^+ - \Sigma^-$. (See [3].)

Gravitational Wave Experiment

How will our findings relate to experiment? In what follows, we are going to show how the electromagnetic field enters the experiment. In particular, we will discuss the instantaneous and the permanent displacement of test masses. For a detailed explanation of the experiment we refer to [6] and for a detailed derivation in the EM case we refer to [3].

Consider a laser interferometer gravitational-wave detector with three test masses. We denote the reference mass by m_0, this is also the location of the beam splitter. The masses m_0, m_1, m_2 are suspended by equal length pendulums of length d_0. The motion of the masses in the horizontal plane can be considered free for timelike scales much shorter than the period of the pendulums. Now one measures the distance of the masses m_1 and m_2 from the reference test mass m_0 by laser interferometry. We observe a difference of phase of the laser light at m_0 whenever the light travel times between m_0 and m_1, m_2, respectively, differ.

The motion of the masses m_0, m_1, m_2 is described by geodesics γ_0, γ_1, γ_2 in spacetime. Denote by T the unit future-directed tangent vectorfield of γ_0 and by t the arch length along γ_0. Let then H_t be for each t the spacelike, geodesic hyperplane through $\gamma_0(t)$ orthogonal to T. At $\gamma_0(0)$ pick an orthonormal frame (E_1, E_2, E_3) for H_0. By parallelly propagating it along γ_0, we obtain the orthonormal frame field (T, E_1, E_2, E_3) along γ_0, where at each t the (E_1, E_2, E_3) is an orthonormal frame for H_t at $\gamma_0(t)$. Then we can assign to a point p in spacetime close to γ_0 and lying in H_t the cylindrical normal coordinates (t, x^1, x^2, x^3).

Supoose that the distance d is much smaller than the time scale in which the curvature of the spacetime varies significantly. Then the geodesic deviation from γ_0, namely the Jacobi equation (6), replaces the geodesic equation for γ_1 and γ_2. Let $R_{k0l0} = R(E_k, T, E_l, T)$, then we write

$$\frac{d^2 x^k}{dt^2} = -R_{k0l0} x^l \tag{6}$$

We can decompose R_{k0l0} into the Weyl curvature and the Ricci curvature

$$R_{k0l0} = W_{k0l0} + \frac{1}{2}(g_{kl}R_{00} + g_{00}R_{kl} - g_{0l}R_{k0} - g_{0k}R_{l0}).$$

From the EM equations (2) we find

$$R_{00} = \frac{1}{2}(\mid \underline{\alpha}(F) \mid^2 + \mid \alpha(F) \mid^2) + \rho(F)^2 + \sigma(F)^2 \tag{7}$$

The component R_{00} observes the term $\mid \underline{\alpha}(F) \mid^2$, where $\underline{\alpha}(F)$ is the electromagnetic field component with worst decay behavior, but entering R_{00} as a quadratic. Hence, R_{00} is of the order $O(r^{-2})$. Whereas the leading order component of the Weyl curvature is of the order $O(r^{-1})$. We give a detailed proof in our paper [3]. Thus, the electromagnetic field does not contribute at highest order to the deviation measured by the Jacobi equation. As a consequence, it does only change at lower order the instantaneous displacement of the test masses. However, we are going to see that it does change the nonlinear memory effect.

Using the relations from theorem 1 and our theorem 2 as well as the fact that $\Xi \to 0$ for $u \to \infty$ and taking the limit $t \to \infty$, we conclude that the test masses experience permanent displacements after the passage of a wave train. In particular, this overall displacement of the test masses is described by $\Sigma^+ - \Sigma^-$

$$\Delta x^A_{(B)} = -\frac{d_0}{r}(\Sigma^+_{AB} - \Sigma^-_{AB}) \tag{8}$$

where from our theorem 2 one sees that the right hand side of (8) includes the electromagnetic field terms at highest order.

Let us now derive formula (8). We will use $L = T - E_3$ and $\underline{L} = T + E_3$. Then we write the leading components of the curvature $\underline{\alpha}_{AB}(W)$ and of the electromagnetic field $\underline{\alpha}_A(F)$ as follows:

$$\begin{aligned} \underline{\alpha}_{AB}(W) &= R\ (E_A,\ \underline{L},\ E_B,\ \underline{L}) = \frac{A_{AB}(W)}{r} + o\ (r^{-2}) \\ \underline{\alpha}_A(F) &= F(E_A, \underline{L}) = \frac{A_A(F)}{r} + o\ (r^{-2}) \end{aligned}$$

Let $x^k_{(A)}$ with $A = 1, 2$ denote the k^{th} Cartesian coordinate of the mass m_A. From [6] and [3] one sees that there is no acceleration to leading order in the vertical direction. One starts with m_1, m_2 being at rest at equal distance d_0 from m_0 at right angles from m_0. Thus to leading order it is

$$\ddot{x}^A_{(B)} = -\frac{1}{4}r^{-1}d_0 A_{AB} \tag{9}$$

In particular, the initial conditions are as $t \to -\infty$:
$x^B{}_{(A)} = d_0\delta^B_A$, $\dot{x}^B{}_{(A)} = 0$, $x^3{}_{(A)} = 0$, $\dot{x}^3{}_{(A)} = 0$.

Integrating gives

$$\dot{x}^A{}_{(B)}\ (t) = -\ \frac{1}{4}\ d_0\ r^{-1} \int_{-\infty}^{t} A_{AB}\ (u)\ du\ . \tag{10}$$

From theorem 1 equation $\frac{\partial \Xi}{\partial u} = -\frac{1}{4}A_W$ and $lim_{|u|\to\infty}\Xi = 0$, one substitutes and concludes

$$\dot{x}^A{}_{(B)}\ (t) = \frac{d_0}{r}\ \Xi_{AB}\ (t)\ . \tag{11}$$

As $\Xi \to 0$ for $u \to \infty$, the test masses return to rest after the passage of the gravitational waves. Now, we use theorem 1 equation $\frac{\partial \Sigma}{\partial u} = -\Xi$ and integrate again to obtain

$$x^A_{(B)}\ (t)\ =\ -\ (\frac{d_0}{r})\ (\Sigma_{AB}\ (t)\ -\ \Sigma^-)\ . \tag{12}$$

Finally, by taking the limit $t \to \infty$ one derives that the test masses obey permanent displacements. This means that $\Sigma^+ - \Sigma^-$ is equivalent to an overall displacement of the test masses given by (8):

$$\triangle\ x^A_{(B)}\ =\ -\ (\frac{d_0}{r})\ (\Sigma^+_{AB}\ -\ \Sigma^-_{AB})\ .$$

The right hand side of (8) includes terms from the electromagnetic field at highest order as given in our theorem 2.

In the next subsection, we are going to apply our results to astrophysical data for binary neutron star mergers.

Binary neutron star mergers

We compute the electromagnetic Christodoulou memory effect for typical sources, that is for different constellations of binary neutron star (BNS) mergers.

In a binary neutron star or binary black hole system, the two objects are orbiting each other. In Newtonian physics, they would stay like that forever. However, according to the theory of general relativity such a system must radiate away energy. Therefore, the radius of the orbits must shrink and finally the objects will merge.

As binary neutron star systems are much more frequent than binary black hole systems, it is very likely that gravitational waves as well as the nonlinear memory effect of gravitational waves will be detected first from the former systems. The magnetic fields produced and radiated away during the merger of two neutron stars are among the largest magnetic fields known in astrophysics. In fact, in the electromagnetic Christodoulou memory effect that we derived, the magnetic field enlarges the nonlinear displacement of (non-charged) test masses significantly. As we are going to show in this subsection, the contribution from the magnetic field is very important, as it is very big for a large part of the known constellations.

Astrophysical data gives for typical neutron star binaries a range of possible constellations which allow the mass and the magnetic field to vary within given boundaries. Typically, the mass of a neutron star is around slightly more than $1M_\odot$ and the radius of a neutron star is 3 - 30 km. Thus, the typical mass for a BNS system ranges between 2.6 and $2.8M_\odot$ [1]. In such a system, as the neutron stars are spiraling around each other, they are radiating away gravitational and magnetic energy. The inspiral goes with increasing speed and the BNS system emits an increasing amount of electromagnetic and gravitational energy, which becomes extremely large when the orbit radius is about 10 - 100km. For the detection of the electromagnetic Christodoulou effect, the largest contribution will come from the last phase of the inspiral, starting when the orbit radius is about 10 times the neutron star radius. In the literature, we find that the merger times range from a few milli-seconds up to 1000 ms. We would like to compare the amount of gravitational energy radiated away during the

[1] $M_\odot\ = 1$ solar mass $\approx 1.9891 \cdot 10^{33}$ g

merger to the amount of magnetic energy radiated away. On the one hand, the amount of gravitational energy radiated away is well-known. In general, about 1% of the initial mass is radiated away during a merger. This is about 10^{52} erg.[2] On the other hand, the amount of magnetic energy radiated away could vary drastically depending on different constellations. Typically, the rate of change for the magnetic field is $\frac{dB}{dt} \approx 10^9 - 10^{17}\,\mathrm{G(ms)}^{-1}$ and the magnetic field produced in the merger is about $10^9 - 10^{17}\,\mathrm{G}$.[3]

Comparing the energy from the radiated mass, i.e. purely gravitational, and from the magnetic field, we observe that during the merger of BNS very large magnetic fields are produced and radiated away in certain scenarios.

Consider the following data. Assume: Total mass of BNS is initially $2M_\odot$, 1% of the total mass will be radiated away during the (whole) merger, radius of each neutron star is 10 km. Under the assumption, the gravitational energy radiated away is about 3.56×10^{52} erg.

In the physics literature, one finds many linearized models. However, the Einstein equations being nonlinear, the main information usually gets lost in linearized models. As we do investigate the nonlinear problem here, and as the results of [6] and [3] show the Christodoulou memory effect of gravitational waves to be a nonlinear phenomenon, we consider a corresponding nonlinear model for the neutron star binary mergers. Thus, we use the results of Zipser's global stability work [14] and [15] for the initial value problem in spacetimes satisfying the Einstein-Maxwell equations. We assume that outside the neutron star, the magnetic field decays like $r^{-5/2}$. Such decay at spatial infinity is suggested by the decay obtained in [14], [15]. One might want to consider situations with a slightly different decay of the magnetic field. This would not affect the main picture, as one finds during the computations that the decay of the magnetic field does not play a role here. Thus, we work with the nonlinear model explained in the following paragraph.

Now, consider such a BNS system with the magnetic field B initially being $B = 10^{13}$ G and $\frac{dB}{dt} = 10^{13}\,\mathrm{G(ms)}^{-1}$ on the surface of the neutron star. Assume that the merger time is 1000 ms. We estimate the total magnetic energy radiated away using the following model. We assume that through the merger, the matter of the neutron star stays in a ball of radius 10 km. We compute the contribution from the magnetic field outside the support of the matter of the neutron star. As a result, we simply use the vacuum magnetic constant when computing the magnetic energy density. Moreover, we assume that outside the neutron star, the magnetic field decays at the rate of $r^{-5/2}$. Using this model, the energy radiated away from the magnetic field is about $4.78 \cdot 10^{49}$ erg. In this case, the addition of a magnetic field has a small contribution to the memory effect.

Next, consider a BNS system with the above data, but where the magnetic field B is initially $B = 10^{15}\,\mathrm{G}$ and $\frac{dB}{dt} = 10^{15}\,\mathrm{G(ms)}^{-1}$ on the surface of the neutron star. Assume that the merger time is 1000 ms. We compute that the total magnetic energy radiated away is about $4.78 \cdot 10^{53}$ erg. This will be one order of magnitude higher than the gravitational energy radiated away. This situation is consistent with astrophysical data. Also, in the numeric simulation in [9], [10] and [11], it is observed that the magnetic field could increase by two orders of magnitude during merger when one starts with magnetic fields around 10^9 to 10^{12} G. When we start with a stronger magnetic field, the merger would take longer and allow

[2] 1 erg $= 1\mathrm{g} \cdot \mathrm{cm}^2\mathrm{s}^{-2}$ and $1M_\odot \approx 1.78 \cdot 10^{54}$ erg
[3] 1 Gauss: $1G = 10^{-4}\,\mathrm{kg} \cdot \mathrm{C}^{-1}\mathrm{s}^{-1} = 10^{-1}\,\mathrm{g} \cdot \mathrm{C}^{-1}\mathrm{s}^{-1}$

more time for the magnetic field to build up. Magnetic fields of similar initial strength are used in the simulation of [12]. Their simulations suggest that the addition of the magnetic field cause observable differences in the dynamics and gravitational waveforms. Moreover, it is noticed that the most important role of magnetic fields are on the long term evolution. This is similar to our conclusion from theorem 1 and 2. Namely, the addition of a magnetic field does not change the system instantaneously but it does contribute to the nonlinear long-term permanent change.

To compare, note that the amount of energy emitted in a binary black hole merger is expected to be as follows: a binary black hole system with equal mass and no spin would lose about 4% of the mass during merger. However, BNS mergers occur more often than black hole mergers.

Conclusions: We find that among the variety of different constellations of BNS systems there is a large part for which the magnetic field contributes to the Christodoulou effect at the same highest order as the purely gravitational term.

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