

NON-MINIMAL HIGHER-SPIN DS_4/CFT_3

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Abstract

We conjecture that the level k $U(N)$ Chern-Simons theory coupled to free anticommuting scalar matter in the fundamental is dual to non-minimal higher-spin Vasiliev gravity in dS_4 with parity-violating phase $\theta_0 = \frac{\pi N}{2k}$ and Neumann boundary conditions for the scalar. Related conjectures are made for fundamental commuting spinor matter and critical theories. This generalizes a recent conjecture relating the minimal Type A Vasiliev theory in dS_4 to the $Sp(N)$ model with fundamental real anti-commuting scalars.

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1 Introduction and Summary

The dS/CFT conjecture posits that that quantum gravity on de Sitter space (dS) is holographically dual to a conformal field theory (CFT) living on the spacelike boundary of dS at future infinity [1, 2, 3]. Development of this conjecture was hindered by the absence of any concrete example. Recently, an example was proposed: Vasiliev type A theory in dS_4 is conjecturally dual to the $Sp(N)$ CFT_3 with anti-commuting scalars [4]. This opened the door to detailed investigations of the structure of higher-spin dS holography *e.g.*[23]-[31].

Vasiliev actually constructed families of classical higher-spin gravity theories in dS_4 labelled by a parity-violating phase θ_0 and, at the quantum level, a loop counting

parameter g_{dS}^2 . In this paper we generalize the construction of [4] and propose duals for this family of theories. They are level k $U(N)$ Chern-Simons theories coupled to wrong-statistics scalars or spinors with $N = g_{dS}^{-2}$ and 't Hooft coupling $\lambda = \frac{N}{k}$ a certain real function of θ_0 . We rely heavily on the analysis of [15, 16, 19] in which the analogous question was addressed in AdS_4 . The dualities are displayed in table 1.

The existence of such dualities was anticipated in [23, 26].¹ Because the parity-violating Vasiliev theories do not exist in Euclidean signature, our construction necessarily involves an $\text{AdS}_4 \rightarrow \text{dS}_4$ continuation, in contrast to the $\text{EAdS}_4 \rightarrow \text{dS}_4$ continuation employed in [4].

Table 1: **CONJECTURED dS_4/CFT_3 DUALITIES**

VASILIEV dS_4 THEORY	BOUNDARY CFT_3	SPECTRUM
Non-minimal; $\theta_0 = \frac{\pi}{2}\lambda$ Neumann scalar	$U(N)_k$ Chern-Simons; free anticommuting scalar	All integer spins
Non-minimal; $\theta_0 = \frac{\pi}{2}(1 - \lambda)$ Dirichlet scalar	$U(N)_k$ Chern-Simons; free commuting spinor	All integer spins
Non-Minimal; $\theta_0 = 0$ Neumann scalar	$U(N)_\infty$ Chern-Simons free anticommuting scalar	All integer spins
Non-minimal; $\theta_0 = \frac{\pi}{2}$ Dirichlet scalar	$U(N)_\infty$ Chern-Simons; free commuting spinor	All integer spins
Minimal; $\theta_0 = \frac{\pi}{2}\lambda$ Neumann scalar	$Sp(N)_k$ Chern-Simons; free anticommuting scalar	Even spins
Minimal; $\theta_0 = \frac{\pi}{2}(1 - \lambda)$ Dirichlet scalar	$Sp(N)_k$ Chern-Simons; free commuting spinor	Even spins
Minimal $\theta_0 = 0$ Neumann scalar	$Sp(N)_\infty$ Chern-Simons; free anticommuting scalar	Even spins
Minimal; $\theta_0 = \frac{\pi}{2}$ Dirichlet scalar	$Sp(N)_\infty$ Chern-Simons; free commuting spinor	Even spins

$g_{dS}^2 = \frac{1}{N}$; $\lambda = \frac{N}{k}$; Dirichlet (Neuman) bulk scalars have $\Delta = 2$ ($\Delta = 1$).

Free (critical) anticommuting scalars are dual to critical (free) commuting spinors.

The outline of this paper is as follows. In Section 2, we review the Chern-Simons theories coupled to bosonic scalar or fermionic spinor matter in the fundamental of $U(N)$. We also discuss the statistics-reversed versions of these theories, namely Chern-Simons coupled to fundamental anticommuting scalar or commuting spinor matter, as well as Wick rotation from Minkowski to Euclidean space. In Section 3, we review the parity-violating Vasiliev theories in the AdS_4 and dS_4 vacua. In section 4, we present

¹The complexification of k considered in [26] is not realized in our construction.

an analytic continuation that relates them. In particular, we show how the n -point correlation functions in AdS_4 and in dS_4 are related by this analytic continuation. In section 5, higher-spin bulk duals are conjectured for the various wrong-statistics Chern-Simons-matter theories. Formulae are given relating the bulk coupling constants and boundary conditions to the boundary level, gauge group and interactions. Spinor conventions are in the appendix.

2 $U(N)$ Chern-Simons theories

In this section, we briefly review the Chern-Simons scalar and Chern-Simons fermion theories in three dimensions, which are conjectured to be dual to the parity-violating Vasiliev theories in AdS_4 [16, 19]. We also discuss the statistics reversed versions of these theories which are conjectured, in Section 4, to be dual to the parity-violating Vasiliev theories in dS_4 .

2.1 Spinors

The action for a complex anticommuting fermion ψ in the fundamental representation of $U(N)$ coupled to a gauge field A_i with a level k Chern-Simons interaction in three Lorentzian dimensions is

$$S = \frac{k}{4\pi} \int \text{tr} \left(AdA + \frac{2}{3} A^3 \right) + \int d^3x \bar{\psi} \gamma^i D_i \psi. \quad (2.1)$$

According to [16], the spectrum of primary operators in this theory consists of a spin- s single trace operator $J^{(s)}$ for each $s > 0$, which take the schematic form

$$J_{i_1 \dots i_s}^{(s)} = i^s \bar{\psi} \gamma_{i_1} D_{i_2} \dots D_{i_s} \psi + \dots. \quad (2.2)$$

$J_{i_1 \dots i_s}^{(s)}$ are almost conserved, in the sense that the violation of current conservation is suppressed by a power of $1/N$, and the conformal dimensions of these operators are given by the unitarity bound up to $1/N$ corrections, i.e. $\Delta = s + 1 + \mathcal{O}(1/N)$. In addition to the spin- s primary operators, there is a spin zero primary operator:

$$J^{(0)} = \bar{\psi} \psi, \quad (2.3)$$

with conformal dimension $\Delta = 2 + \mathcal{O}(1/N)$. All other primaries are products of these “single-trace” operators. This theory is conjectured by [16] to be dual to a Vasiliev theory with parity-violating phase θ_0 .² The higher-spin currents $J^{(s)}$ are dual to the

²We review parity-violating Vasiliev theories in section 3.

higher-spin gauge fields in the bulk, and the spin zero operator $J^{(0)}$ is dual to a bulk scalar with $\Delta = 2$ (Dirichlet) boundary condition. The 't Hooft coupling

$$\lambda = \frac{N}{k} \tag{2.4}$$

is mapped by the duality to the parity-violating phase θ_0 by

$$\theta_0 = \frac{\pi}{2}(1 - \lambda). \tag{2.5}$$

The planar three-point functions for the spin-0, 1 currents have been computed in [21], which exactly match with the tree level correlation functions in the bulk Vasiliev theory. In general, the N dependence of a Feynman diagram is given by N^{2-2g-h} , where h is the number of fermion loops, and g is the genus of the diagram. More explicitly, the n -point function takes the form as $N^{2-2g-h} f_{g,h}(\lambda, x_k^i)$, where x_k^i for $k = 1, \dots, n$ are the positions of the n points.

Now consider the same theory, but with opposite statistics for the spinors. The action for the theory is

$$S = \frac{k}{4\pi} \int \text{tr} \left(AdA + \frac{2}{3} A^3 \right) + \int d^3x \bar{\xi} \gamma^i D_i \xi, \tag{2.6}$$

where ξ is a *commuting* Dirac spinor in the fundamental representation of $U(N)$. In the 't Hooft large- N limit, the spectrum of single-trace primary operators contains the spin- s operators $J^{(s)}$ for each $s \geq 0$. These take the schematic form:

$$J_{i_1 \dots i_s}^{(s)} = i^s \bar{\xi} \gamma_{i_1} D_{i_2} \dots D_{i_s} \xi + \dots, \text{ for } s > 0, \text{ and } J^{(0)} = \bar{\xi} \xi. \tag{2.7}$$

By the same argument as in [16], these spin- s operators are almost conserved and have an anomalous dimension of order $1/N$. The correlation functions of these operators can be computed by the exact same diagrams as in the theory (2.1) with the anticommuting spinors. The only change is that there is an extra minus sign associated with every independent matter loop by Bose statistics. As a result, the correlation functions with h matter loops at genus g take the form $(-1)^h N^{2-2g-h} f_{g,h}(\lambda, x_k^i)$, where $f_{g,h}(\lambda, x_k^i)$ is the same function as in the theory with anticommuting spinors. So to obtain the current correlation functions in the reversed statistics theory, we simply have to flip the sign of N , while keeping λ fixed.

2.2 Scalars

The Lorentzian action for a three-dimensional complex scalar ϕ in the fundamental representation of $U(N)$ coupled to a gauge field A_i with a Chern-Simons interaction at

level k is

$$S = \frac{k}{4\pi} \int \text{tr} \left(AdA + \frac{2}{3}A^3 \right) + \int d^3x \left(|D_i\phi|^2 + \frac{\lambda_6}{3!N^2}(\phi^\dagger\phi)^3 \right), \quad (2.8)$$

where $D_i = \partial_i + A_i$, and $k \in \mathbb{Z}$. We are interested in the 't Hooft large- N limit, keeping $\lambda = \frac{N}{k}$ and λ_6 fixed. According to [19], conformality constrains the parameter λ_6 to be a function of λ . The spectrum of operators in the theory includes a single primary operator for each integer spin $s \geq 0$. Each $J^{(s)}$ can be written as a symmetric, traceless tensor that is schematically given by

$$J_{i_1 \dots i_s}^{(s)} = i^s \phi^\dagger D_{i_1} \dots D_{i_s} \phi + \dots \quad (2.9)$$

As in the fermion case, the $J_{i_1 \dots i_s}^{(s)}$ are almost conserved currents. This theory is conjectured [19] to be dual to a parity-violating Vasiliev theory with $\Delta = 1$ (Neumann) boundary condition for the bulk scalar, and the higher-spin operators $J^{(s)}$ are dual to the higher-spin gauge fields in the bulk. The planar three-point functions for spin-0, 1, 2 currents have been computed in [20], which exactly match with the tree level correlation functions in the bulk Vasiliev theory with $\theta_0 = \frac{\pi}{2}\lambda$. As in the previous subsection, we want to have a formula for the N dependence of general Feynman diagrams. For this purpose, it is convenient to introduce the auxiliary fields D and σ . The equivalent action with the auxiliary fields is given by

$$S = \frac{k}{4\pi} \int \text{tr} \left(AdA + \frac{2}{3}A^3 \right) + \int d^3x \left(|D_i\phi|^2 + \phi^\dagger(\sigma^2 - D)\phi + \sqrt{\frac{3!N^2}{\lambda_6}} \text{tr}(D\sigma) \right). \quad (2.10)$$

In this form it is evident that the N dependence of the Feynman diagrams with h matter loops at genus g is given by N^{2-2g-h} .

Now consider the same theory, but with opposite statistics for the scalar field. The action for the theory is

$$S = \frac{k}{4\pi} \int \text{tr} \left(AdA + \frac{2}{3}A^3 \right) + \int d^3x \left(|D_i\chi|^2 + \frac{\lambda_6}{3!N^2}(\chi^\dagger\chi)^3 \right), \quad (2.11)$$

where χ is an anticommuting scalar in the fundamental representation of $U(N)$. In the 't Hooft large- N limit, the spectrum of single-trace primary operators contains the spin- s operators $J^{(s)}$ for each $s \geq 0$. These take the schematic form:

$$J_{i_1 \dots i_s}^{(s)} = i^s \chi^\dagger D_{i_1} \dots D_{i_s} \chi + \dots \quad (2.12)$$

By the same argument as in [19], these spin- s operators are almost conserved and have an anomalous dimension of order $1/N$. The correlation functions of these operators can be computed by the exact same diagrams as in the theory with the commuting scalar. The only change is that there is an extra minus sign associated with every matter loop by Fermi statistics. The net effect is to flip the sign of N while keeping λ fixed.

2.3 Wick rotation

The future boundary of dS_4 has Euclidean signature, so we are interested in Euclidean CFT_{3s} . Let us consider the analytic continuation of the statistics reversed $U(N)$ Chern-Simons spinor and Chern-Simons scalar theories from Lorentzian signature to Euclidean signature. This can easily be done by an analytic continuation of the coordinate of the function $f_{g,h}(\lambda, x_a^i)$ from x_a^0 to $-ix_a^3$. More explicitly, the analytic continuation of the higher-spin currents are

$$J_{i_1 \dots i_s}^{(s),E} = i^n J_{i_1 \dots i_s}^{(s),L} \Big|_{x^0 \rightarrow -ix^3}, \quad (2.13)$$

where the superscripts L, E distinguish the operators in Lorentzian or Euclidean signature, and n is the number of the indices of $J_{i_1 \dots i_s}^{(s),L}$ that are 0. It is convenient to define generating functions

$$\begin{aligned} J_L^{(s)}(x|y) &= \sum_{i_1, \dots, i_s} J_{i_1 \dots i_s}^{(s),L}(y\sigma_L^{i_1}y) \cdots (y\sigma_L^{i_s}y), \\ J_E^{(s)}(x|y) &= \sum_{i_1, \dots, i_s} J_{i_1 \dots i_s}^{(s),E}(y\sigma_E^{i_1}y) \cdots (y\sigma_E^{i_s}y), \end{aligned} \quad (2.14)$$

where y^α is an auxiliary bosonic spinor variable, $y\sigma_L^i y = y_\alpha(\sigma_L^i)^\alpha_\beta y^\beta$, $y\sigma_E^i y = y_\alpha(\sigma_E^i)^\alpha_\beta y^\beta$, $(\sigma_E^i)^\alpha_\beta = (\sigma^1, \sigma^3, \sigma^2)$ are Pauli matrices, and $(\sigma_L^i)^\alpha_\beta = (\sigma^1, \sigma^3, i\sigma^2)$. In terms of the generating functions, the analytic continuation of the higher-spin currents can be simply stated as

$$J_E^{(s)}(x|y) = J_L^{(s)}(x|y) \Big|_{x^0 \rightarrow -ix^3}, \quad (2.15)$$

which accounts for the i^n in (2.13). The analytic continuation of the correlators is simply

$$\left\langle J_E^{(s_1)}(x_1|y) \cdots J_E^{(s_n)}(x_n|y) \right\rangle = \left\langle J_L^{(s_1)}(x_1|y) \cdots J_L^{(s_n)}(x_n|y) \right\rangle \Big|_{x_k^0 \rightarrow -ix_k^3}. \quad (2.16)$$

3 Vasiliev theories in AdS_4 and dS_4

In this section, we review the Vasiliev theory in AdS_4 and dS_4 backgrounds [5, 6, 7, 18, 4]. We will start with a background independent formalism, and then specify the vacuum solutions and reality conditions. The fields in the Vasiliev theory are functions of bosonic variables $(x, Y, Z) = (x^\mu, y^\alpha, z^\alpha, \bar{y}^{\dot{\alpha}}, \bar{z}^{\dot{\alpha}})$. Here x^μ are an arbitrary set of coordinates on the four dimensional spacetime manifold with signature $(-, +, +, +)$. $(y^\alpha, z^\alpha, \bar{y}^{\dot{\alpha}}, \bar{z}^{\dot{\alpha}})$ are commuting $SO(1, 3)$ spinors. Our spinor conventions for AdS_4 and dS_4 are given in the Appendix A. The Vasiliev master fields consist of an x -space 1-form

$$W = W_\mu dx^\mu, \quad (3.1)$$

a Z -space 1-form

$$S = S_\alpha dz^\alpha + S_{\dot{\alpha}} d\bar{z}^{\dot{\alpha}}, \quad (3.2)$$

and a scalar B , all of which depend on all the bosonic variables introduced above. The master fields are truncated by the condition

$$\pi\bar{\pi}(W) = W, \quad \pi\bar{\pi}(S) = S, \quad \pi\bar{\pi}(B) = B, \quad (3.3)$$

where the π -action is defined as $\pi : (y, z, dz, \bar{y}, \bar{z}, d\bar{z}) \mapsto (-y, -z, -dz, \bar{y}, \bar{z}, d\bar{z})$, and the $\bar{\pi}$ -action is given by the π -action with exchanging the barred and unbarred variables. It is easy to check that the equation of motion is consistent with the truncation (3.3).

The master field equation of motion is [18, 17]:

$$d_x \hat{\mathcal{A}} + \hat{\mathcal{A}} * \hat{\mathcal{A}} = \left(\frac{1}{4} + B * K e^{i\theta_0} \right) dz^2 + \left(\frac{1}{4} + B * \bar{K} e^{-i\theta_0} \right) d\bar{z}^2, \quad (3.4)$$

where $\hat{\mathcal{A}} = W + S - \frac{1}{2}zdz$ and $K = e^{zy}$, $\bar{K} = e^{\bar{z}\bar{y}}$ and θ_0 is a coupling constant and d_x is the exterior derivative with respect to spacetime coordinate x^μ . Here the Vasiliev's *-product is defined by

$$f * g = f(Y, Z) \exp \left[\epsilon^{\alpha\beta} \left(\overleftarrow{\partial}_{y^\alpha} + \overleftarrow{\partial}_{z^\alpha} \right) \left(\overrightarrow{\partial}_{y^\beta} - \overrightarrow{\partial}_{z^\beta} \right) + \epsilon^{\dot{\alpha}\dot{\beta}} \left(\overleftarrow{\partial}_{y^{\dot{\alpha}}} + \overleftarrow{\partial}_{z^{\dot{\alpha}}} \right) \left(\overrightarrow{\partial}_{y^{\dot{\beta}}} - \overrightarrow{\partial}_{z^{\dot{\beta}}} \right) \right] g(Y, Z). \quad (3.5)$$

In the parity invariant A-type and B-type theories, θ_0 takes the values $\theta_0 = 0$ and $\theta_0 = \frac{\pi}{2}$, respectively. Parity is not conserved for generic θ_0 . In addition to θ_0 , the quantum Vasiliev theory has an additional coupling constant g which measures the strength of quantum corrections. For the Vasiliev theory on AdS_4 and dS_4 background, we will denote this coupling as g_{AdS} or g_{dS} , respectively.

The Vasiliev master fields are, a priori, complex-valued fields. There are several different consistent reality conditions that can be imposed on the master fields. Different reality conditions preserve different vacuum solutions. In the following two subsections, we review the Vasiliev theory on AdS_4 and dS_4 backgrounds, and specify the reality conditions that preserve these two backgrounds.

3.1 AdS_4

Let us consider the Vasiliev theory with the spacetime signature $(+, +, +, -)$, with coordinates denoted $x^\mu = (z, x^1, x^2, x^0)$. The AdS_4 vacuum solution is

$$W = W_0 = \omega_0(x|Y) + e_0(x|Y), \quad B = 0, \quad S = 0, \quad (3.6)$$

where

$$\begin{aligned}\omega_0(x|Y) &= -\frac{1}{8} \frac{dx_i}{z} (y\sigma_{AdS}^{iz}y + \bar{y}\sigma_{AdS}^{iz}\bar{y}), \\ e_0(x|Y) &= -\frac{1}{4} \frac{dx_\mu}{z} y\sigma_{AdS}^\mu\bar{y},\end{aligned}\tag{3.7}$$

and the σ -matrices are defined in the (A.2). The metric or vielbein are not fundamental quantities in Vasiliev theory. They can be extracted from the vacuum solution W_0 . The vielbein e_{AdS}^a can be extracted from $e_0(x|Y)$ by

$$e_0(x|Y) = -\frac{1}{4}\eta_{ab}e_{AdS}^a(y\sigma_{AdS}^b\bar{y}),\tag{3.8}$$

and the AdS_4 metric is then given by $g_{\mu\nu}^{AdS}dx^\mu dx^\nu = \eta_{ab}e_{AdS}^a e_{AdS}^b = (\eta_{\mu\nu}/z^2)dx^\mu dx^\nu$.

The reality condition on the Vasiliev's master fields that preserves the AdS_4 vacuum solution is³

$$-\iota(\hat{\mathcal{A}})^* = \hat{\mathcal{A}}, \quad \iota\pi(B)^* = B,\tag{3.9}$$

where the reality condition on the auxiliary variables (Y, Z) are defined in appendix A, and the ι -action is defined as $\iota : (y, \bar{y}, z, \bar{z}, dz, d\bar{z}) \mapsto (iy, i\bar{y}, -iz, -i\bar{z}, -idz, -id\bar{z})$. It follows that the ι -action would reverse the $*$ -product, i.e.

$$\iota(f(Y, Z) * g(Y, Z)) = \iota(g(Y, Z)) * \iota(f(Y, Z)).\tag{3.10}$$

At the linear level, after an appropriate gauge fixing and eliminations of the auxiliary fields, the Vasiliev's equation of motion on the background (3.1) reduces to the Fronsdal's equation of motion [11, 13, 14, 15]. The Fronsdal equation in the traceless gauge is,

$$-(\square - m^2)\varphi_{\mu_1\cdots\mu_s}^{AdS} + s\nabla_{(\mu_1}\nabla^{\nu}\varphi_{\mu_2\cdots\mu_s)\nu}^{AdS} - \frac{s(s-1)}{2(d+2s-3)}g_{(\mu_1\mu_2}^{AdS}\nabla^{\nu_1}\nabla^{\nu_2}\varphi_{\mu_3\cdots\mu_s)\nu_1\nu_2}^{AdS} = 0,\tag{3.11}$$

where $m^2 = s(s-2) - 2$, and $\varphi_{\mu_1\cdots\mu_s}^{AdS}$ is traceless symmetric spin- s gauge field. It appears in the components of the Vasiliev master fields W, B . More explicitly, the spin- s higher-spin gauge field $\varphi_{\mu_1\mu_2\cdots\mu_s}^{AdS}$ is an expansion coefficient of the master field W (equation (3.59) in [14])

$$W(x, Y, Z = 0)\Big|_{y^{s-1}, \bar{y}^{s-1}} \propto (iz)^{s-1}\varphi_{\mu_1\mu_2\cdots\mu_s}^{AdS}(y\sigma_{AdS}^{\mu_2}\bar{y}) \cdots (y\sigma_{AdS}^{\mu_s}\bar{y}).\tag{3.12}$$

The spin zero field φ_{AdS} sits in the (Y, Z) independent part of master field B as

$$B(x|Y, Z)\Big|_{Y=Z=0} = \varphi_{AdS}.\tag{3.13}$$

³The $\iota(W)^*$ is to be understood as first acting the ι on W then taking the complex conjugate.

At the nonlinear level, one can, in principle, extract the corrections to the right hand side of the linear equation (3.11) from the Vasiliev equation order by order in the number of higher spin gauge fields. A systematic procedure for this was discussed in [9, 14].

The reality condition (3.9) for the master fields, hence, gives the reality condition on the physical higher-spin gauge fields. More explicitly, by imposing the reality condition on equation (3.12), we find

$$(\varphi_{\mu_1 \dots \mu_s}^{AdS})^* = \varphi_{\mu_1 \dots \mu_s}^{AdS}. \quad (3.14)$$

The scalar field, on the other hand, is the bottom component of B according to (3.13). Imposing the reality condition on (3.13) gives $\varphi_{AdS}^* = \varphi_{AdS}$ for the spin-0 field.

The scalar has mass square $m^2 = -2$. Depending on the boundary condition for this scalar, its dual operator has either dimension $\Delta = 1$ or $\Delta = 2$, classically. We will refer to the two different boundary conditions as $\Delta = 1$ (Neuman) and $\Delta = 2$ (Dirichlet) boundary conditions, respectively.

3.2 dS₄

In dS₄, we label the coordinates by (η, x^1, x^2, x^3) with the signature $(-, +, +, +)$. The dS₄ vacuum solution to Vasiliev's equation of motion is given by

$$W = W_0 = \omega_0(x|Y) + e_0(x|Y), \quad B = 0, \quad S = 0, \quad (3.15)$$

and

$$\begin{aligned} \omega_0(x|Y) &= -\frac{1}{8} \frac{dx_i}{\eta} (y \sigma_{dS}^{in} y + \bar{y} \sigma_{dS}^{in} \bar{y}), \\ e_0(x|Y) &= -\frac{i}{4} \frac{dx_\mu}{\eta} y \sigma_{dS}^\mu \bar{y}, \end{aligned} \quad (3.16)$$

where the $\sigma_{dS}^\mu, \sigma_{dS}^{in}$ are the σ -matrices defined in (A.5). The vielbein e_{dS}^a is extracted from $e_0(x|Y)$ according to

$$e_0(x|Y) = -\frac{1}{4} \eta_{ab}^{dS} e_{dS}^a (y \sigma_{dS}^b \bar{y}). \quad (3.17)$$

The metric is then $g_{\mu\nu}^{dS} dx^\mu dx^\nu = \eta_{ab} e_{dS}^a e_{dS}^b = -(\eta_{\mu\nu}/\eta^2) dx^\mu dx^\nu$. A reality condition on the Vasiliev's master fields that preserves the dS₄ vacuum solution (3.16) is

$$\pi(\hat{\mathcal{A}})^* = \hat{\mathcal{A}}, \quad \pi(B)^* = B, \quad (3.18)$$

which is also compatible with the equation of motion (3.1) and truncation (3.3).⁴

⁴This reality condition agrees with [18] when reduced to the minimal theory.

As in the AdS_4 case, the linearized Vasiliev equation of motion on the dS_4 background (3.16) is reduced to the Fronsdal equation (3.11) with all the subscripts and superscripts AdS replaced by dS . The spin- s higher-spin gauge field $\varphi_{\mu_1 \dots \mu_s}^{\text{dS}}$ is the expansion coefficient of the master fields W and B :

$$\begin{aligned} W(x, Y, Z = 0) \Big|_{y^{s-1}, \bar{y}^{s-1}} &\propto \eta^{s-1} \varphi_{\mu_1 \mu_2 \dots \mu_s}^{\text{dS}} (y \sigma_{\text{dS}}^{\mu_2} \bar{y}) \cdots (y \sigma_{\text{dS}}^{\mu_s} \bar{y}), \\ B(x|Y, Z) \Big|_{Y=Z=0} &= \varphi_{\text{dS}}. \end{aligned} \tag{3.19}$$

The reality condition (3.18) implies

$$(\varphi_{\mu_1 \dots \mu_s}^{\text{dS}})^* = (-1)^s \varphi_{\mu_1 \dots \mu_s}^{\text{dS}}. \tag{3.20}$$

Note that, for the odd spin gauge fields, this reality condition differs from the reality condition (3.14) in AdS_4 . However we will find below that they are mapped into one another by our analytic continuation procedure.

4 Analytic continuation from AdS_4 to dS_4

In this section, we describe the analytic continuation of Vasiliev theory from AdS_4 to dS_4 .⁵ Let us start with the Vasiliev equation - with any value of θ_0 - for the master fields expanded about the AdS_4 background. Before imposing reality conditions on either the auxiliary spinor variables or the master fields, the analytic continuation of the coordinates:

$$(z, x^1, x^2, x^0)_{\text{AdS}} = (-i\eta, x^1, x^2, -ix^3)_{\text{dS}}, \tag{4.1}$$

maps the AdS_4 background solution (3.7) to the dS_4 background solution (3.16). This gives the first-order Vasiliev master field equations expanded about the dS_4 background. It follows that the second-order equation of motion for the physical higher-spin component fields in dS_4 obtained by continuing the second-order Vasiliev equation in AdS_4 will match the second-order equation obtained directly from the Vasiliev equation expanded about the dS_4 background. Hence the AdS_4 and dS_4 theories are simply related by analytic continuation. We will describe our prescription for the analytic continuation of the higher-spin gauge fields in subsection 4.1, and of the correlation functions in subsection 4.2.

⁵We could have obtained the dualities for the parity invariant Type A and B models by analytically continuing the corresponding results from $E\text{AdS}_4$ instead of AdS_4 . This was done for the minimal Type A model in [4]. But because of the $(4, 0)$ signature of $E\text{AdS}_4$ we cannot impose reality conditions on the auxiliary Weyl spinors (y, \bar{y}, z, \bar{z}) . As noted in [18] this means that the reality conditions on the master fields in $E\text{AdS}_4$ are not compatible with Vasiliev's equation for the parity-violating models. Here we have circumvented this Euclidean problem by directly continuing from Lorentzian AdS_4 to Lorentzian dS_4 .

4.1 Fields

In this subsection, we give the analytic continuation of higher-spin gauge fields. First of all, applying the analytic continuation (4.1) to the background solutions (3.7) and (3.16), we find that the AdS₄ metric $g_{\mu\nu}^{AdS}$ is indeed related to the dS₄ metric $g_{\mu\nu}^{dS}$ by

$$g_{\mu\nu}^{dS} dx^\mu dx^\nu = g_{\mu\nu}^{AdS} dx^\mu dx^\nu \Big|_{z \rightarrow -i\eta}^{x^0 \rightarrow -ix^3}. \quad (4.2)$$

The prescription for continuing the higher-spin fields is

$$\varphi_{\mu_1 \dots \mu_s}^{dS}(\eta, x^1, x^2, x^3) = i^n \varphi_{\mu_1 \dots \mu_s}^{AdS}(-i\eta, x^1, x^2, -ix^3), \quad (4.3)$$

where n is the total number of 0 and z indices. By the reality conditions (3.14) and (3.20), the odd spin fields are pure imaginary on the left hand side, while the odd spin fields are real on the right hand side.

At first sight, this analytic continuation might seem to lead to bunch of unwanted i 's in the Vasiliev equation of motion, but this is actually not the case. Note that there are no explicit indices in Vasiliev's equation of motion, that is, every free index on a higher-spin gauge field must be contracted with $y\sigma_{AdS}^\mu \bar{y}$, $y\sigma_{AdS}^{\mu\nu} \bar{y}$, $\bar{y}\sigma_{AdS}^{\mu\nu} y$ or similar terms with y, \bar{y} replaced by z, \bar{z} . When we perform the analytic continuation from AdS₄ to dS₄, the σ -matrices for AdS₄ absorb the i 's and turn into the σ -matrices for dS₄. More explicitly, the first and last components of σ_{AdS}^μ in $\sigma_{AdS, \alpha\beta}^\mu = (i\mathbf{1}, \sigma^1, \sigma^3, i\sigma^2)$ absorb an $-i$ and turn into the first and last components of σ_{dS}^μ in $\sigma_{dS, \alpha\beta}^\mu = (\mathbf{1}, \sigma^1, \sigma^3, \sigma^2)$. This suggests that we focus on the generating functions:

$$\Phi_{AdS}^s(x|y, \bar{y}) = \varphi_{\mu_1 \dots \mu_s}^{AdS}(y\sigma_{AdS}^{\mu_1} \bar{y}) \cdots (y\sigma_{AdS}^{\mu_s} \bar{y}), \quad (4.4)$$

and

$$\Phi_{dS}^s(x|y, \bar{y}) = \varphi_{\mu_1 \dots \mu_s}^{dS}(y\sigma_{dS}^{\mu_1} \bar{y}) \cdots (y\sigma_{dS}^{\mu_s} \bar{y}). \quad (4.5)$$

For these the analytic continuation procedure takes the very simple form

$$\Phi_{dS}^s(\eta, x^1, x^2, x^3|Y) = \Phi_{AdS}^s(-i\eta, x^1, x^2, -ix^3|Y). \quad (4.6)$$

4.2 Correlators

Now we give the prescription for analytically continuing the AdS₄ correlators to the dS₄ ones. In order to go from the classical equations of motion to the quantum ones one must specify an additional coupling (essentially \hbar), which we have denoted g_{AdS} (g_{dS}) for AdS₄ (dS₄). These couplings may be defined as the coefficient of the singularity in the scalar two point function: More explicitly, one needs to associate a factor g_{AdS}^{-2}

with each internal or external line, and a factor g_{AdS}^2 with each (cubic) vertex. This gives a factor $g_{AdS}^{2n+2\ell-2}$ for the ℓ -loop, n -point function. For example, the bulk scalar two point function takes the form

$$\langle \varphi_{AdS}^0(x_1^\mu) \varphi_{AdS}^0(x_2^\mu) \rangle_{AdS} \approx g_{AdS}^2 \frac{z_1 z_2}{-(x_1^0 - x_2^0)^2 + (x_1^1 - x_2^1)^2 + (x_1^2 - x_2^2)^2 + (z_1 - z_2)^2}, \quad (4.7)$$

$$\langle \varphi_{dS}^0(x_1^\mu) \varphi_{dS}^0(x_2^\mu) \rangle_{dS} \approx g_{dS}^2 \frac{\eta_1 \eta_2}{-(\eta_1 - \eta_2)^2 + (x_1^1 - x_2^1)^2 + (x_1^2 - x_2^2)^2 + (x_1^3 - x_2^3)^2}, \quad (4.8)$$

in the limit when the two points are very close to each other, i.e. $(z_1, \vec{x}_1) \rightarrow (z_2, \vec{x}_2)$. Once this normalization is specified, the dependence of higher point correlators on the coupling is determined by unitarity. By our analytic continuation procedure, the $z_1 z_2$ in the numerator of (4.9) becomes $-\eta_1 \eta_2$ in the numerator of (4.8). Hence, as in [4], the bulk coupling constant must continue as $g_{AdS}^2 \rightarrow -g_{dS}^2$ at the same time to maintain the positivity of the kinetic term.

We now examine the short distance singularity in the two point function for fields of higher spin gauge fields. For $s > 0$, the two point functions for the physical transverse components of two higher spin gauge fields have the singularity

$$\begin{aligned} \langle \varphi_{i_1 \dots i_s}^{AdS}(x_1^\mu) \varphi_{i_1 \dots i_s}^{AdS}(x_2^\mu) \rangle_{AdS} &\approx g_{AdS}^2 \frac{(z_1 z_2)^{-s+1}}{-(x_1^0 - x_2^0)^2 + (x_1^1 - x_2^1)^2 + (x_1^2 - x_2^2)^2 + (z_1 - z_2)^2}, \\ \langle \varphi_{i_1 \dots i_s}^{dS}(x_1^\mu) \varphi_{i_1 \dots i_s}^{dS}(x_2^\mu) \rangle_{dS} &\approx g_{dS}^2 \frac{(-1)^s (\eta_1 \eta_2)^{-s+1}}{-(\eta_1 - \eta_2)^2 + (x_1^1 - x_2^1)^2 + (x_1^2 - x_2^2)^2 + (x_1^3 - x_2^3)^2}, \end{aligned} \quad (4.9)$$

in the limit when the two points are very close to each other, where $i_1, i_2, \dots, i_s = 1, 2$. They are related by our analytic continuation procedure and the analytic continuation of coupling constant: $g_{AdS}^2 \rightarrow -g_{dS}^2$. Recalling that the reality condition implies that the odd spin component fields are purely imaginary in dS_4 , the important factor of $(-)^s$ in the second line of (4.9) implies positivity of the kinetic term (in terms of real fields) is maintained by the analytic continuation.

The rule for the analytic continuation of the bulk correlation function is

$$\langle \Phi_{dS}^{s_1}(x_1^\mu|Y) \cdots \Phi_{dS}^{s_n}(x_n^\mu|Y) \rangle_{dS} = \langle \Phi_{AdS}^{s_1}(x_1^\mu|Y) \cdots \Phi_{AdS}^{s_n}(x_n^\mu|Y) \rangle_{AdS} \Big|_{\substack{g_{AdS}^2 \rightarrow -g_{dS}^2 \\ x^0 \rightarrow -ix^3, z \rightarrow -i\eta}}. \quad (4.10)$$

The boundary correlation functions can be extracted from the bulk correlation functions by taking the scaled boundary limit [14, 15]⁶:

$$J_{AdS}^{(s)}(\vec{x}|y) = \lim_{z \rightarrow 0} \frac{1}{g_{AdS}^2 z} \Phi_{AdS}^s(x^\mu|y, \bar{y} = -i\sigma_{AdS}^z y), \quad (4.11)$$

⁶Notice that $-iy\sigma_{AdS}^\mu \sigma_{AdS}^z y = \delta_{\mu,i} y \sigma_L^i y$, and $y\sigma_{dS}^\mu \sigma_{dS}^\eta y = \delta_{\mu,i} y \sigma_E^i y$.

and similarly

$$J_{dS}^{(s)}(\vec{x}|y) = \lim_{\eta \rightarrow 0} \frac{1}{i g_{dS}^2 \eta} \Phi_{dS}^s(x^\mu|y, \bar{y} = \sigma_{dS}^\eta y). \quad (4.12)$$

Therefore, we have

$$\left\langle J_{dS}^{(s_1)}(\vec{x}_1|y) \cdots J_{dS}^{(s_n)}(\vec{x}_n|y) \right\rangle_{dS} = \left\langle J_{AdS}^{(s_1)}(\vec{x}_1|y) \cdots J_{AdS}^{(s_n)}(\vec{x}_n|y) \right\rangle_{AdS} \Big|_{x^0 \rightarrow -ix^3}^{g_{AdS}^2 \rightarrow -g_{dS}^2}. \quad (4.13)$$

5 dS₄/CFT₃

In section 2, we showed that for both the Chern-Simons scalar and Chern-Simons fermion theories, the net effect of reversing the statistics of the matter fields is flipping the sign of N while keeping λ fixed. Correlators of the statistics reversed theories can be further transformed to the corresponding correlators in Euclidean signature by analytic continuation $x^0 \rightarrow -ix^3$. In section 3, we showed that the correlators in the Vasiliev theory in AdS₄ and dS₄ are related by the analytic continuation (4.13). In particular, the correlators in dS₄ are given by the correlators in AdS₄ by flipping the sign of the squared coupling, i.e. $g_{AdS}^2 \rightarrow -g_{dS}^2$ together with the analytic continuation on the coordinates. Using the conjectures in [16, 19], the parity-violating Vasiliev theory in AdS₄, with the $\Delta = 1$ or $\Delta = 2$ boundary condition for the scalar, is dual to the Chern-Simons scalar or Chern-Simons fermion theory, respectively, with $N = g_{AdS}^{-2}$. For the case $\Delta = 1$ ($\Delta = 2$), the bulk parity-violating phase θ_0 and boundary 'tHooft coupling $\lambda = \frac{N}{k}$ are related by $\theta_0 = \frac{\pi}{2}\lambda$ ($\theta_0 = \frac{\pi}{2}(1 - \lambda)$). Hence, if the conjectures in [16, 19] are correct, the parity-violating Vasiliev theory in dS₄, with either boundary condition, is dual to the statistics reversed Chern-Simons scalar or Chern-Simons spinor theories, respectively, with $N = g_{dS}^{-2}$, with θ_0 and λ obeying the same boundary-condition-dependent relation as in the AdS₄ theory.

In the special case $k \rightarrow \infty$ of our conjecture, we obtain that the Type A theory in dS₄ with $\Delta = 1$ boundary condition is dual to the free $U(N)$ anticommuting scalar theory, and the Type B theory in dS₄ with $\Delta = 2$ boundary condition is dual to the free $U(N)$ commuting spinor theory. Our conjecture can also be generalized to the $Sp(N)$ Chern-Simons anticommuting scalar or commuting spinor theories.⁷ The bulk dual of these theories is the Vasiliev theory in dS₄ background with minimal truncation: $-\iota(\hat{A}) = \hat{A}, \iota\pi(B) = B$. The Chern-Simons critical scalar and Chern-Simons critical spinor theories are also dual to the parity-violating Vasiliev theory with the $\Delta = 2$ or $\Delta = 1$ boundary conditions, respectively. On the CFT side, by the bosonization duality [20], the Chern-Simons critical scalar theory is dual to the Chern-Simons non-critical

⁷The correlators in the $Sp(2N)$ Chern-Simons theory with wrong-statistics matter are equal to the correlators in the $SO(2N)$ Chern-Simons matter theory with N replaced by $-N$. [32]

spinor theory, and the Chern-Simons critical spinor theory is dual to the Chern-Simons non-critical scalar theory. We expect the bosonization duality still holds after reversing the statistics of the matter fields.

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A Conventions

In this appendix, we give our conventions for the σ -matrices and the auxilliary spinor variables (y, \bar{y}, z, \bar{z}) for the theories in dS_4 and AdS_4 .

A.1 AdS_4

AdS_4 has signature $(+, +, +, -)$. It is parametrized by the coordinate (z, x^1, x^2, x^0) in Poincare patch. The σ -matrices in AdS_4 are defined by

$$(\sigma_{AdS}^\mu)_{\alpha\dot{\beta}} (\sigma_{AdS}^\nu)^{\gamma\dot{\beta}} + (\sigma_{AdS}^\nu)_{\alpha\dot{\beta}} (\sigma_{AdS}^\mu)^{\gamma\dot{\beta}} = 2\delta_\alpha^\gamma \eta_{AdS}^{\mu\nu}, \quad (A.1)$$

where $\eta_{AdS}^{\mu\nu} = \text{diag}(1, 1, 1, -1)$. An explicit representation⁸ of the σ -matrices is

$$(\sigma_{AdS}^\mu)_{\alpha\dot{\beta}} = (i\mathbf{1}, \sigma^1, \sigma^3, i\sigma^2), \quad (A.2)$$

where the $\sigma^1, \sigma^2, \sigma^3$ are Pauli matrices. In this representation the complex conjugate of the σ -matrices are given by

$$(\sigma_{AdS, \alpha\dot{\beta}}^\mu)^* = -(\sigma_{AdS}^\mu)^{\beta\dot{\alpha}}. \quad (A.3)$$

The reality condition for the bosonic spinor variables (Y, Z) is defined as

$$(y^\alpha)^* = \bar{y}_{\dot{\alpha}}, \quad (\bar{y}_{\dot{\alpha}})^* = y^\alpha, \quad (z^\alpha)^* = \bar{z}_{\dot{\alpha}}, \quad (\bar{z}_{\dot{\alpha}})^* = z^\alpha, \quad (A.4)$$

such that $y\sigma_{AdS}^\mu\bar{y}$ is real and $(y\sigma_{AdS}^\mu\sigma_{AdS}^\nu y)^* = \bar{y}\sigma_{AdS}^\mu\sigma_{AdS}^\nu\bar{y}$. We also define $(\sigma_{AdS}^{\mu\nu})_{\alpha\beta} = (\sigma_{AdS}^{\mu\nu})_{\alpha\dot{\gamma}} (\sigma_{AdS}^{\nu\dot{\gamma}\beta})$.

⁸This is not the conventional representation for the σ -matrices, but it is related by analytic continuation to the conventional representation for the σ -matrices in dS_4 .

A.2 dS₄

dS₄ has signature $(-, +, +, +)$. It is parametrized by the coordinate (η, x^1, x^2, x^3) in Poincare patch. Our definition of the σ -matrices in dS₄ is different from the σ -matrices in AdS₄. Hence, we denote the σ -matrices in dS₄ by σ_{dS} to avoid confusion. The σ -matrices in dS₄ are defined by the same algebra as in AdS₄:

$$(\sigma_{dS}^\mu)_{\alpha\dot{\beta}}(\sigma_{dS}^\nu)^{\gamma\dot{\beta}} + (\sigma_{dS}^\nu)_{\alpha\dot{\beta}}(\sigma_{dS}^\mu)^{\gamma\dot{\beta}} = 2\delta_\alpha^\gamma\eta_{dS}^{\mu\nu}, \quad (\text{A.5})$$

however, with a different representation:

$$(\sigma_{dS}^\mu)_{\alpha\dot{\beta}} = (\mathbf{1}, \sigma^1, \sigma^3, \sigma^2), \quad (\text{A.6})$$

and $\eta_{dS}^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. In this representation, the complex conjugate of the σ -matrices is given by

$$(\sigma_{dS, \alpha\dot{\beta}}^\mu)^* = (\sigma_{dS}^\mu)_{\beta\dot{\alpha}}. \quad (\text{A.7})$$

The reality condition for the auxiliary variables is defined as

$$(y_\alpha)^* = \bar{y}_{\dot{\alpha}}, \quad (\bar{y}_{\dot{\alpha}})^* = y_\alpha, \quad (z_\alpha)^* = \bar{z}_{\dot{\alpha}}, \quad (\bar{z}_{\dot{\alpha}})^* = z_\alpha, \quad (\text{A.8})$$

such that $y\sigma_{dS}^\mu\bar{y}$ is real and $(y\sigma_{dS}^\mu\sigma_{dS}^\nu y)^* = \bar{y}\sigma_{dS}^\mu\sigma_{dS}^\nu\bar{y}$. We also define $(\sigma_{dS}^{\mu\nu})_{\alpha\beta} = (\sigma_{dS}^\mu)_{\alpha\dot{\gamma}}(\sigma_{dS}^\nu)^{\dot{\beta}\gamma}$.

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