Disordered Vector Models: From Higher Spins to Incipient Strings

Chi-Ming Chang,[1,2,*] Sean Colin-Ellerin,[3,†] Cheng Peng,[4,‡] and Mukund Rangamani[3,§]
[1]*Yau Mathematical Science Center (YMSC), Tsinghua University, Beijing 100084, China*
[2]*Beijing Institute of Mathematical Sciences and Applications (BIMSA), Beijing 101408, China*
[3]*Center for Quantum Mathematics and Physics (QMAP), Department of Physics and Astronomy, University of California, Davis, California 95616, USA*
[4]*Kavli Institute for Theoretical Sciences (KITS), University of Chinese Academy of Sciences, Beijing 100190, China*

(Received 22 January 2022; revised 19 May 2022; accepted 7 June 2022; published 29 June 2022)

We present a one-parameter family of large N disordered models, with and without supersymmetry, in three spacetime dimensions. They interpolate from the critical large N vector model dual to a classical higher spin theory toward a theory with a classical string dual. We analyze the spectrum and operator product expansion data of the theories. While the supersymmetric model is always well-behaved the nonsupersymmetric model is unitary only over a small parameter range. We offer some speculations on the origin of strings from the higher spins.

DOI: 10.1103/PhysRevLett.129.011603

Introduction.—The planar expansion of large N gauge theories [1] is suggestive of string perturbation theory, and motivates the holographic AdS/CFT correspondence [2]. Large N vector models, which capture criticality in a wide class of physical systems, eg., liquid-vapor, superfluid, and Curie transition in ferromagnets [3–6], on the other hand, are dual to the higher spin gravity in an anti-de Sitter spacetime [7–9]. Theories interpolating between the two limits, e.g., $\mathcal{N}=4$ supersymmetric Yang-Mills [10,11] or Chern-Simons matter theories [12], are at best understood at the two extremes.

Models with intermediate behavior, like the Sachdev-Ye-Kitaev (SYK) model [13–16] and its cousins [17–23], characterized by melonic diagrams dominating the large N limit, offer new perspectives. Quantum mechanical examples ($d=1$) capture features of semiclassical gravity [24,25], while $d\geq 2$ examples have classical finite tension string duals [18,20,22], owing to the lack of sparsity in the spectrum and submaximal Lyapunov exponent [26]. We construct herein a one-parameter family of solvable three-dimensional (3D) theories (cf., [20] for 2D examples) where the higher spin symmetry gets Higgsed as we turn on the deformation. The higher spin states, however, remain in the spectrum, suggesting an emergent string theory with finite tension.

We consider examples with two sets of fields transforming as vectors under $\mathrm{O}(N)$ and $\mathrm{O}(M)$, respectively (indexed by $i,j=1,\ldots,N$ and $a,b=1,\ldots,M$). We will discuss in parallel two sets of models: (i) an $\mathcal{N}=2$ supersymmetric (susy) model with chiral superfields $\mathfrak{p}^i$ and $\mathfrak{s}^a$; and (ii) a bosonic (bos) model with fields ϕ^i and σ^a, obtained from the above, by retaining just the real parts of the bottom component of $\mathfrak{p}^i$ and the top component of $\mathfrak{s}^a$.

The dynamics of the two models is characterized by the (Euclidean) Lagrangian densities [29]

$$\mathcal{L}_{\rm susy}=-\int d^2\theta d^2\bar{\theta}[\bar{\mathfrak{p}}_i(y^\dagger)\mathfrak{p}^i(y)+\bar{\mathfrak{s}}_a(y^\dagger)\mathfrak{s}^a(y)] - \left[\int d^2\theta\frac{1}{2}g_{aij}\mathfrak{s}^a(y)\mathfrak{p}^i(y)\mathfrak{p}^j(y)+\text{c.c.}\right].$$

$$\mathcal{L}_{\rm bos}=\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi^i+\frac{1}{2}g_{aij}\sigma^a\phi^i\phi^j-\frac{1}{4}(\sigma^a)^2. \tag{1}$$

The couplings g_{aij} are Gaussian random variables with zero mean and variance

$$\langle g_{aij}g_{bkl}\rangle=\frac{2J}{N^2}\delta_{ab}\delta_{i(k}\delta_{l)j},\qquad [J]_{\rm classical}=1. \tag{2}$$

The bosonic model has a positive semidefinite Hamiltonian; classically integrating out the auxiliary field σ^a results in a vector model with a random quartic potential:

$$V(\phi)=\frac{1}{4}\sum_{a=1}^{M}\left(\sum_{i,j=1}^{N}g_{aij}\phi^i\phi^j\right)^2. \tag{3}$$

When $M=1$ the random coupling g_{1ij} can be absorbed by a $\mathrm{GL}(N,\mathbb{R})$ field redefinition reducing to the critical vector model or its $\mathcal{N}=2$ susy cousin [30–33]. We will solve the models to leading order in the $1/N$ expansion while holding *'t Hooft coupling* λ fixed:

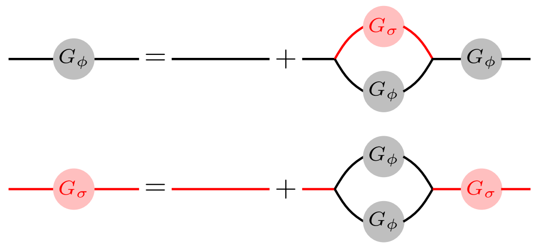


FIG. 1. Diagrammatic representation of the Schwinger-Dyson equations.

$$N \to \infty, \qquad \lambda \equiv \frac{M}{N}, \qquad \text{fixed.} \tag{4}$$

The $\lambda \to \infty$ limit is a variant of the bosonic 3D SYK model with $q = 4$ ($b\mathrm{SYK}^{3\mathrm{D}}_{q=4}$) [34]. The $\mathcal{N} = 2$ susy 3D SYK model [22] is obtained for $\lambda = \frac{1}{2}$.

The IR fixed points.—The models can be solved analogously to the SYK model [14,15] by realizing the Schwinger-Dyson equations truncate [35]. We illustrate the calculations for the bosonic model with the susy case generalizing straightforwardly by working with superfields, cf., [22]. Some details are given in the Supplemental Material [36].

The two-point functions $\langle \phi^i(x)\phi^j(0)\rangle = \delta^{ij} G_\phi(x)$ and $\langle \sigma^a(x)\sigma^b(0)\rangle = \delta^{ab} G_\sigma(x)$ are obtained by iterating melonic diagrams (Fig. 1) leading to

$$G_\phi(p) = \frac{1}{p^2 - \Sigma_\phi(-p)}, \qquad G_\sigma(p) = -\frac{1}{\frac{1}{2} + \Sigma_\sigma(-p)},$$
$$\Sigma_\phi(x) = \lambda J G_\phi(x) G_\sigma(x), \qquad \Sigma_\sigma(x) = \frac{1}{2} J G_\phi(x)^2. \tag{5}$$

At scales below that set by J we can ignore the bare propagators. Picking a conformal ansatz

$$G_\phi(x) = \frac{b_\phi}{|x|^{2\Delta_\phi}}, \qquad G_\sigma(x) = \frac{b_\sigma}{|x|^{2\Delta_\sigma}}, \tag{6}$$

we solve for the scaling dimensions and one combination of the normalization coefficients. We find [40]

$$\Delta_\sigma = 3 - 2\Delta_\phi, \qquad \Delta_{\mathfrak{s}} = 2 - 2\Delta_{\mathfrak{p}},$$
$$\lambda_{\mathrm{bos}} = \frac{(\Delta_\phi - 2)(2\Delta_\phi - 3)[1 + \sec(2\pi\Delta_\phi)]}{4(2\Delta_\phi(4\Delta_\phi - 5) + 3)},$$
$$\lambda_{\mathrm{susy}} = \frac{(\Delta_{\mathfrak{p}} - 1)[1 + \sec(2\pi\Delta_{\mathfrak{p}})]}{2(2\Delta_{\mathfrak{p}} - 1)}. \tag{7}$$

For a fixed 't Hooft coupling λ, the susy model has an unique solution satisfying the unitarity bound $\Delta_{\mathfrak{p}}, \Delta_{\mathfrak{s}} \geq \frac{1}{2}$, while the bosonic model has multiple solutions of the dimensions Δ_ϕ and Δ_σ. We focus on the branch continuously connected to the $\lambda = 0$ theory [41].

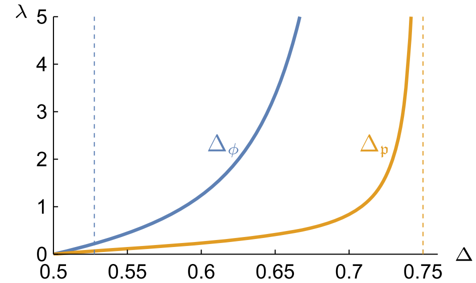


FIG. 2. Scaling dimensions Δ_ϕ and $\Delta_{\mathfrak{p}}$ as we vary λ. The bosonic model is unitary for $\Delta_\phi \in [0.5, 0.52765)$ (region left of the dashed vertical line).

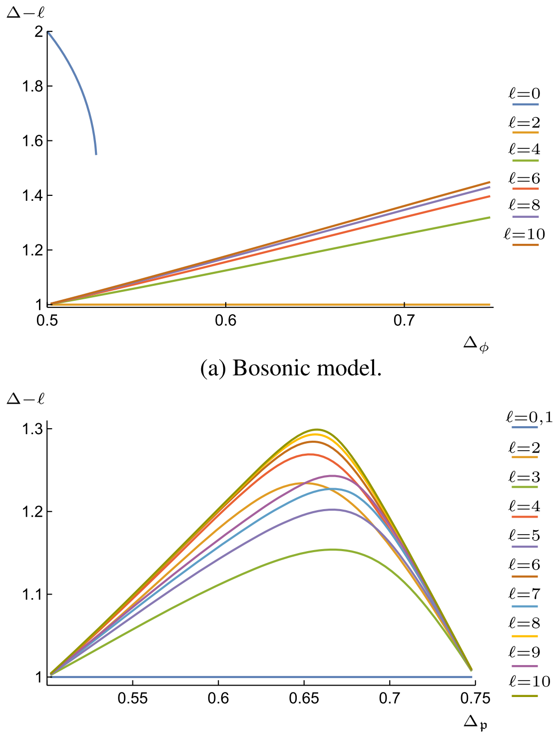
In Fig. 2, we plot the scaling dimension of the O(N) vectors in the two models. Some salient features of interest are (i) $\lambda = 0$ in both cases corresponds to the critical O(N) vector models with ϕ^i and $\mathfrak{p}^i$ having free field dimensions while $\Delta_\sigma = 2$ and $\Delta_{\mathfrak{s}} = 1$. (ii) The bosonic model limits $b\mathrm{SYK}^{3\mathrm{D}}_{q=4}$ as $\lambda \to \infty$ with $(\Delta_\phi, \Delta_\sigma) \to (3/4, 3/2)$. In the susy model we find $(\Delta_{\mathfrak{p}}, \Delta_{\mathfrak{s}}) \to (3/4, 1/2)$, whence $\mathfrak{s}$ becomes a free field. (iii) The intermediate value $\lambda = \frac{1}{2}$ gives $(\Delta_{\mathfrak{p}}, \Delta_{\mathfrak{s}}) = (2/3, 2/3)$, related to the fixed point of [22]. The bosonic theory is related to $b\mathrm{SYK}^{3\mathrm{D}}_{q=3}$ with $(\Delta_\phi, \Delta_\sigma) = (1, 1)$ but lies on a different branch of solutions.

Single-trace operatory spectrum.—An advantage of the disordered models is that one can obtain the spectrum of single-trace operators and OPE coefficients. To do so we look at four-point functions, the connected contribution to which, denoted $\mathcal{F}$, is obtained by summing over the ladder diagrams and suitably diagonalizing the space of four-point correlators (see Supplemental material [36]). We focus here for simplicity on the singlet channel, which can be motivated by averaging over the external operators (as is common in the SYK literature). There also are nonsinglet channels from the tensor product of two vector representations [of $O(N)$ or $O(M)$] in the theory, which are qualitatively similar; cf., [22]. We also note that there is no hierarchical separation between the singlets and the nonsinglets in the large N limit.

Expanding in the (super)conformal partial wave basis one can write $\mathcal{F}$ in terms of a contour integral involving the (super)conformal blocks, a spectral function $\rho(\Delta, \ell)$, and a ladder kernel $k(\Delta, \ell)$, as in various earlier explorations [15,18]. We can schematically write

$$\mathcal{F} = \frac{1}{N} \sum_{\ell} \oint \frac{d\Delta}{2\pi i} \frac{\rho(\Delta, \ell)}{1 - k(\Delta, \ell)} \mathcal{G}_{\Delta, \ell}. \tag{8}$$

The contour of integration for Δ is along the principal series line for (super)conformal representations, i.e., along $\Delta = (3/2) + i\mathbb{R}$ for the bosonic, and $\Delta = \frac{1}{2} + i\mathbb{R}$ for the susy model and closing toward $\Delta \to +\infty$. This picks up the residues at the poles dictated by



(a) Bosonic model.

(b) Susy model.

FIG. 3. The dimensions of the leading twist operators for (a) the bosonic model, and (b) the supersymmetric model. The spin-0 spectrum for the bosonic model is complex for $\Delta_\phi > 0.528$ ($\lambda > 0.222$).

$$k(\Delta, \ell) = 1 \quad \text{for} \begin{cases} \Re(\Delta_{\text{bos}}) > \frac{3}{2}, \\ \Re(\Delta_{\text{susy}}) > \frac{1}{2}, \end{cases} \tag{9}$$

giving thence the spectrum of the single-trace operators. The residues at the poles are the squares of the OPE coefficients. We discuss the two models in turn below.

Bosonic model: For the bosonic model, we consider correlators involving both ϕ^i and σ^a and obtain the ladder kernel entering Eq. (8). In the limit $\lambda \to \infty$ ($\Delta_\phi \to (3/4)$), the ladder kernel at generic value [42] of Δ coincides with the one of $b\text{SYK}^{3D}_{q=4}$ [43]:

$$\lim_{\lambda \to \infty} k(\Delta, \ell) = k_{b\text{SYK}^{3D}_{q=4}}(\Delta, \ell). \tag{10}$$

The spectrum is organized into Regge trajectories,

$$\Delta = 2\Delta_\alpha + \ell + 2n + \gamma_\alpha(\ell, n), \qquad \alpha \in \{\phi, \sigma\}, \tag{11}$$

for $\ell \in 2\mathbb{Z}_{\geq 0}$ and $n \in \mathbb{Z}_{\geq 0}$. The operators on the leading Regge trajectory (leading twist) have twist $\Delta - \ell$ behaving as depicted in Fig. 3(a). The $\ell = 0$ trajectory terminates at $\Delta_\phi = 0.52765$ because the conformal dimension of the operator becomes complex on the principal series $\Delta = (3/2) + i\nu$ when $\Delta_\phi > 0.52765$ ($\lambda \gtrsim 0.222$). This signals that the model becomes nonunitary beyond this point. Such behavior was also observed in the bosonic SYK model and tensor models in [44] and is consistent with the limiting behavior noted in Eq. (10) [45]. The $\ell = 2$ line with the constant unit twist corresponds to the stress tensor. The twists of the higher spin operators ($\ell > 2$) increase along with Δ_ϕ [48].

As $\lambda \to 0$, the spectrum of the leading twist operators approaches that of the critical O(N) model as

$$\Delta \to \begin{cases} 2 & \text{for } \ell = 0, \\ \ell + 1 + \frac{16}{3\pi^2}\frac{\ell-2}{3+2(\ell-2)}\lambda & \text{for } \ell = 2, 4, \ldots. \end{cases} \tag{12}$$

In addition, besides the double twist operators with $\Delta \to \ell + 4 + 2n$ in the $\sigma\sigma \to \sigma\sigma$ channel, operators in the subleading and higher Regge trajectories decouple from the spectrum, as their OPE coefficients approach zero, verifying indeed that as $\lambda \to 0$ we revert to the critical O(N) model.

In the large spin and large twist limits, the anomalous dimensions γ_ϕ and γ_σ scale as

$$\lim_{\ell \gg 1} \gamma_{\phi,\sigma}(\ell, n) \sim \frac{1}{\ell^{2\Delta_\phi}}, \qquad \lim_{n \gg 1} \gamma_{\phi,\sigma}(\ell, n) \sim \frac{1}{n^{4\Delta_\phi}}, \tag{13}$$

consistent with the large spin analytic bootstrap [49,50]. The central charge of the theory can be obtained from the $\phi^i \phi^i T^{\mu\nu}$ OPE coefficient. We find

$$C_T \to N\left(\frac{3}{2} - \frac{20}{3\pi^2}\lambda + \cdots\right) \quad \text{as } \lambda \to 0, \tag{14}$$

as expected for a system of N free bosons.

Beside the spectral information, the Lyapunov exponent λ_L^{hyp} of the out-of-time-order (OTO) four-point function in hyperbolic space is also encoded in $k(\Delta, \ell)$ [18],

$$\lambda_L^{\text{hyp}} = \ell_* - 1, \qquad k\left(\frac{3}{2}, \ell_*\right) = 1. \tag{15}$$

The behavior of λ_L^{hyp} is shown in Fig. 4(a). At the two extreme ends $\Delta_\phi \to \{1/2, 3/4\}$ we find λ_L^{hyp} attains the value in critical O(N) model and $b\text{SYK}^{3D}_{q=4}$, respectively.[51].

Susy model: The analysis of the susy model is similar though we work directly with superconformal blocks as in [22,31]. The results for the leading twist spectrum and hyperbolic chaos exponent are plotted in Figs. 3(b) and 4(a), respectively. The single-trace spectrum is again organized into two Regge trajectories:

$$\Delta = 2\Delta_{\mathfrak{a}} + \ell + 2n + \gamma_{\mathfrak{a}}(\ell, n), \qquad \mathfrak{a} \in \{\mathfrak{p}, \mathfrak{s}\} \tag{16}$$

with $\ell, n \in \mathbb{Z}_{\geq 0}$.

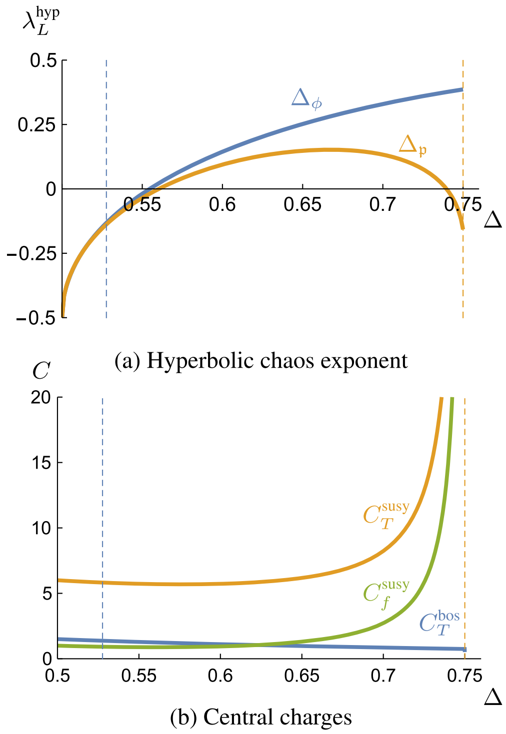


FIG. 4. Hyperbolic chaos exponent λ_L^{hyp} and central charges C_T and C_f for (a) the bosonic model, and (b) the supersymmetric model the two models.

In limiting case $\Delta_\mathfrak{p} \to \frac{1}{2}$, we recover the $\mathcal{N} = 2$ susy O(N) model [30–33] with $\lambda(\Delta_\mathfrak{p})$. The spectrum simplifies: in the $\mathfrak{p}\bar{\mathfrak{p}} \to \mathfrak{p}\bar{\mathfrak{p}}$ channel we find a tower of higher spin currents at leading twist

$$\Delta \to \ell + 1 + \frac{8}{\pi^2}\frac{2\ell - 1 + (-1)^\ell}{2\ell + 1}\lambda. \qquad (17)$$

The higher twist operators in this channel decouple in the limit and only operators with $\Delta \to \ell + 2 + 2n$ from the $\mathfrak{s}\bar{\mathfrak{s}} \to \mathfrak{s}\bar{\mathfrak{s}}$ channel survive.

At the other end, as $\Delta_\mathfrak{p} \to (2/3)$ we encounter the $\mathcal{N} = 2$ susy 3D SYK model studied in [22]. The two theories have identical $\lambda_L^{\text{hyp}} = 0.15207$ with spectrum of the latter model being contained in ours. We, however, have two flavors of fields and thus also have a $U(1)_f$ flavor symmetry wherein $q(\mathfrak{s}) = -2q(\mathfrak{p})$ in addition to a $U(1)_R$ R symmetry. The current multiplets are the $\ell = 0$, 1 trajectories in Fig. 3(b). Computing the OPE coefficients of the current multiplets, we find the central charges for the models consistent with results obtained using supersymmetric localization (see Supplemental Material [36]).

Discussion.—We have at hand a one-parameter family of disordered models smoothly interpolating from the large N critical O(N) vector model, breaking the higher spin symmetry as $\lambda > 0$. The bosonic model is unitary for a small window $\lambda \in [0, 0.222)$, but the susy model is sensible for $\lambda \in \mathbb{R}_{\geq 0}$.

For $\lambda > 0$ the higher spin operators pick up nonvanishing anomalous dimensions, cf., Fig. 3, leading one to expect a classical string dual description resulting from this Higgsing. In conventional AdS/CFT examples, the free field limit has been analyzed in several works [10,12,52,53] with recent constructions of the worldsheet string description [11,54,55] but it is as yet unclear how to connect them to the supergravity description at strong coupling. While we do not yet have an explicit dual, the tractability of the models and the $\lambda = 0$ limit being dual to higher spin AdS gravity offers tantalizing possibilities.

The higher spin states are always in the spectrum, so one expects a dual with a finite string tension. Moreover, their anomalous dimensions exhibit a power-law behavior seen in analytic bootstrap [49,50] and not the logarithmic growth expected from semiclassical strings [56]. As explained in [57], this may be attributed to the fact that vector models have operators with twists close to the unitarity bound.

So how may we expect strings to emerge? A speculation we can offer is the following: at the higher spin limit the bulk degrees of freedom are the bilocal collective fields [58,59], the two-point functions $G(x_1, x_2)$. Let us visualize these bilocal objects to be one-dimensional with end points given by the two operators; we simply have a free Fock space of these collective fields. However, as we turn on λ we should anticipate some linking between different bilocals, leading to a two-dimensional structure, an incipient worldsheet. The glue binding these worldsheets is not as strong as in planar gauge theories so we do not quite make it to the supergravity point. It remains to be seen how to flesh out these ideas, but having analytic control over the field theory is a promising starting point for a perturbative analysis for small λ.

We have focused on the IR fixed point vacuum, but real-time thermal dynamics, be it retarded response, or OTO observables, should give us clues about the nature of stringy black holes duals, through connections to quasinormal modes and Lyapunov exponents. A promising avenue would be to understand the mean-field description of OTO correlators [60] to glean clues about the stringy dual.

It is a pleasure to thank Micha Berkooz and Adar Sharon for discussions on disordered systems, and Xinan Zhou for discussions on anomalous dimensions. C. C. is partly supported by National Key R&D Program of China (No. 2020YFA0713000). S. C. E. was supported by U.S. Department of Energy Grant No. DE-SC0019480 under the HEP-QIS QuantISED program and funds from the University of California. C. P. is supported by the Fundamental Research Funds for the Central Universities, by funds from the University of Chinese Academy of Science, and NSFC No. 12175237. M. R. was supported by U.S. Department of Energy Grant No. DE-SC0009999 and by funds from the University of California.

*Corresponding author.
cmchang@tsinghua.edu.cn
†Corresponding author.
scolinellerin@ucdavis.edu
‡Corresponding author.
pengcheng@ucas.ac.cn
§Corresponding author.
mukund@physics.ucdavis.edu

[1] G. 't Hooft, A planar diagram theory for strong interactions, Nucl. Phys. **B72**, 461 (1974).
[2] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. **2**, 231 (1998).
[3] E. Brezin, D. J. Wallace, and K. Wilson, Feynman-graph expansion for the equation of state near the critical point, Phys. Rev. B **7**, 232 (1973).
[4] K. G. Wilson and J. B. Kogut, The renormalization group and the epsilon expansion, Phys. Rep. **12**, 75 (1974).
[5] A. Pelissetto and E. Vicari, Critical phenomena and renormalization group theory, Phys. Rep. **368**, 549 (2002).
[6] M. Moshe and J. Zinn-Justin, Quantum field theory in the large N limit: A review, Phys. Rep. **385**, 69 (2003).
[7] I. R. Klebanov and A. M. Polyakov, AdS dual of the critical O(N) vector model, Phys. Lett. B **550**, 213 (2002).
[8] E. Sezgin and P. Sundell, Holography in 4D (super) higher spin theories and a test via cubic scalar couplings, J. High Energy Phys. 07 (2005) 044.
[9] S. Giombi and X. Yin, Higher spin gauge theory and holography: The three-point functions, J. High Energy Phys. 09 (2010) 115.
[10] P. Haggi-Mani and B. Sundborg, Free large N supersymmetric Yang-Mills theory as a string theory, J. High Energy Phys. 04 (2000) 031.
[11] M. R. Gaberdiel and R. Gopakumar, String Dual to Free $N = 4$ Supersymmetric Yang-Mills Theory, Phys. Rev. Lett. **127**, 131601 (2021).
[12] C.-M. Chang, S. Minwalla, T. Sharma, and X. Yin, ABJ triality: From higher spin fields to strings, J. Phys. A **46**, 214009 (2013).
[13] S. Sachdev and J. Ye, Gapless Spin Fluid Ground State in a Random, Quantum Heisenberg Magnet, Phys. Rev. Lett. **70**, 3339 (1993).
[14] A. Kitaev, A simple model of quantum holography, in *Proceedings of the KITP Strings Seminar and Entanglement 2015 Program* (2015), https://online.kitp.ucsb.edu/online/entangled15/kitaev/.
[15] J. Maldacena and D. Stanford, Remarks on the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 106002 (2016).
[16] A. Kitaev and S. J. Suh, The soft mode in the Sachdev-Ye-Kitaev model and its gravity dual, J. High Energy Phys. 05 (2018) 183.
[17] W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, Supersymmetric Sachdev-Ye-Kitaev models, Phys. Rev. D **95**, 026009 (2017); Erratum, Phys. Rev. D **95**, 069904 (2017).
[18] J. Murugan, D. Stanford, and E. Witten, More on supersymmetric and 2d analogs of the SYK model, J. High Energy Phys. 08 (2017) 146.
[19] K. Bulycheva, $\mathcal{N} = 2$ SYK model in the superspace formalism, J. High Energy Phys. 04 (2018) 036.
[20] C. Peng, $\mathcal{N} = (0, 2)$ SYK, chaos and higher-spins, J. High Energy Phys. 12 (2018) 065.
[21] C.-M. Chang, S. Colin-Ellerin, and M. Rangamani, On melonic supertensor models, J. High Energy Phys. 10 (2018) 157.
[22] C.-M. Chang, S. Colin-Ellerin, C. Peng, and M. Rangamani, A 3d disordered superconformal fixed point, J. High Energy Phys. 11 (2021) 211.
[23] M. Berkooz, A. Sharon, N. Silberstein, and E. Y. Urbach, The onset of quantum chaos in disordered CFTs, arXiv:2111.06108.
[24] J. Maldacena, D. Stanford, and Z. Yang, Conformal symmetry and its breaking in two dimensional nearly anti-de-Sitter space, Prog. Theor. Exp. Phys. **2016**, 12C104 (2016).
[25] K. Jensen, Chaos in AdS_2 Holography, Phys. Rev. Lett. **117**, 111601 (2016).
[26] It has been argued in [27] that a necessary and sufficient criterion for a field theory to have a semiclassical gravity dual is that it have a sensible large N limit and a sparse low-lying spectrum. One expects such theories to have maximal Lyapunov exponent [28].
[27] I. Heemskerk, J. Penedones, J. Polchinski, and J. Sully, Holography from conformal field theory, J. High Energy Phys. 10 (2009) 079.
[28] J. Maldacena, S. H. Shenker, and D. Stanford, A bound on chaos, J. High Energy Phys. 08 (2016) 106.
[29] We are employing Einstein summation convention for repeated indices and $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$ is a chiral superspace coordinate. Our 3D $\mathcal{N} = 2$ SUSY conventions are as in [22].
[30] N. Bobev, S. El-Showk, D. Mazac, and M. F. Paulos, Bootstrapping the Three-Dimensional Supersymmetric Ising Model, Phys. Rev. Lett. **115**, 051601 (2015).
[31] N. Bobev, S. El-Showk, D. Mazac, and M. F. Paulos, Bootstrapping SCFTs with four supercharges, J. High Energy Phys. 08 (2015) 142.
[32] S. M. Chester, S. Giombi, L. V. Iliesiu, I. R. Klebanov, S. S. Pufu, and R. Yacoby, Accidental symmetries and the conformal bootstrap, J. High Energy Phys. 01 (2016) 110.
[33] S. M. Chester, L. V. Iliesiu, S. S. Pufu, and R. Yacoby, Bootstrapping $O(N)$ vector models with four supercharges in $3 \leq d \leq 4$, J. High Energy Phys. 05 (2016) 103.
[34] J. Liu, E. Perlmutter, V. Rosenhaus, and D. Simmons-Duffin, d-dimensional SYK, AdS loops, and $6j$ symbols, J. High Energy Phys. 03 (2019) 052.
[35] While for the SUSY model one can use nonrenormalization theorems to argue that the renormalization group flow does not induce operators that invalidate the melonic iteration, we are, strictly speaking, assuming such is also true in the bosonic model.
[36] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.129.011603 for technical details regarding the evaluation of the spectrum and OPE coefficients in the large N limit, which includes Refs. [37–39].
[37] D. Simmons-Duffin, D. Stanford, and E. Witten, A spacetime derivation of the Lorentzian OPE inversion formula, J. High Energy Phys. 07 (2018) 085.

[38] T. Nishioka and K. Yonekura, On RG flow of τ_{RR} for supersymmetric field theories in three-dimensions, J. High Energy Phys. 05 (2013) 165.

[39] D. Gang and M. Yamazaki, Expanding 3d $\mathcal{N}=2$ theories around the round sphere, J. High Energy Phys. 02 (2020) 102.

[40] In the SUSY model, $\Delta_{\mathfrak{p}}$ and $\Delta_{\mathfrak{s}}$ refer to the conformal dimension of the chiral multiplet, which is also that of the scalar field in the bottom component.

[41] The existence of the second branch of solutions signals potentially distinct degenerate vacua of Eq. (1). Along this branch $\Delta_\phi \in (3/4, 5/4)$ naively contradicting the quartic interaction picture of Eq. (3). One can motivate this by promoting σ^a to a dynamical field and fine-tune its mass term away as we flow down to attain the critical point.

[42] There are isolated special points where the limit, Eq. (10), does not hold—for example, at $\Delta = 3$ and $\ell = 0$.

[43] Curiously, along the branch not connected to the free theory for $\Delta_\phi = 1$, we encounter a simple relation to the kernel of $b\mathrm{SYK}^{3\mathrm{D}}_{q=3}$: $[1-k(\Delta,\ell)] = [1-k_{b\mathrm{SYK}^{3\mathrm{D}}_{q=3}}(\Delta,\ell)][1+\frac{1}{2}k_{b\mathrm{SYK}^{3\mathrm{D}}_{q=3}}(\Delta,\ell)]$.

[44] S. Giombi, I. R. Klebanov, and G. Tarnopolsky, Bosonic tensor models at large N and small ϵ, Phys. Rev. D **96**, 106014 (2017).

[45] A similar feature is seen in bifundamental multiscalar models; see, for example, [46] (and also [47] which we received as this article was in preparation).

[46] H. Osborn and A. Stergiou, Seeking fixed points in multiple coupling scalar theories in the ϵ expansion, J. High Energy Phys. 05 (2018) 051.

[47] S. Kapoor and S. Prakash, Bifundamental multiscalar fixed points in $d = 3 - \epsilon$, arXiv:2112.01055.

[48] The trajectories are smooth through $\Delta_\phi = (3/4)$ attaining maxima at $\Delta_\phi = 1$, and then decreasing back to 1 at $\Delta_\phi = (5/4)$.

[49] A. L. Fitzpatrick, J. Kaplan, D. Poland, and D. Simmons-Duffin, The analytic bootstrap and AdS superhorizon locality, J. High Energy Phys. 12 (2013) 004.

[50] Z. Komargodski and A. Zhiboedov, Convexity and liberation at large spin, J. High Energy Phys. 11 (2013) 140.

[51] We also recover the result for $b\mathrm{SYK}^{3\mathrm{D}}_{q=3}$ at $\Delta_\phi = 1$.

[52] A. Mikhailov, Notes on higher spin symmetries, arXiv:hep-th/0201019.

[53] R. Gopakumar, From free fields to AdS, Phys. Rev. D **70**, 025009 (2004).

[54] L. Eberhardt, M. R. Gaberdiel, and R. Gopakumar, The worldsheet dual of the symmetric product CFT, J. High Energy Phys. 04 (2019) 103.

[55] M. R. Gaberdiel and R. Gopakumar, The worldsheet dual of free super Yang-Mills in 4D, J. High Energy Phys. 11 (2021) 129.

[56] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, A semi-classical limit of the gauge/string correspondence, Nucl. Phys. **B636**, 99 (2002).

[57] L. F. Alday and A. Zhiboedov, Conformal bootstrap with slightly broken higher spin symmetry, J. High Energy Phys. 06 (2016) 091.

[58] S. R. Das and A. Jevicki, Large-N collective fields and holography, Phys. Rev. D **68**, 044011 (2003).

[59] O. Aharony, S. M. Chester, and E. Y. Urbach, A derivation of AdS/CFT for vector models, J. High Energy Phys. 03 (2021) 208.

[60] Y. Gu, A. Kitaev, and P. Zhang, A two-way approach to out-of-time-order correlators, J. High Energy Phys. 03 (2022) 133.