High order WENO and DG methods for time-dependent convection-dominated PDEs: a brief survey of several recent developments[1]

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ABSTRACT

For solving time-dependent convection-dominated partial differential equations (PDEs), which arise frequently in computational physics, high order numerical methods, including finite difference, finite volume, finite element and spectral methods, have been undergoing rapid developments over the past decades. In this article we give a brief survey of two selected classes of high order methods, namely the weighted essentially non-oscillatory (WENO) finite difference and finite volume schemes and discontinuous Galerkin (DG) finite element methods, emphasizing several of their recent developments: bound-preserving limiters for DG, finite volume and finite difference schemes, which address issues in robustness and accuracy; WENO limiters for DG methods, which address issues in non-oscillatory performance when there are strong shocks, and inverse Lax-Wendroff type boundary treatments for finite difference schemes, which address issues in solving complex geometry problems using Cartesian meshes.

Key Words: high order schemes; time-dependent convection-dominated partial differential equations; finite difference schemes; finite volume schemes; discontinuous Galerkin method; bound-preserving limiters; WENO limiters; inverse Lax-Wendroff boundary treatments.

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1 Introduction

High order numerical methods are attractive in computational physics because of their potential in achieving desired resolution with a small number of degrees of freedom, which often leads to savings in computational cost. Another advantage of high order schemes is that, on machines with a fixed capacity (memory), using high order methods allows better achievable resolution and accuracy. This is why the benchmark results in direct numerical simulation of turbulent flows are often obtained by spectral methods, high order compact schemes, or other high order methods.

This article is mainly concerned with the numerical solution of time-dependent convection-dominated partial differential equations (PDEs), which arise frequently in computational physics. Examples include high Reynolds number Navier-Stokes equations, Maxwell equations and other linear or nonlinear wave equations, magneto-hydrodynamics (MHD), relativistic hydrodynamics (RHD), etc. For such problems, solutions often contain sharp gradient regions or discontinuities, making their numerical simulation more challenging, especially for high order schemes. Nevertheless, because of their good potential in efficiency, high order methods for solving time-dependent convection-dominated PDEs have been actively pursued in the literature in the last decades. Many types of numerical methods have been investigated. On structured meshes over regular geometry, finite difference schemes (including compact schemes) and spectral methods are most popular, because of their simplicity and cost-effectiveness. On unstructured meshes over complex geometry, finite volume schemes and finite element methods (including discontinuous Galerkin methods) are more popular, because of their flexibility on arbitrary meshes. Other types of methods such as meshless methods over arbitrary point clouds have also been designed.

This article gives a brief survey of two classes of high order methods for solving time-dependent convection-dominated PDEs, namely the finite difference and finite volume weighted essentially non-oscillatory (WENO) schemes, and the discontinuous Galerkin (DG) methods, emphasizing a few of their recent developments. We are not attempting to be comprehensive

in this survey and do not cover all available classes of high order methods, and within the two classes of high order methods which we do cover, we do not attempt to discuss all of their recent developments. Rather, we concentrate only on a few selected cases of recent developments of these methods, in order to highlight their utility and values. We will concentrate on the spatial discretizations, and will use mostly the explicit strong-stability-preserving (SSP) Runge-Kutta or multi-step methods [203, 207, 82, 81] as examples of time discretization.

The structure of this article is as follows. In section 2 we describe two classes of high order methods for solving time-dependent convection-dominated PDEs, namely the finite difference and finite volume WENO schemes, and DG and related methods. In sections 3 to 5, we discuss three different topics of recent developments, including bound-preserving limiters, WENO limiters for DG schemes, and inverse Lax-Wendroff type boundary conditions for finite difference schemes, respectively. Concluding remarks are given in section 6.

2 Two classes of high order methods

In this section we give a brief discussion of two classes of high order methods for solving time-dependent convection-dominated PDEs, namely the finite difference and finite volume WENO schemes, and DG and related methods.

2.1 Finite difference and finite volume WENO schemes

A finite difference scheme approximates the point values of the exact solution on a given grid. Let us use the one-dimensional conservation law

$$u_t + f(u)_x = 0 \tag{2.1}$$

to show the idea. The finite difference solution u_j approximates the exact solution $u(x_j, t)$ at the grid point x_j, in a semi-discrete (time continuous) version. To effectively compute weak solutions (which may contain discontinuities) of the conservation law (2.1), we would use conservative difference to approximate the derivative $f(u)_x$ at the grid point x_j:

$$\frac{d}{dt}u_j + \frac{1}{\Delta x}\left(\hat{f}_{j+1/2} - \hat{f}_{j-1/2}\right) = 0 \tag{2.2}$$

where Δx is the spatial mesh size (assumed to be uniform here for simplicity), and $\hat{f}_{j+1/2}$ is the numerical flux, which typically is a Lipschitz continuous function of several neighboring values u_i. In [208], a general procedure to form the numerical flux based on the point values of the solution to guarantee accuracy on uniform meshes is described, which provides formulas for all high order conservative finite difference schemes and have been used widely.

The WENO schemes [153, 120] were based on the earlier work of essentially non-oscillatory (ENO) schemes [93, 207]. Let us use the simple case with $f'(u) \geq 0$ and fifth order finite difference WENO scheme in [120] as an example to demonstrate the basic idea. The fifth order numerical flux is given by

$$\hat{f}_{j+1/2} = w_1 \hat{f}^{(1)}_{j+1/2} + w_2 \hat{f}^{(2)}_{j+1/2} + w_3 \hat{f}^{(3)}_{j+1/2} \tag{2.3}$$

where $\hat{f}^{(i)}_{j+1/2}$, for $i = 1, 2, 3$, are three third order fluxes on three different small stencils given by

$$\begin{aligned}
\hat{f}^{(1)}_{j+1/2} &= \frac{1}{3} f(u_{j-2}) - \frac{7}{6} f(u_{j-1}) + \frac{11}{6} f(u_j), \\
\hat{f}^{(2)}_{j+1/2} &= -\frac{1}{6} f(u_{j-1}) + \frac{5}{6} f(u_j) + \frac{1}{3} f(u_{j+1}), \\
\hat{f}^{(3)}_{j+1/2} &= \frac{1}{3} f(u_j) + \frac{5}{6} f(u_{j+1}) - \frac{1}{6} f(u_{j+2}),
\end{aligned} \tag{2.4}$$

and the nonlinear weights w_i are given by

$$w_i = \frac{\tilde{w}_i}{\sum_{k=1}^{3} \tilde{w}_k}, \qquad \tilde{w}_k = \frac{\gamma_k}{(\varepsilon + \beta_k)^2}, \tag{2.5}$$

with the linear weights γ_k given by

$$\gamma_1 = \frac{1}{10}, \qquad \gamma_2 = \frac{3}{5}, \qquad \gamma_3 = \frac{3}{10}, \tag{2.6}$$

and the smoothness indicators β_k given by

$$\begin{aligned}
\beta_1 &= \frac{13}{12} \left(f(u_{j-2}) - 2f(u_{j-1}) + f(u_j) \right)^2 + \frac{1}{4} \left(f(u_{j-2}) - 4f(u_{j-1}) + 3f(u_j) \right)^2 \\
\beta_2 &= \frac{13}{12} \left(f(u_{j-1}) - 2f(u_j) + f(u_{j+1}) \right)^2 + \frac{1}{4} \left(f(u_{j-1}) - f(u_{j+1}) \right)^2 \\
\beta_3 &= \frac{13}{12} \left(f(u_j) - 2f(u_{j+1}) + f(u_{j+2}) \right)^2 + \frac{1}{4} \left(3f(u_j) - 4f(u_{j+1}) + f(u_{j+2}) \right)^2 .
\end{aligned} \tag{2.7}$$

Finally, ε is a parameter to avoid the denominator to become 0 and is usually taken as $\varepsilon = 10^{-6}$ in the computation. This construction of WENO fluxes can be generalized to the case of $f'(u)$ changing sign (using flux splitting such as the Lax-Friedrichs flux splitting), to the case of one-dimensional systems (using local characteristic decomposition), and to the case of two and more spatial dimensions (fluxes for each coordinate direction, corresponding to $f(u)_x$, $g(u)_y$, etc., are computed using the one-dimensional formulae (2.2)-(2.7) with grid points in that direction only, namely in a dimension-by-dimension fashion), we refer to, e.g., [120, 204] for more details. WENO schemes are available for arbitrarily high order accuracy [8]. The review paper [205] summarizes the development and application of WENO schemes until 2006.

Even though most of the finite difference WENO schemes in applications use the numerical flux construction in (2.2)-(2.7), which reconstructs directly from the physical fluxes $f(u_i)$, the earlier paper [207] actually provided an alternative formulation of the numerical flux, which could rely on the WENO interpolation directly on the point values u_i. This alternative approach has been explored more recently in [122, 123]. It is slightly more expensive than the one in [208, 120], as outlined in (2.2)-(2.7) above, but it has a few distinct advantages, including its flexibility to use arbitrary monotone fluxes or approximate Riemann solvers [122], its narrower effective stencil with a Lax-Wendroff time discretization [122], and its ability to preserve free-stream solution exactly on curvilinear time-dependent meshes [123]. It is well-known that free-stream preserving is difficult for finite difference schemes on curvilinear time-dependent meshes, see, e.g. [220, 269, 231, 171, 211] for discussions on this issue. The approach in [123] is one of the very few high order finite difference schemes which can maintain exactly free-stream solutions on time-dependent meshes when the nonlinear WENO (or other nonlinear limiting) procedure is used in the spatial discretization.

A particular class of finite difference schemes, for which the numerical flux $\hat{f}_{j+1/2}$ relies on all grid points via a banded matrix inversion, is called compact schemes [139]. For the same order of accuracy, compact schemes usually give much smaller dispersion and/or

dissipation errors, hence they are more suitable for problems where marginally resolved waves are important, such as turbulence simulations. The WENO idea can be combined with compact schemes, resulting in weighted compact schemes which can improve robustness for shocked flows, e.g. in [54, 121, 179, 191, 271, 154, 213].

In one space dimension, finite volume schemes are very similar to finite difference schemes. The major difference is that a finite volume scheme approximates the cell averages $\bar{u}(x_j,t) = \frac{1}{\Delta x_j}\int_{x_{j-1/2}}^{x_{j+1/2}} u(x,t)dx$ of the exact solution on a given mesh. For the one-dimensional conservation law (2.1), the finite volume solution $\bar{u}_j$ approximates the cell average of the exact solution $\bar{u}(x_j,t)$ in the cell $I_j = (x_{j-1/2}, x_{j+1/2})$. The scheme is still conservative

$$\frac{d}{dt}\bar{u}_j + \frac{1}{\Delta x_j}\left(\hat{f}_{j+1/2} - \hat{f}_{j-1/2}\right) = 0$$

where the numerical flux $\hat{f}_{j+1/2}$ now depends on several neighboring cell averages $\bar{u}_i$. The computation of the numerical flux $\hat{f}_{j+1/2}$ for a finite volume scheme depends on a monotone flux or an exact or approximate Riemann solver, namely $\hat{f}_{j+1/2} = \hat{f}(u^-_{j+1/2}, u^+_{j+1/2})$, where $u^{\pm}_{j+1/2}$ are reconstructed from the cell averages $\bar{u}_i$ nearby. The reconstruction procedure from $\bar{u}_i$ to $u^-_{j+1/2}$ is completely the same as (2.2)-(2.7) in finite difference, except that $f(u_i)$ is now replaced by $\bar{u}_i$ and the final product $\hat{f}_{j+1/2}$ is replaced by $u^-_{j+1/2}$. The construction of $u^+_{j+1/2}$ is mirror symmetric to that for $u^-_{j+1/2}$ with respect to $x_{j+1/2}$. Therefore, in one space dimension, the codes and computational costs for finite difference and finite volume WENO schemes are almost identical, and they produce similar computational results. However, in multi-dimensions, finite volume schemes are much more costly than finite difference schemes, since a reconstruction from multi-dimensional cell averages to point values needs to be performed and numerical quadratures must be used for the integration to get numerical fluxes. We refer to [204, 25, 270] for the discussion of multi-dimensional finite volume schemes and their comparison with the finite difference counter-parts. However, finite volume schemes have a significant advantage over finite difference schemes, in that they can be designed on non-smooth and unstructured meshes without losing their high order accuracy and conservation properties, see, for example, [1, 72, 103, 66, 283] for high order finite volume ENO and

WENO schemes on two- and three-dimensional unstructured meshes.

Efforts have been made in the literature to improve the accuracy, dispersion or dissipation quality, or the efficiency of WENO schemes, via modifying the linear or nonlinear weights, via modifying the smoothness indicators, or via hybridizing WENO with other types of schemes. For example, an improved WENO scheme (in terms of accuracy and resolution) is provided in [18], which is further developed in [26] as the WENO-Z scheme. Accuracy issues regarding WENO schemes are discussed and improvements made in [96, 57]. Efforts to improve the dissipation and / or dispersion properties of WENO schemes for high frequency wave computations are made in [241, 180, 161, 107, 118, 260].

One research direction regarding WENO schemes which has attracted a lot of attention recently is adaptivity, aiming at reducing computational cost to achieve desired resolution at places where it is needed. For example, an adaptive mesh refinement (AMR) WENO finite difference scheme for multi-scale simulations is developed in [200] and an AMR WENO finite difference solver for multi-dimensional detonation is developed in [234]. A wavelet-based adaptive WENO algorithm is constructed for Euler equations in [144]. A space-time adaptive ADER-WENO finite volume scheme is constructed in [65], and an adaptive mesh WENO method is developed in [104].

Applications of WENO schemes can be found in many areas of computational physics and computational engineering. We list below only a few examples over the past year (since 2015). Recent applications of WENO schemes can be found in the simulations of astrophysics and geophysics [55, 83, 137, 140, 162, 172, 215, 223], atmospheric and climate science [70, 181, 225], batch chromatographic separation [101], biomolecular solvation [286], bubble clusters in fluids [222], combustion [11, 15, 24, 164, 214, 268], detonation waves [92, 114, 145, 233], elastic-plastic solids [173], flame structure [261], granular gas [3], hypersonic flows [109], infectious disease models [209], laser welding [174], magnetohydrodynamics [20, 166], mathematical finance for solving the Black-Scholes equation [90], multiphase and multispecies flows [13, 91, 108, 110, 159, 160, 239], networks and blood flows [168], ocean

waves [28, 125], oil storage process [196], rarefied gas flow [147], rotor aerodynamic performance [117], semiconductor device and other computational electronics [59, 80, 165], shallow water equations [29, 128, 129, 138], special relativistic hydrodynamics [244, 264], supersonic flows [267, 280], and turbulent flows [74, 75, 131, 150, 193, 258]. This very incomplete list over just one year period clearly demonstrates the wide-spread influence of the WENO technique in computational science and engineering.

2.2 Discontinuous Galerkin and related schemes

Similar to a finite volume scheme, a discontinuous Galerkin (DG) method for a conservation law such as (2.1) also approximates an integral version of it. Instead of integrating the PDE (2.1) over the interval $I_j = (x_{j-1/2}, x_{j+1/2})$ directly, we first multiply it by a test function $v(x)$, then integrate over the interval I_j and integrate by parts to obtain

$$\int_{I_j} u_t(x,t)v(x)dx - \int_{I_j} f(u(x,t))v_x(x)dx + f(u(x_{j+1/2},t))v(x_{j+1/2}) - f(u(x_{j-1/2},t))v(x_{j-1/2}) = 0. \tag{2.8}$$

The numerical solution for a DG scheme is represented by a piecewise polynomial function, still denoted as $u(x,t)$, which is a polynomial of degree k in each cell I_j. The test function $v(x)$ is also taken as a piecewise polynomial of degree k. If we would like to convert (2.8) into a scheme, we would need to interpret $f(u(x_{j\pm1/2},t))$ and $v(x_{j\pm1/2})$ suitably, as both the solution $u(x,t)$ and the test function $v(x)$ are discontinuous at the interface $x_{j\pm1/2}$. Based on the similarity with finite volume schemes, we can take the piecewise constant case and attempt to recover the traditional first order monotone scheme. This will determine the choices of the interface terms: $f(u(x_{j\pm1/2},t))$ should be replaced by a numerical flux $\hat{f}(u(x^-_{j\pm1/2},t), u(x^+_{j\pm1/2},t))$, and $v(x_{j\pm1/2})$ should be replaced by its traces inside the cell I_j, namely by $v(x^-_{j+1/2})$ at $x_{j+1/2}$ and by by $v(x^+_{j-1/2})$ at $x_{j-1/2}$. The DG scheme is thus obtained: we would like to find a piecewise polynomial of degree k, still denoted by $u(x,t)$, such that the following equality holds for all test functions $v(x)$ which are piecewise polynomials of

degree k:

$$\int_{I_j} u_t(x,t)v(x)dx - \int_{I_j} f(u(x,t))v_x(x)dx + \hat{f}(u(x^-_{j+1/2},t),u(x^+_{j+1/2},t))v(x^-_{j+1/2}) \\ -\hat{f}(u(x^-_{j-1/2},t),u(x^+_{j-1/2},t))v(x^+_{j-1/2}) = 0. \tag{2.9}$$

Similar to finite volume schemes, the DG method (2.9) can also be evolved explicitly by any ODE solver, without the need to solve any global system. The generalization of the DG method to multi-dimensional unstructured meshes is straightforward, as there is no reconstruction needed as in finite volume schemes. On the same mesh, in comparison with finite volume schemes of the same order of accuracy, the DG method saves in the computational cost of reconstruction, but has a much larger memory requirement and also higher cost in evolving all these extra degrees of freedom in time. On balance, whether a finite volume scheme or a DG scheme is more cost-effective to reach the same level of errors depends on the specific PDE, the complexity of the geometry and meshes, and the computer platform. A comparison of the relative efficiency between the finite volume and DG schemes is given in [287]. There are also intermediate methods between DG and finite volume schemes, which have more than one degrees of freedom per cell yet not enough for the full k-th degree polynomial, hence still require a reconstruction which however has a smaller stencil than that for regular DG schemes, such as the Hermite-type finite volume and finite difference schemes [184, 185, 151], the recovery-type DG schemes [226, 227], and the P^nP^m type methods [63, 62]. If the high memory requirement of DG is a concern, these intermediate methods might be good options. We would like to mention that the DG method is extremely local in data communications. The evolution of the solution in each cell needs to communicate only with its immediate neighbors, regardless of the order of accuracy. The methods thus have excellent parallel efficiency, usually more than 99% for a fixed mesh, and more than 80% for a dynamic load balancing with adaptive meshes which change often during time evolution, see, e.g. [16, 189, 4, 12]. The DG method is also very friendly to the GPU environment [130].

The first DG method was introduced in 1973 by Reed and Hill [188], in the framework of neutron transport, i.e. a time independent linear hyperbolic equation. It was later developed for solving nonlinear hyperbolic conservation laws by Cockburn et al. in a series of papers [44, 43, 40, 45], in which the authors have established a framework to easily solve nonlinear time dependent problems, such as the Euler equations in compressible gas dynamics, using explicit, nonlinearly stable high order Runge-Kutta time discretizations [207] and DG discretization in space described above. Generalizations to convection-diffusion equations were carried out in, e.g. [9, 46, 10], and to PDEs with higher order derivatives in, e.g. [254, 255, 251, 33].

For the history and development of the DG method, we refer to the survey paper [41], and other papers in that Springer volume, which contains the conference proceedings of the First International Symposium on Discontinuous Galerkin Methods held at Newport, Rhode Island in 1999. The lecture notes [39] is a good reference for many details, as well as the extensive review paper [47]. The review paper [252] covers the local DG method for partial differential equations (PDEs) containing higher order spatial derivatives, such as Navier-Stokes equations. There are three special issues devoted to the discontinuous Galerkin methods [48, 50, 49], which contain many interesting papers in the development of the method in all aspects including algorithm design, analysis, implementation and applications. There are also a few recent books and lecture notes [56, 100, 126, 141, 194, 206] on DG methods.

One advantage of the DG framework is that it can easily accommodate local approximation spaces, which do not have to be polynomials, with special properties, such as locally divergence-free or curl-free elements, or spaces fitting particular properties of the PDE solutions. This flexibility can be explored to design structure-preserving DG methods which can better approximate the relevant PDE with less computational cost. See for example the locally divergence-free DG method for the Maxwell equations [42] and for the MHD equations [142], and the locally curl-free DG method for the Hamilton-Jacobi equations [143] and for

the multiscale modeling of dynamics of crystalline solids [237]. Exponential or trigonometric local spaces have been studied in [262, 288], plain wave local spaces are used to solve the Helmholtz equation in [27, 115, 76], and suitable multiscale basis functions obtained from asymptotic analysis are used for multiscale problems in [263, 238, 236, 69, 281, 58].

Another advantage of the DG framework is its easiness to accommodate arbitrary *h-p* adaptivity. Of course, one would need suitable error indicators to guide local decisions on whether to refine or coarsen the mesh, or to increase or decrease the polynomial degree. Various superconvergence results and a posteriori error estimates are pursued towards this goal, see for example [5, 6, 22, 23, 34, 35, 105, 127, 163, 256].

There are a few classes of numerical methods studied in the literature which are closely related to discontinuous Galerkin methods. Examples include the spectral finite volume (SV) method [240], spectral finite difference (SD) method [156], staggered-grid (SG) multi-domain spectral method [132], and correction procedure via reconstruction (CPR) method [116, 242, 89]. Many of these methods can be considered as discontinuous Galerkin methods with suitable and clever choices of numerical integration quadratures, and they often reduce to discontinuous Galerkin methods for linear problems with suitable choices of parameters. These methods may provide certain conceptual simplicity over DG methods, for example the CPR methods can be defined on solution points and hence they look like finite difference methods, similar to the so-called nodal DG methods [100] which uses numerical quadrature and solution values at quadrature points as degrees of freedom.

Applications of DG methods can be found in many areas of computational physics and computational engineering. We list below only a few examples over the past year (since 2015). Recent applications of DG methods can be found in the simulations of the Cahn-Hilliard-Brinkman system [85], compressible flow in the transonic axial compressor [190], computational astrophysics [197], computational geosciences [221], elastodynamics [53], flow instabilities [51], Fokker-Planck equations [152], fractional PDEs [111, 243], front propagation with obstacles [17], functionalized Cahn-Hilliard equation [87], interfaces [278], magnetohy-

drodynamics [265], moment closures for kinetic equations [2], multi-phase flow and phase transition [52, 169], Navier-Stokes and Boussinesq equations [64, 224], nonlinear Schrodinger equation [86, 149, 158], ocean waves [192], population models [112], porous media [84], rarefied gas [212], semiconductor device simulation [155], shallow water equations [73], thin film epitaxy problem [247], traffic flow and networks [21], three-dimensional flows [175], turbulent flows [246], underwater explosion [235], viscous surface wave [245], and wavefield modeling [95]. This very incomplete list over just one year period clearly demonstrates the wide-spread application of the DG method in computational science and engineering.

3 Bound-preserving limiters

In many convection dominated problems, the physical quantities have desired bounds which are satisfied by the exact solutions of the PDEs. For example, for two-dimensional incompressible Euler or Navier-Stokes equations written in a vorticity-streamfunction formulation, the vorticity satisfies a maximum principle. For Euler equations of compressible gas dynamics, density and pressure remain positive (non-negative) when their initial values are positive. It would certainly be desirable if numerical solutions obey the same bounds. If the numerical solution goes out of the bounds because of spurious oscillations, it would either be non-physical (e.g. negative density, negative internal energy, a percentage of a component which goes below zero or above one), or worse still, it could lead to nonlinear instability and blowups of the code because the PDE might become ill-posed (e.g. the Euler equations of compressible gas dynamics become ill-posed for negative density or pressure).

Most limiters in the literature are designed to control spurious oscillations. Not all of them can enforce the bound-preserving property. When they do, they might degenerate the order of accuracy of the original scheme in smooth regions.

Recently, a general framework is established by Zhang and Shu [272, 273] to preserve strict bounds (maximum principle for scalar problems and positivity of relevant quantities for scalar problems or systems) for DG and finite volume schemes, while maintaining provable

high order accuracy of the original schemes. These techniques apply to multi-dimensions in general unstructured triangulations as well [279]. For earlier work that this Zhang-Shu approach is based upon, we refer to [177].

We will not repeat here the details of this general framework and refer the readers to the references. We will only summarize here the main ingredients:

1. We start from a first order base DG or finite volume scheme (on first order level they are the same), using piecewise polynomials of degree zero (piecewise constants), which can be proved to be bound-preserving under certain CFL conditions for Euler forward time discretization.

   For scalar hyperbolic conservation laws, the first order DG or finite volume scheme using any monotone numerical flux would satisfy a maximum principle. For Euler equations of compressible gas dynamics, several first order schemes, including the Godunov scheme [68], the Lax-Friedrichs scheme [177, 273], the Harten-Lax-van Leer (HLLE) scheme [94], and the Boltzmann type kinetic scheme [176], among others, are positivity-preserving for density and pressure. Modified HLLC first order solvers for multi-material compressible flow in the Lagrangian framework are shown to be positivity-preserving for density and internal energy for general equations of states in [31, 229, 230]. For relativistic hydrodynamics (RHD), the first order Lax-Friedrichs scheme is positivity-preserving for density and pressure, and maintains the upper bound of speed by that of the light [244, 182].

2. We then apply a simple scaling limiter to the high order DG polynomial, or the reconstructed polynomial for a finite volume scheme, at time level n. This scaling limiter is completely local, namely it uses information only within the current cell, not even its immediate neighbors. Also, its computation only involves the evaluation of the unlimited solution polynomial at a few pre-determined quadrature points, hence the cost is minimal. The limiter maintains the original cell average, thus ensuring conservation.

3. We then evolve the solution by the Euler forward time discretization, or by TVD or SSP high order Runge-Kutta or multi-step time discretization [203, 207, 82, 81].

This procedure can be applied in arbitrary triangular meshes. Amazingly, this simple process guarantees bound-preserving under a fixed fraction of the CFL condition for the first order base scheme, as long as the bound-invariance region is convex in the phase space, and it leads to this mathematically provable bound-preserving property without degenerating the high order accuracy of the DG or finite volume schemes.

For scalar nonlinear conservation laws, passive convection in a divergence-free velocity field, and 2D incompressible Euler equations in the vorticity-streamfunction formulation, high order DG schemes maintaining maximum principle have been designed in [272, 279]. For Euler equations of gas dynamics, high order DG schemes maintaining positivity of density and pressure (or internal energy) have been designed in [273, 274, 275, 276, 279]. For shallow water equations, high order DG schemes maintaining non-negativity of water height have been designed in [248]. Positivity-preserving semi-Lagrangian DG schemes have been designed in [183, 195]. Realizability-preserving DG and WENO schemes are constructed in [2, 198] for entropy based moment closures of linear kinetic equations. Positivity-preserving semi-implicit DG scheme is developed for solving the extended magnetohydrodynamics equations in [284]. Positivity-preserving Lagrangian schemes for multi-material compressible flows are designed in [31, 229, 230]. The positivity-preserving technique has allowed stable simulations of turbulent cosmology flows in [293]. Simulations for problems with δ-function singularities, which are very difficult for high order methods, have been made possible by this positivity-preserving technique [257, 259]. Bound-preserving high order DG methods have been designed for ideal MHD equations in [32] and for relativistic hydrodynamics (RHD) in [182]. The methodology can also be extended to schemes related to DG methods, such as the correction procedure via reconstruction (CPR) method [60, 61].

While the Zhang-Shu approach mentioned above is simple and effective, it works most well for DG or finite volume schemes solving conservation laws. For finite difference schemes,

this approach can also apply to, e.g. Euler equations to maintain positivity for density and pressure [277], but it has restrictions in order to keep the original high order accuracy. For convection-diffusion equations, this approach works for general DG methods to second order accuracy on arbitrary triangulations [282], and to third order accuracy for a special class of DG methods (the direct DG, or DDG methods) [30].

Another approach to achieve bound-preserving schemes is through the traditional flux-correction method, namely to put an explicit restriction on the high order numerical flux, often to make a convex combination of the high order flux and a first order bound-preserving flux, and choose the parameter carefully so that bound-preserving is ensured. The difficult task for this approach is to prove that the original high order accuracy is not compromised by this flux correction, and this is usually done via explicit and complicated algebraic verifications, thus limiting the scope that the proof can be applied. Among this class, we mention the high order parametrized maximum-principle-preserving and positivity-preserving finite difference schemes in [253, 148, 249], and their extension to unstructured meshes in [37]. The limiters in [106] also belong to this class. Such parametrized positivity-preserving WENO schemes have been generalized to solve MHD equations in [38]. In [250], the limiter is extended to DG schemes for convection-diffusion equations, in [124], it is applied to high order finite difference WENO schemes solving correlated random walk with density-dependent turning rates, in [88], it is applied to DG schemes for nonlinear parabolic equations with blow-up solutions, and in [244], it is applied to finite difference WENO schemes for special relativistic hydrodynamics.

Although such parametrized maximum-principle-preserving and positivity-preserving limiters can also be applied to finite volume and DG schemes for conservation laws, their real advantage is for finite difference schemes solving conservation laws and schemes for solving convection-diffusion equations, for which the Zhang-Shu approach has rather severe restrictions as mentioned above.

4 WENO limiters for DG schemes

Even though the DG method satisfies a cell-entropy inequality for the square entropy for both scalar nonlinear equations and symmetric nonlinear systems on arbitrary meshes [119, 102], which implies an L^2 or energy stability, this stability is not strong enough to prevent spurious oscillations or even blow-ups of the numerical solution in the presence of strong discontinuities. Therefore, nonlinear limiters are often needed to control such spurious oscillations.

Many limiters for the DG method exist in the literature, often adapted from those originally designed for finite volume schemes. Examples include the minmod type total variation bounded (TVB) limiter [202], applied to DG methods in [44, 43, 40, 45], the moment-based limiter [16] and the improved moment limiter [19]. Although these limiters can control spurious numerical oscillations near discontinuities, they tend to degrade accuracy when mistakenly used in smooth regions of the solution. It is usually difficult to design limiters to achieve both high order accuracy and a non-oscillatory property near discontinuities.

The limiters based on the weighted essentially non-oscillatory (WENO) methodology are designed with the objective of maintaining the original high order accuracy even if the limiters take effect in smooth cells. These limiters are based on the WENO methodology for finite volume schemes [153, 120], and involve nonlinear reconstructions of the polynomials in troubled cells using the information of neighboring cells. The WENO reconstructed polynomial has the same cell average and the same high order of accuracy as the original polynomial when the solution is smooth, and it is (essentially) non-oscillatory near discontinuities. In earlier years, Qiu and Shu [186] and Zhu et al. [289] designed WENO limiters using the usual WENO reconstruction based on cell averages of neighboring cells as in [120, 103, 201], to reconstruct the values of the solutions at certain Gaussian quadrature points in the target cells, and then rebuild the solution polynomials from the original cell average and the reconstructed values at the Gaussian quadrature points through a numerical integration for the moments. This limiter needs to use the information from not only the immediate neighboring cells but also neighbors' neighbors, making it complicated to implement in multi-dimensions,

especially for unstructured meshes [289, 103, 283]. It also destroys the local data structure of the base DG scheme (which needs to communicate only with immediate neighbors), thus affecting its parallel efficiency. The effort in [184, 185] attempts to construct Hermite type WENO approximations, which use the information of not only the cell averages but also the lower order moments such as slopes, to reduce the spread of reconstruction stencils. However for higher order methods the information of neighbors' neighbors is still needed.

More recently, Zhong and Shu [285] developed a new WENO limiting procedure for Runge-Kutta DG methods on structured meshes. The idea is to reconstruct the entire polynomial, instead of reconstructing point values or moments in the classical WENO reconstructions. That is, the entire reconstruction polynomial on the target cell is a convex combination of polynomials on this cell and its immediate neighboring cells, with suitable adjustments for conservation and with the nonlinear weights of the convex combination following the classical WENO procedure. The main advantage of this limiter is its simplicity in implementation, as it uses only the information from immediate neighbors and the linear weights are always positive. This simplicity is more prominent for multi-dimensional unstructured meshes, which is studied in [290] for two-dimensional unstructured triangular meshes. Further improvements of this limiter are carried out in [291, 292]. This WENO limiter has also been adapted to the CPR schemes in [60, 61], which are closely related to DG schemes.

The WENO limiters are typically applied only in designated "troubled cells", in order to save computational cost and to minimize the influence to accuracy in smooth regions. Therefore, a troubled cell indicator is needed, to correctly identify cells near discontinuities in which the limiters should be applied. Qiu and Shu in [187] have compared several troubled cell indicators. In practice, the TVB indicator [202] and the KXRCF indicator [136] are often the best choices. We would also like to mention the recent troubled cell indicators based on wavelets and outlier detectors [232].

A novel sub-cell limiting which breaks the DG cell into subcells and then use WENO

ideas for limiting is developed in [67], see also [265, 266].

5 Inverse Lax-Wendroff type boundary conditions for finite difference schemes

When a high order finite difference scheme with a wide stencil is used to solve partial differential equations, the inner schemes cannot be directly used near the boundary. Special treatments near the boundaries are needed in order to maintain accuracy and stability. There exist two difficulties when imposing numerical boundary conditions. Firstly, the points used in these schemes which lie outside the computational domain, namely the "ghost points", should be evaluated properly. Secondly, the grid points may not coincide with the physical boundary exactly, especially when Cartesian meshes are used to solve problems in complex geometry.

For hyperbolic conservation laws, classical Lagrangian extrapolation to evaluate ghost point values near the outflow boundary usually leads to stable approximations [77, 228, 146]. However, it is a challenge to obtain stable and accurate numerical boundary conditions near the inflow boundary. This is especially the case when the physical boundary does not coincide with but is very close to the first grid point, which is referred to as the "cut-cell" problem in the literature, see e.g. [14]. The inverse Lax-Wendroff (ILW) procedure, first introduced in [216], can overcome this difficulty. The simplified ILW (SILW) procedure, which is an extension of the ILW procedure and can save in algorithm complexity and computational cost, is introduced in [219]. For earlier related work, see [78, 79, 97, 113].

We would like to briefly discuss several different approaches in handling numerical boundary conditions for high order finite difference schemes, in order to put the ILW and SILW methods into proper perspective. One commonly used method to deal with complex geometry is to generate a boundary fitted mesh which allows us to impose boundary conditions directly in the algorithm. When employing this method, the governing equations are generally transformed into a new differential form on a curvilinear coordinate system (see e.g.

[123]). If the domain is simple enough, a smooth mapping can be used for the transformation of the entire domain. But if the domain is more complex, composite overlapping meshes are usually generated to fit the physical boundaries, while these meshes are connected via interpolation (see e.g. [36, 99, 98, 199]). The disadvantage of this method is the complexity of generating the body-conformal grids. Another approach is to use Cartesian grids which do not conform to the physical boundary. The embedded boundary method is developed to solve the wave equation with Dirichlet or Neumann boundary conditions by using finite difference methods on Cartesian grids in [134, 135, 133, 170]. In [210] the authors applied this method to hyperbolic conservation laws and obtained a second order accurate scheme. Baeza et al. [7] extended the approach from second order to fifth order using Lagrange extrapolation with a filter for the detection of discontinuities. A third approach is the so called immersed boundary method, which makes a modification of the original partial differential equations by introducing a forcing function at the physical boundary, and using it to reproduce the effects of boundary conditions, see, e.g. [178, 167].

The inverse Lax-Wendroff (ILW) method is similar to the immersed boundary method, but without introducing any forcing function to alter the original equations. In [216, 217, 218], Tan and Shu developed this high order accurate boundary treatment for hyperbolic conservation laws, based on the ILW procedure for the inflow boundaries and high order extrapolation for the outflow boundaries. This boundary treatment allows us to compute hyperbolic conservation laws defined on an arbitrary domain with a Cartesian mesh to arbitrary order of accuracy.

The main idea of the ILW procedure is repeatedly using the partial differential equation to convert the normal derivatives into the time derivatives and tangential derivatives at the physical boundary. With the giving inflow boundary condition and these normal derivatives, we can use Taylor expansions to assign values to grid points or ghost points near the physical boundary. A simplified ILW procedure which uses the ILW process only for the first few normal derivatives and then the less expensive high order extrapolation for the remaining

ones is developed in [219], with good numerical results. Stability analysis for both the ILW and the simplified ILW procedures is given in [228, 146], proving that the method remains stable under the same CFL condition as that for problems without boundaries, regardless of the location of the first grid point relative to the location of the physical boundary. This indicates that the "cut cell" problem [14], which refers to the instability or the requirement of the extremely small CFL condition when the first grid point does not coincide with but is very close to the physical boundary, is effectively removed by the ILW or the simplified ILW procedure.

Let us use simple examples to show the ideas of the ILW procedure. Consider the hyperbolic equation

$$\begin{aligned} u_t + u_x &= 0, \qquad 0 \le x < \infty \\ u(0,t) &= g(t) \end{aligned} \tag{5.1}$$

with a suitable initial condition. We take uniform mesh points as $0 < x_1 < x_2 < \cdots$ where $x_{j+1} - x_j = \Delta x$ and $x_1 = a\Delta x$ where $0 < a < 1$. Note that we have deliberately put the physical boundary $x = 0$ not coinciding with the closest grid point x_1. Suppose we are using a fifth order upwind-biased scheme to approximate the derivative u_x, which involves a 6-point stencil $x_{j-3}, x_{j-2}, \cdots, x_{j+2}$, then we would need to treat the first three points x_1, x_2, x_3 specially, or put values to the three ghost points x_0, x_{-1} and x_{-2} suitably, in order to apply the scheme. The ILW procedure to put values in these points or ghost points starts with a Taylor expansion

$$u_j = \sum_{\ell=0}^{4} \frac{u^{(\ell)}(0)}{\ell!} (x_j)^\ell \tag{5.2}$$

which is fifth order accurate, for $j = 1, 2, 3$ (if we decide to put values to the first 3 grid points inside the domain) or for $j = 0, -1, -2$ (if we decide to put values to the 3 ghost points outside the computational domain). The question is how to obtain the spatial derivatives $u^{(\ell)}(0)$ at the boundary $x = 0$. We know that $u^{(0)}(0) = g(t)$ is given. Using the PDE (5.1),

we can convert the first order spatial derivative $u^{(1)}(0)$ to first order time derivative

$$u^{(1)}(0) = u_x(0,t) = -u_t(0,t) = -g'(t).$$

Likewise, by taking the time derivative of (5.1), we can convert the second order spatial derivative $u^{(2)}(0)$ to second order time derivative

$$u^{(2)}(0) = u_{xx}(0,t) = u_{tt}(0,t) = g''(t),$$

etc. Continuing in this process, we can convert all spatial derivatives at the physical boundary $x = 0$ to time derivatives through repeatedly applying the PDE, hence we can evaluate the values of the grid points or ghost points (5.2) by using only the given boundary condition $u(0,t) = g(t)$. This is very similar to the traditional Lax-Wendroff scheme, in which the time derivatives are rewritten in terms of spatial derivatives through repeatedly using the PDE. Here the roles of time and space are reversed, hence the method was referred to as the inverse Lax-Wendroff procedure.

This procedure can be applied in the general case, up to multi-dimensional nonlinear systems, with the ILW procedure coupled with high order extrapolation for outflow boundary conditions. Stability can be proved [228, 146]. Because the algebra may become very complicated for multi-dimensional nonlinear systems, a simplified ILW procedure, in which only the first few spatial derivatives are obtained by the ILW procedure and the remaining ones by high order extrapolation, is used in [219], with very good numerical results for the difficult problems of detonation computation. In [228, 146], stability analysis has been performed to exactly indicate how many spatial derivatives must be computed by the ILW procedure for different schemes in order to guarantee stability, regardless of the relative location of the first grid point to the physical boundary. This boundary treatment has been extended also to problems with moving boundaries [217] with very good performance.

So far, the ILW method has been mostly used on hyperbolic equations including conservation laws [216, 217, 218, 146] and Boltzmann type models [71], which involve only first order spatial derivatives in the equations. Very recently, Lu et al. extended this methodology

to convection-diffusion equations [157]. It turns out that this extension is highly non-trivial, as totally different boundary treatments are needed for the diffusion-dominated and the convection-dominated regimes. A careful combination of these two boundary treatments has been designed in [157], in order to obtain a stable and accurate boundary condition for high order finite difference schemes when applied to convection-diffusion equations, regardless of whether they are convection or diffusion dominant.

The ILW and SILW methods have enabled simple finite difference schemes on Cartesian meshes to be used for solving complex geometry problems with shocked solutions [216, 217, 219, 157]. These methods hold a good potential in wider applications.

6 Concluding remarks

In this survey article we concentrate on two selected classes of high order methods, namely the weighted essentially non-oscillatory (WENO) finite difference and finite volume schemes and discontinuous Galerkin (DG) finite element methods, for solving time-dependent convection-dominated partial differential equations (PDEs). We first briefly review these methods with an indication of their general development and recent applications, and then describe three topics of their recent developments. The first is the development of bound-preserving limiters for DG, finite volume and finite difference schemes, emphasizing the ability of high order schemes to maintain physical bounds without compromising their high order accuracy. The second is the WENO limiters for DG methods, which address issues in non-oscillatory performance when there are strong shocks. Finally, the third topic is an inverse Lax-Wendroff type boundary treatments for finite difference schemes on Cartesian meshes to solve PDEs in complex geometry. While details have been omitted to save space, references are given for readers interested in these developments to find them. We hope this survey paper gives evidence that high order accuracy methods are among the good choices for solving convection-dominated PDEs in science and engineering, and would thus encourages more researchers to study and apply these methods.

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