Erratum to "The primes contain arbitrarily long polynomial progressions"

by

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Erratum

We wish to issue an erratum for the paper [1]. As the paper is currently written, there is an error just before equation (73) of page 269, in which it is claimed that $O(M^{O(1/\eta_1)})$ is negligible compared to $\sqrt{N}$, which is not the case due to the way M and η_1 are constructed. However, this can be fixed by a careful tracking of the degrees of all the polynomials involved (and a corresponding modification of the polynomial forms and polynomial correlation conditions defined in Definition 3.6, page 231, and in Definition 3.9, page 232, respectively), which ends up replacing $O(M^{O(1/\eta_1)})$ by $O(M^{d_*}W^{O(1/\eta_1)})$, where d_* is the largest degree of the original polynomials $P_1, ..., P_k$.

More specifically, the following corrections need to be made:

- After the sentence following (5) on page 221, the following paragraph should be added: "Throughout this paper, we set d_* to be the largest degree of the polynomials $P_1, ..., P_k$.". On the same page, in the bullet point defining η_0, replace "$1/2d$, where d is the largest degree of the polynomials $P_1, ..., P_k$" by "$1/2d_*$". In the third bullet point, replace "$c\eta_1/d$" by "$c\eta_1/d_*$".

- In Definition 3.6 on page 231, replace "total degree at most $1/\eta_1$" by "total degree at most d_*". In Remark 3.8, delete "degree" from "degree, dimension, number and size", and replace "four parameters" by "three parameters".

- In Definition 3.9, page 232, replace "degree at most $1/\eta_1$" by "degree at most d_*".

- In Theorem 4.5 on page 238, replace "of degree $O(1)$" by "of degree at most d_*".
- In Theorem 4.7 on page 239, replace "of degree $O(1)$" by "of degree at most d_*".
- In Definition 5.1 on page 242, add the restriction that each of the polynomials R_α has degree at most d_*.
- In Proposition 5.9 on page 244, add the conclusion that Q has degree at most d_*.
- After Definition 5.11 on page 245, add the remark that in this paper, one only needs to consider weight vectors with $w_i=0$ for all $i>d_*$.
- Before the equation (73) on page 269, replace $M^{O(1/\eta_1)}$ by $M^{d_*}W^{O(1/\eta_1)}$ (two occurrences).

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References

[1] TAO, T. & ZIEGLER, T., The primes contain arbitrarily long polynomial progressions. *Acta Math.*, 201 (2008), 213–305.

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