

THE RISK-AVERSE NEWSVENDOR GAME WITH COMPETITION ON DEMAND

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ABSTRACT. This paper studies the effect of risk-aversion in the competitive newsvendor game. Multiple newsvendors with risk-averse preferences face a random demand and the demand is allocated proportionally to their inventory levels. Each newsvendor aims to maximize his expected utility instead of his expected profit. Assuming a general form of risk-averse utility function, we prove that there exists a pure Nash equilibrium in this game, and it is also unique under certain conditions. We find that the order quantity of each newsvendor is decreasing in the degree of risk-aversion and increasing in the initial wealth. Newsvendors with moderate preferences of risk-aversion make more profits compared with the risk-neutral situation. We also discuss the joint effect of risk-aversion and competition. If the effect of risk-aversion is strong enough to dominate the effect of competition, the total inventory level under competition will be lower than that under centralized decision-making.

1. Introduction. The newsvendor problem is an extensively-studied problem, which considers a newsvendor selling a single product with random demand during a short selling season (see [16], [21]). The newsvendor has a single order opportunity before the selling season, and he must decide the order quantity to balance the cost of ordering too many and the cost of ordering too little.

A critical feature of the problem is that the decision maker (newsvendor) faces a risky income which is associated with demand uncertainty. In the classical model, the decision maker is assumed to be risk-neutral and will decide his order quantity to maximize his expected profit. However, decision maker's risk attitude towards income uncertainty plays an important role in the decisions (e.g. [32], [33]). In reality, not all decision makers are risk-neutral. Some evidences (see e.g. [3], [11], [13], [23]) suggest that the manager's decision may not be consistent with expected

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profit maximization. Many of them are willing to avoid risky outcomes at the expense of lower expected profits. For instance, 3M Co. would manufacture its new products in their original plants through a complicated, time-consuming and more expensive process, rather than set up a new plant, to avoid the risk that the products may not be accepted by the market [13]. Another example is Zara, a fast fashion retailer, which designed a hyper fast supply chain at a great cost to reduce demand risk [11]. The propensity to prefer a certain but possibly less desirable outcome over an uncertain but potentially greater outcome is called risk-aversion. Schweitzer and Cachon [23] suggest that for high-profit products, decision makers exhibit risk-averse behavior in a newsvendor experiment. Holt and Laury [12] design a laboratory experiment to examine decision makers' risk attitudes which shows strong evidence for risk-aversion.

In view of this, many researchers establish newsvendor models under the assumption of risk-aversion. Keren and Pliskin [15] obtain the first-order optimality conditions of the risk-averse newsvendor problem. They get a closed form solution of a special newsvendor problem in which the demand follows a uniform distribution, and suggest that the closed form solution can be used to assess the newsvendor utility function parameters. Wang et al. [28] find that the risk-averse newsvendor's order quantity can be arbitrarily small while the retail price is high enough. Eeckhoudt et al. [7] study a risk-averse newsvendor model in which the newsvendor has an additional order opportunity. They examine how the price, cost and some other parameters affect the newsvendor's decision. Agrawal and Seshadri [2] extend Eeckhoudt's model to a more general case in which a risk-averse newsvendor facing a random and price-dependent demand has to decide the inventory level and the selling price of a product. They find that under different price-dependent demand assumptions the newsvendor's decisions on the price and order quantity might have quite distinct properties. Chen et al. [6] incorporate risk-aversion into a multi-period inventory model. They depict the structure of the optimal order policy of this problem. The aforementioned papers all concentrate on the case with a single newsvendor. But many consumer goods, like electronics, food, fashion clothes, are sold by multiple newsvendors. In this paper, we attempt to investigate the case with multiple competing risk-averse newsvendors and discuss the effect of competition.

Competition between newsvendors commonly exists in our daily life, and becomes fierce nowadays, for example, Whole Foods Market Inc., who runs the specialty-foods business, has newer competitors such as Sprouts Farmers Market Inc.. Its natural and organic supermarket chain lowered the annual sales projection four times in nine months as it struggles within intensifying competition [10]. One of the commonly existing forms of competition is that dependent on inventory level. When there are multiple newsvendors selling competitive products, the market share of each newsvendor can be affected by others' inventory levels. Thus, lots of papers devote to investigating competitive newsvendor problem. Parlar [20] first discusses this topic and establishes a two-firm competitive newsvendor model in which two firms face independent random demands and unsatisfied demand will be reallocated between them. He shows that the expected profit in a centralized system exceeds that in the decentralized one. Lippman and McCardle [17] investigate a competitive newsvendor problem in which the random demand is allocated among multiple newsvendors with certain demand splitting rules. They find that the competition never leads to a decrease in inventory level if all the unsatisfied demand is reallocated. Cachon [5] studies a competition model among risk-neutral newsvendors

where demand is allocated proportionally to their inventory levels. He points out that due to the demand-stealing effect (i.e. ordering more means the other newsvendors' demands stochastically decrease), competition induces the newsvendors to order more. Wang [27] extends Cachon's [5] model to the loss-averse situation and investigates the joint effect of loss-aversion and competition. Liu et al. [18] examine the influence of loss-aversion in the game between two newsvendors selling substitutable products. They show the existence and uniqueness of equilibrium and explain how the equilibrium changes as loss-aversion coefficient or substitution rate increases. Wu et al. [30] investigate the quantity and price competition among newsvendors under the conditional value-at-risk (CVaR) criterion. In our paper, we focus on the effects of risk-aversion of newsvendors on the optimal decisions under the expected utility (EU) framework.

As aforementioned, the newsvendor's risk attitude is a factor that could not be ignored. Many experimental evidences show that most people are risk-averse when they face uncertain profits. In view of this, we introduce risk-aversion into the competitive newsvendor model in this research, and concern with the the following questions: (1) What are the optimal order strategies of the risk-averse newsvendors competing on demand? (2) How do the newsvendors' decisions vary with their degree of risk aversion? (3) How does the competition affect the order quantities of risk-averse newsvendors? In order to answer these questions, we establish a game theoretical model with multiple risk-averse newsvendors among whom the demand is split proportionally to their inventory levels. The risk-aversion is modeled within the EU framework, and we adopt a general form of utility function. We demonstrate the existence of Nash equilibrium, which is unique under a fairly general condition. We further analyze the impact of risk-aversion on the newsvendors' order decisions and show that a higher degree of risk-aversion or lower initial wealth will reduce newsvendors' order quantities. We also observe the demand-stealing effect which increases newsvendors' total order quantity in equilibrium in our model. The joint effect of risk-aversion and demand-stealing is also discussed in this paper. If the risk-aversion effect which decreases newsvendors' total order quantity is strong enough to dominate the demand-stealing effect, the total order quantity under competition is lower than that under centralized decision-making.

The rest of this article is organized as follows. In Section 2, we establish the competitive newsvendor model in which the newsvendors are risk-averse and show the existence and uniqueness of Nash equilibrium. The impacts of some key factors on the equilibrium, such as competition and degree of risk-aversion, will be discussed in Section 3. We conclude this article in Section 4.

2. The basic model. We consider $n(n \geq 2)$ identical, competing and risk-averse newsvendors with initial wealth $z_0 \geq 0$. The newsvendors order a short-life-circle product from an outside supplier at a unit wholesale price w before the selling season and sell the product at a unit retail price $p(p > w)$. The salvage value of one unit unsold inventory at the end of the selling season is v . Without loss of generality we assume $v = 0$. The market demand X is a random variable with probability density function (PDF) $f(x)$ and cumulative density function (CDF) $F(x)$. The supports of $F(x)$ and $f(x)$ are assumed to be contained in $[0, \infty)$. $F(x)$ is differentiable and increasing. Denote $\bar{F}(x) = 1 - F(x)$. We assume the expected value of the demand to be finite, i.e., $E(X) < \infty$. Denote Q_i and Q_{-i} as the order quantity (inventory level) of the i th newsvendor and sum of other $n - 1$ newsvendors' order quantities, respectively. The total order quantity $Q = Q_i + Q_{-i}$.

We assume that the demand X is divided among the n newsvendors proportionally to their order quantities, i.e., newsvendor i 's demand $X_i = (Q_i/Q) \cdot X$. This allocation rule is known as the proportional demand allocation rule, under which an increase in one newsvendor's inventory raises his allocated demand and the probability of reaching a certain volume of sales. This is consistent with the fact observed in practice, i.e., large amounts of products on display stimulates demand and increases the volume of sales. For example, Wolfe [26] indicates that within the selling season of fashion items, such as women's dresses, unit sales of each item are proportional to the amount of inventory displayed. Generally, there is a positive correlation between the displays and the inventories [31], thus higher inventory levels stimulate sales of retail items [25]. Moreover, Cachon [5] points out that this rule is reasonable when customers have low search costs, for example, online shopping. This rule is also widely seen in the literature (e.g. [5], [27], [30], [31]).

Under this rule, X_i 's PDF is denoted as

$$f_i(x_i) = \frac{Q_i + Q_{-i}}{Q_i} f\left(\frac{Q_i + Q_{-i}}{Q_i} x_i\right).$$

The i th newsvendor's profit $\Pi_i(\mathbf{Q})$ is a function of all the newsvendors' order quantities $\mathbf{Q} = (Q_1, Q_2, \dots, Q_n)$,

$$\Pi_i(\mathbf{Q}) = p \min(Q_i, X_i) - wQ_i.$$

All the newsvendors are risk-averse, and their utility function $u(z)$ with respect to wealth z is concave and monotone increasing. For analytical ease, $u(z)$ is assumed to be thrice differentiable, satisfying $u'(z) > 0$ and $u''(z) < 0$. Let $r(z) = -u''(z)/u'(z)$ denote the coefficient of absolute risk aversion and $r^*(z) = zr(z)$ denote the coefficient of relative risk aversion [19]. $r(z)$ measures an individual's degree of risk-aversion with respect to wealth z . $r^*(z)$ measures the degree of risk-aversion with regard to risky assets that are proportional to the wealth. If $r(z)$ is decreasing in z , $u(z)$ is called to satisfy the property of decreasing absolute risk aversion (DARA). If $r(z)$ is independent of z , $u(z)$ is called to satisfy the property of constant absolute risk aversion (CARA). If $r^*(z)$ is increasing in z , $u(z)$ is called to satisfy the property of increasing relative risk aversion (IRRA). If $r^*(z)$ is independent of z , $u(z)$ is called to satisfy the property of constant relative risk aversion (CRRA). Since the newsvendors' demands are perfectly correlated with their inventories [5], the expected utility function of newsvendor i is given by

$$U_i(\mathbf{Q}) = \int_0^Q u\left(z_0 + \frac{Q_i}{Q_i + Q_{-i}} px - wQ_i\right) f(x) dx + \int_Q^\infty u(z_0 + pQ_i - wQ_i) f(x) dx. \tag{1}$$

Lemma 2.1. *For any given $\mathbf{Q}_{-i} = (Q_1, Q_2, \dots, Q_{i-1}, Q_{i+1}, \dots, Q_n) \geq \mathbf{0}$, $U_i(\mathbf{Q})$ is continuous and concave in Q_i , and there exists a unique optimal order quantity $Q_i^*(\mathbf{Q}_{-i})$ satisfying the following first-order optimality condition*

$$(p - w) \bar{F}(Q_i^*(\mathbf{Q}_{-i}) + Q_{-i}) u'(z_0 + (p - w) Q_i^*(\mathbf{Q}_{-i})) + \int_0^{Q_i^*(\mathbf{Q}_{-i}) + Q_{-i}} \left\{ u'(z_0 + \frac{Q_i^*(\mathbf{Q}_{-i})}{Q_i^*(\mathbf{Q}_{-i}) + Q_{-i}} px - wQ_i^*(\mathbf{Q}_{-i})) \cdot f(x) \left(\frac{pQ_{-i}x}{(Q_i^*(\mathbf{Q}_{-i}) + Q_{-i})^2} - w \right) \right\} dx = 0. \tag{2}$$

Proof. According to (1), we have

$$\begin{aligned} \frac{dU_i(\mathbf{Q})}{dQ_i} &= (p - w)\bar{F}(Q_i + Q_{-i})u'(z_0 + pQ_i - wQ_i) \\ &+ \int_0^{Q_i+Q_{-i}} u'(z_0 + \frac{Q_i}{Q_i + Q_{-i}}px - wQ_i)f(x)(p\frac{Q_{-i}x}{(Q_i + Q_{-i})^2} - w)dx, \end{aligned}$$

and

$$\begin{aligned} \frac{d^2U_i(\mathbf{Q})}{dQ_i^2} &= (p - w)^2\bar{F}(Q_i + Q_{-i})u''(z_0 + pQ_i - wQ_i) \\ &- p\frac{Q_i}{Q_i + Q_{-i}}u'(z_0 + pQ_i - wQ_i)f(Q_i + Q_{-i}) \\ &+ \int_0^{Q_i+Q_{-i}} u''(z_0 + \frac{Q_i}{Q_i + Q_{-i}}px - wQ_i)f(x)(p\frac{Q_{-i}x}{(Q_i + Q_{-i})^2} - w)^2dx \\ &- \frac{2p}{(Q_i + Q_{-i})^3} \int_0^{Q_i+Q_{-i}} u'(z_0 + \frac{Q_i}{Q_i + Q_{-i}}px - wQ_i)f(x)xdx. \end{aligned}$$

It's easy to see that $\frac{d^2U_i(\mathbf{Q})}{dQ_i^2} < 0$ since $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Thus $U_i(\mathbf{Q})$ is concave in Q_i . So the optimal order quantity $Q_i^*(\mathbf{Q}_{-i})$ satisfies the first-order optimality condition (2). \square

Lemma 2.1 characterizes the best response function of newsvendor i . A vector of order quantities $\mathbf{Q}^* = (Q_1^*, Q_2^*, \dots, Q_n^*)$ is a Nash equilibrium of the risk-averse competitive newsvendor game if each newsvendor's order quantity is a best response to others' order quantities, i.e., $Q_i^* = Q_i^*(\mathbf{Q}_{-i})$ for all i .

Theorem 2.2. *There exists at least one pure strategy Nash equilibrium \mathbf{Q}^* in the risk-averse competitive newsvendor game.*

Proof. Newsvendors' strategies are their order quantities $Q_i \in [0, M]$ where M is a large enough upper bound which never constrains the newsvendors. Strategy space in this game can be written as a compact and convex set $[0, M] \times [0, M] \times \dots \times [0, M]$. By Lemma 1, $U_i(Q_i, \mathbf{Q}_{-i})$ is continuous and concave in $Q_i (i = 1, 2, \dots, n)$. Thus, there exists at least one pure Nash equilibrium by Theorem 1.2 in Fudenberg and Tirole [8]. \square

As the game between newsvendors discussed in this paper is a symmetric game, we now concentrate on the symmetric Nash equilibrium. We first introduce one basic assumption and a notation.

Assumption 1. *The utility function $u(z)$ satisfies the property of DARA or CARA.*

This assumption means that the newsvendors become less risk-averse when they own more money, that is, they feel like to pay less for insurance against a given risk when they become wealthier [19]. Arrow [1] first notices that the absolute risk aversion should be decreasing, and his assertion is supported by lots of studies aiming to measure individual risk attitude. Wik et al. show strong evidence for DARA by using an experimental gambling approach to measure risk attitudes of households in Northern Zambia [29]. An empirical study aiming to explore attitudes towards risk among farmers in Nepal also provides support for the property of DARA [14]. Saha et al. find that the absolute risk aversion of some Kansas wheat farmers decreases in their wealth [22]. The most commonly used utility functions

in the literature, like $\ln z, z^\alpha (0 < \alpha < 1), 1 - e^{-rz} (r > 0)$ all satisfy Assumption 1. This assumption is not too restrictive [4], for the reason that it is consistent with everyday observation as well as experimental studies.

Define function $A(Q, n, z_0)$ as follows:

$$A(Q, n, z_0) = (p - w)\bar{F}(Q) + \int_0^Q \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p - w)\frac{Q}{n})} f(x)(p\frac{(n - 1)x}{nQ} - w)dx. \tag{3}$$

Theorem 2.3. *Under Assumption 1, there exists a unique symmetric pure strategy Nash equilibrium $Q^* = (Q_1^*, Q_2^*, \dots, Q_n^*)$, where $Q_1^* = Q_2^* = \dots = Q_n^* = Q^*/n$, and Q^* satisfies the following equation*

$$(p - w)\bar{F}(Q^*)u'(z_0 + (p - w)\frac{Q^*}{n}) + \int_0^{Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n - 1)x}{nQ^*} - w)dx = 0, \tag{4}$$

or equally $A(Q^*, n, z_0) = 0$.

Proof. For a symmetric equilibrium $Q_i^* = Q_j^*$ for any $i, j = 1, 2, \dots, n$, we have $Q^* = nQ_i^*$ and $Q_{-i}^* = (n - 1)Q_i^*$. Replacing $Q_i^*(Q_{-i}^*)$ and Q_{-i} with Q_i^* and Q_{-i}^* in (2), we obtain equation (4). As $u'(\cdot) > 0$, $A(Q^*, n, z_0) = 0$ is equivalent to (4). To complete the proof, we need to show that there exists a unique $Q^* \in (0, \infty)$ satisfying $A(Q^*, n, z_0) = 0$ for fixed n and z_0 . Denote

$$B(Q) = (p - w)\bar{F}(Q)u'(z_0 + (p - w)\frac{Q}{n}) + \int_0^Q u'(z_0 + \frac{px}{n} - \frac{wQ}{n})f(x)(p\frac{(n - 1)x}{nQ} - w)dx.$$

First we claim that there is at least one root of equation $B(Q) = 0$ in $(0, \infty)$. $\lim_{Q \rightarrow 0} B(Q) > 0$ is trivial. Then we show $\lim_{Q \rightarrow \infty} B(Q) < 0$. Since

$$\int_{\frac{wQ}{p}}^{\frac{nwQ}{(n-1)p}} u'(z_0 + \frac{px}{n} - \frac{wQ}{n})f(x)(p\frac{(n - 1)x}{nQ} - w)dx \leq 0,$$

we have

$$B(Q) \leq (p - w)\bar{F}(Q)u'(z_0 + (p - w)\frac{Q}{n}) + (\int_0^{\frac{wQ}{p}} + \int_{\frac{nwQ}{(n-1)p}}^Q)u'(z_0 + \frac{px}{n} - \frac{wQ}{n})f(x)(p\frac{(n - 1)x}{nQ} - w)dx.$$

Since $u'(z)$ is a decreasing function,

$$B(Q) \leq (p - w)\bar{F}(Q)u'(z_0 + (p - w)\frac{Q}{n}) + u'(z_0) \int_0^{\frac{wQ}{p}} f(x)(p\frac{(n - 1)x}{nQ} - w)dx + u'(z_0 + \frac{wQ}{n(n - 1)}) \int_{\frac{nwQ}{(n-1)p}}^Q f(x)(p\frac{(n - 1)x}{nQ} - w)dx.$$

We find that

$$\begin{aligned}
\lim_{Q \rightarrow \infty} B(Q) &\leq \lim_{Q \rightarrow \infty} (p-w)\bar{F}(Q)u'(z_0 + (p-w)\frac{Q}{n}) \\
&\quad + \lim_{Q \rightarrow \infty} u'(z_0) \int_0^{\frac{wQ}{p}} f(x)(p\frac{(n-1)x}{nQ} - w)dx \\
&\quad + \lim_{Q \rightarrow \infty} u'(z_0 + \frac{wQ}{n(n-1)}) \int_{\frac{nwQ}{(n-1)p}}^Q f(x)(p\frac{(n-1)x}{nQ} - w)dx \\
&= u'(z_0) \lim_{Q \rightarrow \infty} \int_0^{\frac{wQ}{p}} f(x)(p\frac{(n-1)x}{nQ} - w)dx \\
&= -wu'(z_0) < 0.
\end{aligned}$$

Thus the equation $B(Q) = 0$ has a root in $(0, \infty)$, which implies that there exists a $Q^* > 0$ satisfying $A(Q^*, n, z_0) = 0$. We still need to claim that Q^* is unique. To show the uniqueness, we first claim $\partial A(Q^*, n, z_0)/\partial Q < 0$ for any Q^* satisfying $A(Q^*, n, z_0) = 0$.

According to (3), we have

$$\begin{aligned}
\frac{\partial A(Q, n, z_0)}{\partial Q} &= \int_0^Q \left\{ \left(\frac{w}{n}r(z_0 + \frac{px}{n} - \frac{wQ}{n}) + \frac{p-w}{n}r(z_0 + (p-w)\frac{Q}{n}) \right) \right. \\
&\quad \cdot \left. \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x)(p\frac{(n-1)x}{nQ} - w) \right\} dx \\
&\quad - \frac{(n-1)p}{nQ^2} \int_0^Q \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x)xdx - \frac{p}{n}f(Q).
\end{aligned}$$

If $nw \geq (n-1)p$, $\partial A(Q^*, n, z_0)/\partial Q < 0$ for the reason that $u'(\cdot) > 0$ and $r(\cdot) > 0$. Then we consider the situation in which $nw < (n-1)p$.

Since $r(z)$ is non-increasing with respect to z , if $x \in [0, \frac{nw}{(n-1)p}Q]$, we have $p\frac{(n-1)x}{nQ} - w < 0$ and

$$\begin{aligned}
&\frac{w}{n}r(z_0 + \frac{px}{n} - \frac{wQ}{n}) + \frac{p-w}{n}r(z_0 + (p-w)\frac{Q}{n}) \\
&\geq \frac{w}{n}r(z_0 + \frac{wQ}{n(n-1)}) + \frac{p-w}{n}r(z_0 + (p-w)\frac{Q}{n}).
\end{aligned} \tag{5}$$

If $x \in [\frac{nw}{(n-1)p}Q, Q]$, we have $p\frac{(n-1)x}{nQ} - w \geq 0$ and

$$\begin{aligned}
&\frac{w}{n}r(z_0 + \frac{px}{n} - \frac{wQ}{n}) + \frac{p-w}{n}r(z_0 + (p-w)\frac{Q}{n}) \\
&\leq \frac{w}{n}r(z_0 + \frac{wQ}{n(n-1)}) + \frac{p-w}{n}r(z_0 + (p-w)\frac{Q}{n}).
\end{aligned} \tag{6}$$

Combining (5) with (6), we obtain

$$\begin{aligned}
\frac{\partial A(Q, n, z_0)}{\partial Q} &\leq \int_0^Q \left\{ \left(\frac{w}{n}r(z_0 + \frac{px}{n} - \frac{wQ}{n}) + \frac{(p-w)}{n}r(z_0 + (p-w)\frac{Q}{n}) \right) \right. \\
&\quad \cdot \left. \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x)(p\frac{(n-1)x}{nQ} - w) \right\} dx
\end{aligned}$$

$$\begin{aligned} &\leq \left(\frac{w}{n}r\left(z_0 + \frac{wQ}{n(n-1)}\right) + \frac{p-w}{n}r\left(z_0 + (p-w)\frac{Q}{n}\right)\right) \\ &\quad \cdot \int_0^Q \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x) \left(p\frac{(n-1)x}{nQ} - w\right) dx. \end{aligned}$$

Since Q^* satisfies $A(Q^*, n, z_0) = 0$, we have

$$\int_0^{Q^*} \frac{u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})}{u'(z_0 + (p-w)\frac{Q^*}{n})} f(x) \left(p\frac{(n-1)x}{nQ^*} - w\right) dx = -(p-w)\bar{F}(Q^*),$$

and then

$$\begin{aligned} \frac{\partial A(Q^*, n, z_0)}{\partial Q} &\leq -(p-w)\bar{F}(Q^*) \\ &\quad \cdot \left(\frac{w}{n}r\left(z_0 + \frac{wQ^*}{n(n-1)}\right) + \frac{p-w}{n}r\left(z_0 + (p-w)\frac{Q^*}{n}\right)\right) \\ &< 0. \end{aligned}$$

$\partial A(Q^*, n, z_0)/\partial Q < 0$ always holds. It's ready to show the uniqueness of Q^* . If $A(Q^*, n, z_0) = 0$ has more than one root, denote $Q_s^* < Q_l^*$ as two adjacent roots. Since $\partial A(Q^*, n, z_0)/\partial Q < 0$, there exist a $Q_s \in (Q_s^*, Q_s^* + \varepsilon)$ and a $Q_l \in (Q_l^* - \varepsilon, Q_l^*)$, s.t. $A(Q_s, n, z_0) < 0$ and $A(Q_l, n, z_0) > 0$, where $\varepsilon > 0$ is a sufficiently small real number. Thus we must have a $Q_m^* \in (Q_s, Q_l)$ satisfying $A(Q_m^*, n, z_0) = 0$, which contradicts with the fact that Q_s^* and Q_l^* are adjacent roots. This completes the proof. \square

Theorem 2.3 characterizes the unique symmetric equilibrium $(Q_1^*, Q_2^*, \dots, Q_n^*)$, and the total order quantity in equilibrium Q^* of all newsvendors in the risk-averse newsvendor game.

3. Statics analysis. In this section, we examine the effects of degree of risk-aversion, the initial wealth and number of newsvendors on total order quantity in equilibrium, newsvendors' expected profits and utilities.

3.1. Effect of degree of risk-aversion. As aforementioned, newsvendors' risk attitudes play a vital role in their decisions. Therefore we investigate how the degree of risk-aversion affect newsvendors' decisions in this part. A person is said to be more risk-averse than another implies that he has greater coefficient of absolute risk aversion, i.e. he has larger $r(z)$ at all z .

Proposition 1. *Under Assumption 1, the total order quantity in equilibrium Q^* decreases while the newsvendors become more risk-averse.*

Proof. An increase in degree of risk-aversion is equivalent to a concave transformation of the utility function [19]. Consider $\hat{u}(z) = k(u(z))$ where $k(\cdot)$ is a concave function. Q^* and \hat{Q}^* are the equilibrium total order quantities with utility functions $u(z)$ and $\hat{u}(z)$ respectively. To complete the proof, we need to demonstrate $\hat{Q}^* \leq Q^*$.

Denote $G(Q)$ as follows

$$\begin{aligned} G(Q) &= (p-w)\bar{F}(Q)\hat{u}'\left(z_0 + (p-w)\frac{Q}{n}\right) \\ &\quad + \int_0^Q \hat{u}'\left(z_0 + \frac{px}{n} - \frac{wQ}{n}\right) f(x) \left(p\frac{(n-1)x}{nQ} - w\right) dx \end{aligned}$$

$$\begin{aligned}
 &= (p-w)\bar{F}(Q)k'(u(z_0 + (p-w)\frac{Q}{n}))u'(z_0 + (p-w)\frac{Q}{n}) \\
 &+ \int_0^Q k'(u(z_0 + \frac{px}{n} - \frac{wQ}{n}))u'(z_0 + \frac{px}{n} - \frac{wQ}{n})f(x)(p\frac{(n-1)x}{nQ} - w)dx.
 \end{aligned}$$

According to Theorem 2.3, there is a unique \hat{Q}^* satisfying $G(\hat{Q}^*) = 0$. If $G(Q^*) \leq 0$, we can conclude that $Q^* \geq \hat{Q}^*$ due to the following two facts (1) $\forall Q < \hat{Q}^*$, $G(Q) > 0$, (2) $\forall Q > \hat{Q}^*$, $G(Q) < 0$.

Thus we only need to prove $G(Q^*) \leq 0$. To finish it, we discuss the following 2 cases.

Case 1. $nw \geq (n-1)p$.

$$\begin{aligned}
 G(Q^*) &\leq k'(u(z_0 + (p-w)\frac{Q^*}{n}))\{(p-w)\bar{F}(Q^*)u'(z_0 + (p-w)\frac{Q^*}{n}) \\
 &+ \int_0^{Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx\} \\
 &= 0.
 \end{aligned}$$

Case 2. $nw < (n-1)p$.

According to the second mean value theorem for integrals, there exist $x_1 \in [0, \frac{nw}{(n-1)p}Q^*)$ and $x_2 \in [\frac{nw}{(n-1)p}Q^*, Q^*]$, s.t.

$$\begin{aligned}
 &\int_0^{\frac{nw}{(n-1)p}Q^*} k'(u(z_0 + \frac{px}{n} - \frac{wQ^*}{n}))u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx \\
 &= k'(u(z_0 + \frac{px_1}{n} - \frac{wQ^*}{n})) \int_0^{\frac{nw}{(n-1)p}Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx,
 \end{aligned} \tag{7}$$

and

$$\begin{aligned}
 &\int_{\frac{nw}{(n-1)p}Q^*}^{Q^*} k'(u(z_0 + \frac{px}{n} - \frac{wQ^*}{n}))u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx \\
 &= k'(u(z_0 + \frac{px_2}{n} - \frac{wQ^*}{n})) \int_{\frac{nw}{(n-1)p}Q^*}^{Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx.
 \end{aligned} \tag{8}$$

By (7) and (8), we can rewrite $G(Q^*)$ as

$$\begin{aligned}
 G(Q^*) &= (p-w)\bar{F}(Q^*)k'(u(z_0 + (p-w)\frac{Q^*}{n}))u'(z_0 + (p-w)\frac{Q^*}{n}) \\
 &+ k'(u(z_0 + \frac{px_1}{n} - \frac{wQ^*}{n})) \\
 &\cdot \int_0^{Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx \\
 &+ [k'(u(z_0 + \frac{px_2}{n} - \frac{wQ^*}{n})) - k'(u(z_0 + \frac{px_1}{n} - \frac{wQ^*}{n}))] \\
 &\cdot \int_{\frac{nw}{(n-1)p}Q^*}^{Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx.
 \end{aligned}$$

Since $x_1 < x_2 < Q^*$, we have $k'(u(z_0 + \frac{px_1}{n} - \frac{wQ^*}{n})) > k'(u(z_0 + \frac{px_2}{n} - \frac{wQ^*}{n})) > k'(u(z_0 + \frac{px}{n} - \frac{wQ^*}{n}))$. Then we obtain

$$\begin{aligned}
G(Q^*) &\leq (p-w)\bar{F}(Q^*)u'(z_0 + (p-w)\frac{Q^*}{n}) \\
&\quad \cdot [k'(u(z_0 + (p-w)\frac{Q^*}{n})) - k'(u(z_0 + \frac{px_1}{n} - \frac{wQ^*}{n}))] \\
&\quad + k'(u(z_0 + \frac{px_1}{n} - \frac{wQ^*}{n}))\{(p-w)\bar{F}(Q^*)u'(z_0 + (p-w)\frac{Q^*}{n}) \\
&\quad + \int_0^{Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx\} \\
&\leq k'(u(z_0 + \frac{px_1}{n} - \frac{wQ^*}{n}))\{(p-w)\bar{F}(Q^*)u'(z_0 + (p-w)\frac{Q^*}{n}) \\
&\quad + \int_0^{Q^*} u'(z_0 + \frac{px}{n} - \frac{wQ^*}{n})f(x)(p\frac{(n-1)x}{nQ^*} - w)dx\} \\
&= 0.
\end{aligned}$$

We have therefore proved the proposition. \square

Proposition 1 indicates that when the newsvendors become more conservative, they would like to lower their order quantities to avoid more risky payoffs. Thus the supplier tends to cooperate with less risk-averse newsvendors.

Denote Q_0 as the total order quantity in equilibrium when newsvendors are risk-neutral. By Theorem 2.3, Q_0 satisfies

$$(p-w)\bar{F}(Q_0) + \int_0^{Q_0} f(x)(p\frac{(n-1)x}{nQ_0} - w)dx = 0.$$

We can infer from Proposition 1 that $Q^* \leq Q_0$, i.e., the risk-averse newsvendors order less than the risk-neutral ones, which is consistent with the result of Eeckhoudt et al. [7].

Now we discuss how the newsvendors' degrees of risk-aversion affect their profits. Denote $E\Pi_i(Q)$ as the expected profit of newsvendor i ,

$$E\Pi_i(Q) = \int_0^Q (\frac{px}{n} - \frac{wQ}{n})f(x)dx + \int_Q^\infty \frac{(p-w)Q}{n}f(x)dx.$$

It's concave with respect to Q and maximized at $Q_c = F^{-1}(\frac{p-w}{p})$. Moreover it is decreasing if $Q < Q_c$ and increasing if $Q > Q_c$. We can easily derive the following two facts, $Q_0 > Q_c$ and existence of a $\hat{Q}_0 \in (0, Q_c)$ s.t. $E\Pi_i(\hat{Q}_0) = E\Pi_i(Q_0)$. Thus we have $E\Pi_i(Q) > E\Pi_i(Q_0)$ if $Q \in (\hat{Q}_0, Q_0)$, and $E\Pi_i(Q) < E\Pi_i(Q_0)$ if $Q \in (0, \hat{Q}_0)$.

The above analysis shows that the newsvendors' expected profits increase firstly and then decrease with respect to their degrees of risk-aversion. Moreover, comparing with the risk-neutral situation, the newsvendors benefit from their moderate preferences of risk-aversion (which lead to $Q^* \in (\hat{Q}_0, Q_0)$). The reason is that in a competitive environment, risk-neutral newsvendor tends to keep higher inventory than Q_c to occupy market share (demand-stealing effect), and his expected profit becomes lower than optimal one. However, higher inventory means higher income uncertainty which the risk-averse newsvendor is unwilling to take. Thus he reduce his order quantity appropriately due to moderate preference of risk-aversion and obtain more profits. But when newsvendors are excessively risk-averse, their total order quantity is less \hat{Q}_0 and expected profits become lower than the risk-neutral newsvendors, for the reason that they miss too many potential gains.

Corollary 1. *If $u(z) = 1 - \exp(-rz)$ where the coefficient of absolute risk aversion r is a constant, i.e. satisfying CARA, the expected utility of i th newsvendor $U_i(Q^*)$ increases in r .*

Proof. By (1) and Theorem 2.3, the expected utility of newsvendor i can be rewrite as

$$\begin{aligned} &U_i(Q^*) \\ &= \int_0^{Q^*} u(z_0 + \frac{px - wQ^*}{n})f(x)dx + \int_{Q^*}^{\infty} u(z_0 + \frac{(p-w)Q^*}{n})f(x)dx \\ &= \int_0^{Q^*} (1 - \frac{u'(z_0 + \frac{px-wQ}{n})}{r})f(x)dx + \int_{Q^*}^{\infty} (1 - \frac{u'(z_0 + \frac{(p-w)Q^*}{n})}{r})f(x)dx \\ &= 1 - \frac{1}{r} (\int_0^{Q^*} u'(z_0 + \frac{px - wQ^*}{n})f(x)dx + \int_{Q^*}^{\infty} u'(z_0 + \frac{(p-w)Q^*}{n})f(x)dx) \\ &= 1 - \frac{1}{r(p-w)} (\int_0^{Q^*} (p - \frac{p(n-1)x}{nQ^*})u'(z_0 + \frac{px - wQ^*}{n})f(x)dx). \end{aligned}$$

The last equal sign holds for the reason that Q^* satisfies equation (4). Denote $M(Q) = \int_0^Q (p - \frac{p(n-1)x}{nQ})u'(z_0 + \frac{px-wQ}{n})f(x)dx$. Now we show that $M(Q)$ increases in Q .

$$\begin{aligned} M'(Q) &= \frac{1}{n}u'(z_0 + \frac{(p-w)Q}{n})f(Q) + \int_0^Q u'(z_0 + \frac{px - wQ}{n})f(x) \frac{(n-1)x}{nQ^2} dx \\ &\quad - \frac{w}{n} \int_0^Q u''(z_0 + \frac{px - wQ}{n})f(x)(1 - \frac{(n-1)x}{nQ})dx > 0. \end{aligned}$$

Thus $M(Q)$ increases in Q . Further we have $M(Q^*)$ decreases in r by Proposition 1. Now we can conclude that $U_i(Q^*) = 1 - \frac{p}{r(p-w)}M(Q^*)$ increases in r . This corollary is proved. □

Corollary 1 shows that when newsvendors’ degrees of risk-aversion are insensitive to wealth, more conservative newsvendors are more likely to be satisfied, although their expected profits become less when their degrees of risk-aversion are high enough. This is because that higher inventory level induces a more risky profit with higher expected value. Those more conservative newsvendors express strong willingness to avert risk at the price of lower expected profit. Thus they tend to pay more for insurance against the risky incomes, and choose order quantities associating with lower expected profits. Avoiding risky incomes provides newsvendors with greater satisfactions, and increases their utilities.

3.2. Effect of initial wealth. The newsvendor’s degrees of risk-aversion is relevant to their wealth which represents his economic strength. We study how the initial wealth impacts on his order decision in this subsection.

Proposition 2. *If the utility function satisfies DARA, the total order quantity in equilibrium Q^* increases in initial wealth z_0 . If the utility function satisfies CARA, Q^* is independent of z_0 .*

Proof. According to (3), we have

$$\begin{aligned} \frac{\partial A(Q, n, z_0)}{\partial z_0} &= \int_0^Q \left\{ r\left(z_0 + (p-w)\frac{Q}{n}\right) - r\left(z_0 + \frac{px}{n} - \frac{wQ}{n}\right) \right. \\ &\quad \left. \cdot \frac{u'\left(z_0 + \frac{px}{n} - \frac{wQ}{n}\right)}{u'\left(z_0 + (p-w)\frac{Q}{n}\right)} f(x) \left(p\frac{(n-1)x}{nQ} - w\right) \right\} dx. \end{aligned}$$

If the utility function $u(z)$ satisfies CARA, then $\partial A(Q, n, z_0)/\partial z_0 = 0$; hence Q^* is independent of z_0 .

If the utility function satisfies DARA, higher initial wealth z_0 implies lower degree of risk-aversion [19]. By Proposition 1, we come to the conclusion that Q^* increases in z_0 . \square

If the property of CARA is satisfied, newsvendors' order quantities are independent of their wealth since their preferences of risk-aversion are irrelevant to wealth. Under the assumption of DARA, the wealthier newsvendors order more for the reason that they are less risk-aversion. From the perspective of the supplier, he prefers to cooperate with economically strong newsvendors for their high order quantities.

If $\lim_{z \rightarrow \infty} r(z) = 0$, i.e. newsvendors tend to be risk-neutral when their wealth go to infinity, we have $\lim_{z_0 \rightarrow \infty} Q^* = Q_0$. Hence there exists a z_0^* s.t. $Q^*|_{z_0=z_0^*} = Q_c$. Recall that Q_c maximizes the expected profit of each newsvendor, then we have that $E\Pi_i(Q^*)$ increases in z_0 if $z_0 < z_0^*$ and decreases in z_0 if $z_0 > z_0^*$. For those newsvendors with weak economic strength, more initial wealth induces higher expected profits, and the whole supply chain results in a win-win outcome. However, newsvendors do not always benefit from their economic strength. When newsvendors are wealthy enough, more wealth leads to lower expected profits since they order too much to gain market shares, and the whole supply chain results in a win-lose outcome.

3.3. Effect of the level of competition. The number of competitors indicates that how intense the competition is. The level of competition among the newsvendors affects their demand and further affects their order decisions. This effect is studied in this subsection. We first introduce the following assumption.

Assumption 2. *The utility function $u(z)$ satisfies the property of IRRA or CRRA.*

The property of IRRA or CRRA means that the relative risk aversion should be non-decreasing, which is consistent with the hypothesis of Arrow [1] — if both wealth and the size of bet are increased in the same proportion, the willingness to accept the bet should not increase. This hypothesis is supported by many studies aiming to measure individual risk attitude. Friend and Blume [9] empirically show that CRRA is a fairly accurate description for wealthy households in U.S.. Siegel and Hoban [24] present another empirical study among households in U.S. and their results suggest that IRRA is a reasonable assumption in theoretical models. Saha et al. also find evidence for IRRA using the data of Kansas wheat farmers [22]. This assumption is also widely used in theoretical models. The most common utility functions, such as $\ln z$, z^α ($0 < \alpha < 1$), $1 - e^{-rz}$ ($r > 0$), satisfy the property of IRRA or CRRA.

Proposition 3. *Under Assumption 1 and Assumption 2, the total order quantity in equilibrium Q^* increases in n .*

Proof. We treat n as a continuous variable defined on $[0, \infty)$ in this proof. Then

$$\begin{aligned} \frac{\partial A(Q, n, z_0)}{\partial n} &= \int_0^Q \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x) \frac{px}{n^2 Q} dx \\ &+ \int_0^Q \left\{ \left(\frac{px-wQ}{n^2} r(z_0 + \frac{px}{n} - \frac{wQ}{n}) - \frac{pQ-wQ}{n^2} r(z_0 + \frac{pQ}{n} - \frac{wQ}{n}) \right) \right. \\ &\quad \left. \cdot \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x) \left(p \frac{(n-1)x}{nQ} - w \right) \right\} dx. \end{aligned}$$

Let $H(x, Q, n) = \frac{px-wQ}{n^2} r(z_0 + \frac{px}{n} - \frac{wQ}{n})$, we find that

$$\frac{\partial H(x, Q, n)}{\partial x} = \frac{p(px-wQ)}{n^3} r'(z_0 + \frac{px}{n} - \frac{wQ}{n}) + \frac{p}{n^2} r(z_0 + \frac{px}{n} - \frac{wQ}{n}).$$

Under assumption of Proposition 3, we have $r^{*'} \geq 0$, i.e.

$$\left(z_0 + \frac{px-wQ}{n} \right) r'(z_0 + \frac{px}{n} - \frac{wQ}{n}) + r(z_0 + \frac{px}{n} - \frac{wQ}{n}) \geq 0,$$

which further implies $\partial H(x, Q, n)/\partial x \geq 0$. Hence for all $x \in [0, Q]$, $H(x, Q, n) \leq H(Q, Q, n)$. Fixing z_0 , denote $Q(n)$ satisfying $A(Q(n), n, z_0) = 0$. We need to prove that $Q(n)$ is monotone increasing in n . To finish the proof, we discuss the following 2 cases.

Case 1. $n \leq \frac{p}{p-w}$, then $nw \geq (n-1)p$.

$$\begin{aligned} \frac{\partial A(Q, n, z_0)}{\partial n} &\geq \int_0^Q \left\{ \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} (H(x, Q, n) - H(Q, Q, n)) \right. \\ &\quad \left. \cdot f(x) \left(p \frac{(n-1)x}{nQ} - w \right) \right\} dx \\ &\geq 0. \end{aligned}$$

Thus $\forall n_1 < n_2 < \frac{p}{p-w}$, $A(Q(n_1), n_1, z_0) = 0 < A(Q(n_1), n_2, z_0)$. Similar to the proof of Proposition 1, we have $Q(n_1) < Q(n_2)$.

Case 2. $n > \frac{p}{p-w}$, then $nw < (n-1)p$.

For all $x \in [0, \frac{nw}{(n-1)p}Q]$,

$$H(x, Q, n) \left(p \frac{(n-1)x}{nQ} - w \right) > H\left(\frac{nw}{(n-1)p}Q, Q, n \right) \left(p \frac{(n-1)x}{nQ} - w \right). \tag{9}$$

For all $x \in [\frac{nw}{(n-1)p}Q, Q]$,

$$H(x, Q, n) \left(p \frac{(n-1)x}{nQ} - w \right) \geq H\left(\frac{nw}{(n-1)p}Q, Q, n \right) \left(p \frac{(n-1)x}{nQ} - w \right). \tag{10}$$

By (9) and (10), we have

$$\frac{\partial A(Q, n, z_0)}{\partial n} \geq \int_0^Q \left\{ (H(x, Q, n) - H(Q, Q, n)) \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} \right.$$

$$\begin{aligned}
 & \cdot f(x)(p\frac{(n-1)x}{nQ} - w)\}dx \\
 = & \left\{ \int_0^{\frac{nw}{(n-1)p}Q} + \int_{\frac{nw}{(n-1)p}Q}^Q \right\} \{ (H(x, Q, n) - H(Q, Q, n)) \\
 & \cdot \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x)(p\frac{(n-1)x}{nQ} - w)\}dx \\
 \geq & (H(\frac{nw}{(n-1)p}Q, Q, n) - H(Q, Q, n)) \\
 & \cdot \int_0^Q \frac{u'(z_0 + \frac{px}{n} - \frac{wQ}{n})}{u'(z_0 + (p-w)\frac{Q}{n})} f(x)(p\frac{(n-1)x}{nQ} - w)dx.
 \end{aligned}$$

Since $A(Q(n), n, z_0) = 0$, we find that

$$\begin{aligned}
 & \int_0^{Q(n)} \frac{u'(z_0 + \frac{px}{n} - \frac{wQ(n)}{n})}{u'(z_0 + (p-w)\frac{Q(n)}{n})} f(x)(p\frac{(n-1)x}{nQ(n)} - w)dx \\
 & = -(p-w)\bar{F}(Q(n)) < 0.
 \end{aligned}$$

Hence for a fixed n , there exists a neighborhood $U(n)$, s.t. for any $n_0 \in U(n)$,

$$\int_0^{Q(n)} \frac{u'(z_0 + \frac{px}{n_0} - \frac{wQ(n)}{n_0})}{u'(z_0 + (p-w)\frac{Q(n)}{n_0})} f(x)(p\frac{(n_0-1)x}{n_0Q(n)} - w)dx < 0.$$

Therefore

$$\begin{aligned}
 \frac{\partial A(Q(n), n_0, z_0)}{\partial n} & \geq (H(\frac{nw}{(n-1)p}Q(n), Q(n), n_0) - H(Q(n), Q(n), n_0)) \\
 & \cdot \int_0^{Q(n)} \frac{u'(z_0 + \frac{px}{n_0} - \frac{wQ(n)}{n_0})}{u'(z_0 + (p-w)\frac{Q(n)}{n_0})} f(x)(p\frac{(n_0-1)x}{n_0Q(n)} - w)dx \\
 & > 0.
 \end{aligned}$$

$A(Q(n), n_0, z_0)$ is monotone increasing in $n_0 \in U(n)$. Thus for $n_0(\in U(n)) > n$, $A(Q(n), n_0, z_0) > A(Q(n), n, z_0) = 0$. Similar to the proof of Proposition 1, we have $Q(n) < Q(n_0)$. Since n is arbitrary, we obtain $Q(n_1) > Q(n_2)$ for any $n_1 > n_2 > \frac{p}{p-w}$.

Combining with the result of Case 1, we finish the proof of Proposition 3. \square

The Proposition 3 indicates that the total order quantity in equilibrium increases due to the demand-stealing effect while competition is heating up. From the perspective of the supplier, he has motivation to cooperate with more newsvendors since their total order quantity will be higher.

When the number of newsvendors $n \rightarrow \infty$, the total order quantity in equilibrium $Q(n) \rightarrow Q_\infty$, where Q_∞ satisfies $(p-w)\bar{F}(Q_\infty) + \int_0^{Q_\infty} f(x)(p\frac{x}{Q_\infty} - w)dx = 0$. To investigate how the degree of risk-aversion and the number of newsvendors affect the total order quantity in equilibrium, we compare the order quantity under competition with that under centralized decision-making. Following from the standard newsvendor model we know that Q_c is also the optimal inventory level under centralized decision-making. We have $Q_c < Q_\infty$ for the fact that $(p-w)\bar{F}(Q_c) + \int_0^{Q_c} f(x)(p\frac{x}{Q_c} - w)dx > 0$. For $n = 1$, we can easily get $Q(1) < Q_c$.

Thus there must exist a n_c satisfying

$$(p-w)\bar{F}(Q_c) + \int_0^{Q_c} \frac{u'(z_0 + \frac{px}{n_c} - \frac{wQ_c}{n_c})}{u'(z_0 + (p-w)\frac{Q_c}{n_c})} f(x) (p\frac{(n_c-1)x}{n_cQ_c} - w) dx = 0,$$

where n_c is a threshold number of newsvendors. If $n < n_c$, the total order quantity in equilibrium is lower than the optimal order quantity under centralized decision-making and the effect of competition is more significant than the effect of risk-aversion, and vice versa. When $n \approx n_c$, the effect of risk-aversion on total order quantity is neutralized by the effect of competition, then the decentralized decision system is coordinated, and its expected profit achieves the maximum. From the proofs of Proposition 1 and Proposition 3, we can derive that n_c is increasing in r . That is, in order to achieve the optimal order quantity under centralized decision-making, higher level of competition is needed to offset the effect of increasing degree of risk-aversion.

When competition is not intense enough to dominate the effect of risk-aversion, more competitors means higher total expected profit of all the newsvendors since $Q^* < Q_c$. Considering the situation that competition is fierce enough to dominate the effect of risk-aversion, if a new newsvendor enters the market, the total order quantity in equilibrium will deviate further from Q_c and the total expected profit of all the newsvendors decreases, then each newsvendor's expected profit becomes lower. In the intense competition environment, newsvendors' interests are harmed by the heating-up competition.

4. Conclusion. In this paper, we investigate a newsvendor game in which risk-averse newsvendors compete on demand. We show that the Nash equilibrium exists if the demand is allocated proportionally to their order quantities, and it is unique if the newsvendors' preferences exhibit the commonly assumed property DARA or CARA. The total order quantity in equilibrium decreases in the newsvendors' degrees of risk-aversion and increases in the initial wealth or number of competitors. Comparing with the risk-neutral situation, moderate preferences of risk-aversion are beneficial to newsvendors, but excessive preferences of risk-aversion reduce their profits. We analyze the interplay between competition and risk-aversion, and for a given degree of risk-aversion, if the number of newsvendors is smaller than a threshold, the effect of risk-aversion dominates the effect of risk-aversion and the total order quantity in equilibrium is lower than that under centralized decision-making. However if the number of newsvendors is larger than the threshold, the total order quantity in equilibrium is higher than that under centralized decision-making, and all the newsvendors become worse off as the competition becomes more intense. Our conclusions are provided for a large class of utility functions since we adopt a general form of the utility function.

There are many directions in which this line of research can be extended. In this paper, we assume that the newsvendors are identical. Future research could also consider the situation in which the competitors are heterogeneous. Additionally, it may be interesting to study some other demand splitting rules like those discussed by Lippman and McCardle [17]. Our single period model can also be extended to multiple periods.

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