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This paper focuses on pricing and vertical cooperative advertising decisions in a two-tier supply chain. Using a Stackelberg game model where the manufacturer acts as the game leader and the retailer acts as the game follower, we obtain closed-form equilibrium solution and explicitly show how pricing and advertising decisions are made. When market demand decreases exponentially with respect to the retail price and increases with respect to national and local advertising expenditures in an additive way, the manufacturer benefits from providing percentage reimbursement for the retailer's local advertising expenditure when demand price elasticity is large enough. Whether the manufacturer benefits from cooperative advertising is also closely related to supply chain member's relative advertising efficiency. In the decision for adopting coop advertising strategy, it is critical for the manufacturer to identify how market demand depends on national and local advertisements. The findings from this research can enhance our understanding of cooperative advertising decisions in a two-tier supply chain with price-dependent demand.

Keywords: cooperative advertising; pricing; game theory; supply chain management

1. Introduction

When market demand of a certain product is advertising dependent, both national advertising and local advertising play a role. National advertising is usually brand name oriented and aims at enlarging potential client base, whereas local advertising is end customer oriented and is basically used to stimulate short-term sales. In a typical two-tier supply chain, national advertising is usually undertaken by the upstream member (the manufacturer) and local advertising is usually undertaken by the downstream member (the retailer). Vertical cooperative (coop) advertising is an arrangement in which the manufacturer shares a portion of the retailer's local advertising cost. The fraction shared is referred to as the (manufacturer's) participation rate. In the absence of coop advertising, the retailer would typically advertise less than that desired by the manufacturer. Thus, participation rates, as well as supply chain members' advertising expenditures, are fundamental decisions in a supply chain. Many studies about coop advertising and various extensions have been presented in the literature (Berger 1972; Jørgensen and Zaccour 1999, 2003; Huang and Li 2001; Karray and Zaccour 2007; Wang et al. 2011; He et al. 2011, 2012; Ahmadi-Javid and Hoseinpour 2012; Zhang et al. 2013; Aust and Buscher 2014a; Gou et al. 2014; Karray and Amin 2015). Recently, Jørgensen and Zaccour (2014) and Aust and Buscher (2014b) provide comprehensive reviews of researches on coop advertising.

Pricing is another fundamental decision in a supply chain. It typically includes decisions for the manufacturer's wholesale price and the retailer's retail price. Both pricing and coop advertising are significant determinants of market demand and hence profits of both supply chain members. However, analytical models that simultaneously deal with coop advertising and pricing decisions are relatively sparse. Karray and Zaccour (2006) allow price competition in their model and address the coop advertising as an efficient counterstrategy for the manufacturer in the presence of the retailer's private label. Yet, the manufacturer's national advertising decision is not included in their model. Yue et al. (2006) incorporate demand price elasticity in the customer demand function, while the manufacturer, bypassing the retailer, directly gives the consumer a price deduction from the suggested retail price. However, their model takes neither wholesale price nor retail price as supply chain member's decision variables.

More recently, there is increasing research interest in analytical models that simultaneously deal with coop advertising and pricing decisions. Assuming that market demand decreases exponentially with respect to the retail price and increases with respect to national and local advertising expenditures in a multiplicative way, Szmerekovsky and Zhang

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(2009) show that it is optimal for the manufacturer not to provide reimbursement for the retailer's local advertising expenditure. Xie and Neyret (2009), Xie and Wei (2009) and Yan (2010) assume that the demand decreases linearly with respect to the retail price and find that the manufacturer usually benefits from providing percentage reimbursement for the retailer's local advertising expenditure. SeyedEsfahani, Biazaran, and Gharakhani (2011) introduce a demand function with a new parameter that can induce either a convex or a concave demand curve. Aust and Buscher (2012) extend the work by relaxing restrictions on the ratio between the manufacturer's and the retailer's profit margins. A common assumption made in these studies is that the demand function is multiplicatively separable in advertisement and price. Besides, all these studies adopt a deterministic and static game model, which implicitly assumes that players (firms) make decisions for a single-period problem. There are also stochastic and dynamic models that deal with the advertising goodwill evolution. For example, He, Prasad, and Sethi (2009) propose a stochastic Stackelberg differential game and provide in the feedback form the optimal advertising and pricing policies for the manufacturer and the retailer.

This paper focuses on the deterministic and static game model only. Noticing that both the linear demand function (Xie and Neyret 2009; Xie and Wei 2009; Yan 2010) and the non-linear demand function (SeyedEsfahani, Biazaran, and Gharakhani 2011; Aust and Buscher 2012) in the abovementioned literature imply a non-constant price elasticity, while the exponential demand function (Szmerekovsky and Zhang 2009) reflects a constant price elasticity, we are interested in the following question: What's the impact of demand price elasticity on the manufacturer's decision on cooperative advertising? Specifically, assuming market demand decreases exponentially with respect to the retail price and increases with respect to national and local advertising expenditures in an additive way, we find that the manufacturer benefits from providing percentage reimbursement for the retailer's local advertising when demand price elasticity is larger than a certain value. Whether the manufacturer benefits from cost sharing is also related to supply chain members' relative advertising efficiency. These results can enhance our understanding about the coop advertising and pricing decisions in a two-tier supply chain.

The remainder of this paper is organised as follows: Section 2 presents the model and assumptions. Section 3 solves the unique equilibrium for the Stackelberg game. Section 4 does sensitive analysis regarding impacts of different parameters on the equilibrium pricing and coop advertising decisions. Section 5 concludes the paper and proposes future research directions.

2. Model and assumptions

Consider a two-tier supply chain consisting of a single manufacturer and a single retailer, in which they play a sequential Stackelberg game. At the first stage of the game, the manufacturer acts as the game leader and decides the wholesale price w ($w > 0$), the national advertising expenditure A ($A \geq 0$) and the participation rate t ($0 \leq t < 1$) simultaneously. At the second stage of the game, the retailer acts as the game follower and decides the local advertising expenditure a ($a \geq 0$) and meanwhile sets the retail price p ($p \geq w$).

Assume market demand V depends jointly on a, A and p as follows:

$$V(p, a, A) = p^{-e}(k_r\sqrt{a} + k_m\sqrt{A}) \tag{1}$$

where e is the demand price elasticity and k_r and k_m are positive parameters taking account of the different effectiveness of local and national advertising expenditures. Equation (1) implies that market demand depends on pricing effect (p^{-e}) and advertising effect ($k_r\sqrt{a} + k_m\sqrt{A}$) in a multiplicative pattern, which is in accord with most of the existing studies (e.g. Xie and Wei 2009; Aust and Buscher 2012). As in Szmerekovsky and Zhang (2009), we assume $e > 1$, so the demand is exponentially decreasing with respect to p. In order to simplify expositions, we define the advertising ratio as $k = k_m^2/k_r^2$.

Let the manufacturer's unit production cost be C ($C > 0$), then we can write the profit functions for the manufacturer and the retailer as follows:

$$\Pi_m = (w - C)p^{-e}(k_r\sqrt{a} + k_m\sqrt{A}) - ta - A, \tag{2}$$

$$\Pi_r = (p - w)p^{-e}(k_r\sqrt{a} + k_m\sqrt{A}) - (1 - t)a. \tag{3}$$

3. Stackelberg equilibrium

In this section, we solve the Stackelberg game and obtain the equilibrium solution by backward induction. At the second stage of the game, with the variables A, w and t being given, the retailer faces the following decision problem:

$$\begin{array}{c} \text{Max } \Pi_r = (p-w)p^{-e}(k_r\sqrt{a}+k_m\sqrt{A})-(1-t)a \\ \text{s.t.} \quad p \geq w \text{ and } a \geq 0. \end{array} \tag{4}$$

Although the objective function Π_r may not be jointly concave with respect to the decision variables a and p, the particular function form allows to obtain a closed-form solution to the optimisation problem. Specifically, it is straightforward to calculate

$$\partial\Pi_r/\partial p = p^{-e-1}(k_r\sqrt{a}+k_m\sqrt{A})[ew-(e-1)p]. \tag{5}$$

with the assumption $e > 1$, the retailer's profit Π_r will increase in p when $w \leq p \leq ew/(e-1)$ and decrease in p when $p \geq ew/(e-1)$. This justifies that the optimal retail price for the retailer should be

$$p = ew/(e-1), \tag{6}$$

for all $a \geq 0$, $A \geq 0$, $w \geq C$ and $0 \leq t < 1$.

Equation (6) indicates that the retail price (p) should be set proportionally to the wholesale price (w), with the proportional coefficient $e/(e-1)$ being a decreasing function of the demand price elasticity (e). But it does not explicitly depend on the variables A, t and a and the parameters k_r and k_m.

It is easy to check whether Π_r is a concave function with respect to a. By setting $\partial\Pi_r/\partial a = 0$, we can obtain the retailer's optimal local advertising expenditure as

$$a = k_r^2 p^{-2e}(p-w)^2/4(1-t)^2. \tag{7}$$

Equation (7) holds for all $p \geq w$. Substituting (6) into (7), we have

$$a = k_r^2[ew/(e-1)]^{2-2e}/4e^2(1-t)^2. \tag{8}$$

Equation (8) indicates that the local advertising expenditure (a) should be set as a decreasing function of the wholesale price (w),and as an increasing function of the participation rate (t). But the national advertising expenditure (A) does not explicitly impact the decision for the local advertising expenditure (a).

Therefore, at the first stage of the game, the manufacturer faces the following decision problem:

$$\begin{array}{c} \text{Max } \Pi_m = (w-C)p^{-e}(k_r\sqrt{a}+k_m\sqrt{A})-ta-A \\ \text{s.t.} \quad 0 \leq t < 1, w \geq C, \text{ and } A \geq 0, \end{array} \tag{9}$$

where the variables p and a satisfy (6) and (8), respectively.

According to (9), the Stackelberg equilibrium (w^*, A^*, t^*, p^*, a^*) can be characterised as in Proposition 1 (see also Table 1, and the proof is provided in Appendix).

Proposition 1. *The Stackelberg game has a unique equilibrium* (w^*, A^*, t^*, p^*, a^*) *expressed as:*

When $e > 2 + 2/k$, $w^* = w_1$, $t^* = t_1$, *where*

$$w_1 = \frac{C\left\{(2e-1)[2e-1+2(e-1)k]+\sqrt{(2e-1)^2+4(e-1)^2k+4(e-1)^2k^2}\right\}}{(2e-1)^2+4(e-1)^2k}, \tag{10}$$

Table 1. Equilibriums under two different cases.

$e > 2 + 2/k$	$1 < e \leq 2 + 2/k$
$w^* = w_1$	$w^* = w_2$
$t^* = t_1 = [(2e-3)w_1 - 2(e-1)C]/[(2e-1)w_1 - 2(e-1)C]$	$t^* = t_2 = 0$
$p^* = p_1 = ew_1/(e-1)$	$p^* = p_2 = ew_2/(e-1)$
$A^* = A_1 = k_m^2(w_1-C)^2\left(\frac{ew_1}{e-1}\right)^{-2e}/4$	$A^* = A_2 = k_m^2(w_2-C)^2\left(\frac{ew_2}{e-1}\right)^{-2e}/4$
$a^* = a_1 = k_r^2\left(\frac{ew_1}{e-1}\right)^{-2e}((2e-1)w_1-2(e-1)C)^2/16(e-1)^2$	$a^* = a_2 = k_r^2(ew_2/(e-1))^{2-2e}/4e^2$

$$t_1 = \frac{(2e-3)w^* - 2(e-1)C}{(2e-1)w^* - 2(e-1)C}. \tag{11}$$

When $e \le 2 + 2/k$, $w^* = w_2$, $t^* = t_2$, *where*

$$w_2 = \frac{C\left\{(2e-1)[1+(e-1)k] + \sqrt{(2e-1)^2 + 2(e-1)k + (e-1)^2k^2}\right\}}{2(e-1)[2+(e-1)k]}, \tag{12}$$

$$t_2 = 0. \tag{13}$$

Under both cases, after the values for (w^*, t^*) *are determined, the values for* (A^*, p^*, a^*) *can be calculated as:*

$$A^* = k_m^2(w^* - C)^2\left(\frac{ew^*}{e-1}\right)^{-2e}/4, \tag{14}$$

$$p^* = ew^*/(e-1), \tag{15}$$

$$a^* = k_r^2[ew^*/(e-1)]^{2-2e}/4e^2(1-t^*)^2. \tag{16}$$

Proposition 1 reveals that under our assumptions for the demand function, the manufacturer has no incentive to share the retailer's local advertising expenditure when the price elasticity is smaller than a certain value. However, when the price elasticity is larger than a certain value, the manufacturer benefits from providing percentage reimbursement for the retailer's local advertising expenditure. This result is different from that in Szmerekovsky and Zhang (2009). Assuming the demand function takes the form of $V(p, a, A) = p^{-e}(\alpha - \beta a^{-\gamma}A^{-\delta})$ ($e > 1$ and α, β, γ, δ are positive constants), they find that for all values of price elasticity greater than 1, the optimal strategy for the manufacturer is not to provide any subsidy to the retailer's local advertisement. This difference is due to the different assumptions about the effect of advertisements on the demand. National and local advertisements are assumed to increase market demand in a multiplicative way in their model, while they are assumed to work in an additive way in our model. These two different findings complement each other and suggest that when the manufacturer considers whether coop advertising strategy should be adopted, it is critical to identify how the demand depends on national and local advertisements.

Usually, products with many substitutes or whose purchase can be easily postponed, or that are considered luxuries other than essential to everyday living, have higher price elasticities. For example, the demand for a specific brand soft drink would likely be highly elastic, and its price elasticity can be more than 4 (Ayers and Collinge 2005). Similarly, goods and services include specific-model automobiles, fresh tomatoes and long-run foreign travels. (Gwartney et al. 2014). For these industries, if national advertising and local advertising increase market demand in an additive way as in our model, our result suggests that coop advertising strategy will be very attractive to the goods manufacturers (or service providers).

Now we focus on the case where the price elasticity is large enough (i.e. $e > 2 + 2/k$), so that the manufacturer's optimal strategy is to provide a positive reimbursement to the retailer's local advertising expenditure. If the manufacturer makes a wrong decision and provides no reimbursement for the retailer's local advertising expenditure, his optimal strategy is to use $t^* = t_2 = 0$, and the associated optimal wholesale price is obtained by $w^* = w_2$, as expressed in (12). However, when he provides a positive reimbursement to the retailer's local advertising expenditure, his optimal strategy is to use $t^* = t_1$ and $w^* = w_1$, as expressed in (10) and (11). Obviously, the reimbursement from the manufacturer will induce the retailer to invest more on local advertising, i.e. $a_1 > a_2$. The following proposition reveals the impact of the decision mistake on other decision variables (the wholesale price, the retail price and the manufacturer's national advertising expenditure).

Proposition 2. *Suppose* $e > 2 + 2/k$, *then* $w_1 > w_2$, $A_1 > A_2$, $p_1 > p_2$, $a_1 > a_2$.

Proposition 2 concludes that when $e > 2 + 2/k$, if the manufacturer provides no reimbursement for the retailer's local advertising expenditure, the retailer will invest less on national advertising and the manufacturer will also invest less on national advertising. Thus, the demand generated from both advertising will be lower. In order to maximise his profit, the retailer has to announce a lower retail price to induce more sales. Similarly, the manufacturer should also charge a lower wholesale price. This leads to low profit margins for both the manufacturer and the retailer, resulting in a lose-lose situation for both supply chain members. On the contrary, by adopting coop advertising, the demand generated from advertising will be higher and both the supply chain members can increase their profit margins and improve the performance of the supply chain.

4. Effects of demand parameters on the pricing and coop advertising decisions

In this section, we concentrate our discussions on the case where the price elasticity is large enough (i.e. $e > 2 + 2/k$), so that the manufacturer's optimal strategy is to provide a positive reimbursement to the retailer's local advertising cost. Proposition 3 summaries how the manufacturer's and retailer's decision variables will be influenced by the parameters of the model.

Proposition 3. *Suppose* $e > 2 + 2/k$, *then*

$$\text{(i)}\ \frac{\partial w_1}{\partial k} > 0, \frac{\partial w_1}{\partial e} < 0;$$

$$\text{(ii)}\ \frac{\partial p_1}{\partial k} > 0, \frac{\partial p_1}{\partial e} < 0;$$

$$\text{(iii)}\ \frac{\partial t_1}{\partial k} > 0, \frac{\partial t_1}{\partial e} > 0;$$

$$\text{(iv)}\ \frac{\partial a_1}{\partial k_r} > 0, \frac{\partial a_1}{\partial k_m} < 0;$$

$$\text{(v)}\ \frac{\partial A_1}{\partial k_r} < 0, \frac{\partial A_1}{\partial k_m} > 0.$$

Parts (i) and (ii) of Proposition 3 mean that both the manufacturer's wholesale price (w_1) and the retailer's selling price (p_1) increase with the advertising ratio k and decrease with the price elasticity e. They suggest that the supply chain sees a high level of wholesale price (and retail price) if either the local advertising is poorly effective compared with the national advertising or the market demand is highly sensitive with the unit retail price changes. This observation is reasonable. We take $\partial\ w_1/\partial e < 0$ for example. Because e represents the sensitivity of the sales volume to the retail price as defined in (4), we can expect that the more sensitive the sales volume to the retail price is, the lower the wholesale price will be. Part (iii) of Proposition 3 indicates that the manufacturer's cost sharing percentage t_1 will increase with both the advertising ratio k and the pricing elasticity e. This is intuitively understandable. On the one hand, when the local advertising is not very effective, the retailer has little incentive to impose massive local advertising. So the manufacturer will have to raise the participation rate and induce a higher level of local advertising expenditure. On the other hand, when the market demand is less price sensitive, the retailer's pricing instrument has limited power in generating end customer demand. So the manufacturer can raise its participation rate in the hope that the retailer's non-pricing promotion (local advertising) induces more sales. Finally, Parts (iv) and (v) of Proposition 3 show that both members' advertising expenditures will increase with respect to their own advertising efficacy and decrease with respect to their counterparty's advertising efficacy. This is also intuitively understandable, and it coincides with the findings in Xie and Wei (2009).

Please note that Proposition 3 does not say anything about the monotonicity of a_1 and A_1 with respect to the price elasticity e. In fact, when $e > 2 + 2/k$ and e increase, a_1 and A_1 can either increase or decrease. In order to display how the demand price elasticity will impact the advertising decisions for both parties, we provide the following numerical example.

Example. Fix $k_r = 1$ and $k_m = 2$ (thus $k = 4$). For two different levels of unit production costs $C = 0.83$ and $C = 1.00$, Figure 1 plots how a_1 and A_1 change with e ($e > 2 + 2/k = 2.5$) in a non-monotonic way.

Figure 1 reveals the following observations. (1) With different unit production costs, the monotonicity for the equilibrium advertising expenditures with respect to the price elasticity does not travel well. Take a_1 for example. With a higher level of $C = 1.00$, a_1 decreases in e for all $e > 2.5$; this decreasing trend, however, does not remain with a lower level $C = 0.83$. When e is relatively high, a_1 even goes in totally opposite directions with e. Specifically, when e is greater than a certain critical value, a_1 will increase in e with $C = 0.83$ and decrease in e with $C = 1.00$. (2) Even with the same unit production cost, there can be no simple monotonic relations between the equilibrium advertising expenses and the price elasticity. Taking A_1 for example, when $C = 0.83$, A_1 will decrease in e when e is low, but will increase in e when e is relatively high.

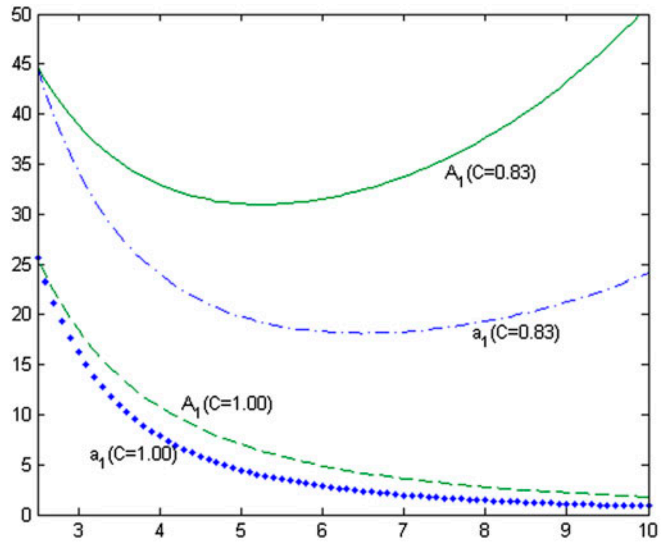

Figure 1. How a_1 and A_1 change with e.

5. Conclusions and future researches

With the help of a game theoretical model, this paper shows that the manufacturer benefits from providing percentage reimbursement for the retailer's local advertising expenditure when the price elasticity is larger than a certain value. In case that the manufacturer makes a wrong decision by choosing a zero participation rate, both national and local advertising expenditures and both wholesale and retail prices will systematically drop to lower levels. Sensitivity analyses show that (i) both the wholesale price and the retail price will increase with the advertising ratio and decrease with the price elasticity; (ii) the manufacturer's optimal participation rate increases with both the advertising ratio and the price elasticity; and (iii) both members' advertising expenditures will increase with respect to their own advertising efficacy and decrease with respect to their counterparty's advertising efficacy.

Our contributions to the coop advertising literature are twofold. Qualitatively, we find that the manufacturer benefits from coop advertising when demand price elasticity is high if national and local advertising expenditures influence the demand in an additive way. This result, along with existent studies (e.g. Szmerekovsky and Zhang 2009; Xie and Wei 2009), can enhance our understanding about the coop advertising and pricing decisions in a two-tier supply chain. In the decision for coop advertising, it is critical for the manufacturer to identify how market demand depends on the retail price and national and local advertisements. Quantitatively, we managed to identify the unique Stackelberg equilibrium for supply chain members' pricing and coop advertising decisions in all closed-form solutions.

There are some directions that our model can be extended. First, considering competition between manufacturers and retailers may lead to more interesting results. Second, our findings are made under a very special and specific demand function. It is not yet clear for which type of demand functions does the manufacturer benefit from providing a positive participation rate, or, as imposed conversely, does there exist a broad class of demand functions under which the manufacturer's optimal participation strategy is not to participate? Finally, taking an empirical investigation is always a good idea. The associated managerial insights mentioned in this article may serve as the baseline for possible hypothesis testing with industrial data.

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References

Ahmadi-Javid, A., and P. Hoseinpour. 2012. “On a Cooperative Advertising Model for a Supply Chain with One Manufacturer and One Retailer.” *European Journal of Operational Research* 219: 458–466.

Aust, G., and U. Buscher. 2012. “Vertical Cooperative Advertising and Pricing Decisions in a Manufacturer–Retailer Supply Chain: A Game-theoretical Approach.” *European Journal of Operational Research* 223: 473–482.

Aust, G., and U. Buscher. 2014a. “Vertical Cooperative Advertising in a Retailer Duopoly.” *Computers & Industrial Engineering* 72: 247–254.

Aust, G., and U. Buscher. 2014b. “Cooperative Advertising Models in Supply Chain Management: A Review.” *European Journal of Operational Research* 234: 1–14.

Ayers, R., and R. Collinge. 2005. *Microeconomics: Explore and Apply*. Upper Saddle River, NJ: Pearson Prentice Hall.

Berger, P. D. 1972. “Vertical Cooperative Advertising Ventures.” *Journal of Marketing Research* 9: 309–312.

Gou, Q., J. Zhang, L. Liang, Z. Huang, and A. Ashley. 2014. “Horizontal Cooperative Programmes and Cooperative Advertising.” *International Journal of Production Research* 52 (3): 691–712.

Gwartney, J., R. Stroup, R. Sobel, and D. Macpherson. 2014. *Economics: Private and Public Choice*. Mason, OH: Cengage Learning.

He, X., A. Prasad, and S. P. Sethi. 2009. “Co-op Advertising and Pricing in a Stochastic Supply Chain: Feedback Stackelberg Strategies.” *Production and Operations Management* 18 (1): 78–94.

He, X., A. Krishnamoorthy, A. Prasad, and S. P. Sethi. 2011. “Retail Competition and Cooperative Advertising.” *Operations Research Letters* 39 (1): 11–16.

He, X., A. Krishnamoorthy, A. Prasad, and S. P. Sethi. 2012. “Co-op Advertising in Dynamic Retail Oligopolies.” *Decision Sciences* 43 (1): 73–106.

Huang, Z., and S. X. Li. 2001. “Coop Advertising Models in a Manufacturer–Retailer Supply Chain: A Game Theory Approach.” *European Journal of Operational Research* 135 (3): 527–544.

Jorgensen, S., and G. Zaccour. 1999. “Equilibrium Pricing and Advertising Strategies in a Marketing Channel.” *Journal of Optimization Theory and Applications* 102: 111–125.

Jorgensen, S., and G. Zaccour. 2003. “Channel Coordination Over Time: Incentive Equilibria and Credibility.” *Journal of Economic Dynamics and Control* 27: 801–822.

Jorgensen, S., and G. Zaccour. 2014. “A Survey of Game-theoretical Models of Cooperative Advertising.” *European Journal of Operational Research* 237: 1–14.

Karray, S., and S. H. Amin. 2015. “Cooperative Advertising in a Supply Chain with Retail Competition.” *International Journal of Production Research* 53 (1): 88–105.

Karray, S., and G. Zaccour. 2006. “Could Co-op Advertising be a Manufacturer’s Counterstrategy to Store Brands?” *Journal of Business Research* 59 (9): 1008–1015.

Karray, S., and G. Zaccour. 2007. “Effectiveness of Coop Advertising Programs in Competitive Distribution Channels.” *International Game Theory Review* 9 (2): 151–167.

SeyedEsfahani, M. M., M. Biazaran, and M. Gharakhani. 2011. “A game Theoretic Approach to Coordinate Pricing and Vertical Co-op Advertising in Manufacturer–Retailer Supply Chains.” *European Journal of Operational Research* 211 (2): 263–273.

Szmerekovsky, J. G., and J. Zhang. 2009. “Pricing and Two-tier Advertising with One Manufacturer and One Retailer.” *European Journal of Operational Research* 192 (3): 904–917.

Wang, S., Y. Zhou, J. Min, and Y. Zhong. 2011. “Coordination of Cooperative Advertising Models in a One-manufacturer Two-retailer Supply Chain System.” *Computers & Industrial Engineering* 61 (4): 1053–1071.

Xie, J., and A. Neyret. 2009. “Co-op Advertising and Pricing Models in Manufacturer–Retailer Supply Chains Co-op Advertising and Pricing Models in Manufacturer–Retailer Supply Chains.” *Computers & Industrial Engineering* 56 (4): 1375–1385.

Xie, J., and J. C. Wei. 2009. “Coordinating Advertising and Pricing in a Manufacturer–Retailer Channel.” *European Journal of Operational Research* 197 (2): 785–791.

Yan, R. 2010. “Cooperative Advertising, Pricing Strategy and Firm Performance in the E-Marketing Age.” *Journal of the Academy of Marketing Science* 38 (4): 510–519.

Yue, J., J. Austin, M. Wang, and Z. Huang. 2006. “Coordination of Cooperative Advertising in a Two-level Supply Chain When Manufacturer Offers Discount.” *European Journal of Operational Research* 168 (1): 65–85.

Zhang, J., Q. Gou, L. Liang, and X. He. 2013. “Ingredient Branding Strategies in an Assembly Supply Chain: Models and Analysis.” *International Journal of Production Research* 51 (23–24): 6923–6949.

Appendix 1

A.1 *Proof of Proposition 1*

Substituting Equations (6) and (8) into the expression of Π_m, the manufacturer's decision problem (9) becomes

$$\text{Max } \Pi_m = (w-C)\left(\frac{ew}{e-1}\right)^{-e}\left(\frac{k_r^2\left(\frac{ew}{e-1}\right)^{1-e}}{2e(1-t)} + k_m\sqrt{A}\right) - \frac{tk_r^2\left(\frac{ew}{e-1}\right)^{2-2e}}{4e^2(1-t)^2} - A \quad \text{s.t.} \quad 0 \le t < 1, w \ge C \text{ and } A \ge 0. \tag{A.1}$$

In order to determine the unique optimal solution (w^*, t^*, A^*), we first examine the manufacturer's optimal decision on the national advertising expenditure. Noticing $\partial^2\Pi_m/\partial A^2 = -\frac{1}{4}k_m(w-C)\left({ew}/{(e-1)}\right)^{-e}A^{-3/2} \le 0$ and $w \ge C$, we know Π_m is a concave function with respect to A for all $A \ge 0$. Thus, we can solve the first-order condition $\partial\Pi_m/\ \partial A = 0$ and uniquely obtain the optimal national advertising expenditure by

$$A = k_m^2(w-C)^2\left(\frac{ew}{e-1}\right)^{-2e}/4. \tag{A.2}$$

Substituting (A.2) into (A.1), we reduce the manufacturer's maximisation problem to

$$\{\text{Max } \Pi_m(w,t), \text{s.t. } 0 \le t < 1, w \ge C\}, \text{where } \Pi_m(w,t) = (w-C)\left(\frac{ew}{e-1}\right)^{-e}\left[\frac{k_r^2\left(\frac{ew}{e-1}\right)^{1-e}}{2e(1-t)} + \frac{k_m^2(w-C)\left(\frac{ew}{e-1}\right)^{-e}}{2}\right] - \frac{tk_r^2\left(\frac{ew}{e-1}\right)^{2-2e}}{4e^2(1-t)^2} - \frac{k_m^2(w-C)^2\left(\frac{ew}{e-1}\right)^{-2e}}{4} \tag{A.3}$$

Next we consider the manufacturer's optimal decisions on t and w.

(i) Consider the case with $e > 2 + 2/k$.

The first-order derivative of $\Pi_m(w,t)$ with respect to t is given by

$$\frac{\partial\Pi_m(w,t)}{\partial t} = k_r^2\frac{[(1-2e)w + 2C(e-1)]t + (2e-3)w - 2C(e-1)}{4(1-t)^3e^{2e}w^{2e-1}(e-1)^{2-2e}}. \tag{A.4}$$

With the constraints $0 \le t < 1$ and $w \ge C$, the sign of the derivative (A.4) is determined by the formula $f(t) = [(1-2e)w + 2(e-1)C]t + (2e-3)w - 2(e-1)C$, which is a decreasing function in t since the slope $(1-2e)w + 2(e-1)C \le [(1-2e) + 2(e-1)]C = -C < 0$. We define $t_0 = [(2e-3)w - 2(e-1)C]/[(2e-1)w - 2(e-1)C]$ such that $f(t_0) = 0$, then $f(t)$ will be positive for $t < t_0$ and negative for $t > t_0$. Taking the constraint $0 \le t < 1$ into consideration, two subcases will follow as below.

Subcase [I]. If $f(0) > 0$, or equivalently $w > C(2e-2)/(2e-3)$, then $f(t)$ will be positive for $0 \le t < t_0$ and negative for $t_0 < t < 1$, which suggests that $\Pi_m(w,t)$ will first increase in t for $0 \le t < t_0$ and then decrease in t for $t_0 < t < 1$. Thus, the manufacturer's optimal choice of t should be given by

$$t^{[I]}(w) = t_0 = \frac{(2e-3)w - 2(e-1)C}{(2e-1)w - 2(e-1)C}. \tag{A.5}$$

Substituting (A.5) into (A.3), the manufacturer's optimisation problem is now further reduced to
$\{\text{Max } \Pi_m^{[I]}(w), \text{s.t. } w > C(2e-2)/(2e-3)\}$,
where

$$\Pi_m^{[I]}(w) = \frac{(e-1)^{2e-2}}{16(ew)^{2e}}\left\{k_r^2[2(w-C)(e-1) + w]^2 + 4k_m^2(w-C)^2(e-1)^2\right\} \tag{A.6}$$

is a function of the decision variable w only.

Taking $\partial\Pi_m^{[I]}(w)/\partial w = 0$, and making use of $k = k_m^2/k_r^2$, after some algebraic simplifications, we have

$$-[(2e-1)^2 + 4(e-1)^2k]w^2 + 2(2e-1)[(2e-1) + 2(e-1)k]Cw - 4e(e-1)(k+1)C^2 = 0. \tag{A.7}$$

The left-hand side of (A.7) is a quadratic and concave function that has two real roots given as follows:

$$w^{[I]} = \frac{C\{(2e-1)[(2e-1) + 2(e-1)k] + \sqrt{\Delta_1}\}}{(2e-1)^2 + 4(e-1)^2k}, \tag{A.8}$$

$$\bar{w}^{[I]} = \frac{C\left\{(2e-1)[(2e-1) + 2(e-1)k] - \sqrt{\Delta_1}\right\}}{(2e-1)^2 + 4(e-1)^2k}, \tag{A.9}$$

where

$$\Delta_1 = (2e-1)^2 + 4(e-1)^2k + 4(e-1)^2k^2. \tag{A.10}$$

It is easy to verify that $\bar{w}^{[I]} < C < C(2e-2)/(2e-3)$. Furthermore, we have

$$w^{[I]} - 2C(e-1)/(2e-3) = \frac{4C(e-1)[(2e-1)^2 + 4(e-1)^2k][ke-2(k+1)]}{(2e-3)[(2e-1)^2 + 4(e-1)^2k][2k(e-1) + (2e-1)^2 + (2e-3)\sqrt{\Delta_1}]}, \tag{A.11}$$

which is greater than zero given our assumption $e > 2 + 2/k$. So, $\bar{w}^{[I]} < C < C(2e-2)/(2e-3) < w^{[I]}$. It follows that $\partial\Pi_m^{[I]}(w)/\partial w$ takes positive values when $w \in (C(2e-2)/(2e-3), w^{[I]})$ and takes negative values when $w \in (w^{[I]}, +\infty)$, indicating that $\Pi_m^{[I]}(w)$ increases in w when $C(2e-2)/(2e-3) < w < w^{[I]}$ and decreases in w when $w > w^{[I]}$. This argument concludes that the optimal wholesale price under Subcase [I] should be $w^{*[I]} = w^{[I]}$ and the associated optimal participation rate should be $t^{[I]}(w^{*[I]})$.

Subcase [II]. If $f(0) \le 0$, or equivalently $(C \le) w \le C(2e-2)/(2e-3)$, then $f(t) \le 0$ for all t $(0 \le t < 1)$, i.e. $\Pi_m(t, w)$ is decreasing in t for all t. So, the optimal choice of t should be $t^{[II]} = 0$. With $t = t^{[II]} = 0$, the manufacturer's optimisation problem is then reduced to:

Max $\Pi_m^{[II]}(w)$, s.t. $(C \le) w \le C(2e-2)/(2e-3)$,
where

$$\Pi_m^{[II]}(w) = \frac{(w-C)\left(\frac{ew}{e-1}\right)^{-2e}}{4(e-1)}\left[2k_r^2 w + k_m^2(e-1)(w-C)\right]. \tag{A.12}$$

Taking $\partial\Pi_m^{[II]}(w)/\partial w = 0$, and making use of $k = k_m^2/k_r^2$, after some algebraic simplifications, we have

$$-\left\{(e-1)[k(e-1)+2]w^2 - (2e-1)[k(e-1)+1]wC + e(e-1)kC^2\right\} = 0. \tag{A.13}$$

The left-hand side of (A.13) is again a quadratic and concave function of w with two real roots as follows:

$$w^{[II]} = \frac{C\left\{(2e-1)[1+(e-1)k] + \sqrt{\Delta_2}\right\}}{2(e-1)[2+(e-1)k]}, \tag{A.14}$$

$$\bar{w}^{[II]} = \frac{C\left\{(2e-1)[1+(e-1)k] - \sqrt{\Delta_2}\right\}}{2(e-1)[2+(e-1)k]}, \tag{A.15}$$

$$\text{where } \Delta_2 = (2e-1)^2 + 2(e-1)k + (e-1)^2k^2. \tag{A.16}$$

It is easy to check that $\bar{w}^{[II]} < C < C(2e-2)/(2e-3) < w^{[II]}$. Then $\partial\Pi_m^{[II]}(w)/\partial w$ takes positive values for all w in $[C, C(2e-2)/(2e-3)]$, which indicates that $\Pi_m^{[II]}(w)$ is increasing in w for all $C \le w \le C(2e-2)/(2e-3)$. This argument concludes that the optimal wholesale price under Subcase [II] should be $w^{*[II]} = C(2e-2)/(2e-3)$, with the associated optimal participation rate $t^{[II]} = 0$.

In order to find the global optimal solution (w^*, t^*) for $\Pi_m(w,t)$ with the condition $e > 2 + 2/k$, we should compare the manufacturer's optimal profits

$\Pi_m^{[I]}(w^{*[I]}) = \Pi_m(w^{*[I]}, t^{[I]}(w^{*[I]}))$ under Subcase [I] and
$\Pi_m^{[II]}(w^{*[II]}) = \Pi_m(w^{*[II]}, t^{[II]}) = \Pi_m(w^{*[II]}, 0)$ under Subcase [II].

By definition, we have $\Pi_m^{[I]}(w^{*[I]}) = \Pi_m(w^{*[I]}, t^{[I]}(w^{*[I]})) \ge \Pi_m(w, t^{[I]}(w))$ for all $w > 2C(e-1)/(2e-3)$. Since $\Pi_m(w, t)$ is a continuous function in w, when w goes down to the lower bound $2C(e-1)/(2e-3)$, we have

$$\begin{aligned}\Pi_m(w^{*[I]}, t^{[I]}(w^{*[I]})) &\ge \lim_{w \to +2C(e-1)/(2e-3)} \Pi_m(w, t^{[I]}(w)) \\ &= \Pi_m(w^{*[II]}, t^{[I]}(w^{*[II]})) = \Pi_m(w^{*[II]}, 0) = \Pi_m(w^{*[II]}, t^{[II]}),\end{aligned} \tag{A.17}$$

which concludes that $\Pi_m^{[I]}(w^{*[I]}) \ge \Pi_m^{[II]}(w^{*[II]})$.

Therefore, when $e > 2 + 2/k$, the optimal decision for the manufacturer should be $w^* = w^{[I]} = w_1$ and $t^* = t^{[I]} = t_1$, which justifies Equations (10) and (11).

(ii) Now we consider the case with $e \le 2 + 2/k$.

By (A.4), the sign of the first-order derivative $\partial\Pi_m(w,t)/\partial t$ is determined by the formula $f(t) = [(1-2e)w + 2(e-1)C]t + (2e-3)w - 2(e-1)C$, which is decreasing in t for all $w \ge C$. Recalling that t_0 is defined such that $f(t_0) = 0$, we have $t_0 > 0$ if and only if $f(0) = (2e-3)w - 2(e-1)C > 0$. Taking the constraint $0 \le t < 1$ into consideration, we have three independent subcases as follows.

Subcase [III]. If $e \le 3/2$, then $f(0) = (2e-3)w - 2(e-1)C \le 0$. Consequently, $f(t) \le 0$ for all t $(0 \le t < 1)$, i.e. $\Pi_m(w,t)$ is decreasing in t for all given values of $w \ge C$. So the optimal choice of t in Subcase [III] should be $t^{[III]} = 0$.

Subcase [IV]. If $3/2 < e (\le 2 + 2/k)$ and $f(0) = (2e-3)w - 2(e-1)C \le 0$, or equivalently, $w \le C(2e-2)/(2e-3)$, then we still have $f(t) \le 0$ for $0 \le t < 1$, i.e. $\Pi_m(w,t)$ is decreasing in t for all given values of w such that $C \le w \le C(2e-2)/(2e-3)$. For this situation, the optimal choice of t is $t^{[IV]} = 0$.

Subcase [V]. If $3/2 < e (\le 2 + 2/k)$ and $f(0) = (2e-3)w - 2(e-1)C > 0$, or equivalently, $w > C(2e-2)/(2e-3)$, then $f(t)$ takes positive values for $0 \le t < t_0$ and negative values for $t_0 < t < 1$. Therefore, $\Pi_m(w,t)$ will increase in t for $0 \le t < t_0$ and decrease in t for $t_0 < t < 1$. So the manufacturer's optimal choice of t should be $t^{[V]} = t_0$.

Using similar arguments as in Part (i), by comparing all the three subcases listed above, we can show that when $e \le 2 + 2/k$, the manufacturer's (globally) optimal participation rate should be zero, i.e. $t^* = 0 = t_2$. The optimal wholesale price should be

$$w^* = \frac{C\left\{(2e-1)[1+(e-1)k] + \sqrt{(2e-1)^2 + 2(e-1)k + (e-1)^2k^2}\right\}}{2(e-1)[2+(e-1)k]} = w_2, \tag{A.18}$$

which justifies Equations (12) and (13).

This completes the proof.

A.2 *Proof of Proposition 2*

By Equations (14)–(16), $A_1 > A_2$, $p_1 > p_2$, $a_1 > a_2$ can be easily verified if $w_1 > w_2$. Thus, it suffices to prove $w_1 > w_2$. Using expressions of w_1 and w_2 in (10) and (12), we have

$$\frac{C}{w_1} = \frac{[(2e-1)^2 + 2k(e-1)(2e-1)] - \sqrt{(2e-1)^2 + 4k(e-1)^2 + 4k^2(e-1)^2}}{4e(e-1)(1+k)}, \tag{A.19}$$

$$\frac{C}{w_2} = \frac{(2e-1)[1+k(e-1)] - \sqrt{(2e-1)^2 + 2k(e-1) + k^2(e-1)^2}}{2ke(e-1)}. \tag{A.20}$$

In order to show $w_1 > w_2$, we only need to prove $\frac{C}{w_2} > \frac{C}{w_1}$, which, after some algebraic simplifications, is equivalent to

$$\begin{aligned}(2e-1)(2+k) + k\sqrt{(2e-1)^2 + 4(e-1)^2k + 4(e-1)^2k^2} \\ > 2(1+k)\sqrt{(2e-1)^2 + 2(e-1)k + (e-1)^2k^2}\end{aligned}. \tag{A.21}$$

Squaring both sides, this inequality is equivalent to

$$\begin{aligned}(2e-1)(2+k)k\sqrt{(2e-1)^2 + 4(e-1)^2k + 4(e-1)^2k^2} \\ > (2e-1)^2(2+k)k + 4(e-1)k(1+k)^2 + 2(e-1)^2(1+k)k^2.\end{aligned} \tag{A.22}$$

Squaring both sides again, this inequality is equivalent to

$$[3(e-1)k + 8e - 6][(e-2)k - 2] > 0. \tag{A.23}$$

Since $e > 2 + 2/k$, this is obviously true. This completes the proof.

A.3 *Proof of Proposition 3*

(i) According to (A.10), $\Delta_1 = (2e-1)^2 + 4(e-1)^2k + 4(e-1)^2k^2$, thus

$$w_1 = \frac{C\left\{(2e-1)[2e-1+2(e-1)k] + \sqrt{\Delta_1}\right\}}{(2e-1)^2 + 4(e-1)^2k} = \frac{4e(e-1)(1+k)C}{(2e-1)[2e-1+2(e-1)k] - \sqrt{\Delta_1}}. \tag{A.24}$$

After tedious algebraic calculation, we obtain

$$\frac{\partial w_1}{\partial k} = \frac{4e(e-1)C\left[(2e-1)\sqrt{\Delta_1} - 2e^2 + 1 + 2(e-1)^2k\right]}{\sqrt{\Delta_1}\left\{(2e-1)[2e-1+2(e-1)k] - \sqrt{\Delta_1}\right\}^2}. \tag{A.25}$$

Since $\Delta_1 = (2e-1)^2 + 4(e-1)^2k + 4(e-1)^2k^2 > (2e-1)^2$, we have

$$\begin{aligned}\frac{\partial w_1}{\partial k} &> \frac{4e(e-1)C}{\sqrt{\Delta_1}\left\{(2e-1)[2e-1+2(e-1)k] - \sqrt{\Delta_1}\right\}^2}\left[(2e-1)^2 - 2e^2 + 1 + 2(e-1)^2k\right] \\ &= \frac{4e(e-1)C}{\sqrt{\Delta_1}\left\{(2e-1)[2e-1+2(e-1)k] - \sqrt{\Delta_1}\right\}^2}\left[2(e-1)^2(1+k)\right] > 0\end{aligned} \tag{A.26}$$

Similarly, after tedious algebraic calculation, we obtain

$$\begin{aligned}\frac{\partial w_1}{\partial e} &= \frac{4(1+k)C}{\sqrt{\Delta_1}\left\{(2e-1)[2e-1+2(e-1)k] - \sqrt{\Delta_1}\right\}^2}\cdot \\ &\left\{[2e-1-2(e-1)^2k]\sqrt{\Delta_1} - (2e-1)[e^2+(e-1)^2] - 4(e-1)^3(k+k^2)\right\}\end{aligned} \tag{A.27}$$

Since $e > 2 + 2/k$, or equivalently, $k > 2/(e-2)$, we have

$$2e-1-2(e-1)^2k < 2e-1-4(e-1)^2/(e-2) = -[e+2(e-1)^2]/(e-2) < 0. \tag{A.28}$$

Therefore, $\partial w_1/\partial e < 0$.

(ii) Noticing that $p_1 = ew_1/(e-1)$ is a linear function of w_1, and w_1 is an increasing function of k according to Part (i), thus p_1 is also an increasing function of k, which means $\partial p_1/\partial k > 0$. Besides, since $e/(e-1) > 0$ is a decreasing function of e, and w_1 is a decreasing function of e according to Part (i), thus p_1 is also a decreasing function of e, which means $\partial p_1/\partial e < 0$ is true.

(iii) Since $t_1 = [(2e-3)w_1 - 2(e-1)C]/[(2e-1)w_1 - 2(e-1)C]$ is an increasing function of w_1, and w_1 is an increasing function of k according to Part (i), thus t_1 is also an increasing function of e, which means $\partial t_1/\partial k > 0$.

In order to prove $\partial t_1/\partial e > 0$, we calculate

$$\begin{aligned}\frac{\partial t_1}{\partial e} &= \frac{\partial\{1-2/[2e-1-2(e-1)C/w_1]\}}{\partial e} + \frac{\partial\{1-2/[2e-1-2(e-1)C/w_1]\}}{\partial w_1}\cdot\frac{\partial w_1}{\partial e}\\ &= \left[\frac{2C}{(2e-1)w_1-2(e-1)C}\right]^2\left[\left(\frac{w_1}{C}\right)^2 - \frac{w_1}{C} + \frac{e-1}{C}\cdot\frac{\partial w_1}{\partial e}\right]\end{aligned} \tag{A.29}$$

Thus, we only need to show that $(w_1/C)^2 - (w_1/C) + (e-1)/C \cdot \partial w_1/\partial e > 0$.

According to Equations (10) and (A.27), this is equivalent to

$$\begin{aligned}&\left\{\frac{4e(e-1)(1+k)}{(2e-1)[2e-1+2(e-1)k]-\sqrt{\Delta_1}}\right\}^2 - \frac{4e(e-1)(1+k)}{(2e-1)[2e-1+2(e-1)k]-\sqrt{\Delta_1}}\\ &+\frac{4(e-1)(1+k)}{\sqrt{\Delta_1}\left\{(2e-1)[2e-1+2(e-1)k]-\sqrt{\Delta_1}\right\}^2}\cdot\\ &\left\{[2e-1-2(e-1)^2k]\sqrt{\Delta_1}-(2e-1)[e^2+(e-1)^2]-4(e-1)^3(k+k^2)\right\} > 0.\end{aligned} \tag{A.30}$$

After tedious algebraic calculation, we obtain this is equivalent to

$$[(2e^2+(e-1)(1+2k)]\sqrt{\Delta_1} + (2e-1)(e-1) + 4(e-1)^2(k+k^2) > 0. \tag{A.31}$$

This is obviously true; thus, $\partial t_1/\partial e > 0$ is also true.

(iv) From $a_1 = k_r^2(ew_1/(e-1))^{-2e}[(2e-1)w_1-2(e-1)C]^2/16(e-1)^2$, we have

$$\frac{\partial a_1}{\partial w_1} = \frac{k_r^2}{8(e-1)}\left(\frac{ew_1}{e-1}\right)^{-2e}[(2e-1)w_1-2(e-1)C]\left[\frac{2eC}{w_1}-(2e-1)\right]. \tag{A.32}$$

Since $e > 2 + 2/k$ is equivalent to $C/w_1 < (2e-3)/(2e-2)$, we have

$$\frac{2eC}{w_1} - (2e-1) < 2e(2e-3)/(2e-2) - (2e-1) = -1/(e-1) < 0. \tag{A.33}$$

Therefore, $\partial a_1/\partial w_1 < 0$. When k_r increases, k will decrease and w_1 will decrease according to (i); thus, a_1 will increase. Therefore, $\partial a_1/\partial k_r > 0$. However, when k_m increases, k will increase and w_1 will increase according to Part (i); thus, a_1 will decrease. Therefore, $\partial a_1/\partial k_m < 0$.

(v) From $A_1 = k_m^2(w_1-C)^2(ew_1/(e-1))^{-2e}/4$, we have

$$\frac{\partial A_1}{\partial w_1} = \frac{k_m^2}{2}\left(\frac{ew_1}{e-1}\right)^{-2e}(w_1-C)\left[\frac{eC}{w_1}-(e-1)\right]. \tag{A.34}$$

Making use of Part (i), w_1 is increasing with k, thus $w_1 < \lim_{k\to\infty} w_1 = eC/(e-1)$ according to the expression of w_1 in (10). Therefore, we have $eC/w_1 - (e-1) > 0$ and thus $\partial A_1/\partial w_1 > 0$.

When k_r increases, k will decrease and w_1 will decrease according to Part (i); thus, A_1 will increase. Therefore, $\partial A_1/\partial k_r < 0$. However, when k_m increases, k will increase and w_1 will increase according to Part (i); thus, A_1 will increase. Therefore, $\partial A_1/\partial k_m > 0$.

This completes the proof.