Actuarial Modeling of a Children's Protection Insurance

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Abstract

The article is to build an actuarial model of a children's protection insurance product with basic actuarial knowledge and mathematics methods. Combining the annuity insurance and risk of parents' mortality, the product protects the children from harm of accidental death of their parents by compensating the economic loss which equals to the expected income of the dead parent until the children achieve the adult age.

The authors build an independent model based on the assumption that family status has no relation with the mortality of two parents and their child. In this way, the actuarial present value can be easily presented by a fixed proportion of parents' income.

However, some previous investigations indicate that the family status significantly affects the health conditions of the family members and thus directly affects the probability of mortality. The authors, therefore, try to prove the mortality risk is dependent on the family status (the marriage status), introduce another variable (marriage year) and restructure an independent model.

The paper uses the China's 5th population sample census data in 2000 to build the model, calculates the actuarial present value of the future premium and compares the premium results of dependent and independent models.

Key words: Actuarial model, Marriage effect, Family income, Children's protection insurance

1. Introduction and background

According to a research in 2005, there are millions of orphans in China, 30% of whom cannot receive regular economic support and suffer a lot in their childhood or even years after they become adults. The children from single families are in similar situations. Realizing the fact that there are insofar few insurance policies protecting those pathetic children in China, the authors design a specific insurance model to assist those children. So this insurance model will be based on the status of the family (married and complete/widowed/both dead).

1.1 The brief review of established practice and theory

1.1.1 Some actuarial basic application review

There are some basic actuarial concepts from them:

ANDERSON, J.E., LOUIS, T.A., HOLM, N.V. and HARVALD B. (1992) provide some useful survival function estimation to reflect the dependent time variables. FREES, E., CARRIERE, J. and VALDEZ, E. (1996) base on the copula function and estimate annuity actuarial valuation on dependent mortality. Bowers and his cooperators (1997) summarize the actuarial basic concepts and actuarial application method.

1.1.2 Some study on marriage and mortality

Willekens, F. J., Shah, I., Shah, J. M. and Ramachandran, P. (1982) discuss the multi-state on the marital life tables. Thomas J Espenshade and Rachel Eisenberg (1982) also indicate the relationship between marriage and mortality and find some mortality rules on several marriage status. Schrijvers CT (1999) explains that marriage infects the mortality, often lowers the mortality. Dena H. Jaffe (2007) indicates the positive effect that marriage has on mortality in current society.

In China there are few studies focusing on the family status and mortality. Zeng yi (1987) builds the increment-decrement life model for the population in China. Song Jian and Huang Fei (2008) study some marriage characters in demography and find their importance in the family life.

However, few studies combine the actuarial model and family status to design a new type of family-income insurance.

1.2 Object and outline of current study

The object of the article is to design an actuarial model for a specific insurance product. It will respectively discuss the dependent and independent models. The benefit is, different from the studies above, based on the children's family status. It also provides data from the 5th population census to support the model. In the article,

Section 1 is the introduction and background of the insurance product and some previous theory or study review.

Section 2 is the basic purpose and description of the product.

Section 3 is the content of the independent model.

Section 4 is the content of the modified dependent model with the marital factor taken into consideration.

Section 5 is conclusion of the model and the final result.

2. The preparation to build the insurance product model

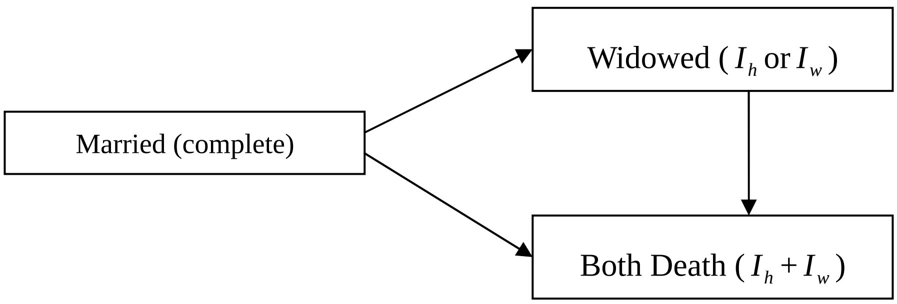
2.1 The purpose of insurance product

The initial purpose is to make sure that a family, either poor or wealthy, can afford to buy our product and benefit from it. Therefore, we set the premium as a fixed proportion of family income. And since the loss of the mortality of a parent is his/her family income, we pay compensation (benefit) equivalent to the income in the form of annuity.

Then after some considerations, the features of the insurance are summarized as follow:

1) The insurance policy holder should be parents from a complete family.
2) The benefit covers the period from the year the family buys the insurance to the year the child achieves his/her adult age.
3) The benefit is an annuity equal to the lost family income.
4) The policy will end if the child dies before 18 years old.
5) The premium should be paid as a fixed proportion of the family income at the beginning of each year unless one or both parents are dead or the child becomes an adult.
6) If death occurs, the annuity will be receivable at the beginning of the next year.

We further illustrate these features in the following picture.



I_h and I_w are the income of husband and wife. They respectively represent the benefit received by widowed family status or by both-dead family status.

2.2 Some basic actuarial notations

See the Appendix A.

3 The model with assumption of independent mortality

As the caption referred, the model is built based on the assumption that mortality is independent on family status (the family status will not significantly impacts the probability of mortality and the probabilities of the mortality of two parents and a child will be independent on each other).

Then suppose husband age is x, wife age y and the child age z. We have

The couples survival function at future time t, ${}_t p_{xy} = {}_t p_x \cdot {}_t p_y$

The family survival function at future time t, ${}_t p_{xyz} = {}_t p_x \cdot {}_t p_y \cdot {}_t p_z$

They all follow the same survival probabilities distribution; in a specific year the family status can only change once, such as from marriage to widow or from widow to death.

So in t+1year, the expected benefit would be

$$B_{t+1} = I_h \cdot P(widow(husband)) + I_w \cdot P(widow(wife)) + (I_h + I_w) \cdot P(Bothdeath)$$

$$= I_h \cdot {}_t q_x \cdot {}_t p_y + I_w \cdot {}_t p_x \cdot {}_t q_y + (I_h + I_w) \cdot ({}_t q_x \cdot {}_t q_y)$$

$$= I_h \cdot {}_t q_x \cdot (1 - {}_t q_y) + I_w \cdot (1 - {}_t q_x) \cdot {}_t q_y + (I_h + I_w) \cdot ({}_t q_x \cdot {}_t q_y)$$

$$= I_h \cdot {}_t q_x + I_w \cdot {}_t q_y$$

The function shows us that B_{t+1}（ the benefit of year t+1）can be divided into a husband part and a wife part, with each part being the product of death rate q and income I.

The actuarial present value would be (all the payment is discrete, i represents interest between each year)

$$A_{xyz} = \sum_{t=0}^{18-z} B_{t+1} \cdot v^{t+1} \cdot {}_t p_z = \sum_{t=0}^{18-z} (I_h \cdot {}_t q_x + I_w \cdot {}_t q_y) \cdot v^{t+1} \cdot {}_t p_z$$

$$= \sum_{t=0}^{18-z} I_h \cdot {}_t q_x \cdot v^{t+1} \cdot {}_t p_z + \sum_{t=0}^{18-z} I_w \cdot {}_t q_y \cdot v^{t+1} \cdot {}_t p_z \qquad \text{where } v = (1+i)^{-1}$$

Now it is clear that the insurance is separated into two joint-life group (x, z) and (y, z) No benefit will be compensated when child z is dead, while the expected income will be paid when father x or mother y is dead. When the survival function is known, the a.p.v of the benefit will be easily calculated.

According to the equivalent actuarial principle, Premium amount = $A_{xyz} / \ddot{a}_{xyz:z-18} = (\sum_{t=0}^{18-z} I_h \cdot {}_t q_x \cdot v^t \cdot {}_t p_z + \sum_{t=0}^{18-z} I_w \cdot {}_t q_y \cdot v^t \cdot {}_t p_z) / \sum_{t=0}^{18-z} v^t \cdot {}_t p_z \cdot {}_t p_y \cdot {}_t p_x$

And the proportion between the premium amount and the family income

$$\alpha = (\frac{I_h}{I_h + I_w} \cdot \sum_{t=0}^{18-z} {}_t q_x \cdot v^t \cdot {}_t p_z + \frac{I_w}{I_h + I_w} \cdot \sum_{t=0}^{18-z} {}_t q_y \cdot v^t \cdot {}_t p_z) / \sum_{t=0}^{18-z} v^t \cdot {}_t p_z \cdot {}_t p_y \cdot {}_t p_x$$

If the ratio of husband income to wife income is fixed, the proportion will remain constant no matter how the actual family income changes.

All the formulas above are built not by actuarial dynamic but deterministic models, are classic discrete actuarial formulas and can be easily applied if provided enough conditions of mortality.

The authors use the mortality life table in China (based on 2000 census population data) to build the insurance model in Excel 2003. Detail assumption and premium results are in Appendix B.

To further demonstrate how the functions work, let's take x=30-60, y=30, z=0, i=2.5%, $I_h = I_w$ as an example.

The premium is shown in the Table 3.1

Table 3.1 Premium as a ratio of income (per year)

age	Income Ratio	age	Income Ratio
30	1.56%	46	4.41%
31	1.63%	47	4.81%
32	1.72%	48	5.25%
33	1.81%	49	5.75%
34	1.91%	50	6.29%
35	2.03%	51	6.87%
36	2.14%	52	7.56%
37	2.28%	53	8.33%
38	2.43%	54	9.20%
39	2.60%	55	10.16%
40	2.78%	56	11.24%
41	2.97%	57	12.48%
42	3.20%	58	13.83%
43	3.45%	59	15.35%
44	3.74%	60	16.98%
45	4.06%		

If a 30-year-old wife and 30-year-old husband buy t

he insurance for their 0-year-old child, the premium just costs 0.65% of their income per year. If the husband age increases, the cost will increase correspondingly, because of the increasing probability of mortality. Therefore the premium seems more attractive to the younger couples to protect their children.

4 The model with dependent mortality assumption

4.1 Basic marriage effect estimate model and method

Many researches indicate that the model with independent mortality assumption is not reliable because in general the family status will affect the family members' mortality.

After analyzing the causes of the main dependency, the authors find it necessary to take marriage, one most significant factor, into account in order to price the insurance product more accurately in the current deterministic model. Comparing to several cases, the marriage period may be a good indicator to reflect the effect of marriage. The authors choose the marriage time (M) to calculate the effect when the family status is married. When family status is widowed or both-dead, M=0 and no marriage effect exists.

M_{ix} is total marriage year at i-th sample point in age x,

Then assume ${}_t p_x^* = {}_t p_x \cdot e^{-\lambda M_{ix}}$ using a common exponential survival function of M_{ix} to form an "independent" survival estimation.

We use the common and simple exponential function rather than some functions like copula and joint function between husband and wife because it would more easily indicate the relationship between marriage group and common group and more properly adjust the mortality rate in applications of the insurance model.

In order to estimate the unknown parameter λ, we structure the likelihood function

$$L = \prod_t \prod_i \left(\frac{S(x+t)^*}{S(x)^*}\right)^{l_{ix+t}} \cdot \left(1 - \left(\frac{S(x+t)^*}{S(x)^*}\right)\right)^{d_{ix+t}}$$

Where l_{ix+t} is the survive people aged x+t and its total marriage year is M_{ix}

d_{ix+t} is the people who dies between (x,x+t) and its total marriage year is M_{ix}

$$\ln(L) = \sum_t \sum_i l_{ix+t} \cdot \ln({}_t p_x \cdot e^{-\lambda M_{ix+t}}) + \sum_t \sum_i d_{ix+t} \cdot \ln(1 - {}_t p_x \cdot e^{-\lambda M_{ix+t}})$$

According to the chain rules,

$$\frac{\partial \ln(L)}{\partial \lambda} = \sum_t \sum_i l_{ix+t} \cdot (-M_{ix+t}) + \sum_t \sum_i \frac{d_{ix+t} \cdot {}_t p_x \cdot e^{-\lambda M_{ix+t}} \cdot M_{ix+t}}{1 - {}_t p_x \cdot e^{-\lambda M_{ix+t}}} = 0$$

According to observed l_{ix+t}, M_{ix+t}, d_{ix+t}, the population survival function ${}_t p_x$, the parameter λ can be calculated in the computer programs.

4.2 Parameter estimate in actual data

Then we use the mean sample data of the population census in 1999-2000 as the average proportion to estimate the death of the people in marriage group and test whether the value of the parameter is appropriate: we set the hypothesis stating there is no difference between the marriage group and common group, using the population assumption that the population expectation death is a good survival function to fit the marriage group.

Choose 25-64 as the common range of age at which the couple would raise a child.

Set $H(0): S(x) = S(x)^{\tau}$ where $S(x)^{\tau}$ is the survival number if the marriage group to use the common mortality.

Calculate the $\chi^2 = \sum (E_j - d_j)^2 / E_j \sim \chi^2(k-1)$ when the survival data is given without estimation. k is the number of the age interval so k =40 and free degree is 39. E_j is the death number at age j estimated by total population death rate and d_j is the actual death number at age j .

After calculation, χ^2 >63.69= $\chi^2_{0.01}(39)$, the probability is less than 0.01. Therefore $H(0)$ is rejected. The survival data from population is not proper to fit the marriage group.

Then we have to estimate λ which will help us to find the marriage effect. From the first hypothesis test, we know the λ is not zero because the survival function of the common group and that of marriage group are different.

For the male group, the maximum likelihood estimation (MLE) $\hat{\lambda}_m$ = -0.000095622

We substitute it into the function and test its accuracy.

Set $H(0): S(x) = S(x)^* = S(x)^{\tau} \cdot e^{-\lambda M_x}$ where $S(x)^*$ is including the marriage effect.

So $\chi^2 = \sum (E_j - d_j)^2 / E_j \sim \chi^2(k - \mathrm{r} - 1)$ r =1 which is the number of estimated parameters.

χ^2 =32.47<50.65= $\chi^2_{0.1}(38)$ It cannot reject $H(0)$ at the 10% significance so we regard the new survival function with parameter λ as a good estimation for the marriage group.

Also for the female we have $\hat{\lambda}_m$ = -0.000067193 and χ^2 =20.65<50.65= $\chi^2_{0.1}(38)$

Two negative parameters that let $S(x)^* > S(x)^\tau$ tell us marriage will decline the mortality of the people.

All the calculation processes and data are put into the Appendix C.

4.3 The modified insurance model and result analysis

Then we go back to our insurance product model.

We use the calculation results to form a new life table. It can be separated into two parts. If the previous year the family is in the complete status, the survival and corresponding mortality rate should be adjusted with the λ and M . If the family status in previous year changed to widow, the marriage effect will be eliminated and the mortality will follow the population assumption.

So the formula for the insurance product is revised below:

The husband and wife survival functions are ${}_t p_x^* = {}_t p_x \cdot e^{-\lambda_m M}$, ${}_t p_y^* = {}_t p_y \cdot e^{-\lambda_f M}$ in marriage status; in widowed status, the functions remain the same.

The couples survival function at future time t, ${}_t p_{xy} = {}_t p_x \cdot {}_t p_y \cdot e^{-(\lambda_m+\lambda_f)M}$

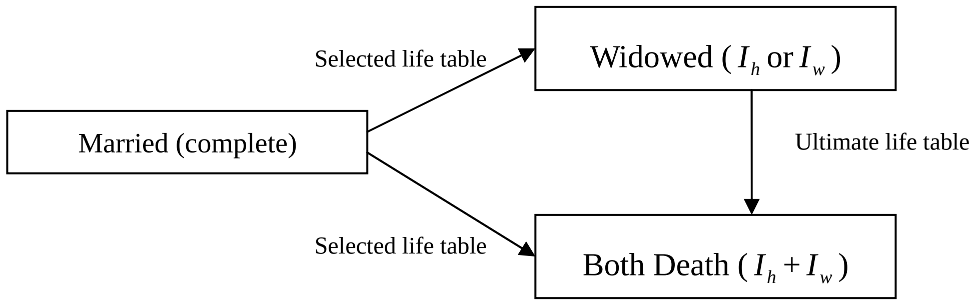
The family survival function at future time t, ${}_t p_{xyz} = {}_t p_x \cdot {}_t p_y \cdot {}_t p_z \cdot e^{-(\lambda_m+\lambda_f)M}$

Then at year t+1, the expected benefit would be

$$\begin{aligned} B_{t+1} &= I_h \cdot P(widow(husband)) + I_w \cdot P(widow(wife)) + (I_h + I_w) \cdot P(Bothdeath) \\ &= I_h \cdot \sum_{u=1}^{t} {}_u p_{xy} \cdot q_{x+u}^* \cdot {}_{t-u} p_{y+u} + I_w \cdot \sum_{u=0}^{t} {}_u p_{xy} \cdot q_{y+u}^* \cdot {}_{t-u} p_{x+u} \\ &\quad +(I_h + I_w) \cdot (\sum_{u=0}^{t} {}_u p_{xy} \cdot q_{y+u}^* \cdot q_{x+u}^* + \sum_{u=0}^{t} {}_u p_{xy} \cdot q_{x+u}^* \cdot \sum_{w=0}^{t-u} {}_w p_{y+u} \cdot q_{y+w+u} + \sum_{u=0}^{t} {}_u p_{xy} \cdot q_{y+u}^* \cdot \sum_{w=0}^{t-u} {}_w p_{x+u} \cdot q_{x+w+u}) \end{aligned}$$

u is the marriage status transition time.

The dependent model is complicated but easy to be set up on the computer (detailed in Appendix C). The marriage period is a selected period, and widow will ultimately use the total population life table (ultimate life table).



The other formulas remain the same except for different B_{t+1} and ${}_t p_{xyz}$.

Then we substitute the estimation of λ above into the model and get the premium result under the dependent condition. (x=30-60, y=30, z=0, i=2.5%, $I_h = I_w$)

Table 4.1 The comparison of the dependent and independent conditions (% of income)

age	Dependent	Independent	Discount
30	1.34%	1.56%	14.49%
31	1.41%	1.63%	13.88%
32	1.49%	1.72%	13.22%
33	1.58%	1.81%	12.57%
34	1.69%	1.91%	11.89%
35	1.80%	2.03%	11.24%
36	1.92%	2.14%	10.64%
37	2.06%	2.28%	10.00%
38	2.20%	2.43%	9.41%
39	2.37%	2.60%	8.82%
40	2.55%	2.78%	8.26%
41	2.74%	2.97%	7.75%
42	2.97%	3.20%	7.21%
43	3.22%	3.45%	6.71%
44	3.51%	3.74%	6.21%
45	3.83%	4.06%	5.73%

It is clear that the values of premium get lower (generally 25% lower) because marriage improves the survival probability when λ <0. The lower premium after modification becomes more reliable and satisfying to parents who want to protect their child.

5. Conclusion

The article designs a specific insurance product related to the family status to protect the children who lose their parent(s) and this type of insurance product can not yet be found in China and may be peculiar in the world. Using the actuarial basic concepts and independent assumptions, we build the actuarial model for this insurance and calculate its premium. After referring to some references we take marriage's impacts on mortality into consideration and restructure a dependent mode with the population sample data in 2000, from which the values of the premium are lower than those from the independent model. Pay attention that the actuarial model we build ignores the widow and divorce group's impact on mortality, which requires further study.

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Appendix A

Some Basic actuarial formulas:

T is the death age of the individual, and x is current age of the individual.
When the individual follow the survival function, then we get the basic actuarial function

The survival probability at age x+1 when survive at age x, the survive number is l_x

$$_t p_x = P(T > x+t \mid T > x) = S(x+t)/S(x) = l_{x+t} / l_x$$

The death number is d_x, the death probability from age x to age x+t, $_t q_x = 1 - {_t p_x} = d_x / l_x$

So for two independent lives, the combined survive function would be

Both survive probability is $_t \overline{p}_{xy} = {_t p_x} \cdot {_t p_y}$

At least one survive probability is $_t p_{xy} = 1 - {_t q_x} \cdot {_t q_y}$;

Then we have the following notation to represent if 1 per annum is certain to gain at each beginning of year in a period t.
In the discrete time as the actual world, the annuity will represent the formulas:
Set $v = 1/(1+i)$

Then $\ddot{a} = \sum_{n=0}^{t-1} v^n = (1-v^n)/(iv)$, $A_x = \sum_{n=0}^{t-1} v^n \cdot {_n p_x} \cdot q_{x+n}$, $\ddot{a}_x = \sum_{n=0}^{t-1} v^n \cdot {_n p_x}$

Combined the survival probability notation above, we build the initial joint family survive model. Set the age of father and mother is at x and y, and then

In the discrete future years t, the family survive probability is $_t \overline{p}_{xy}$;

The widowed probability is $_t p_{xy} - {_t \overline{p}_{xy}}$, while the both death probability is $_t q_x \cdot {_t q_y}$

Appendix B

Assumption is based on the mortality table with current life table 1999-2000

Interest is 2.5%, fixed in following years; Income is fixed when issue.(Variable interest and income is not the main point in the article) Equal income between husband and wife

The premium rate as a proportion of Income (30-45)

Husband

	age	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
	30	1.56%	1.63%	1.72%	1.81%	1.91%	2.03%	2.14%	2.28%	2.43%	2.60%	2.78%	2.97%	3.20%	3.45%	3.74%	4.06%
	31	1.58%	1.65%	1.74%	1.83%	1.93%	2.04%	2.16%	2.30%	2.45%	2.62%	2.80%	2.99%	3.22%	3.47%	3.76%	4.08%
	32	1.61%	1.68%	1.76%	1.85%	1.96%	2.07%	2.19%	2.33%	2.48%	2.64%	2.83%	3.02%	3.24%	3.49%	3.78%	4.11%
	33	1.63%	1.70%	1.78%	1.87%	1.98%	2.09%	2.21%	2.35%	2.50%	2.67%	2.85%	3.04%	3.27%	3.52%	3.81%	4.14%
	34	1.66%	1.73%	1.82%	1.91%	2.01%	2.13%	2.24%	2.39%	2.53%	2.70%	2.89%	3.08%	3.30%	3.55%	3.84%	4.17%
	35	1.70%	1.77%	1.85%	1.94%	2.05%	2.16%	2.28%	2.42%	2.57%	2.74%	2.92%	3.11%	3.34%	3.59%	3.88%	4.21%
Wife	36	1.73%	1.80%	1.89%	1.98%	2.08%	2.20%	2.32%	2.46%	2.61%	2.77%	2.96%	3.15%	3.38%	3.63%	3.92%	4.25%
	37	1.78%	1.85%	1.93%	2.02%	2.13%	2.24%	2.36%	2.50%	2.65%	2.82%	3.01%	3.20%	3.43%	3.68%	3.97%	4.30%
	38	1.83%	1.90%	1.98%	2.07%	2.18%	2.29%	2.42%	2.56%	2.71%	2.87%	3.06%	3.25%	3.48%	3.73%	4.02%	4.35%
	39	1.89%	1.96%	2.04%	2.13%	2.24%	2.35%	2.47%	2.61%	2.77%	2.93%	3.12%	3.31%	3.54%	3.79%	4.09%	4.41%
	40	1.95%	2.02%	2.10%	2.19%	2.30%	2.41%	2.54%	2.68%	2.83%	3.00%	3.18%	3.37%	3.60%	3.86%	4.15%	4.48%
	41	2.01%	2.08%	2.17%	2.26%	2.37%	2.48%	2.60%	2.74%	2.90%	3.06%	3.25%	3.44%	3.67%	3.93%	4.22%	4.55%
	42	2.09%	2.16%	2.25%	2.34%	2.45%	2.56%	2.68%	2.82%	2.98%	3.14%	3.33%	3.53%	3.76%	4.01%	4.31%	4.64%
	43	2.18%	2.25%	2.33%	2.43%	2.53%	2.65%	2.77%	2.91%	3.06%	3.23%	3.42%	3.62%	3.85%	4.10%	4.40%	4.73%
	44	2.27%	2.35%	2.43%	2.52%	2.63%	2.75%	2.87%	3.01%	3.17%	3.34%	3.52%	3.72%	3.95%	4.21%	4.50%	4.84%
	45	2.38%	2.46%	2.54%	2.63%	2.74%	2.86%	2.98%	3.13%	3.28%	3.45%	3.64%	3.83%	4.07%	4.32%	4.62%	4.96%

Appendix C

The data is from 2000 population census (0.95% sample) data the population is estimated by the mortality used in the total population without considering it is marriage. The estimated ones are from the model listed in the article. The data is also combined with the total population result.

Age	Male Death(Est)	Actual Death(act)	Male Population	Female Death(Est)	Actual Death(act)	Female Population
25	7,402	7,478	8,467	5,779	5,836	7,215
26	8,632	8,723	10,425	5,864	5,808	7,974
27	10,031	10,087	12,732	6,291	6,240	9,138
28	10,531	10,619	14,315	5,813	5,778	9,573
29	11,538	11,685	16,748	6,148	6,198	10,968
30	12,381	12,342	18,807	6,022	6,027	11,807
31	11,277	11,317	19,018	5,030	5,056	11,695
32	10,977	10,989	19,415	4,560	4,563	11,716
33	9,227	9,304	18,375	3,258	3,282	10,769
34	10,663	10,770	21,501	3,590	3,619	12,359
35	11,578	11,585	23,321	3,879	3,876	13,239
36	11,296	11,366	24,884	2,687	2,712	13,418
37	11,958	12,070	25,924	2,903	2,895	13,669
38	8,104	8,103	17,989	1,777	1,779	9,451
39	7,031	6,998	15,442	2,020	2,015	8,527
40	9,356	9,265	18,890	2,725	2,745	9,898
41	9,443	9,423	20,498	2,328	2,344	10,511
42	13,319	13,402	27,047	3,819	3,786	14,100
43	12,926	12,837	27,482	4,012	3,973	14,997
44	14,181	14,046	29,132	4,789	4,816	16,169
45	17,461	17,445	33,699	6,420	6,467	18,762
46	17,025	16,931	33,185	6,633	6,600	19,048
47	18,710	18,571	34,994	7,681	7,677	20,279
48	18,683	18,766	34,697	8,204	8,158	20,502
49	20,031	19,965	35,440	9,584	9,623	21,399
50	25,012	24,967	40,877	11,985	11,924	23,808
51	21,800	21,627	36,933	10,346	10,263	21,485
52	23,505	23,425	37,971	11,734	11,800	22,293
53	23,770	23,681	38,002	12,370	12,247	22,702
54	26,396	26,663	39,852	13,674	13,546	23,543
55	27,333	27,203	40,422	14,280	14,228	23,724
57	30,734	30,905	43,142	15,700	15,658	24,498
58	33,779	33,958	46,590	17,382	17,253	26,296
59	41,307	41,811	54,623	21,052	21,241	30,163
60	46,693	46,634	59,211	23,876	23,673	32,281

Dependent Mortality Model Premium Result (% of Income)

Remain the same assumptions with the independent cases but includes the estimated marriage effect.

The premium rate as a proportion of Income (30-45), initial marriage year at age 28

Husband

Wife

age	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
30	1.34%	1.41%	1.49%	1.58%	1.69%	1.80%	1.92%	2.06%	2.20%	2.37%	2.55%	2.74%	2.97%	3.22%	3.51%	3.83%
31	1.36%	1.43%	1.51%	1.60%	1.70%	1.82%	1.94%	2.07%	2.22%	2.39%	2.57%	2.76%	2.99%	3.24%	3.53%	3.85%
32	1.38%	1.45%	1.53%	1.62%	1.73%	1.84%	1.96%	2.10%	2.25%	2.41%	2.60%	2.79%	3.01%	3.26%	3.55%	3.88%
33	1.40%	1.47%	1.56%	1.65%	1.75%	1.87%	1.98%	2.12%	2.27%	2.44%	2.62%	2.81%	3.04%	3.29%	3.58%	3.90%
34	1.43%	1.51%	1.59%	1.68%	1.78%	1.90%	2.02%	2.16%	2.31%	2.47%	2.65%	2.85%	3.07%	3.32%	3.61%	3.94%
35	1.47%	1.54%	1.62%	1.71%	1.82%	1.93%	2.05%	2.19%	2.34%	2.51%	2.69%	2.88%	3.11%	3.36%	3.65%	3.97%
36	1.51%	1.58%	1.66%	1.75%	1.86%	1.97%	2.09%	2.23%	2.38%	2.54%	2.73%	2.92%	3.15%	3.40%	3.69%	4.01%
37	1.55%	1.62%	1.71%	1.80%	1.90%	2.02%	2.14%	2.28%	2.43%	2.59%	2.78%	2.97%	3.20%	3.45%	3.74%	4.06%
38	1.60%	1.67%	1.76%	1.85%	1.95%	2.07%	2.19%	2.33%	2.48%	2.64%	2.83%	3.02%	3.25%	3.50%	3.79%	4.12%
39	1.66%	1.73%	1.81%	1.90%	2.01%	2.12%	2.24%	2.38%	2.53%	2.70%	2.89%	3.08%	3.31%	3.56%	3.85%	4.18%
40	1.72%	1.79%	1.87%	1.96%	2.07%	2.18%	2.31%	2.45%	2.60%	2.76%	2.95%	3.14%	3.37%	3.62%	3.92%	4.25%
41	1.78%	1.85%	1.94%	2.03%	2.14%	2.25%	2.37%	2.51%	2.66%	2.83%	3.02%	3.21%	3.44%	3.69%	3.99%	4.32%
42	1.86%	1.93%	2.02%	2.11%	2.22%	2.33%	2.45%	2.59%	2.74%	2.91%	3.10%	3.29%	3.52%	3.78%	4.07%	4.40%
43	1.95%	2.02%	2.10%	2.20%	2.30%	2.42%	2.54%	2.68%	2.83%	3.00%	3.19%	3.38%	3.61%	3.87%	4.16%	4.50%
44	2.04%	2.12%	2.20%	2.29%	2.40%	2.52%	2.64%	2.78%	2.93%	3.10%	3.29%	3.49%	3.72%	3.97%	4.27%	4.60%
45	2.15%	2.23%	2.31%	2.40%	2.51%	2.63%	2.75%	2.89%	3.05%	3.22%	3.41%	3.60%	3.83%	4.09%	4.39%	4.72%

An illustration of calculation the premium in Excel 2003

To calculate the probability of 3 family status, then complete family will use the new survival probability ${}_t p_x^*, {}_t p_y^*$. It is easily to get complete family probability ${}_t p_x^* \cdot {}_t p_y^*$ for each year t

Then the widowed family, it can be divided into husband survive and wife survive family.

Now set widowed probability

$$w_t^{husband} = w_{t-1}^{husband} \cdot (1-q_{x+t-1}) + {}_t p_x^* \cdot {}_{t-1} p_y^* \cdot q_{y+t-1}^*$$

$$w_t^{wife} = w_{t-1}^{wife} \cdot (1-q_{y+t-1}) + {}_t p_y^* \cdot {}_{t-1} p_x^* \cdot q_{x+t-1}^*$$

Set death probability each year

$$d_t = d_{t-1} + w_{t-1}^{husband} \cdot (1-q_{x+t-1}) + w_{t-1}^{wife} \cdot (1-q_{y+t-1}) + {}_{t-1} p_x^* \cdot {}_{t-1} p_y^* \cdot q_{x+t-1}^* \cdot q_{y+t-1}^*$$

For 30-30 couple their family status probability is in the table below:

age	Married	Widowed	Widowed	Both Death
30	1.00000000			
31	0.99780654	0.00137773	0.00081461	0.00000112
32	0.99580002	0.00264599	0.00154937	0.00000463
33	0.99378030	0.00393468	0.00227420	0.00001082
34	0.99196593	0.00511487	0.00290001	0.00001919
35	0.99008812	0.00634532	0.00353591	0.00003064
36	0.98811753	0.00764525	0.00419143	0.00004579
37	0.98635097	0.00885716	0.00472870	0.00006318
38	0.98445212	0.01016767	0.00529532	0.00008490
39	0.98256932	0.01148798	0.00583232	0.00011038
40	0.98054537	0.01285569	0.00645734	0.00014160
41	0.97821447	0.01445643	0.00714857	0.00018052
42	0.97608715	0.01593839	0.00775128	0.00022319
43	0.97365473	0.01761138	0.00845862	0.00027527
44	0.97126003	0.01923164	0.00917414	0.00033418
45	0.96865037	0.02097240	0.00997358	0.00040365
46	0.96556609	0.02301390	0.01093099	0.00048902
47	0.96234926	0.02511295	0.01195005	0.00058773
48	0.95868395	0.02747661	0.01313243	0.00070702