The Mathematical Model of the Oil Spilling in the Gulf of Mexico Based on the Extended Fay Formulas

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CONTENT

Abstract

This article is primarily based on the fundamental model of fluid mechanics, applying the Newton's Laws of motion and conservation of mass, to work out the new formula for stable but continuous spilling oil, and this formula was promoted from the formulas of Fay. In an ideal situation, using several crucial factors including gravity, density, viscous force, inertia, we can primarily estimate the size of the polluted area in the Gulf of Mexico and evaluate the final damage of this accident.

Initially we used the Fay formulas to build a conventional ellipse model, and approximately graphed the oil spreading area and got the variation of the spreading oil area. However, because this model is not primarily based on the local hydrology and weather condition, it lacks accuracy. Thus, we further discussed the movement of crude oil on the sea surface under the influence of ocean current and wind. Then we innovatively suggested a brand new model called "egg-shape", and simulated a more accurate contour of the spreading oil. Consequently, with the unprecedented "egg-shape" model, we can more accurately predict the direction of the oil spilling, leading to more effective remediation for a better environmental protection, and providing a more comprehensive treatment for the future of the Gulf of Mexico.

本文主要基于流体力学的基本模型，依据牛顿运动定律和质量守恒定律，借鉴 Fay 公式的进一步推广得到的静止点源连续溢油方程，在理想化的前提下，通过重力、表面张力、粘滞力、惯性力这几个决定性因素，对墨西哥湾洋面受污染的面积进行初步的估算，并确定原油泄漏的最终面积。

最初我们利用由 Fay 公式得到的传统椭圆模型对原油扩散进行建模，得到原油扩散的大致图形和面积变化情况，但该模型不符合当地的气候水文条件，故该模型的精准性较差。因此我们进一步讨论了原油在洋流风力影响下的运动，创新地提出了前所未有的蛋形模型，并绘制出了更精准的原油扩散大致轮廓。同时对整个墨西哥湾原油扩展的今后发展方向进行预测，提出有效的整治措施，让环境问题不再抽象。

I. General Information

[i] Background of Research

Recently, more and more people are concerned about environmental issues, in order to protect our common home. And it is widely accepted that the prevention and treatment of environmental disasters are very important. On April 20 this year, the U.S. Gulf of Mexico oil spilled, causing great concern in all parts of the world. It is estimated that a few months of time will be necessary to block the leaking oil wells but much worse than that, the oil pollution clean-up operation may take up to 10 years. In the next 10 years, the Gulf of Mexico may turn out to be a severely damaged sea, with hundreds of billions of dollars economic lost. Serious impact on the global environment is so profound that it is difficult to forecast the consequence.

As world citizens with a strong sense of responsibility, we have attached great importance to this event, so that more people are alerted of the urgency of the protection of the environment. Besides, "oil spilling" was reported to be one of the hottest words in 2010 recently. Therefore, we have decided to establish an accurate model for this oil spilling event, to evaluate and to predict the seriousness of this disaster and to provide a more accurate solution so as to minimize the damage of the oil spilling event, and to raise the concerns of people to environmental problem at the same time.

[ii] Highlights of Research

- We kept the essence of the Fay formulas and abandoned the complicated calculation. In addition, we adopted the experience of other researchers to simplify our model so as to make those abstract parameters more realistic.

- Since this treatise is primarily focused on the result and the ending of this oil spreading, we did not explore deeply into the detail of the oil spreading, but concentrated on the depiction of the shape and the area of the spreading oil.
- We made use of the familiar ellipse formula of parameters to primarily fit our oil spreading model, and thus guaranteed the accuracy of our model on the whole.
- Considering the special geographic location of this oil spilling accident, we put forward a specific model which is more applicable to this very event, thus making the movement of our model and the location of the final oil spreading picture more accurate, and therefore better predicted the spreading of the crude oil.
- In this article, we unprecedentedly put forward an "egg-shape" model. According to the simulation experiments we conducted, we found that this model turned out to be realistic, and therefore this model can specify the depiction of oil spreading.
- We took the advantage of Maple and some other software to make our model visional and graphical; we used it to produce a more accurate final graph combining news and data from the satellites.

II. Basis of Model

[i] Assumptions

To facilitate our modeling, we have made the following assumptions:

- Oil pollution is regional and water condition is the same. We use the same diffusion rate of crude oil, and do not consider the variation of water in different areas;
- The model is formed with continuous variables involved and the parameters in functions are smooth;
- The total volume of water is constant, regardless of rain and snow fall and self-purification process under natural conditions (crude oil in general will not be affected by the natural deposition), and no facilities are on the sea (that includes ships, or other oil-controlling facilities);
- Do not consider the outflow from the drilling platform before the drilling platform sank;
- In a large-scale, oil spreading only includes such parameters as gravity, surface tension, viscous forces which change the nature of diffusion process, such as density, tension, etc, and oil is in equilibrium in the vertical direction.

[ii] Parameters

- H is for the reservoir thickness, and Δh is for the water reservoir thickness. ρ_o=810kg/m^3 is for the oil density, ρ_w=1022kg/m^3 as the density of sea water, and therefore we have Δ=0.2074.
- I is for the expansion of crude oil leaked in diameter, which can be interpreted as a crude measure of all stages of spreading.

- Using δ to be the net surface tension with water-air interface δ_{wa}, oil-air interface δ_{oa}, and oil-water interface δ_{ow}. According to Fay's theory, in the ideal conditions when $\delta \leq 0$, the oil will stop the spreading. So the problem was simplified to the evaluation of the net surface tension coefficient, and thus δ is the amount which we will focus on.
- Local (28°N) acceleration is due to gravity g=9.79m/s^2. T is the time from when the oil started leaking to when the oil stopped leaking; V is the volume of crude oil which had already leaked.
- k_x, k_y, k_z are leaked stage of three empirical parameters, which are also called the crude oil volume attenuation coefficient. The international results are that k_x=1.40, k_y=1.65, k_z =0.60.
- Q is for the oil spreading amount per unit time; S is the constant speed in a single direction of flow, the direction shall be x-axis direction; D is the distance for the x-axis direction.

[iii] Correlated Formulas of the Model

As the spreading of oil can be divided into two stages (divided by 84 days, mainly the time when spilling was stopped) by three effects, we need not take all into account.

The early stage of spreading is mainly due to the spilling of crude oil, supplemented with evaporation; in the later stage of the spreading of crude oil (after blocked), it was the film diffusion and extension and also evaporation which played dominant factors (as the total volume of spilled oil has stopped increasing). Here we will consider the evaporation, degradation, and the oil control measures, all of which are positive factors that help reduce the crude oil spreading. However, the winds and ocean currents are factors which increase the spreading of crude oil, so we call them negative factors.

The leakage of crude oil began from the moment of time t_0, and we can assume the whole spreading as two processes, combined with information[1][2][3] on the hydrodynamic equations:

According to the fluid dynamics equations, in $0<t<t_0$ of the continuous leakage of crude oil, the volume $V(t)$ is as

$$V(t)=\sum_{t=j}^{n} Q\Delta t\left[1-K\left(t-j\Delta t\right)\right] \quad 0<t<t_0 \tag{1}$$

K is for the crude oil volume attenuation coefficient (to be confirmed).

When $0<t<t_0$, the leakage of crude oil changed in three stages:

$$I_{\mathrm{I}}=K_{\mathrm{I}}(\Delta gV)^{\frac{1}{4}}t^{\frac{1}{2}} \tag{2}$$

$$I_{\mathrm{II}}=K_{\mathrm{II}}(\Delta g)^{\frac{1}{6}}V^{\frac{1}{3}}v_w^{-\frac{1}{12}}t^{\frac{1}{4}} \tag{3}$$

$$I_{\mathrm{III}}=K_{\mathrm{III}}\sigma^{\frac{1}{2}}v_w^{-\frac{1}{4}}\rho_w^{-\frac{1}{2}}t^{\frac{3}{4}} \tag{4}$$

In the formulas above, I_{I}, I_{II}, I_{III} are the scales of each stage of the spreading, and K_{I}=1.35, K_{II}=1.60, K_{III}=0.48 are for the spreading coefficient of each stage. From the three types above, we can determine the adjacent two-phase critical points of two-phase t_{12} and t_{23}.

At $t=t_0$, the oil leakage stopped. Suppose that crude oil volume is V_0 (calculated by the (1) style), and oil volume with t is $V(t)$ as

$$V(t)=V_0\left[1-K(t-t_0)\right] \quad t>t_0 \tag{5}$$

Similarly, when $t>t_0$, the oil spreading can be also divided into three stages:

$$I_1=K_1(\Delta gV)^{\frac{1}{4}}t^{\frac{1}{2}} \tag{6}$$

$$I_2=K_2(\Delta g)^{\frac{1}{6}}V^{\frac{1}{3}}v_w^{-\frac{1}{12}}t^{\frac{1}{4}} \tag{7}$$

$$I_3=K_3\sigma^{\frac{1}{2}}v_w^{-\frac{1}{4}}\rho_w^{-\frac{1}{2}}t^{\frac{3}{4}} \tag{8}$$

In the formulas above, K_1, K_2, K_3 are expansion coefficients in consecutive stages of oil spreading after time t_0.

In Fay's theory, under ideal conditions, net surface tension coefficient is less than zero, when the expansion of crude oil stopped,

$$A_f = K_f \left(\frac{\delta^2 V^6}{\rho_w{}^2 v_w D^3 S^6} \right)^{\frac{1}{8}} \tag{9}$$

where S is the oil solubility, D is surfactant for oil spreading coefficient in the water, and K_f is the coefficient which is needed to be determined. However, K_f involves many uncertain factors, which makes it more difficult to determine. According to the information, usually, by the experiments we use the simplified formula:

$$A_f = 10^5 V^{\frac{3}{4}} \tag{10}$$

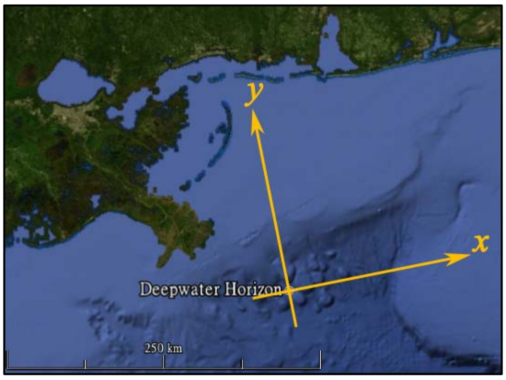
with A_f in units of m^2, V in units of m^3.

Since our model is based on a continuous period of time of oil leakage, the model spans [0, t_0]

III. Analysis & Calculations of Initial Model

[i] Analysis of the Model

On the sea, for the two-dimensional unsteady flow field, the spreading of crude oil can be divided into two directions. We considered the area near the leakage point as a plane, and therefore set the direction of *X*-axis to be eastward, and the direction of *Y*-axis to be northward. In order to facilitate description, we assume *x*-axis as the main diffusion direction of crude oil, with *x*-axis perpendicular to *y*-axis, which was the secondary direction of movement (Graph 1, the direction of *x*-axis will be discussed hereinafter).



Graph 1

The following are the main influence parameter:

- Total volume of crude oil leaked: V=0.76×10^{6}m^3
- Oil leakage was successfully blocked by: t_0=84d=7.26×10^{6}s
- Crude oil in the [0, t_0] flow: Q=9100m^3/d=(9100/86400)m^3/s
- Oil density: ρ_o=810kg/m^3
- The density of sea water: ρ_w=1022kg/m^3
- Wind drift coefficient: K=0.020
- Continuous expansion coefficient of the actual oil spill: K_{I}=1.35, K_{II}=1.60, K_{III}=0.48
- Upper vertical surface viscosity: γ_i=10kg/(m·s)
- Water viscosity: v_w=1.01×10^{6}m^2/s
- Gulf Centre latitude: φ=28°(N)
- The average water depth of Gulf of Mexico : H=1512m
- Net surface tension: σ=71.05×10^{-3}kg/s^2

- Gravity acceleration in Gulf of Mexico: g=9.79m/s^2
- Earth's rotation angular velocity: ω=72.72×10^{-6}rad/s
- The speed and the direction of wind near the leakage point: $|\boldsymbol{U}_{10}|$=5m/s, 315°
- The speed and direction of the fluid near the leakage point: $|\boldsymbol{U}_C|$=0.25m/s, 105°

(All the data cited above are collected from Internet, such as Baidu and Wiki.)

[ii] Calculation Procedure

In the Gulf of Mexico, the spreading scale of oil can be approximately superimposed on the sea of spreading crude oil. We still use the coordinates above. The main direction of movement is the x-axis, supplemented with a different y-axis of the spreading. Okubo et al concluded the data[1], and then provided the mean-square deviation of normal distribution of spreading oil:

$$\delta(t) = 0.001t^{1.17} \tag{11}$$

Then we used the formula for the main movement in the direction of x-axis, d_x, and the subordinate movement in the direction of y-axis, d_y, as follow:

$$d_x(t) = \omega\delta(t) \tag{12}$$

$$d_y(t) = \omega\alpha\delta(t) \tag{13}$$

where α, ω are constants, in general, to take $\alpha = \frac{1}{\sqrt{10}} = 0.316$, $\omega = \sqrt{12} = 3.464$. t unit of s, d_x, d_y unit of m.

According to the information which we obtained, until oil leakage was blocked up, the total amount of the leaked oil to the surface (not including those which were burned) was about 400 million barrels of crude oil which can be expressed by the average leakage rate (flow) around Q=9100m^3/d.

As for the spread of crude oil itself, theoretically (it will be combined with the actual adjustment hereinafter) we have:

$$D_x(t) = d_f(t) + d_x(t) \tag{14}$$

$$D_y(t) = d_f(t) + d_y(t) \tag{15}$$

d_f is for the scale in the isotropic spreading, and d_x, d_y, respectively x, y are the scale direction of the spreading.

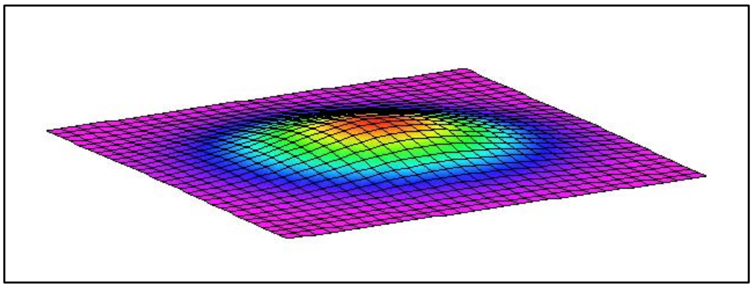
Approximate outline of the spreading crude oil can be regarded as an ellipse, when $\delta<0$ is complete cessation of spreading of crude oil. We can infer that when the thickness h and crude oil spreading of area A (including some less than the area of observation, is the theoretical value) are infinitely close, then when $S=A$, the extended area will be the largest, after $S=A$, due to the edge diffusion, these sections cannot be observed, so A will be greater than S, so there we have: $0<S<A$.

According to the formulas of circle, it can be derived that the extension of the equivalent circle diameter is:

$$d_f(t)=\sqrt{\frac{4A_f(t)}{\pi}} \tag{16}$$

During the initial spreading process, the leakage center is the thickest and symmetrical as well (Graph 2):

$$h(x,y,t)=\frac{V}{2\pi\sigma_3\sigma_n}e^{-\left(\frac{x^2}{2\sigma_3{}^2}+\frac{y^2}{2\sigma_n{}^2}\right)} \tag{17}$$



Graph 2

V is the volume of oil, so the formula h is equal to what we perceived the minimum thickness h_k, and then we have:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{18}$$

where

$$\begin{cases} a^2 = \dfrac{2\sigma_3{}^2 \ln V}{2\pi h_K \sigma_3 \sigma_n} \\ b^2 = \dfrac{2\sigma_n{}^2 \ln V}{2\pi h_K \sigma_3 \sigma_n} \\ S = \dfrac{\pi_{ab} \sigma_3 \sigma_n \ln V}{2\pi h_K \sigma_3 \sigma_n} \end{cases} \tag{19}$$

In summary, we found that in practice there were subtle differences during the spreading, so we have an adjustment as follows:

$$D_x' = \frac{S}{A}(d_f + d_x) \tag{20}$$

$$D_y' = \frac{S}{A}(d_f + d_y) \tag{21}$$

According to this treatise, S≈A, and thus $D_x' \approx D_x$, $D_y' \approx D_y$.

[iii] Data Substitution

From the equations above, we can use the existing data to test the model. In the $0<t<84$d stage,

$$V(t) = Qt \tag{22}$$

Also from (10) and (16), we have the spreading area and diameter of the equivalent circle as follows.

$$A_f(t) = 10^5 (Qt)^{\frac{3}{4}} \tag{23}$$

$$d_f(t) = \sqrt{\frac{4 \times 10^5 (Qt)^{\frac{3}{4}}}{\pi}} \tag{24}$$

From (14) (15) (24) we have

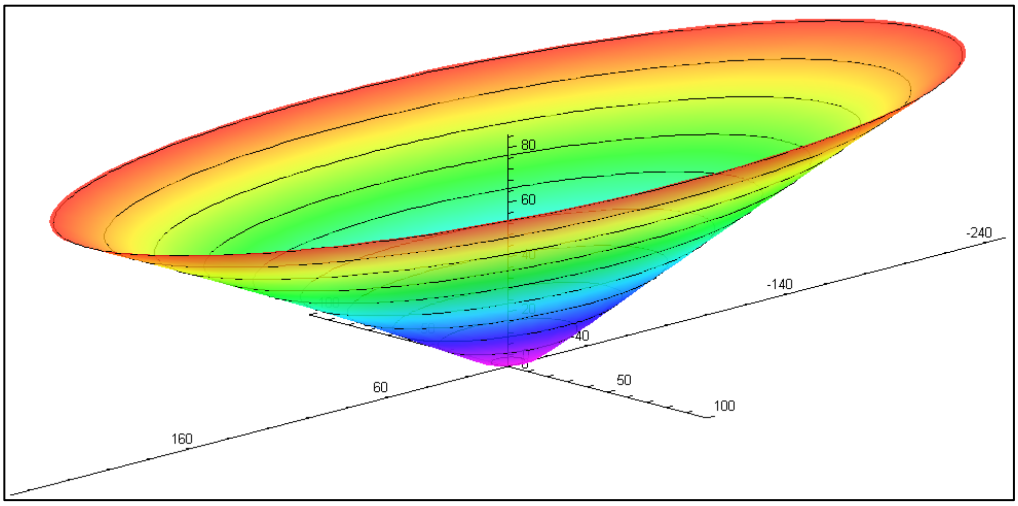
$$D_x(t) = \sqrt{\frac{4 \times 10^5 (Qt)^{\frac{3}{4}}}{\pi}} + 10^{-3}\sqrt{12}t^{1.17} \tag{25}$$

$$D_y(t) = \sqrt{\frac{4 \times 10^5 (Qt)^{\frac{3}{4}}}{\pi}} + \frac{0.001}{\sqrt{10}}t^{1.17} \tag{26}$$

From these two equations, we can draw the parameter equations of crude oil under static sea wind conditions:

$$\begin{cases} x = \dfrac{10^{-3}}{2}\left[\sqrt{4\cdot 10^{5}(Qt)^{\frac{3}{4}}\pi^{-1}} + 10^{-3}t^{1.17}\right]\cos\theta \\ y = \dfrac{10^{-3}}{2}\left[\sqrt{4\cdot 10^{5}(Qt)^{\frac{3}{4}}\pi^{-1}} + 10^{-\frac{7}{2}}t^{1.17}\right]\sin\theta \\ z = \dfrac{t}{86400} \end{cases} \tag{27}$$

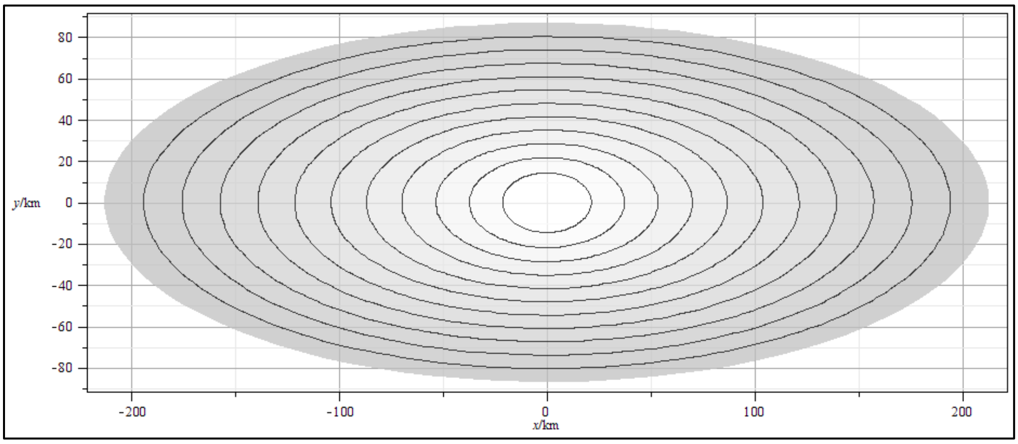
where the unit of t is s. Accordingly, we can draw a contour image of different time (Graph 3):



Graph 3

In the figure above, the long axis is the x-axis (units km), and the short axis is the y-axis (units km), and the vertical axis is the z-axis (timeline, units d).

To make it more understandable, we can replace the image above into the contour map graphic form (Graph 4, the gap between the two neighboring contours shows a week, 7 days):

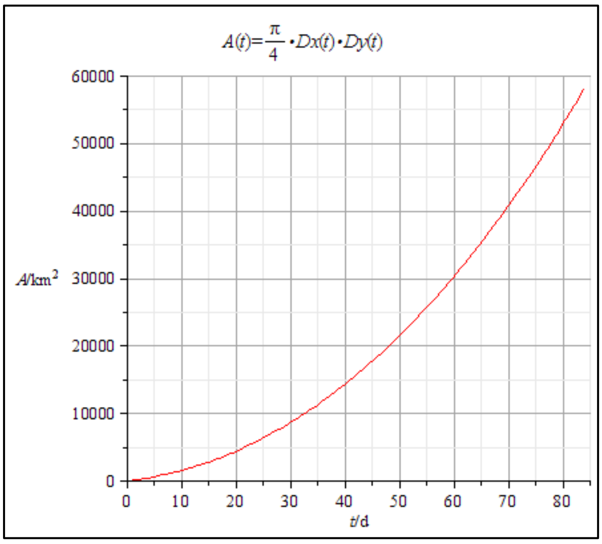


Graph 4

After the correction, the actual area of crude oil

$$A(t)=\frac{\pi}{4}D_x(t)D_y(t) \quad 0\le t\le 84\text{d} \tag{28}$$

In the formula above, the unit of t is sec. When t=84d=84×86400s substitution test, we have $A(84\times86400)=1.17\times10^{10}\text{m}^2$. The change of the oil spreading area according to time is as shown in Graph 5.



Graph 5

IV. Model Optimization

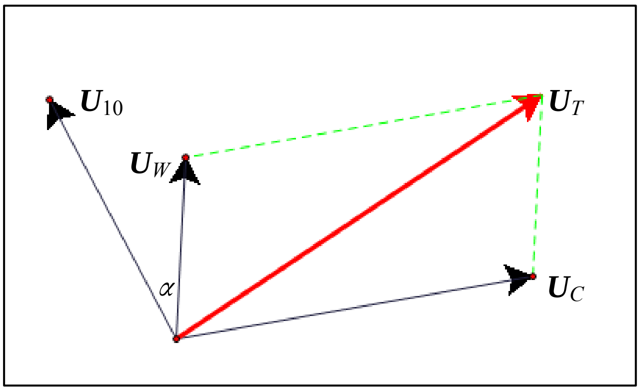
[i] Movement of the Oil on the Sea Surface

According to other researchers[1], the surface film of the spreading oil is determined by bias power (hereinafter referred to Ho by the deflection angle of the decision), velocity of the sea surface wind and velocity of surface flow (tides, currents, etc.) (Graph 6).

$$\boldsymbol{U}_T = \boldsymbol{U}_C + \boldsymbol{U}_W \tag{29}$$

$$|\boldsymbol{U}_W| = K|\boldsymbol{U}_{10}| \tag{30}$$

In the formulas above, $\boldsymbol{U}_T$ is the film transferring velocity vector; $\boldsymbol{U}_C$ is the velocity of surface stream; $\boldsymbol{U}_{10}$ is the velocity of wind 10m above the leaking point; $\boldsymbol{U}_W$ is caused by the drift velocity vector, and K is the wind drift factor. α is Coriolis Deflection Angle (the northern hemisphere to the right side), usually determined by Ekman analysis formula:



Graph 6

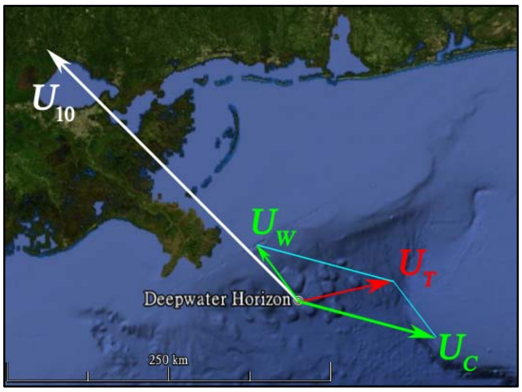
$$\alpha = \arctan\frac{\sinh\frac{H}{D} - \sin\frac{H}{D}}{\sinh\frac{H}{D} + \sin\frac{H}{D}} \tag{31}$$

$D = \pi\sqrt{\frac{\gamma_i}{\Omega}}$ is the depth of the friction effect, and γ_i is the upper ocean vertical viscosity (approximately constant). Ω is the Coriolis coefficient and $\Omega=\omega\sin\varphi$. In this formula, ω is angular velocity of the Earth's rotation, and φ is the local latitude.

According to the data listed on P8, we have α=7.476°, $|\boldsymbol{U}_W|$=0.10m/s, and furthermore according to the law of cosines,

$$|U_T|^2 = |U_C|^2 + |U_W|^2 - 2|U_C||U_W|\cos[180\text{-}(105+45-7.476)]° \quad (32)$$

Through computing, we have $|U_T|$=0.181m/s, $<U_T, U_C>$=19.624°. The direction of U_T is 85.376° (Graph 7).



Graph 7

Since the direction of movement of crude oil leaking on the surface was U_T, while the leakage point is stationary, and the oil spill did not stop at once, so there would be continuously leaking oil in most of the parts in the Gulf of Mexico. The spread profile of crude oil according to the above formulations can be broadly described as an ellipse, and the spread of the main direction which x-axis coincides with, and the direction of the secondary spread of ellipse formed a minor axis, which y-axis coincides with, can be used to described the shape of the ellipse. When $0<t<t_0$, crude oil continued leaking, so that each point in time between the formation of the ellipse stacked, and the original contour would form a shape similar to an egg over time. It can be understood as the first oil drops into a basin, and rapidly begins spreading, but at this time since the drop does not stop, and the first spread formed by the first drop continues to spread in all directions, and therefore the first drop becomes the same as a larger drop. Overlapped with countless similar spreads, taking into account the migration of oil in the sea, it formed an egg-shape contour.

[ii] New Model Establishment

From the description of section 1 of this chapter, the "egg-shape" can be seen in the depiction of the plane to xOy graphics on time points. more specifically, the formation of "egg-shape" at t moments on every bit of edge, according to the stress of fluid mechanics moves in a tiny pace, and then we can get "egg shape" of $(t+\Delta t)$ moment, but this involves difficult fluid mechanics formulas, two-dimensional normal distribution and higher mathematics calculation. Therefore, this is beyond our ability, and we were unfortunately unable to accurately depict the "egg-shape" according to the above information about the time of the accurate equations (neither function parameter equation nor equations.)

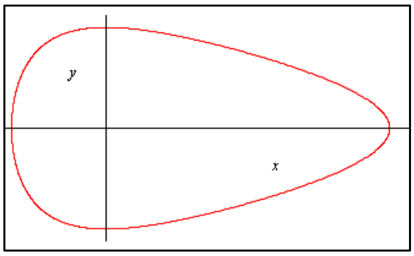
On the other hand, based on previous ideal condition of crude oil spreading ellipse model, we tried to use another way to get the "egg-shape" generating equation. Considering the oil leakage rate Q did not change over time and sea situation did stay calm, we made full use of the isochronic homalographic method to formulate the modeling of the "egg-shape" [5].

$$\left(\frac{x}{a-kx}\right)^2+\left(\frac{y}{b}\right)^2=1 \quad a,b\geq 0,\ k\in(0,1) \tag{33}$$

Also we have the parametric equation:

$$\begin{cases} x=\dfrac{a\cos\theta}{1-k\cos\theta} \\ y=b\sin\theta \end{cases} \quad a,b\geq 0,\ k\in(0,1) \tag{34}$$

In this specific issue, we found a connection between the "egg-shape" model and the ellipse model. According to the previous discussion, the area of the ellipse is A=A(t), and here let us suppose that the area of the "egg-shape" figure is S=S(t). The parameters here about the time are a=a(t) and b=b(t),



Graph 8

using the (33) and (34), then we can get the actual shape of the "egg-shape" figure, whose intersections with the coordinate are (0,b), (0,-b), $\left(\frac{a}{1-k},0\right)$, $\left(-\frac{a}{1+k},0\right)$.

Taking the figure above the x-axis (Graph 8), we can turn it into an explicit function as follow:

$$y=b\sqrt{1-\left(\frac{x}{a+kx}\right)^2} \tag{35}$$

Collectively, we have the area of the "egg-shape" figure:

$$S(t)=2\int_{-\frac{a(t)}{1+k}}^{\frac{a(t)}{1-k}} b(t)\sqrt{1-\left(\frac{x}{a(t)+kx}\right)^2}\,\mathrm{d}x \tag{36}$$

Theoretically, we can integrate the equation (35) over $\left[-\frac{a}{1+k}, \frac{a}{1-k}\right]$ so as to obtain the connection between S and a, b and k, yet this complicated function is beyond our capacity. We therefore analyzed the actual spreading of oil[6], which after 60 days spread to Florida, 400km away from the leaking point. Suppose that U is the speed of spreading caused by the ocean current on the right intersection, and then we have:

$$\frac{a(t)}{1-k} = \frac{D_x(t)}{2} + Ut \tag{37}$$

According to the situation of the day 60:

$$\frac{a\left(\frac{60}{84}t_0\right)}{1-k} = \frac{D_x\left(\frac{60}{84}t_0\right)}{2} + U \cdot \left(\frac{60}{84}t_0\right) = 4\times 10^5\text{m} \tag{38}$$

Approximately solve this formula $U \approx 0.0483$m/s, and we assumed k=0.75.

From (37)

$$a(t) = (1-k)\left[\frac{D_x(t)}{2} + Ut\right] \tag{39}$$

Let the four intersecting point of the coordinate to be the vertex of the ellipse, and then we can use semi-ellipses on both sides of y-axis, discovering that the sum of their area is similar to that of the "egg-shape", so the area of "egg-shape" can be computed by calculating the two semi-ellipse on the both sides of y-axis and then we have

$$S(t) = \frac{\pi b(t)}{2}\left[\frac{a(t)}{1-k} + \frac{a(t)}{1+k}\right] = \frac{\pi a(t) b(t)}{1-k^2} \tag{40}$$

Combine (39) (40)

$$b(t) = \frac{(1-k^2) D_x(t) D_y(t)}{4a(t)} \tag{41}$$

Then we have the formula of parameters a and b from (25) (26) (39) (41):

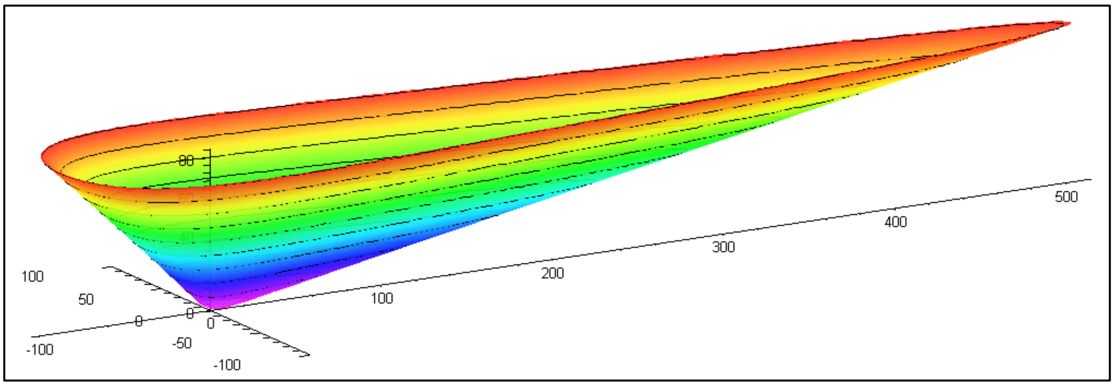
$$a(t) = 4.033\times 10^{-3} t^{1.17} + 19.18 t^{0.375} + 1.223\times 10^{-2} t \tag{42}$$

$$b(t)=\frac{4.150\times10^{-7}t^{2.34}+7.651\times10^{-2}t^{1.545}+2574.684t^{0.75}}{4.330\times10^{-4}t^{1.17}+1.223\times10^{-2}t+19.178t^{0.375}} \tag{43}$$

Combining (34) (42) (43), we established the formula of parameters of the alternation of "egg-shape" in accordance with time:

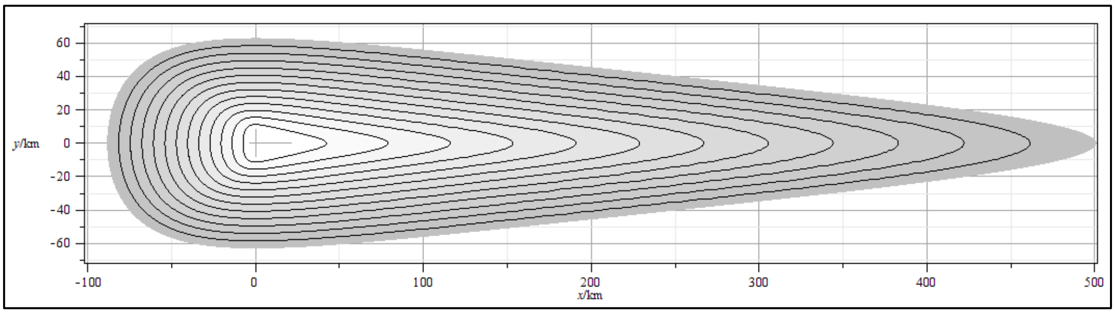
$$\begin{cases} x=\dfrac{4.033\times10^{-3}t^{1.17}+19.18t^{0.375}+1.223\times10^{-2}t}{1-0.7\cos\theta}\cos\theta \\ y=\dfrac{4.150\times10^{-7}t^{2.34}+7.651\times10^{-2}t^{1.545}+2574.684t^{0.75}}{4.330\times10^{-4}t^{1.17}+1.223\times10^{-2}t+19.178t^{0.375}}\sin\theta \\ z=\dfrac{t}{86400} \end{cases} \tag{44}$$

Facilitated with Maple, we graphed the contour of oil spreading area (Graph 9, units and meanings are the same as those in the ellipse model):



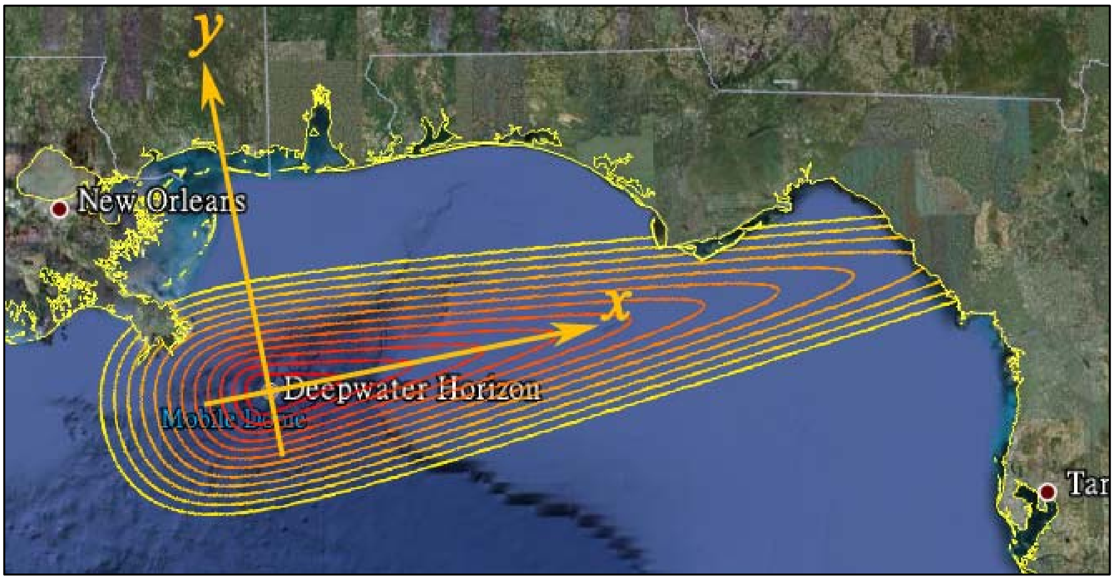
Graph 9

In the same way, we change the graph above into the isochronal plane graph (Graph 10):



Graph 10

According to the $\boldsymbol{U}_T$ direction given hereinbefore, we imposed the "egg-shape" model on the satellite map (Graph 11), and got the final result.

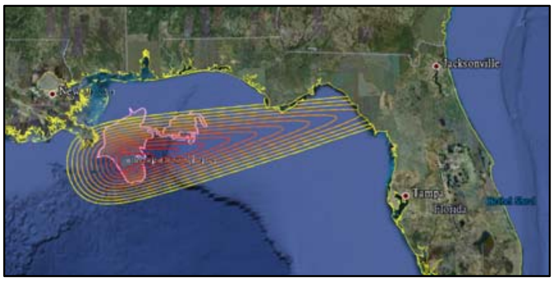


Graph 11

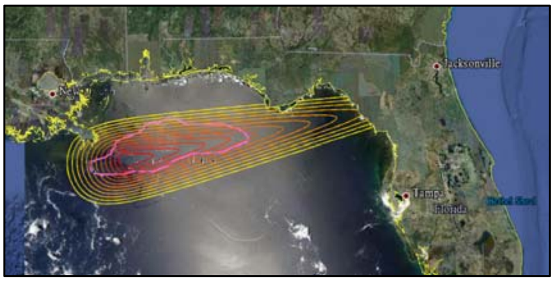
V. Conclusions of Modelling

[i] Tests of Model

According to the data and other information on the Internet[7], we drew the contour of the spreading oil in the third week (Graph 12) and the actual satellite photo in the fifth week (Graph 13).



Graph 12



Graph 13

From the comparison pictures we can see that, although there are some minor differences between the model and the actual contour, the general movement of the spreading oil is in accordance with the fact on the whole. The subtle differences are mainly caused by the oil-control facilities and the various hydrology and weather conditions, including hurricanes.

[ii] Analysis of Result

No matter how hard we have tried to improve our model, the spreading of crude oil still depends on many uncertainties, especially in the expansion of the spreading border which is vulnerable to external influence, resulting in uneven expansion at the edge. According to our model, the ideal conditions, ocean currents and tides have profound impacts to the spreading of crude oil, especially in the Gulf of Mexico, which is the tropical and sub-tropical, hot and rainy place with heavy rainfall, leading to the accumulation of rainwater in Florida, a dominant factor affecting the Bay: ocean current. Although the influence of the tidal waves about 2 meters averagely is relatively small. However in summer, it is a very different scenario; storm waves can be up to 5m. Therefore, the spreading of crude oil is largely due to the ocean current, which results in the x-axis direction of the oil spreading in the direction of the ocean current.

The Comparison between the Two Models

	Ellipse Model	**Egg-Shape Model**
Calculation	Relatively easy	Complex
Theoretical basis	Relatively strong	Relatively weak
Considering factors	Fewer	More
Authenticity	Low	Relatively high
Application area	Relatively narrow	Relatively wide
Development area	Small	Large

The traditional ellipse model is easy to compute, and it can show the basic pattern of oil spreading. The length, the ration, the changing of long and short axis are all similar to the real situation, but it cannot show the influence of the ocean current, so there are some major difference between the fact and the model.

Nonetheless, the innovative "egg-shape" model put forward by us is a little difficult to compute, so we suggested a simplified version and it can show the pattern of oil spreading more accurately. The origin of the coordinates is the point where oil first began to leak, and it can fully show the influence of the Mexico ocean current, so it is much superior to the original ellipse model.

However, during the 84 days of oil spreading, the government of the US has sent ships to stop the oil from spreading, and the hydrology varies from place to place and from time to time, so there are still some differences between the "egg-shape" model and the actual oil spreading area.

[iii] Conclusions

Through the modeling of ellipse and egg-shape, we find that under the idealistic premise, our models can reflect the spreading of crude oil more accurately and generally describe the rough images of spreading crude oil.

Though many factors including ocean currents, in turns of the bias power, tidal waves and oil depletion have been taken into account, there are still some other discrepancies in the model compared with the actual situation. Particularly the potential impact of ocean currents significantly influences the spreading of the spilt crude oil, so this impact brings challenges to our models as well.

So, as mentioned at the end of last section, when crude oil spread is put into a larger picture, by taking the negative impact of factors (such as ocean currents and winds which help the spread of oil) into account, we can roughly find that crude oil concentrates differently. Consequently the spread of crude oil can be predicted easily and accurately in the future. However, because ocean currents in the sea vary quickly, our model needs to be further improved.

[iv] Reasonable Advice

Based on the analysis of both models, we put forward some of our advice as follows:

1. Through the analysis of the variation rate of the area of the spreading oil (Graph 5), we find that $A(t)$ is an increasing concave function, so we suggest that government and authority should stop the oil from spilling as quickly as possible, so as to prevent oil film from expanding faster and faster.
2. We analyzed the movement of crude oil on the sea surface (Graph 11), and it appears that a large number of salvage ships were sent to intercept the oil spreading to the leakage point in the opposite direction of $\boldsymbol{U}_T$ at the beginning of the oil spilling, the spreading of oil could be effectively controlled.

[v] Further Development & Practical Use

Since our innovative "egg-shape" model can fully show the influence of ocean current, it is fit for many broad maritime spaces with complex hydrological condition. As the ellipse model has some limitations in depicting the movement of oil film, it is not fit for water area with currents, but it still applies to broad calm lake (sea) surface.

If more actual conditions are considered, the "egg-shape" model will also be fit for estuaries where the impact of currents is notable. For example, it can be used to predict and graph the oil spreading incident taking place few months ago in the Gulf of Bohai Sea in China or assisting the managers of South China Sea oil Exploration Company to manipulate the oil exploration properly and correctly so as to prevent such accidents from happening again.

Therefore, our models cover most problems taking place on sea surface. In addition, they not only predict the accurate spreading of crude oil, but also graph the contour of oil spreading. Nevertheless, the accuracy of this model needs to be further strengthened.

[vi] Problems to be Solved

- The modeling process is all based on the assumptions citing in the beginning, but some of those assumptions can hardly be satisfied in reality. For example, the leakage rate varied while oil was spilling.
- Because we used the average depth of the Gulf in the Ekman analysis formula in the discussion on the movement and due to the differences of characters between sand and seawater, there are still some limitations (even in the "egg-shape" model) as the crude oil spreads to the coastal areas.
- Since we could not valuate the specific influence of the salvage ships and containment cap on the leakage oil, the computed volume of crude oil on the sea surface is slightly different from that of the actual situation.

- The condition of hydrology and weather in the Gulf of Mexico varies from time to time and from place to place, so there are minor discrepancies when we discussed the movement of spreading oil.

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墨西哥湾原油泄漏事件
在推广 Fay 公式基础上的建模

The Mathematical Model of the Oil Spilling
in The Gulf of Mexico Based on the Extended Fay Formulas

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关键词：　墨西哥湾；原油泄漏；推广 Fay 公式；
数学建模；椭圆；“蛋形”

Keywords: Gulf of Mexico; crude oil spilling;
the extended Fay Formulas;
mathematical model; ellipse; egg-shape

目录　CONTENT

摘要　Abstract

本文主要基于流体力学的基本模型，依据牛顿运动定律和质量守恒定律，借鉴 Fay 公式的进一步推广得到的静止点源连续溢油方程，在理想化的前提下，通过重力、表面张力、粘滞力、惯性力这几个决定性因素，对墨西哥湾洋面受污染的面积进行初步的估算，并确定原油泄漏的最终面积.

最初我们利用由 Fay 公式得到的传统椭圆模型对原油扩散进行建模，得到原油扩散的大致图形和面积变化情况，但该模型不符合当地的气候水文条件，故该模型的精准性较差．因此我们进一步讨论了原油在洋流风力影响下的运动，创新地提出了前所未有的蛋形模型，并绘制出了更精准的原油扩散大致轮廓．同时对整个墨西哥湾原油扩展的今后发展方向进行预测，提出有效的整治措施，让环境问题不再抽象.

This article is primarily based on the fundamental model of fluid mechanics, applying the Newton’s Laws of motion and conservation of mass, to work out the new formula for a stable table but continuous spilling oil which was promoted from the formulas of Fay. In an ideal situation, using several elementary factors including gravity, density, viscous forces, inertia, we can primarily estimate the size of the polluted area in the Gulf of Mexico and evaluate the final damage of this accident.

Initially we used the Fay formulas to build the conventional ellipse model, and approximately graphed the oil spreading area and got the variation of the spreading oil area. However, because this model is not primarily based on the local hydrology and weather condition, it lacks accuracy. Thus, we further discussed the movement of crude oil on the sea surface under the influence of ocean current and wind. Then we innovatively suggested a brand new model called “egg-shape”, and simulated a more accurate contour of the spreading oil. Consequently, with the unprecedented “egg-shape” model, we can more accurately predict the direction of the oil spilling, leading to some more effective remediation for a better environmental protection, and providing a more comprehensive treatment for the future of the Gulf of Mexico.

一 综合信息 General Information

〔I〕研究背景 Background of Research

近年来环境问题备受关注，为了保护我们共有的家园，环境问题的防治显得尤为重要．然而今年 4 月 20 日，美国墨西哥湾原油泄漏，引起了国际社会的高度关注．据估计，有关方面需要花几个月时间封闭油井，油污的清理工作将耗时近十年．墨西哥湾在长达十年的时间里将成为一片废海，造成的经济损失将以数千亿美元计．对全球环境的严重影响，更是不言而喻．

作为一个有责任感的世界公民，我们应该高度重视此事件，让更多的人了解到环境的保护是多么迫在眉睫．不久前，“原油泄漏”一词被评为 2010 年的年度最热门词汇之一．因此，我们应该建立准确的事件模型，对此事件的严重性进行评估及预测，并提出整治措施，将损失降至最小，同时引起大家对环境问题的关注，规避此类事件的再次发生．

〔II〕 研究亮点 Highlights of Research

- 保留传统的 Fay 公式的精华，省去繁杂推导，将经验所得运用到其中，让较难考量的参数具体化，使复杂的模型更简明．
- 由于本文重点考察原油扩散的最终状况，所以不对原油扩散过程作过多深究，而是着重描绘出原油扩散及其形成油膜的形状与面积．
- 利用大家熟悉的椭圆的参数方程对复杂的原油扩散形状进行初步的拟合模拟，在大方向上保证了模型的准确性．
- 考虑到所处环境独特的地理特征，本文提出了更具针对性的办法，让模型的移动以及其最终的位置更准确，并可以依此对原油及油膜的未来走向作出预测．
- 首次提出了极具创造性的蛋形模型，经过我们在学校的模拟实验，发现此图形与实际情况更类似，使得原油扩散的图形更精准．

- 运用 Maple 等计算机软件对建立的模型进行图形化处理，并结合新闻以及卫星图片等资料作出原油扩散的合成图，使结果更直观.

二 建模铺垫 Basis of Model

〔I〕近似化处理 Assumptions

为了方便建模，我们首先做出如下假设：

- 被原油污染的区域水质等情况相同，原油的扩散速度相同，即不考虑水体在不同水域的差别；
- 参与模型变量连续且光滑；
- 海水总体积保持不变，不考虑雨雪渗漏以及自然状况下的自净过程（原油在一般情况下，无法自然消除沉积），海面无任何设施（包括船舶、围油设施）；
- 不考虑钻井平台沉没之前的泄油量；
- 油在大规模扩散仅由重力、表面张力、粘滞力决定，扩散过程中性质不变，如密度、张力等，且在竖直方向处于平衡状态.

〔II〕 选择参数 Choosing Parameters

- H 为油层厚度，Δh 为水上油层厚度，$\Delta = 1\text{-}\frac{\rho_o}{\rho_w}$，其中 ρ_o=810kg/m^3 为油的密度，ρ_w=1022kg/m^3 为海水的密度，因此 Δ=0.2074
- I 为原油泄露的扩展直径，即可理解为原油各阶段的扩展尺度.
- 用 δ 表示净表面张力系数，水—空气界面 δ_{wa}、油—空气界面 δ_{oa}、油—水界面 δ_{ow}. 根据 Fay 的理论，在理想条件下当 $\delta \le 0$ 时，油的扩展将停止. 故问题可简化成对净表面张力系数的估值，即 δ 为我们所关心的量.
- 当地（28°N）重力加速度 g=9.79m/s^2；t 为从原油刚开始泄漏时起算的时间；V 为原油泄露的体积.
- k_x、k_y、k_z 分别为三个泄露阶段的经验参数，即为原油体积衰减系数，参照国内外实验结果，k_x=1.40，k_y=1.65，k_z=0.60.

- Q 为油的溢出量，即单位时间内的扩散量；S 为在单一方向上恒定流场的速度，该方向即为 x 轴方向；D 为 x 轴方向的距离.

〔III〕 模型相关公式 Correlated Formulas of the Model

由于原油扩散主要分为两个阶段（以漏油口被堵住即泄漏第 84 天为分界线），主要受三个作用影响，所以不能顾此失彼，必须综合考虑.

在扩展初期，主要为原油的扩展作用，辅之有蒸发作用；在原油扩展的后期（漏油口堵住后），油膜的扩散及延伸成为主导因素（由于原油泄漏的总体积已不再增加），同时亦有蒸发作用. 这里我们将蒸发、降解、有关控油措施这些帮助减少原油的因素称为积极因素，而风、洋流、地转偏向力这些有助原油扩散的因素，我们称其为消极因素.

记漏油口堵住的时刻距离原油开始泄漏的时间为 t_0，我们以此把整个漏油事件分成两大过程，结合相关资料[1][2][3]上的流体力学方程：

根据物理学中的流体力学方程，在 $0<t<t_0$ 内原油连续泄漏，油的体积 $V(t)$为

$$V(t)=\sum_{t=j}^{n} Q\Delta t\left[1-K\left(t-j\Delta t\right)\right] \quad 0<t<t_0 \tag{1}$$

其中，K 为原油体积衰减系数（待定）.

当 $0<t<t_0$ 时，泄漏原油的变化分为三个阶段：

$$I_{\mathrm{I}}=K_{\mathrm{I}}(\Delta gV)^{1/4}t^{1/2} \tag{2}$$

$$I_{\mathrm{II}}=K_{\mathrm{II}}(\Delta g)^{1/6}V^{1/3}v_w^{-1/12}t^{1/4} \tag{3}$$

$$I_{\mathrm{III}}=K_{\mathrm{III}}\sigma^{1/2}v_w^{-1/4}\rho_w^{-1/2}t^{3/4} \tag{4}$$

其中，I_{I}、I_{II}、I_{III}为各个阶段的扩展尺度，$K_{\mathrm{I}}=1.35$、$K_{\mathrm{II}}=1.60$、$K_{\mathrm{III}}=0.48$ 为各个阶段的扩展系数，由以上三式可以确定相邻两阶段的临界点 t_{12} 和 t_{23}.

在 $t=t_0$ 时，原油泄漏停止. 设到此时已经泄漏到海面的原油体积为 V_0（由(1)式算出），则从泄漏开始起 t 时刻油的体积 $V(t)$为

$$V(t)=V_0\left[1-K(t-t_0)\right] \quad t>t_0 \tag{5}$$

同样，当 $t>t_0$ 时，原油的变化也分为三个阶段：

$$I_1 = K_1(\Delta g V)^{1/4} t^{1/2} \tag{6}$$

$$I_2 = K_2(\Delta g)^{1/6} V^{1/3} v_w^{-1/12} t^{1/4} \tag{7}$$

$$I_3 = K_3 \sigma^{1/2} v_w^{-1/4} \rho_w^{-1/2} t^{3/4} \tag{8}$$

式中，K_1、K_2、K_3 为连续溢油结束后各阶段的扩展系数．

根据 Fay 的理论，理想条件下当净表面张力系数小于等于零时，原油停止扩展，有

$$A_f = K_f \left(\frac{\delta^2 V^6}{\rho_w^{\ 2} v_w D^3 S^6} \right)^{\frac{1}{8}} \tag{9}$$

其中 S 为油的溶解度，D 为石油的表面活化剂在水中的扩展系数，K_f 为待定系数．然而，K_f 涉及的不确定因素很多，较难确定．根据资料，通常情况下，使用由实验得到的简化公式：

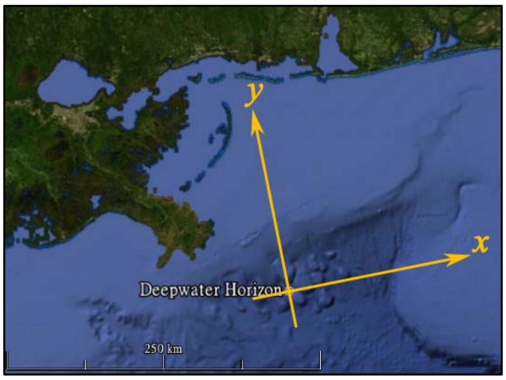
$$A_f = 10^5 V^{\frac{3}{4}} \tag{10}$$

式中 A_f 单位为 m^2，V 单位为 m^3．

由于我们的模型研究的是连续漏油的时段，所以模型跨越的时间范围为 $[0,t_0]$．

三 原始模型的分析和计算 Analysis & Calculations of Initial Model

〔I〕模型分析 Analysis of the Model



Graph 1

在海面上，流场为二维非恒定，由于在海洋中，原油的扩散可以主要分成两个垂直的方向，把漏油点附近区域海面看成平面，以原油初始泄漏的中心为坐标原点，向东为 X 轴，向北为 Y 轴建立平面直角坐标系．但为了叙述方便，现规定 x 轴方向为原油的主要扩散方向，与 x 轴互相垂直的 y 轴定为油膜扩展的次要运动方向（Graph 1，x 轴的方向将会在下文讨论到）．

下面列出主要的影响参数：

- 泄漏原油的总体积：0.76×10^6m^3
- 原油泄漏被成功封堵的时间点：t_0=84d=7.26×10^6s
- 原油在[0, t_0]的流量：Q=9100m^3/d
- 油的密度：ρ_o=810kg/m^3
- 海水的密度：ρ_w=1022kg/m^3
- 风漂流系数：$K\in$[0.016, 0.035]（计算中取 0.020）
- 连续漏油的实际扩展系数：K_{I}=1.35，K_{II}=1.60，K_{III}=0.48
- 海面上层的垂直粘性系数：μ=10kg/m·s
- 海水粘滞系数：v_w=1.01×10^6m^2/s
- 墨西哥湾中心纬度：φ=28°(N)
- 墨西哥湾的平均海水深度：H=1512m
- 净表面张力系数：σ=71.05×10^{-3}kg/s^2
- 墨西哥湾附近的重力加速度：g=9.79m/s^2
- 地球自转角速度：ω=72.72×10^{-6}rad/s
- 泄漏点附近风速：$|\mathbf{U}_W|$=5m/s，吹向 315°

- 泄漏点附近水流流速：$|\boldsymbol{U}_C|$=0.25m/s，流向 105°

（注：以上有关原油泄漏的数据来自互联网的报道，有关墨西哥湾及其他相关参数来自维基和百度）

〔II〕　计算方案 Calculation Procedure

在墨西哥湾上，溢油的扩展尺度和扩散尺度的叠加可近似代表海面上原油的扩延．仍沿用上文中的坐标，主运动方向为 x 轴，辅之有 y 轴上的各异向扩散，Okubo 等人总结若干现场资料[1]，给出了油膜扩散引起的油膜或油膜碎片质量近似成正态分布的均方差为

$$\delta = 0.001t^{1.17} \tag{11}$$

在各向异性海洋中，沿油膜主要运动方向（x）和次要运动方向（y）的油膜扩展尺度 d_x 和 d_y 计算如下：

$$d_x = \omega\delta \tag{12}$$

$$d_y = \omega\alpha\delta \tag{13}$$

其中 α、ω 均为常数，一般情况下取 $\alpha = \dfrac{1}{\sqrt{10}} = 0.316$， $\omega = \sqrt{12} = 3.464$． t 的单位是 s，d_x、d_y 的单位为 m．

查阅资料得知，至漏油口被堵住为止，泄漏到海面上的原油（不包括焚烧掉的）约有 400 万桶，换算得原油平均泄漏速率（流量）约为 Q=9100m^3/d．

考虑原油自身的扩展，理论上（下文会结合实际调整）有：

$$D_x = d_f + d_x \tag{14}$$

$$D_y = d_f + d_y \tag{15}$$

式中 d_f 为油膜各向同性扩展尺度，d_x、d_y 分别为 x、y 方向油膜的扩散尺度．

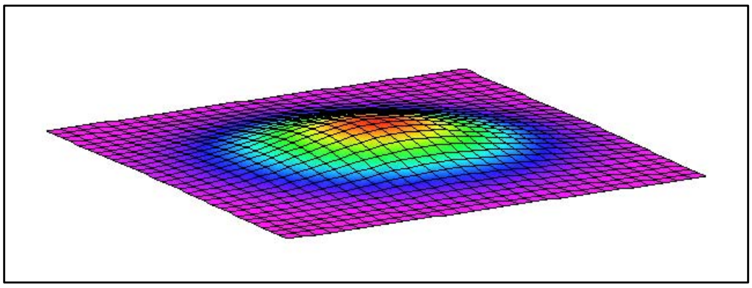
把原油扩散的轮廓近似成椭圆，当 δ<0 时即为原油扩散完全停止时，我们可以推知，当 h 厚度时，所围成的椭圆面积 S（即为实际上可以观测到的面积）与原油的扩展面积 A（包括部分观测不到的面积，乃理论值）无限接近于零，又当 S=A 时，油膜的扩展面积达到最大，当 S=A 后，由于部分油膜继续在边沿扩散，这些部分是观测不到的，故此时 A 将大于 S，因此有：0<S<A．

根据圆的面积公式，可推导出油膜扩展范围的等效圆直径：

$$d_f = \sqrt{\frac{4A_f}{\pi}} \tag{16}$$

在最初的扩展过程中，泄漏中心处油膜最厚，油膜厚度以中心处为对称中心，成正态分布（示意图，Graph 2，Maple 作图）：

$$h(x,y,t) = \frac{V}{2\pi\sigma_3\sigma_n} e^{-\left(\frac{x^2}{2\sigma_3{}^2}+\frac{y^2}{2\sigma_n{}^2}\right)} \tag{17}$$



Graph 2

式中 V 为油的体积，令式中 h 等于人眼所觉察的最小厚度 h_k，则有

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{18}$$

式中，

$$\begin{cases} a^2 = \dfrac{2\sigma_3{}^2 \ln V}{2\pi h_K \sigma_3 \sigma_n} \\ b^2 = \dfrac{2\sigma_n{}^2 \ln V}{2\pi h_K \sigma_3 \sigma_n} \\ S = \dfrac{\pi_{ab} \sigma_3 \sigma_n \ln V}{2\pi h_K \sigma_3 \sigma_n} \end{cases} \tag{19}$$

综上所述，我们发现，实际上油膜的长短与上文中的表达式有细微差别．经调整，可得油膜的实际扩散长短为:

$$D_x' = \frac{S}{A}(d_f + d_x) \tag{20}$$

$$D_y' = \frac{S}{A}(d_f + d_y) \tag{21}$$

在本文讨论范围中，我们认为 S≈A，因此 $D_x' \approx D_x$，$D_y' \approx D_y$.

〔III〕 代入数据 Substituting the Data

由以上的方程，我们可以用代入已有的数据，在 0<t<84d 阶段，有

$$V(t)=Qt \tag{22}$$

又由(10)、(16)式可得原油在无其他作用下的扩展范围等效圆面积和直径

$$A_f(t)=10^5(Qt)^{\frac{3}{4}} \tag{23}$$

$$d_f(t)=\sqrt{\frac{4\times10^5(Qt)^{\frac{3}{4}}}{\pi}} \tag{24}$$

式中 t 的单位是 d，Q=9100m^3/d．由(14)(15)(24)得

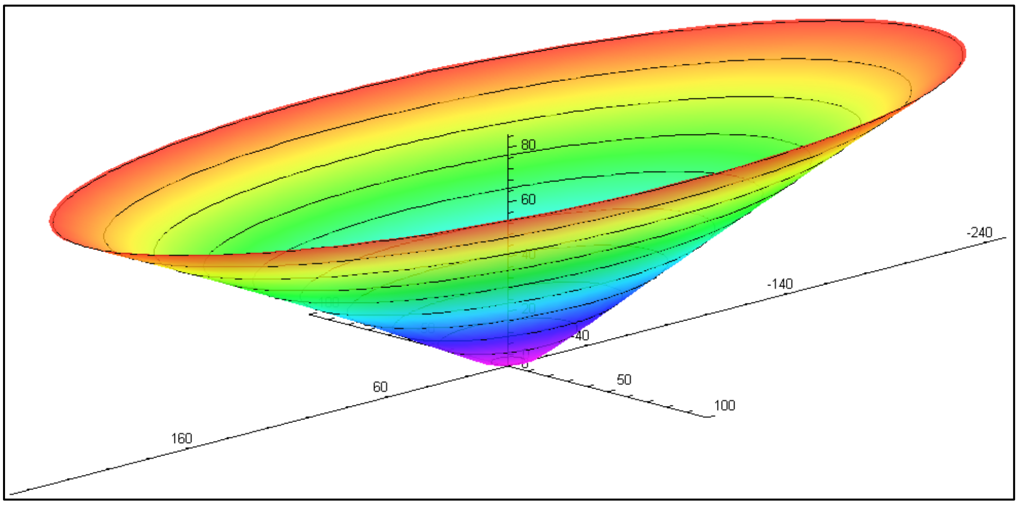
$$D_x(t)=\sqrt{\frac{4\times10^5(Qt)^{\frac{3}{4}}}{\pi}}+10^{-3}\sqrt{12}t^{1.17} \tag{25}$$

$$D_y(t)=\sqrt{\frac{4\times10^5(Qt)^{\frac{3}{4}}}{\pi}}+\frac{0.001}{\sqrt{10}}t^{1.17} \tag{26}$$

由以上两式，可以得出无风状态静止海面上的原油扩散轮廓的参数方程:

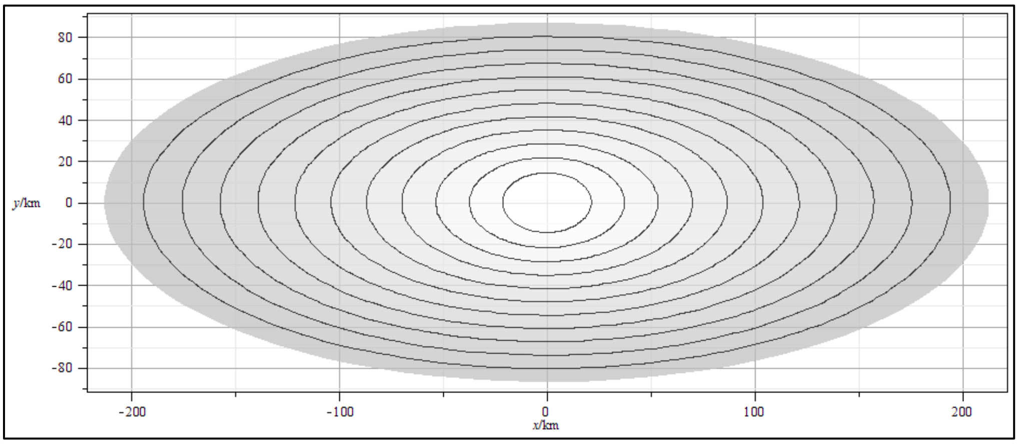
$$\begin{cases} x=\dfrac{10^{-3}}{2}\left[\sqrt{4\cdot10^5(Qt)^{\frac{3}{4}}\pi^{-1}}+10^{-3}t^{1.17}\right]\cos\theta \\ y=\dfrac{10^{-3}}{2}\left[\sqrt{4\cdot10^5(Qt)^{\frac{3}{4}}\pi^{-1}}+10^{-\frac{7}{2}}t^{1.17}\right]\sin\theta \\ z=\dfrac{t}{86400} \end{cases} \tag{27}$$

其中 t 的单位为 s，$Q=\dfrac{9100}{86400}$ m^3/s．据此用 Maple 作出等时轮廓变化图（Graph 3）：



Graph 3

该图中，横轴为 x 轴（长轴，单位 km），纵轴为 y 轴（短轴，单位 km），竖轴为 z 轴（时间轴，单位 d）．为了更直观，我们可以把以上图形换成等时线平面图形式（Graph 4，相邻两轮廓的时间间隔为 7d）：

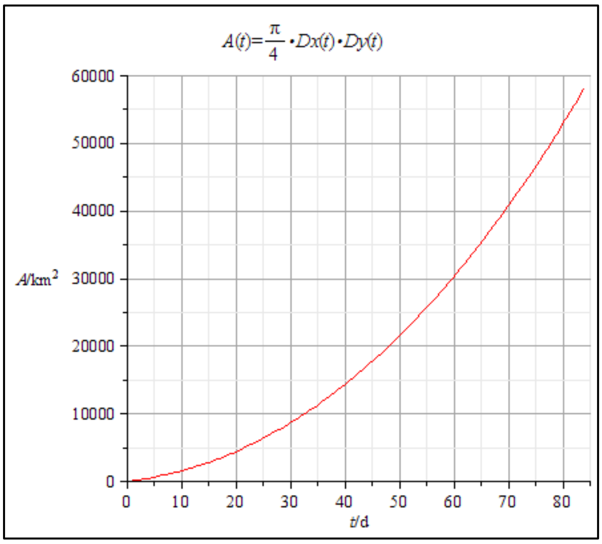


Graph 4

经过修正，原油的实际面积

$$A(t) = \frac{\pi}{4} D_x(t) D_y(t) \quad 0 \le t \le 84\mathrm{d} \tag{28}$$

式中 t 的单位为 s，把 t=84d=84×86400s 代入，得 $A(84\times86400)=5.82\times10^{10}\text{m}^2$．泄漏原油的面积随时间的变化情况如 Graph 5 所示．



Graph 5

四 模型优化 Optimizing the Model

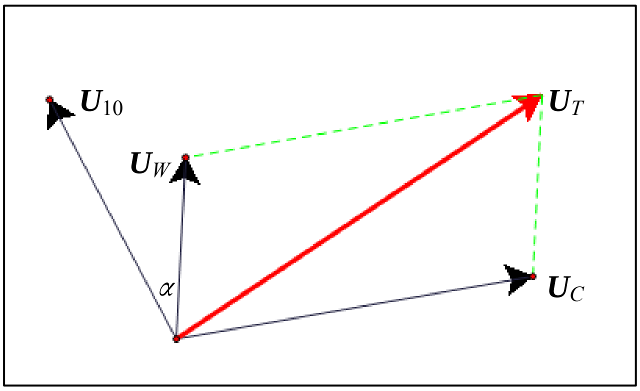
〔I〕海面石油的迁移运动 Movement of the Oil on the Sea Surface

研究表明[1]，水面油膜的迁移与地转偏向力（由下文提及的何氏偏转角决定），海面风向，以及表面流（包括潮汐，洋流等）共同决定（如 Graph 6）.

$$\boldsymbol{U}_T = \boldsymbol{U}_C + \boldsymbol{U}_W \tag{29}$$

$$|\boldsymbol{U}_W| = K|\boldsymbol{U}_{10}| \tag{30}$$

式中，$\boldsymbol{U}_T$是油膜迁移速度矢量，$\boldsymbol{U}_C$是海洋表面流速，$\boldsymbol{U}_W$是由风速 $\boldsymbol{U}_{10}$（水面上 10m 处风速）引起的漂移速度矢量，K 是风漂移系数．图中 α 为柯氏偏转角（北半球为顺时针)，一般采用 Ekman 解析式确定：



Graph 6

$$\alpha = \arctan\frac{\sinh\frac{H}{D} - \sin\frac{H}{D}}{\sinh\frac{H}{D} + \sin\frac{H}{D}} \tag{31}$$

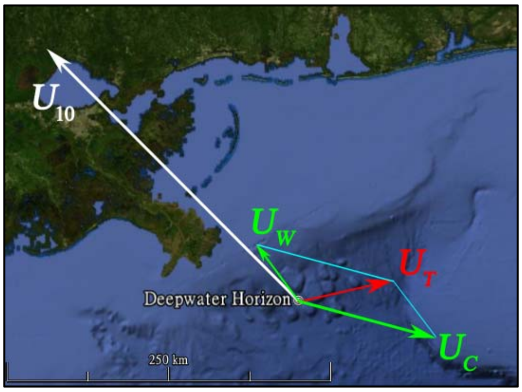
式中 $D=\pi\sqrt{\frac{\gamma_i}{\Omega}}$ 为摩擦影响深度，γ_i是海洋上层的垂直粘滞系数（近似为常数）. Ω 是科氏系数，$\Omega=\omega\sin\varphi$，ω 是地球自转角速度，φ 是纬度.

根据 P7 的相关数据，求得 α=7.476°，$|\boldsymbol{U}_W|$=0.10m/s，进而根据余弦定理有

$$|\boldsymbol{U}_T|^2 = |\boldsymbol{U}_C|^2 + |\boldsymbol{U}_W|^2 - 2|\boldsymbol{U}_C||\boldsymbol{U}_W|\cos[180\text{-}(105+45-7.476)]^\circ \tag{32}$$

得$|\boldsymbol{U}_T|$=0.181m/s，<$\boldsymbol{U}_T$, $\boldsymbol{U}_C$>=19.624°，$\boldsymbol{U}_T$指向 85.376°（Graph 7）.

由于已泄漏到海面上的原油会有 $\boldsymbol{U}_T$方向的运动，但泄漏点是静止状态，且在原油的泄漏未成功阻止前，仍有不间断的原油在往墨西哥湾的海洋里大量倾



Graph 7

泻．在原油泄漏的每一个时间点，原油扩散的轮廓据上文可以被大致描述为椭圆形，原油扩散的主要运动方向形成了椭圆的长轴，与 x 轴重合，次要扩散方向形成了椭圆的短轴，与 y 轴重合．当在 $0<t<t_0$ 这个时间段内，原油的继续外泄，使得每个时间点上形成的椭圆相互堆叠，原有的轮廓会随时间推移形成一个类似鸡蛋的外形．可以理解为第一滴油滴滴入水盆后，迅速扩散开来，然而此时由于仍有油滴不停滴入，第一次扩散开来的油层又类似与一滴大油滴继续扩散开了．因此，在无数次类似叠加后，考虑到原油在海面的迁移运动，便形成了一个类似于鸡蛋纵切面的轮廓（下文简称为“蛋形”）．

〔II〕 建立新模型 Establishing New Model

从本章第一节的描述可以看出，“蛋形”的刻画需要在 xOy 平面上把二维图形的变化对时间积分，具体来说就是把 t 时刻形成的“蛋形”上边缘的每一点按照流体力学的受力情况来进行微小的移动，得到 $t+\Delta t$ 时刻的“蛋形”，但这涉及到难度较大的流体力学公式、二维正态分布（前文提到）和高等数学的计算．因此现阶段凭借已有的知识我们暂时无法依据以上的信息准确刻画出“蛋形”（关于时间变化）的准确方程（隐函数方程或参数方程）．

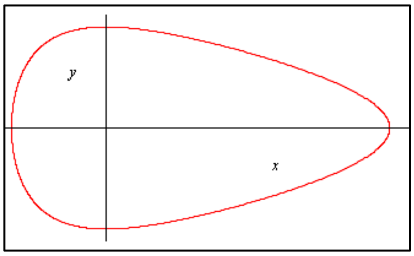
另一方面，基于上一章中已经建立的理想状态下原油会扩散为椭圆的模型，我们尝试用另一种途径得到“蛋形”的大致方程．考虑到原油的泄漏率 Q 不随时间和洋面情况变化而变化，我们想到用等时等面积法来建立这个“蛋”的形状．在互联网上，我们找到了一个比较适合用来描绘“蛋形”的较为简单的方程[5]：

$$\left(\frac{x}{a-kx}\right)^2+\left(\frac{y}{b}\right)^2=1 \quad a,b\geq 0,\ k\in(0,1) \tag{33}$$

除标准方程外，还有参数方程形式：

$$\begin{cases} x=\dfrac{a\cos\theta}{1-k\cos\theta} \\ y=b\sin\theta \end{cases} \quad a,b\ge 0,\ k\in(0,1) \tag{34}$$

在这个具体问题中，我们希望找到联结“蛋形”和椭圆之间的关系．在前文讨论中，椭圆的面积是 $A=A(t)$，所以“蛋形”的面积也应是 $A(t)$，有关参数关于时间的变化情况为 $a=a(t)$，$b=b(t)$，分别运用(33)和(34)式，得到“蛋形”与坐标轴的交点为(0,b)、(0,-b)、$\left(\dfrac{a}{1-k},0\right)$、$\left(-\dfrac{a}{1+k},0\right)$． 取 x 轴上方的一半图形（Graph 8）化成显函数形式，我们有



Graph 8

$$y=b\sqrt{1-\left(\frac{x}{a+kx}\right)^2} \tag{35}$$

综合以上关系式，我们得到“蛋形”面积

$$S(t)=2\int_{-\frac{a(t)}{1+k}}^{\frac{a(t)}{1-k}} b(t)\sqrt{1-\left(\frac{x}{a(t)+kx}\right)^2}\,\mathrm{d}x \tag{36}$$

理论上，我们可以通过对(35)式在$\left[-\dfrac{a}{1+k},\dfrac{a}{1-k}\right]$上进行积分来获得“蛋形”面积与 a、b、k 之间的关系，不过这种复杂函数的积分超出了我们的能力，因此在研究了原油实际扩散情况[6]，包括原油经过 60 天左右到达 400km 外的佛罗里达州，我们设水流带动“蛋形”右顶点处原油运动的速度为 U，则有

$$\frac{a(t)}{1-k}=\frac{D_x(t)}{2}+Ut \tag{37}$$

根据第 60 天的情况

$$\frac{a\left(\frac{60}{84}t_0\right)}{1-k}=\frac{D_x\left(\frac{60}{84}t_0\right)}{2}+U\cdot\left(\frac{60}{84}t_0\right)=4\times10^5\mathrm{m} \tag{38}$$

精确到两位小数近似解得 $U\approx0.0483$m/s，取 $k\approx0.75$．

由(37)得

$$a(t)=(1-k)\left[\frac{D_x(t)}{2}+Ut\right] \tag{39}$$

分别以“蛋形”上下左右四个顶点为椭圆顶点，在 y 轴两侧作出半椭圆，发现其面积与“蛋形”模型相近，故计算“蛋形”面积可近似为计算 y 轴两边的半椭圆面积之和，由此简化计算，于是有

$$S(t)=\frac{\pi b(t)}{2}\left[\frac{a(t)}{1-k}+\frac{a(t)}{1+k}\right]=\frac{\pi a(t)b(t)}{1-k^2} \tag{40}$$

联立(39)(40)可得

$$b(t)=\frac{(1-k^2)D_x(t)D_y(t)}{4a(t)} \tag{41}$$

由(39)(41)式得到参数 a、b 的表达式

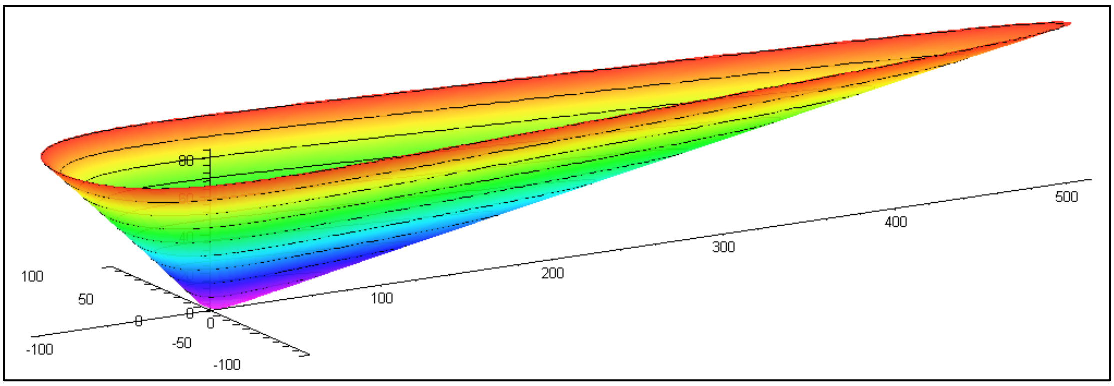
$$a(t)=4.033\times10^{-3}t^{1.17}+19.18t^{0.375}+1.223\times10^{-2}t \tag{42}$$

$$b(t)=\frac{4.150\times10^{-7}t^{2.34}+7.651\times10^{-2}t^{1.545}+2574.684t^{0.75}}{4.330\times10^{-4}t^{1.17}+1.223\times10^{-2}t+19.178t^{0.375}} \tag{43}$$

联立(34)(42)(43)建立“蛋形”关于时间变化的参数方程

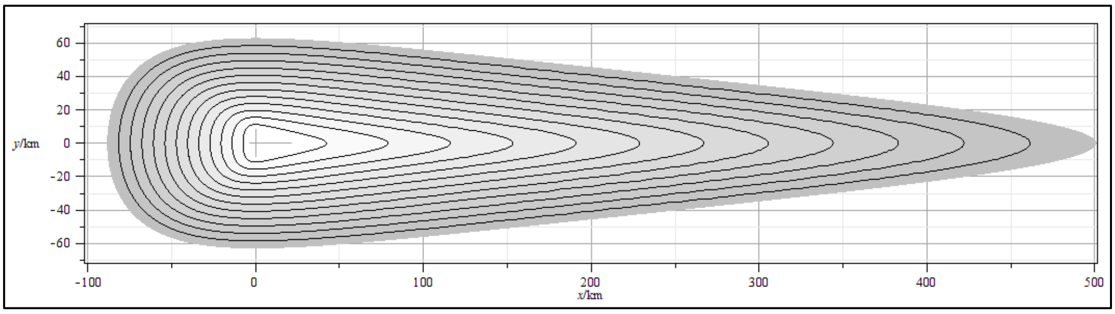
$$\begin{cases} x=\dfrac{a(t)=4.033\times10^{-3}t^{1.17}+19.18t^{0.375}+1.223\times10^{-2}t}{1-0.7\cos\theta}\cos\theta \\ y=\dfrac{4.150\times10^{-7}t^{2.34}+7.651\times10^{-2}t^{1.545}+2574.684t^{0.75}}{4.330\times10^{-4}t^{1.17}+1.223\times10^{-2}t+19.178t^{0.375}}\sin\theta \\ z=\dfrac{t}{86400} \end{cases} \tag{44}$$

据此用 Maple 作出等时轮廓变化图（Graph 9，各轴单位与椭圆模型相同）：



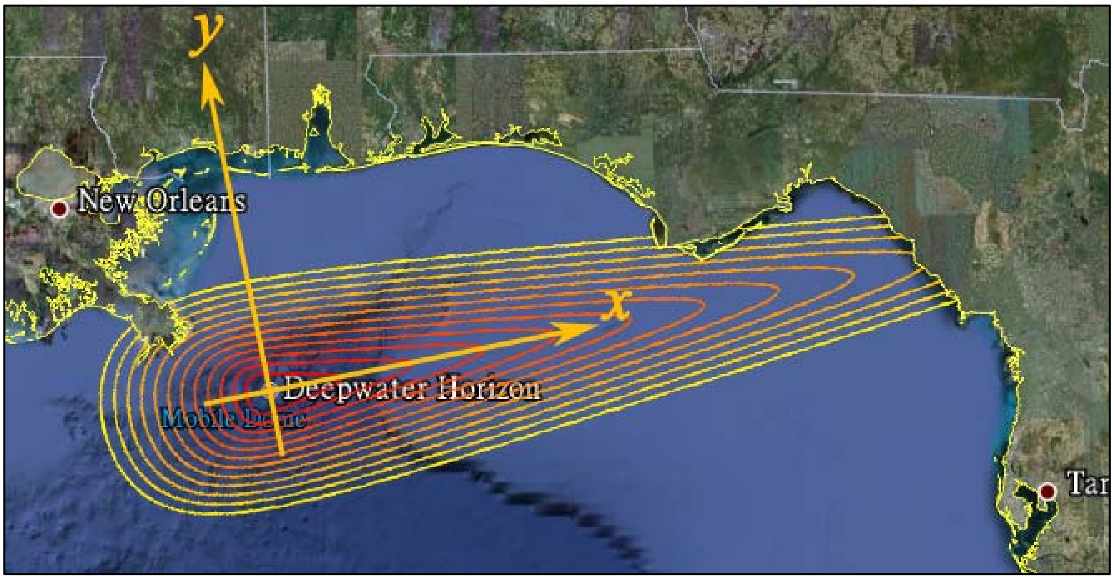
Graph 9

把以上图形换成等时线平面图形式（Graph 10）：



Graph 10

根据本章第一节得出的原油运动方向 $\boldsymbol{U}_T$，在卫星图片上摆放该“蛋形”模型，可以看到最终结果（Graph 11）．

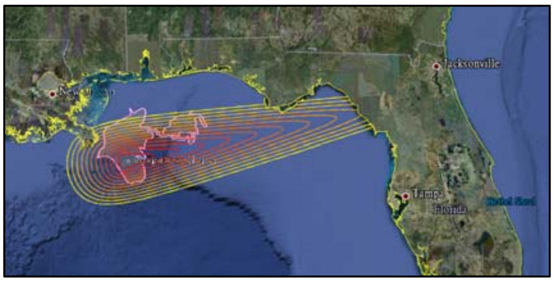


Graph 11

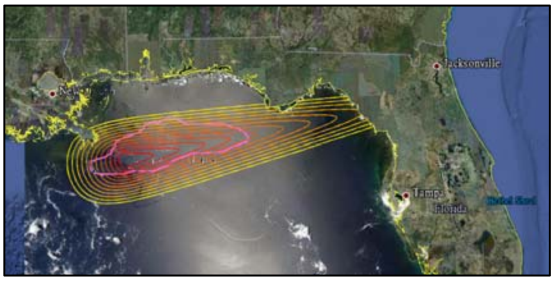
五　建模结论 Conclusions of Modelling

〔I〕模型检验 Tests of Model

根据互联网上的相关资料[7]，我们找到了原油在大约第三周时的扩散轮廓（Graph 12）以及在大约第五周时的卫星照片（Graph 13）.



Graph 12



Graph 13

从对照图中可以看出，尽管模型与真实轮廓之间存在少量的差别，但原油的总体迁移与实际情况仍然比较符合. 这些细微的差别主要是由控油设施和海湾时刻变化的水文气候状况（包括飓风）引起的.

〔II〕　结果分析 Analysis of Result

无论模型如何改进，原油的扩散仍有许多不确定因素，尤其在扩展的边界处极易受外界影响，导致出现扩展边缘参差不齐的现象. 根据我们的模型，在理想化的条件下，洋流与潮汐对原油的扩展有较为深厚的影响，尤其在墨西哥湾，处于热带和亚热带，高温多雨、降水丰富，导致美国佛罗里达聚集的大量雨水从海峡流出，形成影响该湾的主导因素：墨西哥暖流. 虽然其导致的潮差相对较小，约 2m 左右，但是飓风多发的夏季，情景却很不一样，风暴使得水位达到 5m 之多. 因此原油的扩散很大程度上受到洋流的影响，故所得结果中 x 轴方向的主要运动与洋流的走向基本吻合.

两种模型的比较

	椭圆模型	蛋形模型
计算	较简单	复杂
理论依据	较充分	稍显不足
考虑因素	较少	较多
真实度	低	较高
应用范围	较狭窄	较广泛
发展空间	小	大

传统的椭圆模型计算简便,可较易得出原油扩散的大致图形,长短轴的长度、比例、变化情况，与实际相符，可是难体现出洋流等的影响，故与真实轮廓有差别.

我们团队新设想的“蛋形”模型，虽计算繁琐复杂，且面积只能近似估计，但更符合实际，其坐标中心即为漏油点，洋流的作用体现得较准确，明显优于最初的椭圆模型.

不过，在原油泄漏的 84 天内美国政府不时派出船只对原油进行拦截，加之水文条件在海湾表面各点各时间都有差异，故原油的“蛋形”轮廓仍与实际有一定差距.

〔III〕　结论 Conclusions

通过这次建模，我们发现原油泄漏事件在理想化前提下，模型能较精准的反映原油扩展的三个阶段，特别是模型能将原油泄漏的三个阶段的划分准确，且能大致描述出原油扩散的二维图像.

在考虑到多方因素（包括洋流、地转偏向力、潮汐、原油耗减）的情况下，模型与实际情况仍有一些出入，特别是原油的扩散受到洋流的潜在影响，而洋流在各区间上的速度不尽相同，故给我们的模型带来不少挑战.

故本文在上节末提出，在对原油泄漏事件进行宏观讨论时，考虑到消极影响因素，如洋流风这些有助扩散的因素，我们大致可以对原油铺展开时的浓度变化，

并较好的对未来原油扩散的走势进行预报及预测．然而，由于洋流在各海域上的速度不尽相同，故我们的模型仍有待完善．

〔IV〕　合理化建议 Reasonable Advice

基于以上对模型的分析，我们提出了如下几点建议：

1. 从原油面积变化率的分析（Graph 5）中看出，$A(t)$是一个下凸形的增函数，因此我们建议有关部门应该在原油开始泄漏时尽快封堵漏油口，以防止原油在海面扩散得越来越快．

2. 从海面原油的迁移运动（Graph 11）中看出，如果在原油泄漏的初期便派遣大量打捞船沿着 $\boldsymbol{U}_T$ 的反方向往漏油点处阻截原油，那么原油的扩散将会得到有效的控制．

〔V〕　拓展应用 Further Development & Practical Use

由于我们的“蛋形”模型可以体现洋流对原油的作用，所以可以应用到很多水文条件较复杂的宽阔海域．而椭圆模型因为有先天的局限性，虽能简明的反应原油扩散的大致情况，但不适用于水文复杂受洋流等因素影响较大的海面，故只能运用在平静的宽阔湖（海）面上．

今后我们将把我们首创的“蛋形”模型做进一步研究，考虑更多的实际情况，应用到江河出海口、洋流影响显著的海面上，例如数月前发生的渤海湾油管爆炸漏油事件，以及协助南海石油开采方注意规范操作，防止此类事件的再次发生．

因此，我们的模型能基本覆盖海面污染的大多数问题，不仅能对原油的扩散进行准确的预测，还能绘制出原油扩散的大致轮廓．但是这个模型在精确性方面有待加强（主要在近海区域）．

〔VI〕 待解决的问题 Problems to be Solved

- 文中运用 Fay 公式的建模都是基于五点假设，而现实生活中这五点假设很难满足.
- 在原油扩散到近海区域时模型的精确性仍有限制.
- 由于无法获知原油打捞船和控油罩对原油的清除量，所以计算得到泄漏到海面的原油体积略不同于真实情况.
- 墨西哥湾的气候水文状况存在时空上的差异，因此在讨论原油迁移运动时存在偏差.

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