Repeater of Information
Coordinator of Intelligence
Terminator of Ignorance

A Research Paper on Repeater Distribution

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Summary

The VHF radio spectrum involves line-of-sight transmission and reception. This limitation can be overcome by “repeaters,” which pick up weak signals, amplify them, and retransmit them on a different frequency. Thus, using a repeater, low-power users (such as mobile stations) can communicate with one another in situations where direct user-to-user contact would not be possible. However, repeaters can interfere with one another unless they are far enough apart or transmit on sufficiently separated frequencies.

In addition to geographical separation, the “continuous tone-coded squelch system” (CTCSS), sometimes nicknamed “private line” (PL), technology can be used to mitigate interference problems. This system associates to each repeater a separate sub audible tone that is transmitted by all users who wish to communicate through that repeater. The repeater responds only to received signals with its specific PL tone. With this system, two nearby repeaters can share the same frequency pair (for receive and transmit); so more repeaters (and hence more users) can be accommodated in a particular area.

Our paper describes the problem of repeater placement. A major contradiction is that since only a fixed number of 54 Private Lines are available, which means that each repeater can only serve 54 users at the same time, it seems better to have as many repeaters in service as possible, whereas in fact, interference may occur when repeaters are placed in a much-too dense allocation or are not on sufficiently separated frequencies.

We always bear two principles in mind while searching for the optimal tradeoff; one is leaving nowhere uncovered by the system, the other is that minimizing the use of repeaters, which equals minimizing the overlapping service area.

Since circles allocation cannot meet our first principle, our primary goal turns to be finding out the most circle-like regular polygon that fits our two principles. Through further mathematical deduction of several possible solutions, we conclude that hexagon is the best equivalent for the repeater’s perceptual distance, with the repeater on its center, which is a very important conclusion in our models.

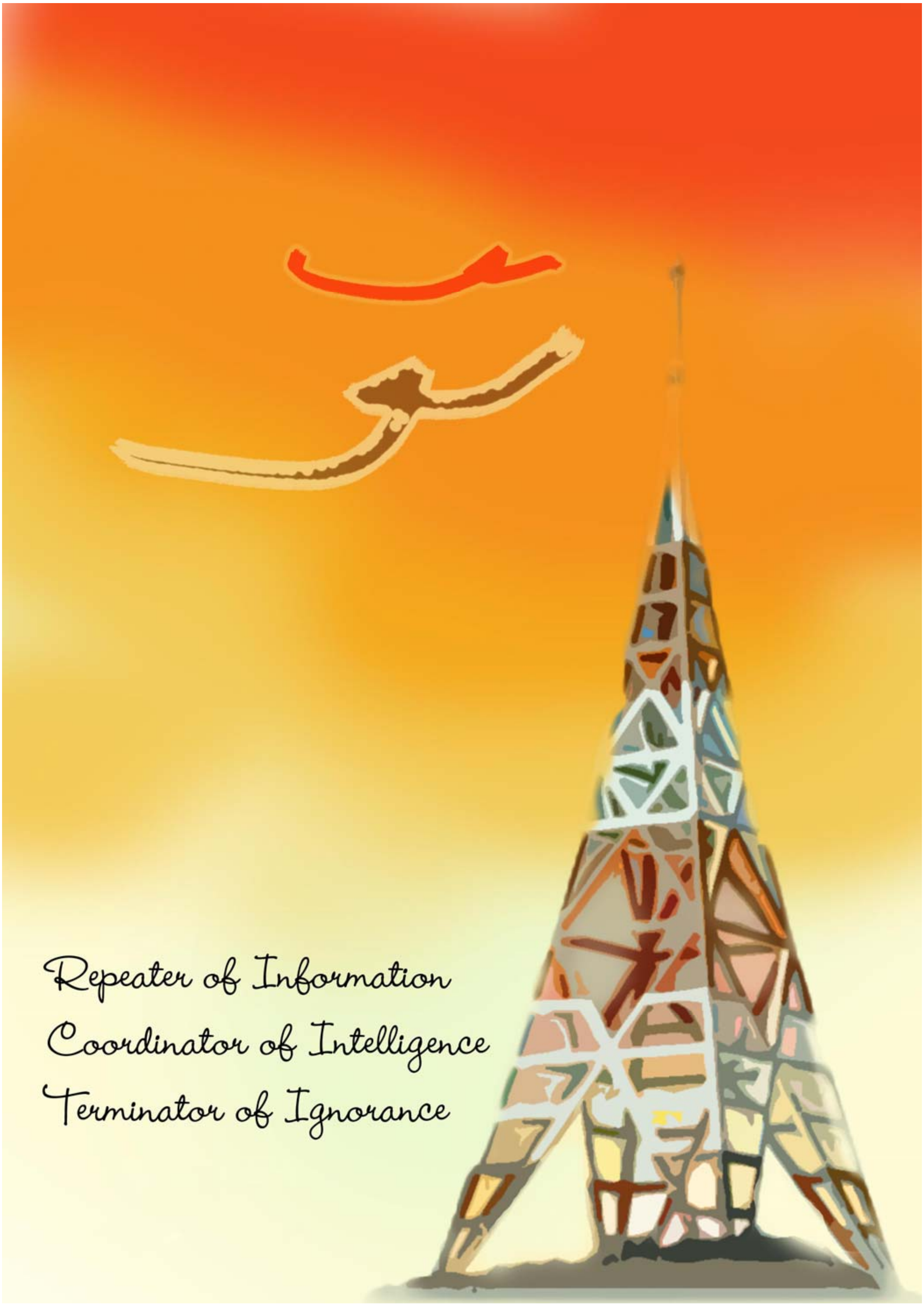
Based on the “hexagon coverage model” above, we established our model for 1000 simultaneous users by covering the area with 19 small regular hexagons connecting into a larger one. Through programming, our model achieves that two persons can communicate in the area no matter where they are.

As for the planning for 10000 users, we consider it the future situation of case 1 when more and more people are using this system. That’s why we decide to base our Model 2 on the former

one. The key to larger capacity is splitting the hexagons. After discussion, dividing one hexagon into twelve smaller ones turns to be the fittest for a user number of 10000, as it allows the least number of repeaters.

While discussing the 3-D mountainous area, taking the block of mountains to signals into consideration, we worked out a series of modeling steps that is suitable to any kind of situations. In this way, we can find out the least number of repeaters which ensure the whole coverage of signal in this area.

Our main assumption is the even population distribution, which can, by some means, be not specific enough. However, in our paper, we have proposed further improvements when our assumption does not work properly. What's more, through our precise mathematical proof and calculation, together with the help of programming, our three models greatly fit the requirements of the problem and come out with satisfying results in our model testing, which makes us convinced that, with the help of our model, a more efficient signal transmitting system can be established.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1. Introduction

The late twentieth century witnessed great leaps in the use of radio technology. The Americans had been attempting to eliminate the wires of a telegraph system while several European pioneers endeavored to develop equipments to generate and receive electromagnetic waves. Experimental wireless stations, also known by this time as amateur radio stations, sprung up across the Western world as radio technology prevailed.

The 1924 allocation of a 5-meter amateur wavelength marked the very beginning of the ultra-high era in amateur radio. Many thousands of highly active experimenters were actively pushing that technology along, through continual improvement and innovation on their own wireless sets, as well as developing more advanced operating skills. Since then, the exploration of VHF, specifically known as Very High Frequencies, has propagated throughout the world.

The technology of VHF has played a significant role in people's lives ever since the commencement of modern radio history. Its common uses include FM radio broadcast, television broadcast, etc. It is because of its prevalence that long range communications can be realized for coping with emergencies, for business and for military. During the transmission of radio signals, repeaters are especially indispensable devices. However, when two frequencies aren't sufficiently separated, interference occurs, impeding the proper transmission and reception of radio signals greatly. In cases like these, CTCSS, also known as Private Line, settles the problem. It allows more signals of the same frequency transmit between repeaters, and thus, enables the existence of more repeaters.

Our team's goal is to determine the minimum number of repeaters employed to accommodate 1,000 simultaneous users. A same task is required to solve under the condition of 10,000 users. We will also discuss the cases where the line-of-sight propagation of VHF is obstructed in mountainous areas. The preconditioned assumptions are: the models are applicable to a circular flat area of radius 40 miles; the spectrum available is 145 to 148 MHz; the transmitter frequency in a repeater is either 600 kHz above or 600 kHz below the receiver frequency; 54 different PL tones are available.

Before we start to resolve any of the tasks above, we have to define how in fact the system, where signals transmit from user to receiver through repeaters, works. On the basis of the several prerequisites, we want this system to be realistic and practical while ensuring the signals of simultaneous users will not interfere with one another. Based on our research and understanding of the problem, the frequencies of the signals transmitted in this system fall into six categories which range from 145 to 148 MHz. There is a 600 kHz difference between every two categories of frequencies, as we assume is sufficient for keeping the radio signals from interfering with one another. Meanwhile, each repeater is assigned to transmit signals with one type of frequency, and all repeaters can only transmit the signals with the frequency that is the same to its own. After being sent from a user, signals reach the receiver through passing along different repeaters. In order to give a better answer to how signals pass along, we propose the terms *user-linked repeaters* and *intermediate repeaters* to distinguish two different transmission processes. Here, *user-linked repeaters* refer to the very first and the very last repeaters, which build up the connection between users and the transferring chain. As a matter of fact, this user-to-repeater transmission is low-powered. In comparison, *intermediate repeaters* are driven by a relatively higher power when passing the radio signals among themselves. While signals are transmitting between *intermediate repeaters*, none of their Private Lines will be occupied, which is also to say that this high-powered transmission does not interfere any of the repeaters' regular operation. 54 Private Lines enables 54 users to connect to the same repeater simultaneously.

Then, after determining how the system operates, we come up with a hypothetical minimum number of repeaters that satisfies Case One, which has 1,000 simultaneous users. Through building up a model that effectively points out the locations of the repeaters, we wish to prove that the minimum number we came up with is correct.

In Case Two, where there is a prominent increase in population density, we aim at exploring new ways of locating repeaters to accommodate the change. How to balance the growth in the density of repeaters while ensuring that interference of signals won't occur is the problem we are likely to lay emphasis on.

2. Assumptions and Justifications

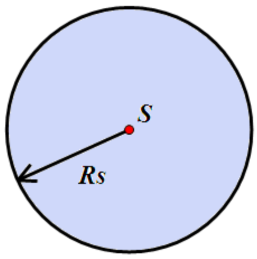
- The population of 1,000 users is distributed evenly over the flat area.
- A 600 kHz difference between two frequencies is sufficient for keeping them from interfering with one another.
- In order to be prevented from interference, adjacent repeaters don't receive the same frequency.

3. Case One - the Minimum Number of Repeaters to accommodate 1,000 users and Its Justification

3.1 No-gap Coverage of the Area

(1) Perceptual coverage and Interconnected coverage

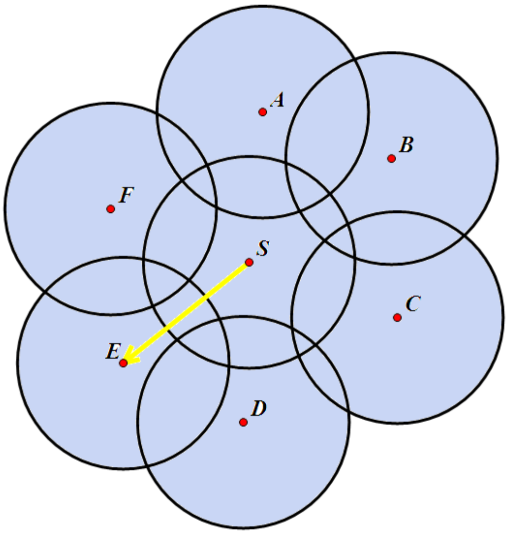
Perceptual coverage refers to the area where a repeater can perceive and transmit signals. The coverage is represented by a circle with repeater *S* as its center and *Rs* as its radius, as is shown in GRAPH-3.1 below.



GRAPH-3.1 The blue area represents the *perceptual coverage*, with repeater *S* as its center and *Rs* as its radius.

Since *Rs* can be variable according to different levels of working power of repeater *S*, the radius is actually determined by the density of population. Based on the assumption that population is evenly distributed, we come to the conclusion that all repeaters share the same *Rs*.

Under an *interconnected coverage*, repeater *S* should be interconnected properly with a certain number of neighboring repeaters, so that radio signals can be perceived in their joint *perceptual coverage*, as is illustrated in GRAPH-3.2.



GRAPH-3.2 The blue area represents the *interconnected coverage*. The yellow arrow points out the direction in which signals are transmitted, in this case the signals are transmitted from repeater *S* to *E*.

How to determine the number of neighboring repeaters of repeater *S* will be discussed in the succeeding sections.

It is noted that maximizing the utility of every repeater means minimizing the overlapping *perceptual coverage*. How to cover the entire flat area while leaving no gaps and minimizing the overlaps is the problem we will discover next.

(2) The "Hexagon Coverage Model"

First of all, it is obvious that we can't use circles to cover an area entirely without overlapping circles and leaving gaps. The optimal alternative solution is to use the minimum number of regular polygons to realize the no-gap coverage with the smallest area of overlaps.

To meet the principal of no-gap coverage, the polygons should altogether form a seamless mosaic. That is to say, a perfect perigon can be created by simply placing the angles of several congruent regular polygons one by one around the same vertex. The interior angle of each polygon is $(n-2)180°/n$, hence x n-sided regular polygons which are placed around one vertex meet the following equation:

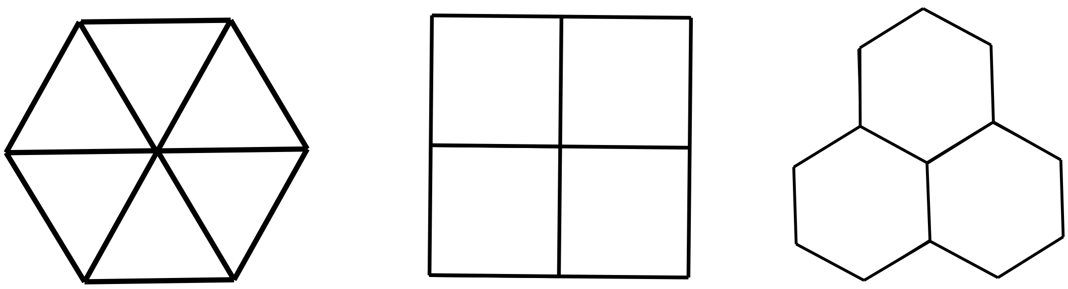
$$x\,\frac{(n-2)180^\circ}{n} = 360^\circ \quad (x, n \in \mathrm{N}^*, n \geq 3)$$

$$\text{Thus} \qquad x = 2 + \frac{4}{n-2}$$

Since x is a positive integer, 3, 4 and 6 becomes the only possible values for n, which leads us to three different potential solutions as are shown below:

$$n = 3, x_3 = 6; \qquad n = 4, x_4 = 4; \qquad n = 6, x_6 = 3,$$

The above figures represent the three ways of covering the entire area with congruent regular triangles, squares, or regular hexagons.



(a) congruent regular triangles (b) congruent squares (c) congruent hexagons

GRAPH-3.3 the No-gap Coverage of Congruent Square Graphs

Let *A* be one of our repeaters, covering a perceptual area of A_l. Let A_s be the area of the inscribed regular polygon of circle *A*. A_c is the difference between area A_s and area A_l, which is also the overlap area of two adjacent circles. Thus we get:

$$A_c = A_l - A_s = \pi r^2 - \frac{nr^2}{2}\sin\frac{2\pi}{n} \qquad ①$$

In ①, n stands for the number of sides of the inscribed polygon.

When $n \to \infty$, $A_c \to 0$. Therefore, we can get the smallest overlap area A_c when $n = 6$, which is to say:

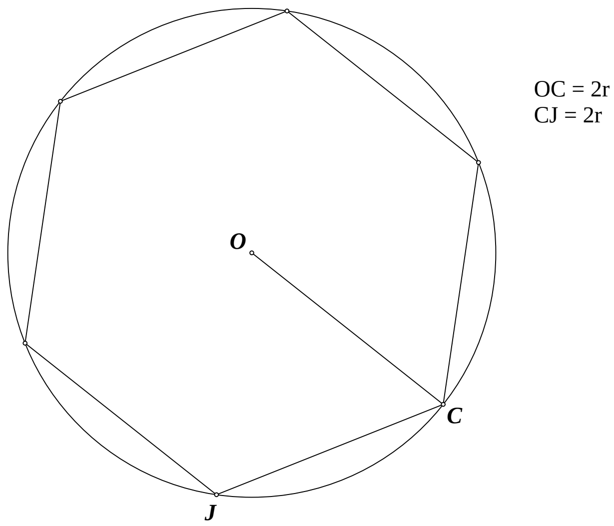
$$A_{C\min} = (\pi - \frac{3\sqrt{3}}{2})r^2$$

In conclusion, to cover an entire area with the inscribed regular hexagons of circles with a *perceptual radius* ***r*** is to realize the no-gap coverage with the smallest area of overlaps. We give this model the name "hexagon coverage model".

3.2 How to determine the Number of Neighboring Repeaters

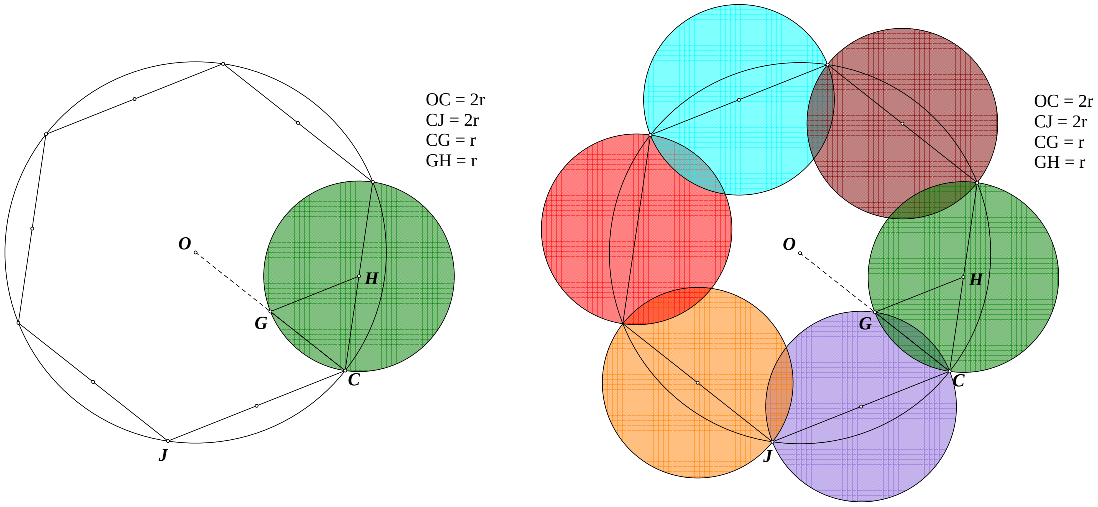
After the set-up of "hexagon coverage model", we proceed to the discussion of how many repeaters and where they shall be set, bearing in mind the principal of attaining a complete coverage and the smallest overlapping area. According to our solution, it is essential that a repeater is being placed with at least six neighbors to ensure the circulation of signals.

Let O be the centre of a circle with a radius of 2***r***, which means the inscribed hexagon has the edge length of 2***r***, as is shown in GRAPH-3.4:



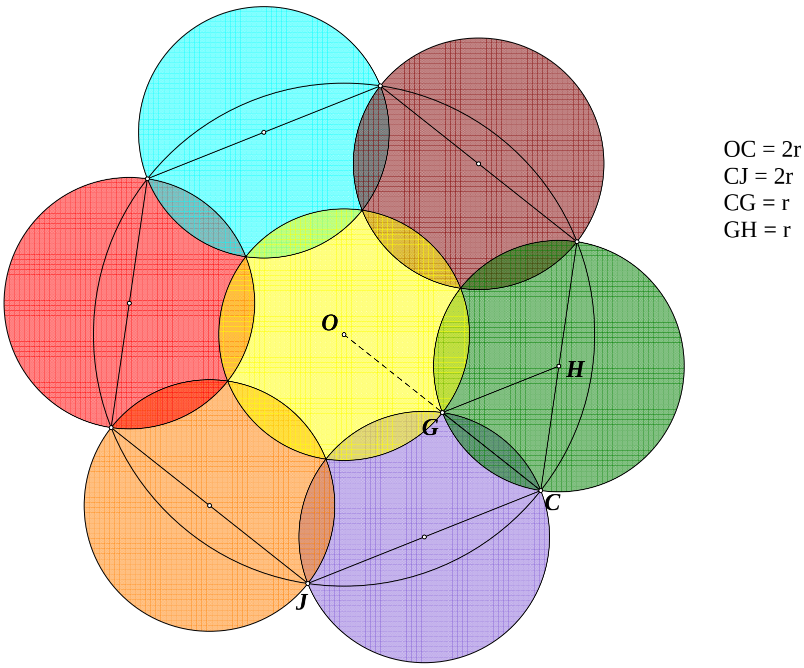
GRAPH-3.4

To cover the entire circumference of ⊙O, we construct a circle of radius of *r*, each based on the midpoint of the one edge of the hexagon O as its centre, as is shown in GRAPH-3.5:



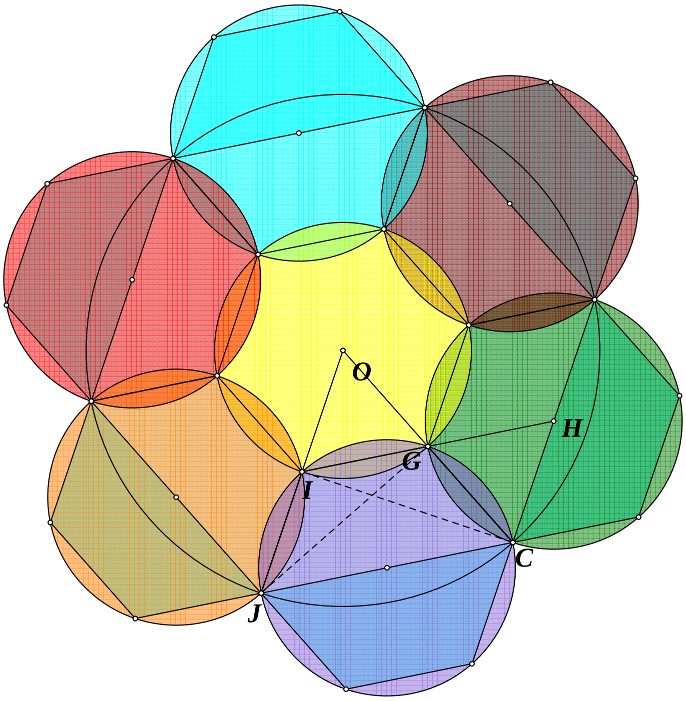
GRAPH-3.5

Finally, lay a circle of the same size with the same centre as ⊙O. Thus, the seven circles complete the no-gap coverage of ⊙O, as is shown in GRAPH-3.6:



GRAPH-3.6

Mark the equilateral triangle △OJC (I is the midpoint of OJ, while J is that of OC). According to the theorem of angle of circumference, ∠JIC=∠CGJ=90°. Besides, △OIG is overlaid by the ⊙O in the center (the yellow circle in GRAPH-3.7).



GRAPH-3.7

In a like manner, we can draw the conclusion that 7 circles of radius r can completely cover a larger one of radius 2***r***, as is shown in GRAPH-3.7. This is to say that a repeater should at least be set with six neighboring ones to create a no-gap coverage, which ensures every user in the area have the access to send and receive radio signals.

On the basis of our “no-gap model” and theory of connection between repeaters, we can easily form a network of repeaters covering the entire area.

3.3 Minimum Number of Repeaters and Its Justification

(1) Conjecture of the Minimum Number

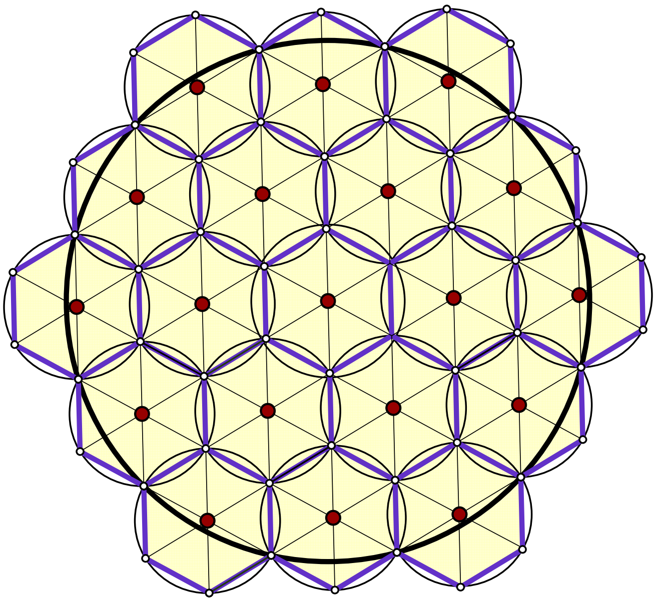
54 Private Lines enables 54 users to connect to the same repeater simultaneously. While 1,000 users are accommodated, interference led by a simultaneous use of the same PL should be avoided. Hence, we intend to divide 1,000 by 54, and then rounded up to the next integer $\left[\frac{1000}{54}\right]+1=19$. According to our conjecture, the final outcome, 19, shall be the minimum number of repeaters that allows no PL to be used

by more than one user simultaneously and meets all other demands of Case One.

(2) Justification—Model 1 (the "Honeycomb Model")

Mere conjecture is surely not enough. On the basis of our "hexagon coverage model" and theory of connection, we intend to build up a model that effectively points out the locations of the repeaters. Through proving that a network of 19 repeaters covers the entire area while satisfying all demands, we wish to prove that the minimum number we came up with is correct.

Recall the radio signal transmission system we mentioned in **Introduction**. Recall the **Assumptions** we set previously. With the models and theories we deduced just now, we can set up an integrated model that simulates the network of repeaters under Case One. GRAPH-3.8 is displayed below.



GRAPH-3.8 Model 1, aka., the "Honeycomb Model".

We now present Model 1, aka., the "Honeycomb Model".

STEP 1

In our model, "1" stands for the repeaters which can only receive and transmit signals with the frequency of 145 MHz, "2" for 145.6 MHz, "3" for 146.2 MHz, "4" for 146.8 MHz, "5" for 147.4 MHz, "6" for 148 MHz, since there are altogether 6 categories of frequencies.

Signals can only be transmitted between repeaters with a difference of 1 in their numbers because whenever a signal passes a repeater, its frequency is automatically added by 600 kHz or subtracted by 600 kHz. For instance, repeaters numbered “3” can only transmit signals along to “4” or “2”.

Frequency sent by one repeater could only reach its adjacent repeaters. This is because only two adjacent repeaters can have an overlapping area of *perceptual coverage*. It is only in this overlapping area that both repeaters can send and receive signals. Therefore, communication of signals between adjacent repeaters is possible.

STEP 2

As is stated in **Assumptions**, we stipulate that adjacent repeaters don’t receive signals with the same frequency. A reflection of this stipulation in Model 1 is that adjacent numbers cannot be the same.

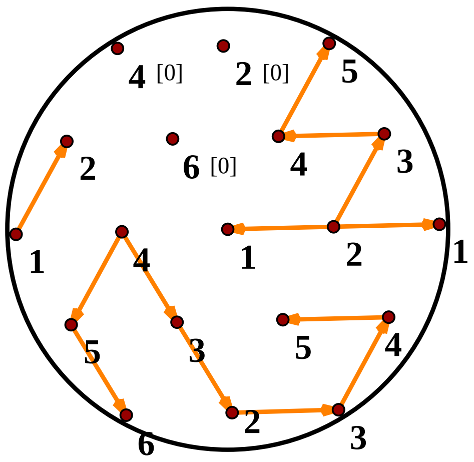
STEP 3

We can obtain a number of different combinations which satisfy our stipulations above. The next thing to do is to screen out the combinations which don’t qualify for a successful transmission of radio signals.

STEP 4

All the spots on the network (representing the location of 19 repeaters) are assigned with Boolean numbers of 0. Let’s start the following process with, namely, the first spot of a combination. Our model searches for spots surround the first spot that have a difference of 1 comparing to the first spot. When such surrounding spots are not detected, the Boolean number of the first spot, which is 0, will not change. If such spots do exist, they will be assigned with the Boolean number of 1 along with the first spot. The model will proceed to repeat the same procedure until all spots are assigned with a 0 value or 1 value Boolean number.

GRAPH-3.9 shows an example of this procedure.



GRAPH-3.9 This is Combination No.1. Since a network with all spots linked together can't be established, successful transmission of radio signals can't be ensured. This combination is impractical.

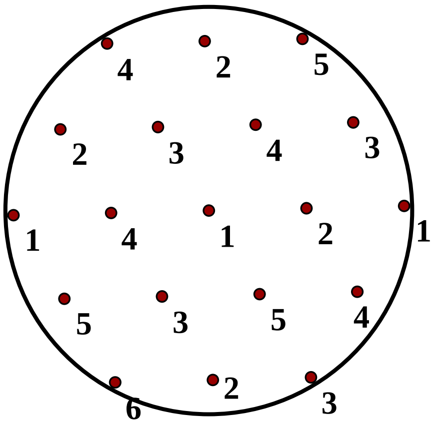
GRAPH-3.9 shows a case where some numbers can't find any of their surrounding numbers to have a 1 value difference. Three spots are left out, and thus a coordinate network of repeaters can't form. The 0 value Boolean number of those three spots will lead to the result that the final product of all Boolean numbers equals 0. Our model will consequently screen out Combination 1.

STEP 5

After this, we multiply all the Boolean numbers together, and see if the result is 0 or 1. Those combinations which have a 0 value product will be screened out because there could be at least one 0-Boolean-number among the spots, which means that there could be at least one repeater that can't pass the frequency along to any of its surrounding repeaters. Under a case like this, the circulation of radio signals is impeded. That's why only the combinations with a 1 value product could possibly stand for feasible networks where 19 repeaters work coordinately.

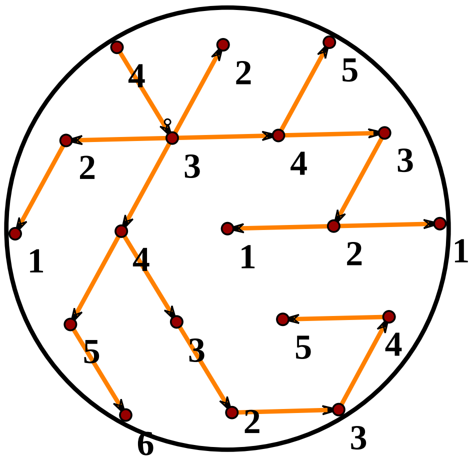
(3) Model 1 Testing and Analysis

A graph-illustrating testing of Model 1 is presented below:



GRAPH-3.9 This is Combination No.2.

GRAPH-3.9 shows one of the 5,000 combinations that are picked out after STEP 2. No pairs of adjacent repeaters are assigned with the same frequency amongst these 5,000 combinations.



GRAPH-3.10 The successful transmission of radio signals indicates that Combination No.2 is one of the practical combinations.

GRAPH-3.10 shows a case where all numbers can find one of their surrounding numbers to have a 1 value difference. This indicates that no matter from which repeater the signal is sent and to which repeater the signal is sent, there will be no obstacle during the process of transmission. Combination No.2 is totally capable of transmitting radio signals successfully.

Much to our delight, such a combination like Combination No.2 does exist. In fact, after the model testing, we find 29 combinations that accord with all stipulations.

In conclusion, the successful discovery of Combination No.2 and the other 28 likewise combinations demonstrates an inevitable rationality and feasibility of our conjecture that 19 is the minimum number of repeaters required to satisfy all our stipulations and assumptions. The programming of Model 1 is included in the **Appendices**.

4. Case Two - The Minimum Number of Repeaters to accommodate 10,000 users and Its Justification

4.1 *Center-placing Method* and *Vertex-placing Method*

(1) Rationale

In a case where 10,000 simultaneous users are to be accommodated, we think it necessary to explore more methods of how to place the repeaters. This is in order to adapt our new model to the noteworthy change in population density comparing to Case One.

All following methods take effect on the basis of our "hexagon coverage model" mentioned before. It should be noted that a "cell" refers to the inscribed hexagon area of a circle. The circle symbolizes a *perceptual coverage* with the repeater as its center.

(2) Definition

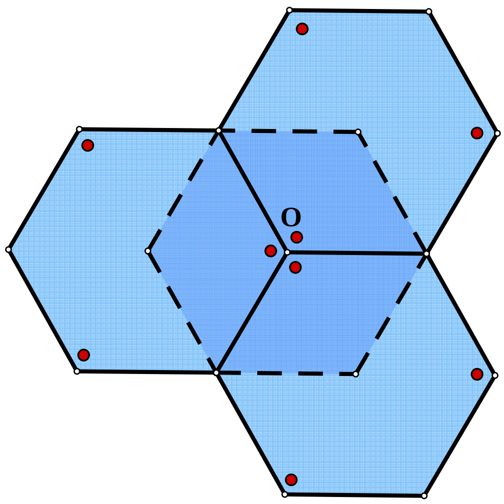
When *Center-placing Method* is used, we

- Place the repeaters at the center of each hexagon area
- Cover the hexagon areas with omni-directional antenna

When *Vertex-placing Method* is used, we

- Place the repeaters at 3 vertices that are not adjacent to each other
- Cover the hexagon areas with directional antenna

(3) A Justified Interconversion between the two methods



GRAPH-4.1

The hexagon in the middle of GRAPH-4.1 with the center O utilizes the *Center-placing Method*. The successful application of this method to Model 1 indicates its feasibility.

When constructing three more hexagons with their vertices gathering at center O, we will find that a new honeycomb based on the *Vertex-placing Method* is formed. We can also find that the total number of repeaters covered in both methods are the same. This is to say that the two commonly-used methods of network coverage can be easily interconverted. The interconversion can be illustrated by GRAPH-4.1.

4.2 Cell Splitting

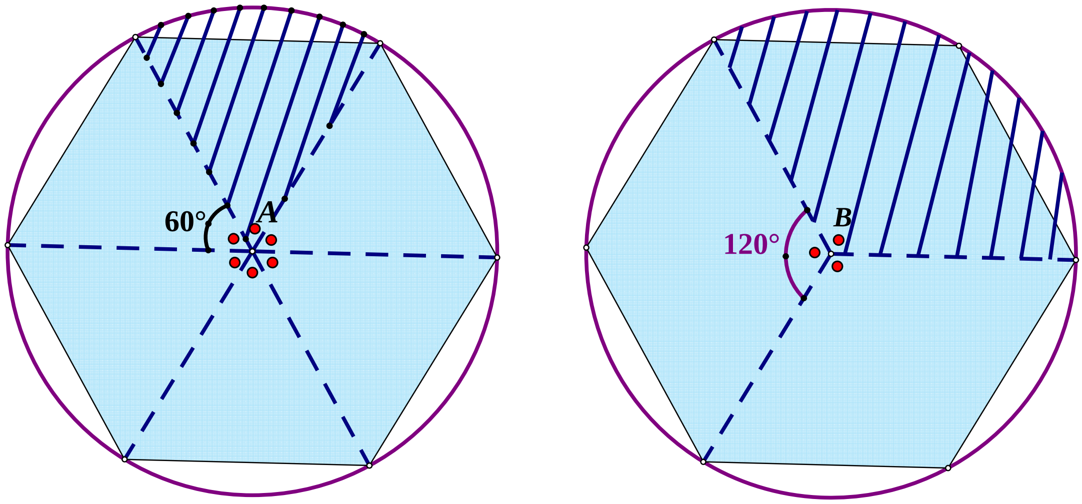
(1) Definition

For a honeycomb-like repeater network, *center-placing method* is usually adopted in an initial stage. When the number of users augments, as is the situation in Case Two, we can meet the increased needs for capacity by shrinking each cell. As for the “hexagon coverage model”, we can achieve this by splitting each hexagon area into several identical parts. This method is the so-called “cell splitting”.

(2) How 1:12 Cell Splitting is developed

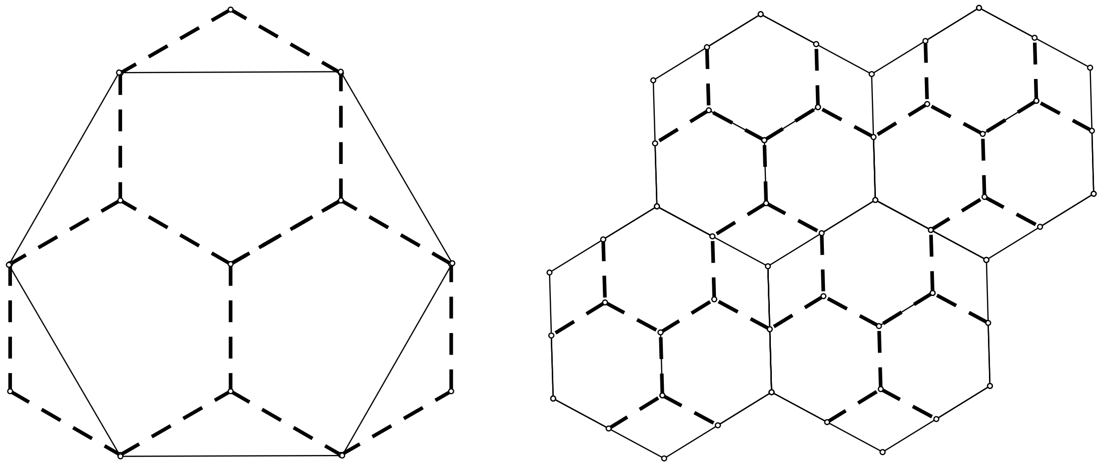
There are obviously quite a few ways to split a hexagon area into identical parts.

We can start by splitting the perigon at the center of a hexagon: When the central perigon is split by 6, the entire hexagon is divided into 6 congruent triangles; when split by 3, it is shattered into 3 diamonds.



GRAPH-4.2 The graph on the left displays a 1:6 cell splitting, a process during which a hexagon (the blue figure) that is split into 6 identical triangles. The shaded sector is an example of the scope and direction to which repeater *A* passes along its frequencies. The graph on the right displays a 1:3 cell splitting, a process during which a hexagon is split into 3 identical diamonds. The shaded sector is an example of the scope to which repeater *B* passes along its frequencies.

The deduction could go on endlessly, but some other interesting discoveries enable us to have a better look into cell splitting. While trying to perform a 1:3 cell splitting of the hexagon, we come up with another way besides dividing the area into diamonds. We attempt to split it into congruent regular hexagons. Since the outcome has a clover-like shape, we refer to this 1:3 splitting as the "clover splitting". Meanwhile however, we can't help but wonder if there is a way of splitting that doesn't follow the same pattern as others. We stop dividing the perigon and hence produced the 1:4 cell splitting. The results prove our creative thoughts to be of great value.



GRAPH-4.3 The graph on the left displays a “clover splitting”, while the graph on the right displays a 1:4 cell splitting.

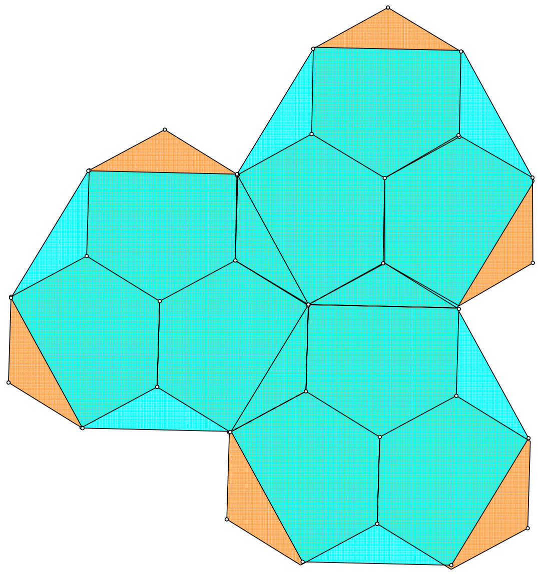
Incorporating the 1:3 “clover splitting”,1:4 splitting, and the *vertex-placing method*, we produce a way in which a 1:12 cell splitting is possible.

Recall Model 1, which places repeaters at the center of each hexagon. We would now propose a new network for locating repeaters for Model 2, but to reach the highest efficiency, we wish not to demolish our previous arrangement of repeaters in Model 1. This is to say that we would add new repeaters in Model 2 on the foundation of Model 1.

Steps for setting up this network are as follows:

STEP 1

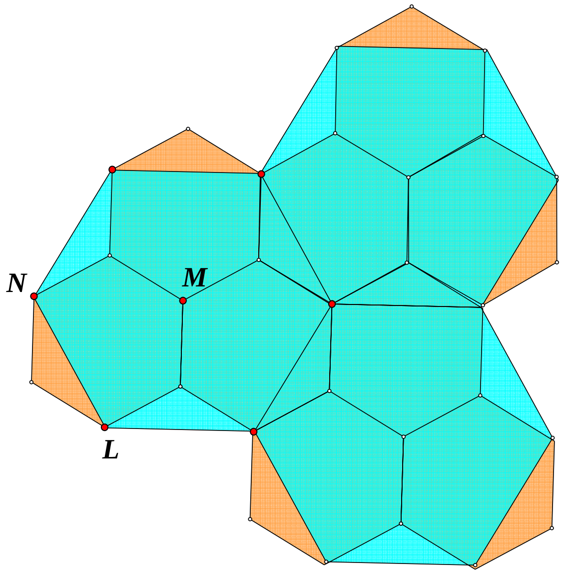
Use the “clover splitting” method to split the joint hexagons, as is shown in GRAPH-4.4.



GRAPH-4.4

STEP 2

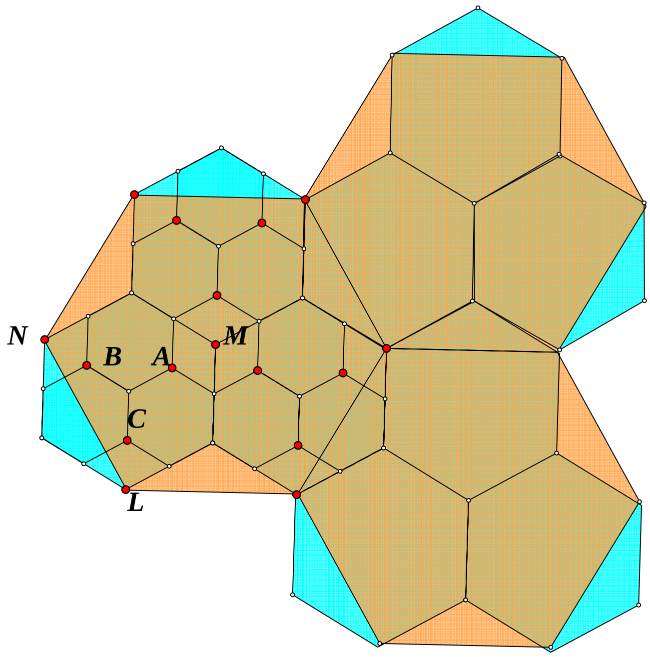
While reserving all repeaters placed in Model 1, use the *vertex-placing method* to add new repeaters, as is shown in GRAPH-4.5.



GRAPH-4.5 The red spots represent repeaters that are located after STEP 2. For instance, repeaters *N* and *L* are added on the basis of repeater *M*, a repeater that has already been set in Model 1.

STEP 3

Use a 1:4 method to further split the new cells. as is shown in GRAPH-4.6.



GRAPH-4.6 Repeaters *A, B, C* are allocated in the above manner so that their directional antenna can cover a range which Repeaters *M, N, L* can't.

Any hexagon that has undergone the above cell splitting process will contain an average of 12 repeaters. These hexagons are previously assigned with 1 repeater located at its center in Model 1, whereas in the new network, their capacities are enlarged to 12. Thus, this network is given the name "1:12 cell splitting network".

(3) Superiority of a 1:12 Cell Splitting network

• The new network saves cost and efforts from demolishing repeaters located in Model 1 by preserving previous arrangements and adding new repeaters to the network.

• Since *vertex-placing method* employs directional antenna, it can considerably restrain the interference between same frequencies. The application of *vertex-placing method* allows the network to multiplex two identical frequencies within a looser distance restriction. In this way, frequency utilization rate is enhanced, network is simplified and cost is reduced.

• The overlapping area in the interconnected coverage of all repeaters is small enough for the network to be less interference-prone.

4.3 the Minimum Number of Repeaters and Its Justification

(1) Conjecture of the Minimum Number

54 Private Lines enables 54 users to connect to the same repeater simultaneously. While 10,000 users are accommodated, interference led by a simultaneous use of the same PL should be avoided. Hence, we intend to divide 10,000 by 54, and then take the integral value of its outcome and add 1 to it. According to our conjecture, the final outcome shall be the minimum number of repeaters that allows no PL to be used by more than one user simultaneously and meets all other demands of Case One. The calculation is as follows:

$$\left[\frac{10000}{54}\right]+1=186$$

Thus we conjecture that 186 repeaters is the least number of repeaters that satisfies 10,000 simultaneous users.

(2) Justification—Model 2

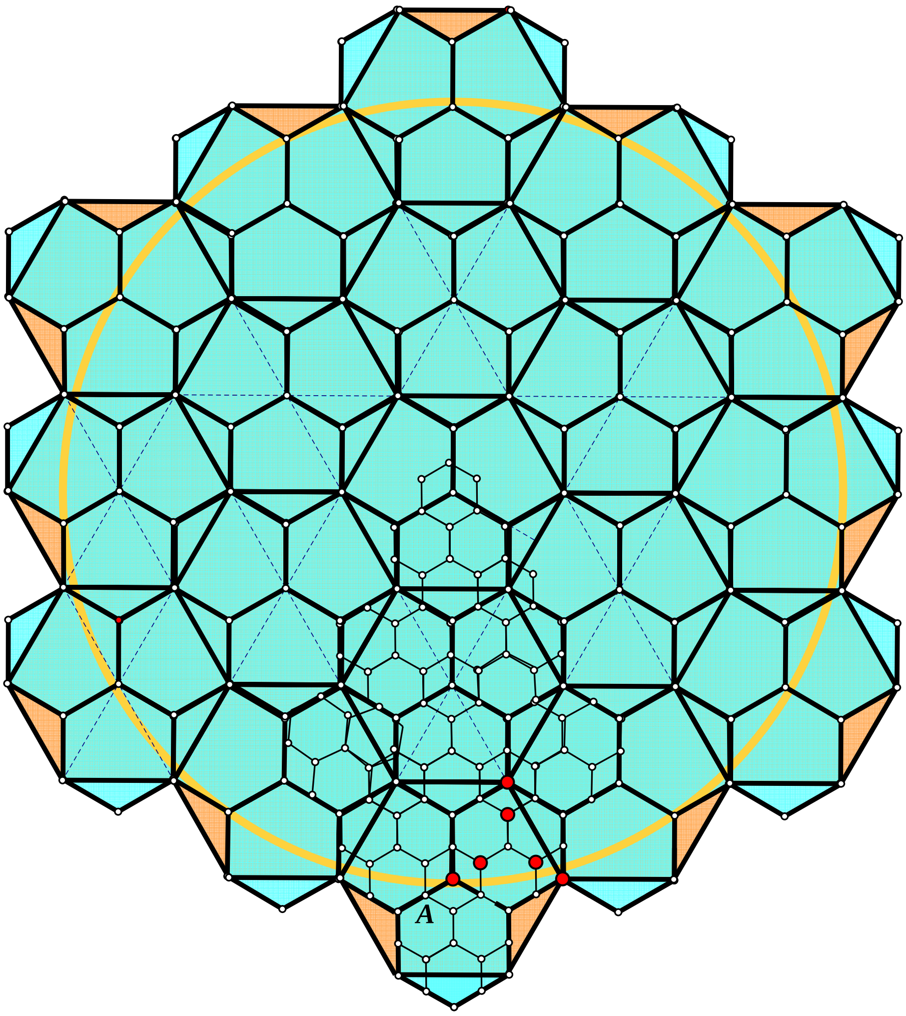
On the basis of our 1:12 cell splitting network, we intend to build up a model that effectively points out the locations of the repeaters. Through proving that a network of 186 repeaters covers the entire area while satisfying all demands, we wish to prove that the minimum number we came up with is correct.

Since a hexagon in a cell splitting network has a capacity of 12 repeaters while the theoretical minimum number of repeaters is 186, the minimum number of hexagons needed to satisfy the needs of all 186 repeaters is:

186 / 12 = 16

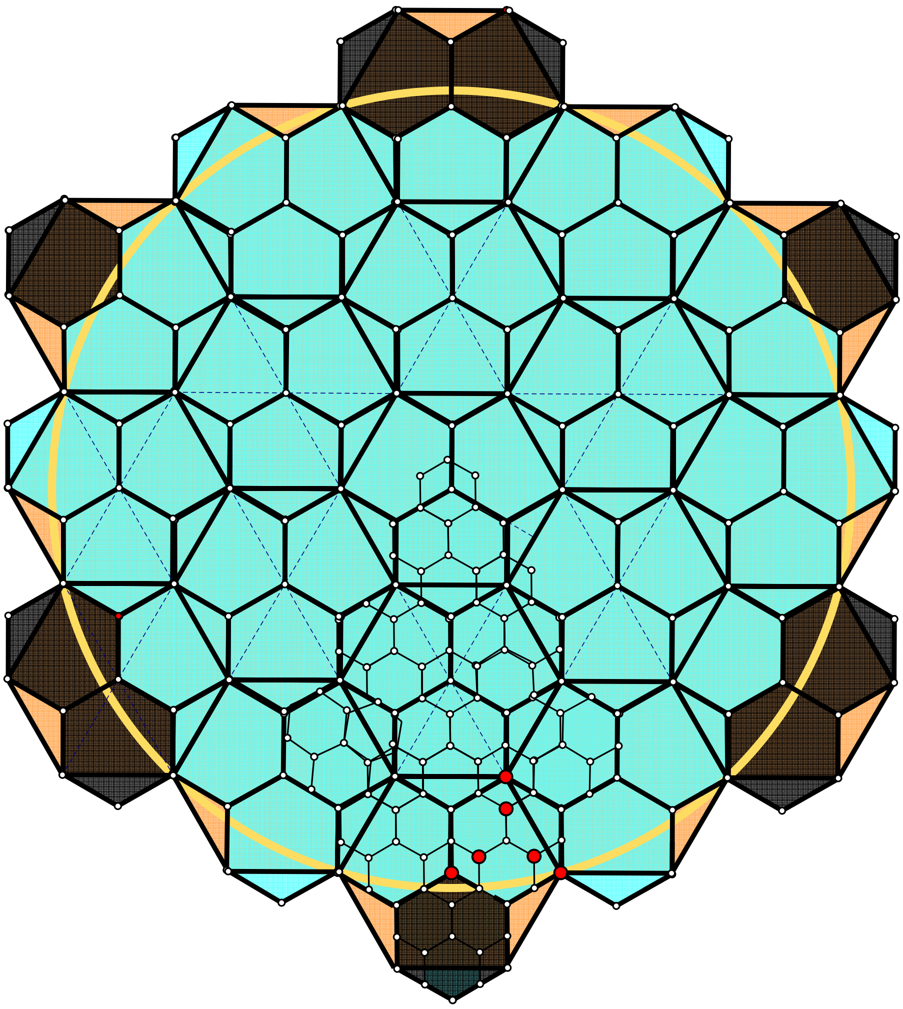
Thus, a total of 16 hexagons will be included in Model 2.

Recall **Model 1**. Recall the **Assumptions** we set previously. With the models and theories we deduced just now, we can set up an integrated model that approximates the network of repeaters under Case Two. GRAPH-4.7 is displayed below.



GRAPH-4.7 The red spots represent part of the repeaters located under Case Two. Repeater *A* is one of the 19 repeaters set previously in Model 1. The yellow circle symbolizes the flat area. The blue area stands for an *interconnected coverage* under which communication of signal between users are available.

In pursuit of higher efficiency, we tend to preserve as many locations of repeaters set in Model 1 as possible. Hence, how to set up 16 hexagons in Model 2 on the original network of 19 hexagons from Model 1 becomes a pressing problem to solve. This means that 3 hexagons are spared in our new network. By subtracting a total area of 3 hexagons from the original network of Model 1, we thus get the new network of Model 2, which is presented below:



GRAPH-4.8 represents the network for locations of repeaters in Model 2. The red spots stand for some of the 186 repeaters. The yellow circle symbolizes the flat area. The 9 small dark hexagons represent the area subtracted from the original network of Model 1 and they altogether have a total area of 3 hexagons. The blue area stands for an *interconnected coverage* under which communication of signal between users are available.

Recall the Justification of Interconversion between *Center-placing Method* and *Vertex-placing Method.* Although the establishment of the above network is comprised of both methods, neither the total number of repeaters nor the *interconnected coverage* will be changed if we unify the comprehensive methods into one kind.

This is to say, the signal transmission network of Model 2 works the same way comparing to that of Model 1. Thus, the programming of Model 2 is identical to that of Model 1, which is included in the **Appendix**.

4.4 Further Improvement

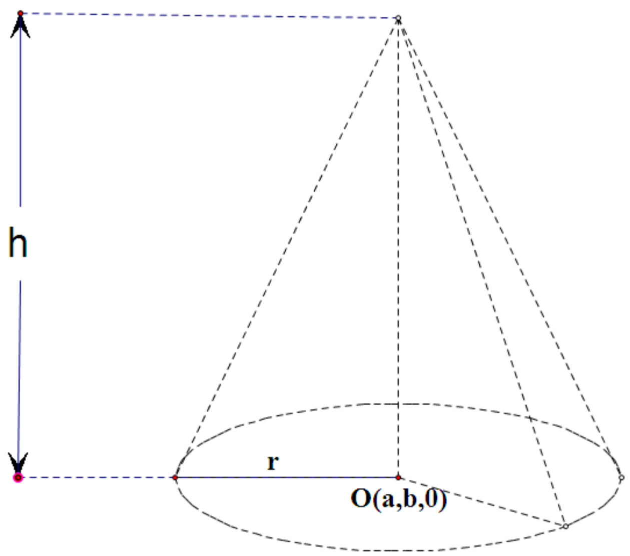
In this case, though population density increases, the assumption of even distribution stays constant, which is usually not the case in reality. For further improvement of this model, we would like to adjust it according to the user density in different areas. Our preliminary plan about this model is to connect regular hexagons of different sizes together. For instance, when dealing with a city model, we can have smaller hexagons in the centre and larger ones on the edges to meet the polarizing user numbers.

Also through this kind of splitting method, we can be able to create a model for almost any number of users, as we can divide the cell into different size of minor ones to satisfy the required number of repeaters.

5. Case Three – the Case with Mountainous Areas

5.1 Analysis Formulas of the Area

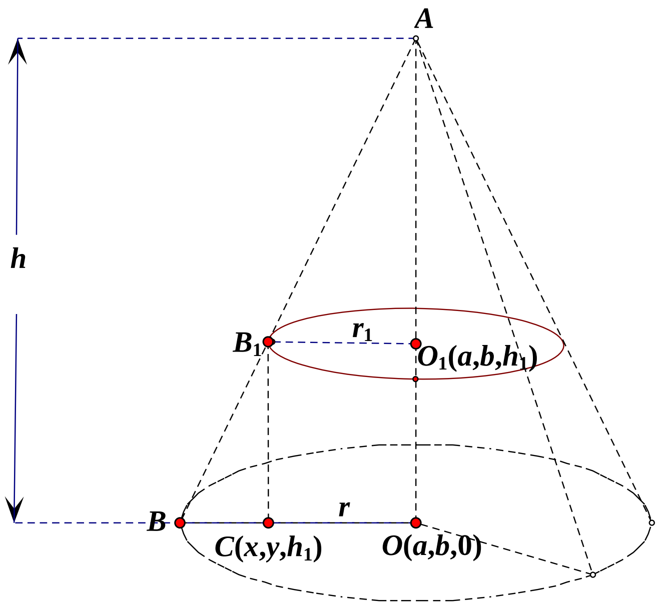
To simplify our model of the mountainous area, we define every mountain as its similar solid figure, a cone, with height h, subface radius r, the center of its subface O(a,b,0). GRAPH-5.1 illustrates our definition.



GRAPH-5.1

Let F(x,y) be the function of a cone with qualities above. That is to say, the equation F(x,y)=z represents that the point (x,y,0) is the projection of (x,y,z) on the subface of the cone. Define O_1(a,b,h1) as a point on the axis AO, whose circle-shaped plane has a radius of r_1, as is shown in GRAPH-5.2. It is quite easy to come to the conclusion that $\triangle AO_1B_1 \backsim \triangle AOB$. Thus,

$$\frac{h-h_1}{h}=\frac{r_1}{r} \qquad \Rightarrow \qquad r_1=\frac{(h-h_1)r}{h}.$$



GRAPH-5.2

On the other hand, since (x,y,h_1) is a point on the surface of the cone, r_1 should also be the distance between (x, y, z) and (a, b, z), that is, by applying the formula of the distance between two points,

$$r_1=\sqrt{(x-a)^2+(y-b)^2+(z-z)^2}$$

Thus,

$$\sqrt{(x-a)^2+(y-b)^2+(z-z)^2}=\frac{(h-h_1)r}{h}$$

$$h_1=-\sqrt{(x-a)^2+(y-b)^2}*h+hr$$

Finally, we can get the function for a cone as below,

$$F(x,y)=\frac{hr-h\sqrt{(x-a)^2+(y-b)^2}}{r}$$

For every mountain in the area, there exists a formula $F_n(x, y)$. By taking the maximum value at each pair of (x, y), we get the function for the whole studied area,

$$G(x,y)=\max[F_1(x,y), F_2(x,y), F3(x,y),\ldots, Fn(x,y)]$$

5.2 Determining the Location of Repeaters

(1) No-gap Coverage

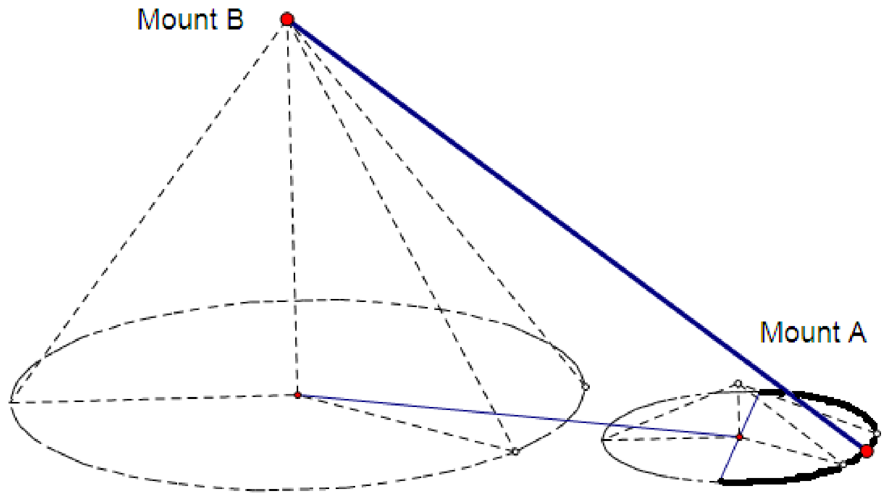
Since we have limited known conditions for this case, in order to achieve our primary goal of completely covering the mountainous area, we would like to firstly simplify the repeater transmission system. It is quite obvious that it is on the top of the mountain where the system can best avoid the defections of line-of-sight propagation. Thus it makes sense that we consider the summit repeater the single spot on a mountain to send and receive signals and all others playing the role of extending the perceptual distance of the top repeater. Furthermore, through this kind of continuous expansion, the perceptual distance can be considered very large or even unlimited as we make the case much easier to discuss. In this simplified network, we are going to further study the”no-gap coverage” as followed

STEP1 “Coverable Mountains”

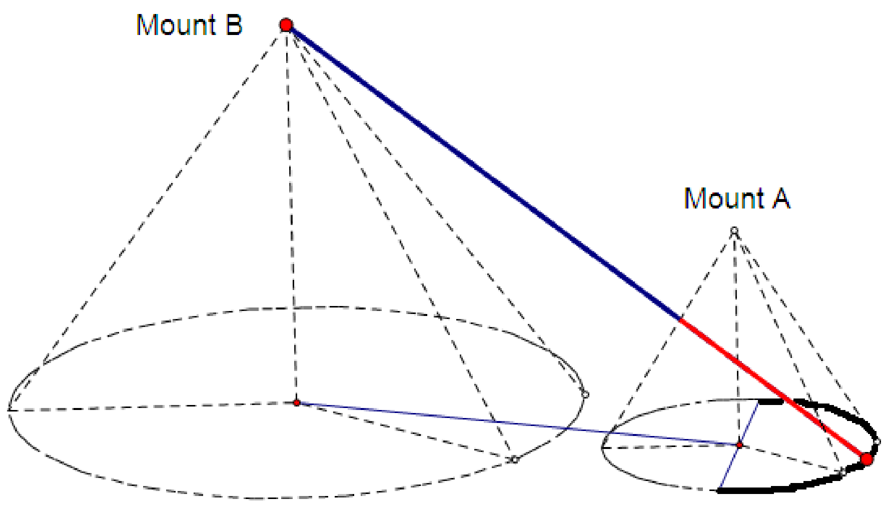
According to our assumption above, every summit repeater’s transmitting distance is not restricted. However, some parts of the area still cannot be covered by a certain repeater due to defects in line-of-sight propagation. Considering the repeater to be on the summit, we transfer the issue of building repeaters into the issue of choosing mountains.

Here we create the concept of Coverable Mountains. If Mount A can be TOTALLY covered by the signal from a repeater on the top of Mount B, Mount A is said to be the Coverable Mountains of Mount B.

To determine whether Mount A is coverable by Mount B, we conclude that if the furthest semicircle of the subface of Mount A from Mount B can be covered, Mount A can be entirely covered. GRAPH-5.3 presents this conclusion. By connecting every point on the semicircle with the summit of Mount B, we can get a function of the line connecting the two points H(x,y). For H(x,y), if H(x,y)=G(x,y) has more than two roots, that is to say, they intersect with each other at some other points, the line-of-sight propagation is blocked, as is the case in GRAPH-5.4. Only if for all points on the semicircle, H(x,y)=G(x,y) has exactly two roots, Mount A is said to be coverable by Mount B.



GRAPH-5.3 Here, Mount A is coverable by Mount B. There are only two intersections represented by points painted red.



GRAPH-5.4 Here, Mount A is uncoverable by Mount B. There are more then two intersections, represented by the points and line painted red.

STEP2 "Full coverage plans"

For every different mountain, it has several coverable mountains, most likely to be different from others'. To meet the principle of the least repeaters, what we are doing is finding the "Full coverage plan" with the least summit repeaters by programming (presented in **Appendix 3**). It is possible that there can exist several plans of this kind, and then we use the "Spread Cost" (defined below in iii)) for further choices.

(2) Least Repeaters

From now, on the basis of our former conclusion, we are going to pay more attention to the details of the network, including the repeaters that are ignored in the discussion above, to determine which one of the "full-coverage plans" is actually the best.

STEP3 "Spread Cost"

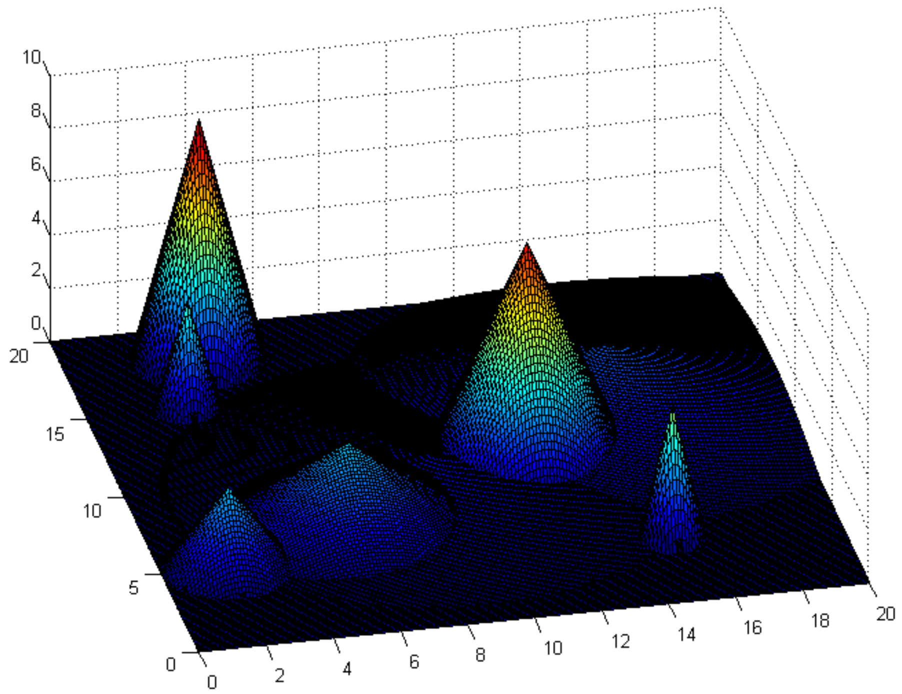
"Spread Cost" refers to the ratio of the length of the slope of a mountain to its radius. Higher the Spread Cost, more unlikely we are going to transmit information on the mountain, which indicates that there need to be more repeaters to ensure the top one's signals can be transferred successfully. While at the same time, a high "spread cost" may also be equivalent to a relatively greater height and more coverable mountains as a result. This kind of contradiction makes it necessary to clearly define the influence of SC during our programming, hence forming an accurate choice of repeater system. When two methods share the same number of repeaters we're going to build, the one with less spread cost will be adopted.

STEP4 Final Distribution of Repeaters

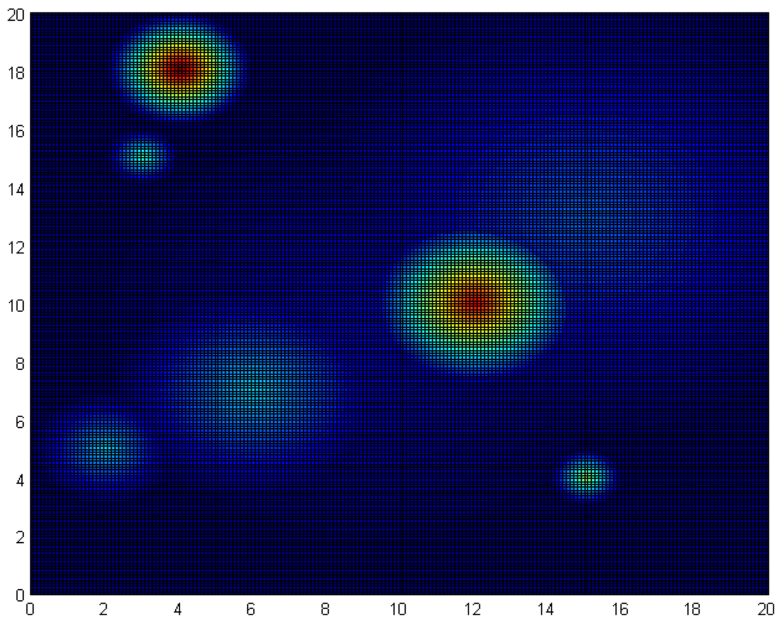
We here take population distribution back into account. Still, we assume that the population is evenly distributed. Due to the population density, every repeater has a limited transmitting radius in order to provide enough PLs for the users under its service area. So we have to transmit the signal along so that every area can receive the signal. The route has its own cost by adding up all the spread cost of the mountains it passes. For the same reason, lower the cost, fewer the repeaters. Thus, we can get the final distribution of repeaters in the mountainous area.

5.3 Model Testing

To illustrate our method more clearly, we generate a model area and apply our method to it as followed,



GRAPH-5.5 the 3-D Topograhical Map of a Simulated Mountainous Area



GRAPH-5.6 The Vertical View of a Simulated Mountainous Area

I. In the model, we build 8 mountains, they are:

Mount Number	O(a,b,0)	r	H
Mount One	(6,7,0)	4	3
Mount Two	(2,5,0)	2	3
Mount Three	(9,9,0)	8	1
Mount Four	(3,15,0)	1	4
Mount Five	(15,4,0)	1	5
Mount Six	(4,18,0)	2	9
Mount Seven	(15,13,0)	8	2
Mount Eight	(12,10,0)	3	8

II. Thus, we calculate the Coverable Mountains:

Mount Number	Coverable Mountains
Mount One	Mount One
Mount Two	Mount Two, Mount Three
Mount Three	Mount Three
Mount Four	Mount Three, Mount Four, Mount Seven
Mount Five	Mount Three, Mount Five Mount Seven
Mount Six	Mount One, Mount Two, Mount Three, Mount Four, Mount Six, Mount Seven
Mount Seven	Mount Three, Mount Seven
Mount Eight	Mount One, Mount Three, Mount Seven, Mount Eight

III. Calculate the Spread Cost

Mount Number	Spread Cost
Mount One	1.25
Mount Two	1.8
Mount Three	1.008
Mount Four	4.123
Mount Five	5.099
Mount Six	4.61
Mount Seven	1.031
Mount Eight	2.848

IV. We get the optimized solution of Mount Five, Mount Six and Mount Eight. And the Spread Costs of Mount One, Two, Three and Seven are relatively low, so it is suggested that repeaters to transmit signals should be built on these mountain areas.

V. Final Determination (exact result isn't available since there is no accurate information about population distribution)

5.4 Further Improvements

For further improvements of case 3, we believe that several points are worth more consideration to make the model more accurate.

(1) By accounting area value, that is, the area's population density, we can adapt our model to uneven population distribution, thus making it closer to reality.
(2) There should be further measures in determining the analysis formula of the area by using more specific function to present every single mountain.
(3) We can make our model easier to handle by using a 2-D method with the evaluating value of each pair of (x,y) instead of a 3-D one.
(4) Rejection and decline of signals by the mountains could be taken into account.

6. Strengths and Weaknesses

6.1 Strengths

(1) Model 1

• The "Hexagon coverage model" achieves no-gap coverage and a smallest area of overlaps.

(2) Model 2

• The introduction of the "clover splitting" and the "1:4 splitting" maximizes the preservation of previous repeaters, reducing the unnecessary cost.
• The combination of the two methods, *Center-placing Method* with omni-directional antennae and *Vertex-placing Method* with directional antennae, enables the same frequency to be used in a shorter distance.
• The ultimate method of "1:12 splitting" can be integrated with the "hexagon coverage model" perfectly, and thus the new model can meet the demands of more

users while being able to inherit the advantages of the previous one

(3) Model 3

• Model 3 is endowed with a high adaptability to varieties of geographic area.

6.2 Weaknesses

• All of the three models apply to the situation where users are evenly distributed, which means they will no longer be the optimal models if there are variations in population density.
• In Model 2, a minor fraction of the circular flat area is not in the coverage of the signal transmission network.
• Geographic influence on population density isn't concerned when constructing Model 3.

7. Conclusion

For Case One, in order to achieve a no-gap coverage and a minimized area of overlaps, we propose the "hexagon coverage model" to be the basic network on which repeaters are allocated. After the exploration of cell-splitting method, which helps to build more repeaters in one area, we apply a 1:12 splitting method to Model 2 and preserve repeaters built previously in Model 1 to meet the increased needs while reducing cost and produce high efficiency. As for Model 3, we test if a hill is "coverable" by a repeater on other hills by looking at the results of the several equations we set. Cases where there are any hills that can't be covered by other repeaters will be eliminated by our program. For the cases that are eligible, we will calculate each of their "spread cost" – the ratio used to define if a hill is suitable for placing repeaters.

We think there is chance for improvement on the following aspects:

(1) The distribution of population could be taken into account
(2) There should be further measures in determining the analysis formula of the area by using more specific function to present every single mountain.
(3) We can make our model easier to handle by using a 2-D method with the evaluating value of each pair of (x,y) instead of a 3-D one.

(4) Rejection and decline of signals by the mountains could be taken into account.

8. References

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10. http://en.wikipedia.org/wiki/Vhf
11. http://en.wikipedia.org/wiki/CTCSS

9. Appendices

9.1 Appendix 1 – the C Programming of Model 1

```
#include <stdio.h>
#include <time.h>
int sp[19]={0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0};
int f[19]={0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}; //Default frequency of repeaters
int r[19][19]={0,1,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
              1,0,1,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
              0,1,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,
              1,0,0,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,
              1,1,0,1,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,
              0,1,1,0,1,0,1,0,0,1,1,0,0,0,0,0,0,0,0,
              0,0,1,0,0,1,0,0,0,0,1,1,0,0,0,0,0,0,0,
              0,0,0,1,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,
```

```
        0,0,0,1,1,0,0,1,0,1,0,0,1,1,0,0,0,0,0,
        0,0,0,0,1,1,0,0,1,0,1,0,0,1,1,0,0,0,0,
        0,0,0,0,0,1,1,0,0,1,0,1,0,0,1,1,0,0,0,
        0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,1,0,0,0,
        0,0,0,0,0,0,0,1,1,0,0,0,0,1,0,0,1,0,0,
        0,0,0,0,0,0,0,0,1,1,0,0,1,0,1,0,1,1,0,
        0,0,0,0,0,0,0,0,0,1,1,0,0,1,0,1,0,1,1,
        0,0,0,0,0,0,0,0,0,0,1,1,0,0,1,0,0,0,1,
        0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,1,0,
        0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,1,
        0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0}; //Relationships between repeaters

judge(int a, int v)    //Judge whether the “potential solution” is legal
{ int k,p;
   k=1;
   p=1;
   for(k=0;k<=18;k++)
   { if (r[a][k]==1 && v==f[k])
      p=0;
   }
   return p;
}

void random()    //Generate a set of legal “potential solution”
{int a,k,p,q;
q=0;
for (a=0;a<=18;a++)
f[a]=0;
for (a=0;a<=18;a++)
{ p=0;
   do
   {
   k=rand()% 6 + 1;
   p++;
   }while(judge(a,k)==0 && p<=10);
```

```
    if (p==11)
    return;
    f[a]=k;
}
}

spread(int k)    //Determine whether every repeater can be covered
{ int i;
    for (i=0;i<=18;i++)
    if (r[k][i]==1 && abs(f[k]-f[i])==1 && sp[i]==0)
    {
    sp[i]=1;
    spread(i);
}
}

main()
{
int a,b,c;
srand((unsigned int) time (NULL));
for (a=1;a<=5000;a++)
{
random();
c=1;
for(b=0;b<=18;b++)
sp[b]=0;
spread(0);
for(b=0;b<=18;b++)
c=c*sp[b];
if (c==1)
{for(b=0;b<=18;b++)
printf("%i ", f[b]);    //Print out the answer
printf("\n");
getch();
}
```

}
}

9.2 Appendix 2 – the MATLAB Programming of Simulated Mountainous Area in Model 3

```
[x,y]=meshgrid(0:0.1:20);
a=6;
b=7;
r=4;
h=3;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
y-b).^2))+1)./2;
surf(x,y,z);hold;

a=2;
b=5;
r=2;
h=3;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
y-b).^2))+1)./2;
surf(x,y,z);

a=9;
b=9;
r=8;
h=1;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
y-b).^2))+1)./2;
surf(x,y,z);

a=3;
b=15;
r=1;
h=4;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
```

```
y-b).^2))+1)./2;
surf(x,y,z);

a=15;
b=4;
r=1;
h=5;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
y-b).^2))+1)./2;
surf(x,y,z);

a=4;
b=18;
r=2;
h=9;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
y-b).^2))+1)./2;
surf(x,y,z);

a=15;
b=13;
r=8;
h=2;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
y-b).^2))+1)./2;
surf(x,y,z);

a=12;
b=10;
r=3;
h=8;
z=(h*r-h*sqrt((x-a).^2+(y-b).^2))/r.*(abs(r-sqrt((x-a).^2+(y-b).^2))./(r-sqrt((x-a).^2+(
y-b).^2))+1)./2;
surf(x,y,z)
```

9.3 Appendix 3 – the C Programming of "Full coverage plan" in Model 3

```
#include <stdio.h>

int
mount[101][101],spread[101],select[101],bselect[101],num_m,num_s,i,j,k,num=0,snu
m=100;

judge()
{    int value=1;
     num=0;
     for (i=1;i<=num_m;i++)
         if (select[i]==1)
             {num++;
             for(j=1;j<=mount[i][100];j++)
             spread[mount[i][j]]=1;
             }
    for (i=1;i<=num_m;i++)
    value=value*spread[i];
    return value;
}

add(int x)
{
    if(select[x]==0)
        select[x]++;
    else
        {select[x]--;
    add(x+1);
}
}

full()
{    int n,val=1;
    for(n=1;n<=num_m;n++)
```

```
   val=val*select[n];
   return val;
}

cal(int a)
{ int val=1;
  for(i=1;i<=a;i++)
    val=val*2;
  return val;
}

main()
{ int count=0;
  scanf("%i",&num_m);
  for(i=1;i<=num_m;i++)
  { scanf("%i",&num_s);
    mount[i][100]=num_s;
  for(j=1;j<=num_s;j++)
    scanf("%i",&mount[i][j]);
  }
do
{
  for(k=1;k<=num_m;k++)
     spread[k]=0;
if(judge()==1)
if(num<snum)
{    snum=num;
  for(k=1;k<=num_m;k++)
    bselect[k]=select[k];
}
add(1);
count++;
}while(count<=cal(num_m)+1);
for (i=1;i<=100;i++)
if (bselect[i]==1)
```

```
printf("%i ",i);
}→
```