From the Snail Trails to Missile Tracks:

A Preliminary Exploration of “Converging Curve” and “Tracing Curve”

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Abstract: This research, initiated by a problem excerpted from Baidu, one of the most famous internet search engines in China, explores two types of movements, “converging curve” and “tracing curve” named by this research group, which really act in a regular pattern. In the research, by applying scores of mathematical methods, we found that the key to converging curves is to spot the invariant elements from the changeful motions. For tracing curves, we adopt estimating equations to get the solution. Due to the limit of time and the relevant knowledge of the research group, many unexpected problems constantly appeared which are quite far beyond our ability. Yet we still tried to apply some promising mathematical thoughts, and have achieved some exciting results. In the process of discovering and rediscovering, we did not succeed in finding a perfect solution to the problem. After all, as a group of middle school students fascinated by mathematics, creating and discovering more are the greatest delight of us all.

摘要：事物的运动是有规律的，本课题从一道在“百度”上发现的题目出发，探讨两种规律性比较强的运动。我们把这两种运动的运动轨迹分别命名为“聚合线”与“追踪线”。在对这两种曲线的研究中，我们尝试了许多数学方法。就“聚合线”而言，我们发现如何找到不变的量，在运动中找到不动的因素是研究的关键。而在“追踪线”的研究中，则采用了估计方程的方法去求得其方程。限于中学的数学工具有限、时间的紧迫，在研究的过程中不断有意想不到的问题出现，也超出了我们的能力范围。但是在研究的过程中我们仍能感受到许多宝贵的数学思想，也得到了一些研究成果。在这个发现问题、完善问题的过程中，虽然不能得到课题的全解。但与队友一道，在数学的世界里尽情探索、发现与创造，这才是我们最大的快乐。

1. Introduction

While surfing in the website "Baidu", we found lots of interesting mathematical problems. By chance, we found a problem in the web (http://tieba.baidu.com/f?kz=70699687), which said:

Four snails, marked A, B, C and D, lie on the four vertices of a square (Fig.1-1). Assume the length of each side is L. Also assume A moves to B, B moves to C, C moves to D, D moves to A, and they all move at the speed of v in uniform motion. Find the distances they cover when they meet each other.

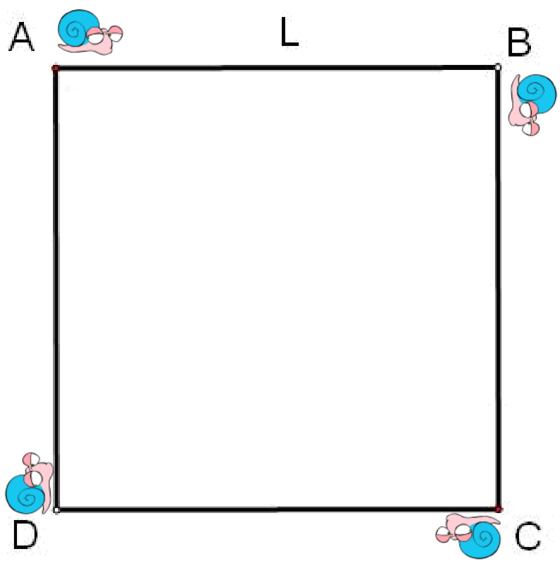


Figure 1-1

This problem seems to be quite complicated, since every snail's action is determined by that of the next one. That is to say, their velocities and moving directions are varying all along until they finally get together. After a rough simulation, we discovered that all these traces were a helix joining the vertex to the center of the square. We call them "the converging curves", which can be defined as follows.

Given n points, denote the points by $a_i\,(1 \le i \le n,\ \ i \in Z)$*, and let* a_i *moves to* a_{i+1}*.*

Specially, when $i = n$*, let* a_i *moves to* a_1*. The trace of each point is called the "converging curve"; the traces of the n points are called the "converging curve group". The point where that all points finally get together is called the "converging point".*

Figure 1-2 is the trace of the "converging curves group" made by the computer.

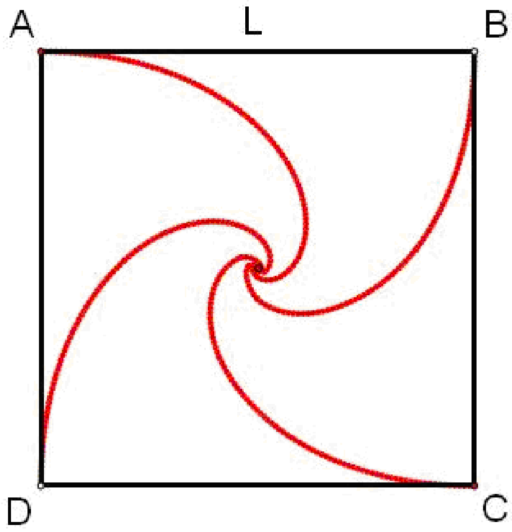


Figure 1-2

Yet, it is not easy to get the movement information about the actions of the snails from the "converging curve". Obviously every snail is affected by another one. Therefore, it seems to be difficult to compute the length of each "converging curve" directly.

Figure 1-3

But, if we observe the situation from another perspective, things are different. For Snail A, it's quite clear that the Snail B is just in front (figure 1-3 is what snail A sees). Wherever B moves to, A would just follow it. That is to say, B is locked as a target by A. Because of this, the distance between A and B is always shrinking. As the four snails start from the four vertices of a square, their directions of speeds are always perpendicular to each other. Hence the motion of B has no effect on the distance between them, and it only affects the direction of the speed of A. Thus the relative speed between every pair of snails is:

$$v' = v$$

That is to say, Snail A is getting closer and closer to Snail B with the velocity of v. At the initial time, all four snails are separated from each other by the length of L. Therefore, the time that one snail spends to arrive at its target snail's position is:

$$t = \frac{L}{v}$$

Since $s = vt$, we can get:

$$s = L$$

By far, the original problem is solved properly. However, the situation is only confined to squares. What would the results be under other circumstances? We find research behind this problem interesting and invaluable. This research intends to explore more moving situations other than the square. Specifically, this study attempts to answer the following questions:

1. Can the same method used in the square situation be applied to solve the moving track problem in a general regular polygon?

2. If at the beginning, the four snails are at the vertices of a rectangle or a rhombus is it possible for them to gather at the same time?
3. If a snail moves straightly with constant velocity, another snail constantly traces it from a given distance away with the same velocity, what will the trace equation be?

The first two questions are extensions of the original problem, while the third is a transformational one. In this paper, we will also study the above three problems and its applications. We can build up the link between it and the application, such as applying the solution to the equation of the locus of tracking missiles.

In this study, the Geometer's Sketchpad V4.06 was used to draw relevant figures.

2. The Extension of the Converging Curves and Their Equations

2.1 The Extension of the Converging Curves

As discussed in the introduction, the distance that every snail covers until it meets another one in the square equals to the side length of the square. It seems complicated locomotion can be presented with such a simple outcome. However, new problems arise. If two snails move to each other on a line at the same velocity, undoubtedly the distance that each snail covers is:

$$s = \frac{L}{2} \qquad (2.1.1)$$

Then it is natural to ask the question for the three-snail case: what is the distance if there are three snails? Is the value L or $\frac{L}{2}$? What about other situations? It seems that we have to further modify this equation.

Now let's consider the three-snail case, at the beginning, we have a profile as shown in figure 2-1-1

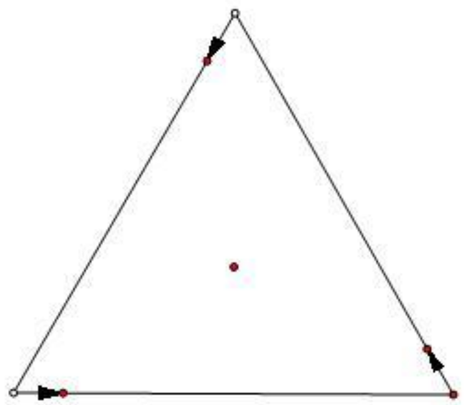


Figure 2-1-1

Later, the motions of three snails begin to change. Since speed is a vector, the relative speeds of three snails are affected by the angle of their speeds, which are determined at the very beginning. In the whole process, the three snails keep in the vertices of a regular triangle as shown in figure

2-1-2. In fact, since a regular triangle has a property of rotational symmetry. If we rotate the three snails by 60°, we find that A becomes B, B becomes C, and C becomes A. But except the change of their names, there is no difference in other properties, including their moving directions and their velocities. So the traces would not change, either. In that case, the distances between the snails and their target will remain the same, and it proves that the snails will be still at the vertices of a regular triangle.

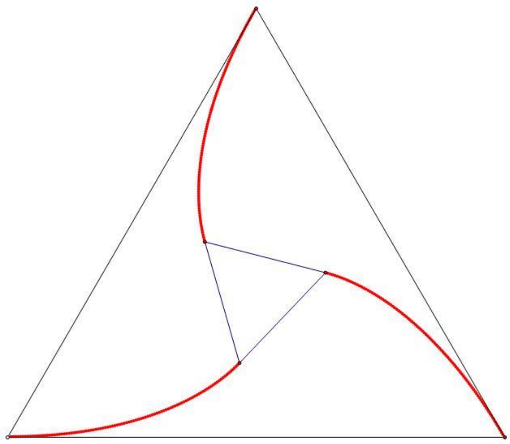


Figure 2-1-2

Now let's consider the two of them, as shown in figure 2-1-3.

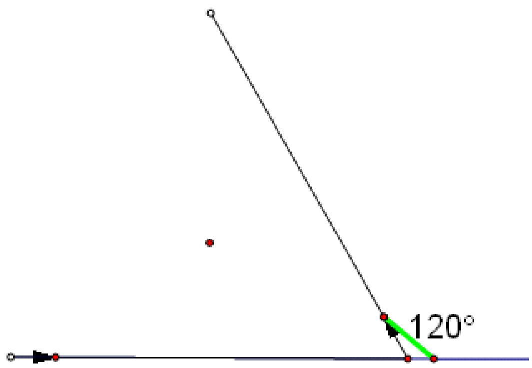


Figure 2-1-3

It is true that the angle of their speeds is always $120°$, so the gathering time is not decided by a snail, but by both of them. In the three-snail case, the relative speed is:

$$v' = v(1+\cos\frac{\pi}{3}) \qquad (2.1.2)$$

So the gathering time is:

$$t = \frac{L}{v(1+\cos\frac{\pi}{3})} \qquad (2.1.3)$$

It is not difficult to extend the equation to the general regular polygon. Since the inner angle of an n regular polygon is $\frac{n-2}{n}\pi$, then the relative speed between two adjacent snails is:

$$v' = v(1+\cos\frac{n-2}{n}\pi) \qquad (2.1.4)$$

So the gathering time is:

$$t = \frac{L}{v(1+\cos\frac{n-2}{n}\pi)} \qquad (2.1.5)$$

Since $s = vt$, the distance that a single snail covers can be presented as:

$$s = \frac{L}{1+\cos\frac{n-2}{n}\pi} \qquad (n \in Z \quad and \quad n \geq 2) \qquad (2.1.6)$$

From formula 2.1.6, we can find that n can not be 0 or 1. It shows that when there is no snail or only one, the problem has no meaning. When n goes to be positive infinite, that is, when there are infinite snails, the distance and angle between their speeds tend to be 0. As their velocities are the same, this implies that they would never get together.

From the above discussion, we know that every snail's movement is affected by the other one, but how? So we need to study the movement of the snails. We also want to know that what the converging curve is. Is it evolvent, or something unknown? In analytic geometry, many curves can be presented by a curtain equation. So is it possible to present the trace by a kind of equation? In order to solve these problems, we need to do some research about the equation of the converging curve.

2.2 The Converging Curve Equation of General Regular Polygons

It seems difficult to show the equation of the converging curve of a general regular polygon, but there is still something invariant. The velocities are invariant, so are the angles.

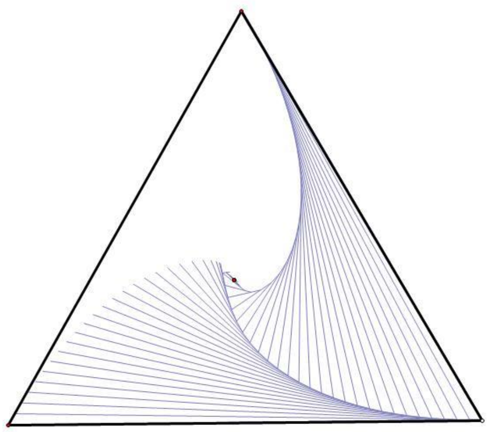
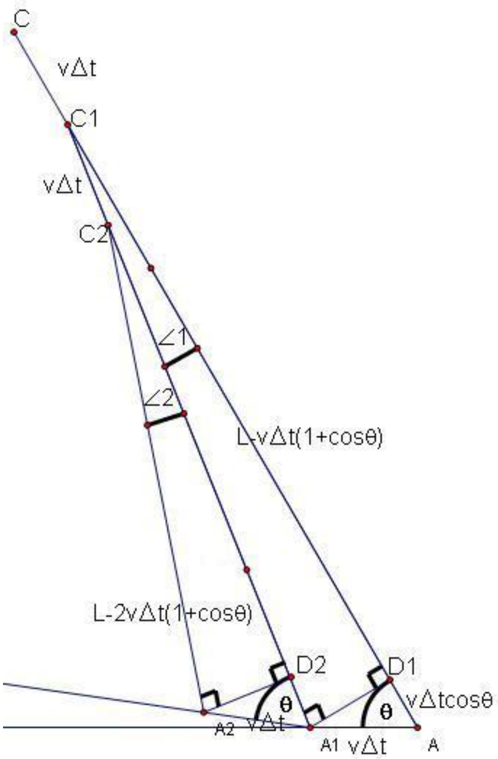


Figure 2-2-1

Take the converging curve of regular triangle as an example. Previous discussion found that the formation of these points resembled the initial pattern, which means the angles of their speeds are invariant.

Supposed that the inner angle of the pattern is θ， the length of the side is L, the velocity of every point is v，and the time is *t*. We divide the time into k parts, ($k \to \infty$) and denote every part by Δt （ $\Delta t \to 0$ ）. It also generates many extremely tiny curved triangles. Then it's possible to find out the related equation (see Fig. 2-2-1).

We enlarge a part of the above figure as follows (see Fig. 2-2-2)



2-2-2

It can be seen that as the time Δt tends to 0, the four angles tends to $90°$, arc AA_2 can be presented by the segments AA_1 and AA_2, segments A_1D_1 and A_2D_2 approach the two associated arcs, $\angle 1$ can be presented by $\frac{A_1D_1}{C_1D_1}$, therefore, we have the following equation:

$$\angle 1 = \frac{v\Delta t \sin\theta}{L - v\Delta t(1+\cos\theta)}$$

$$\angle 2 = \frac{v\Delta t \sin\theta}{L - 2v\Delta t(1+\cos\theta)}$$

Similarly:

$$\angle k = \frac{v\Delta t \sin\theta}{L - kv\Delta t(1+\cos\theta)}$$

Choose a coordinate with A as the origin and AC as the positive direction of x-axis as shown in figure 2-2-3.

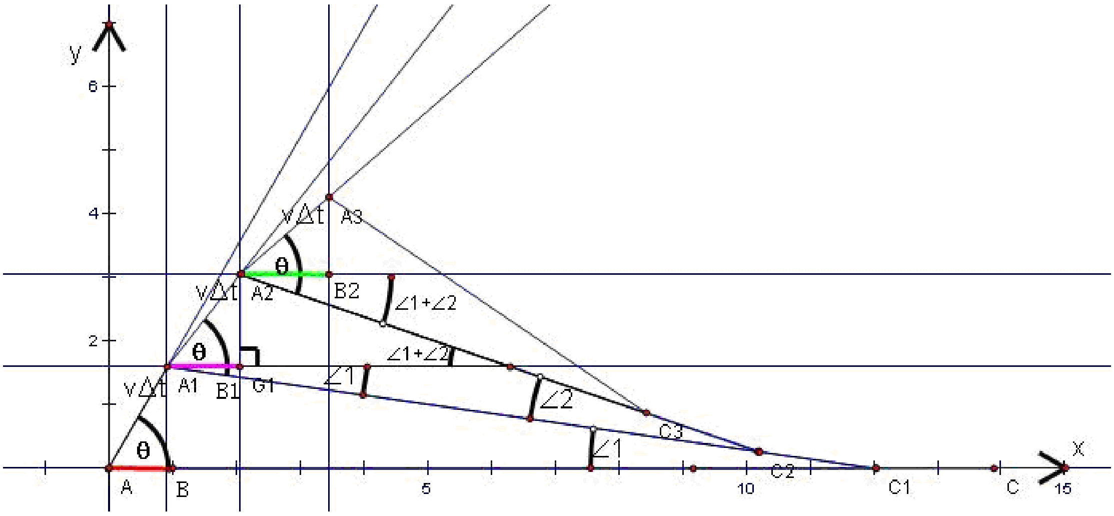


Figure 2-2-3

Segment AB, A1B1, and A2B2 (Fig. 2-2-3) can be presented by:

$$v\Delta t\cos\theta$$

$$v\Delta t\cos(\theta-\angle 1)$$

$$v\Delta t\cos(\theta-\angle 1-\angle 2)$$

The rest may be deduced by analogy:

$$v\Delta t\cos[\theta-\angle 1-\angle 2-\cdots-\angle(k-1)]$$

Since the x-coordinate of A is 0, the x-coordinates of A1 and A2 are:

$$v\Delta t\cos\theta$$

$$v\Delta t\cos\theta+v\Delta t\cos(\theta-\angle 1)$$

Finally the x-coordinate of Ak is:

$$\lim_{\Delta t\to 0} v\Delta t\cos\theta+v\Delta t\cos(\theta-\angle 1)+v\Delta t\cos(\theta-\angle 1-\angle 2)+\cdots+v\Delta t\cos(\theta-\sum_{i=1}^{k-1}\angle i)$$

Similarly, we can deal with the y-coordinate as shown in figure 2-2-4.

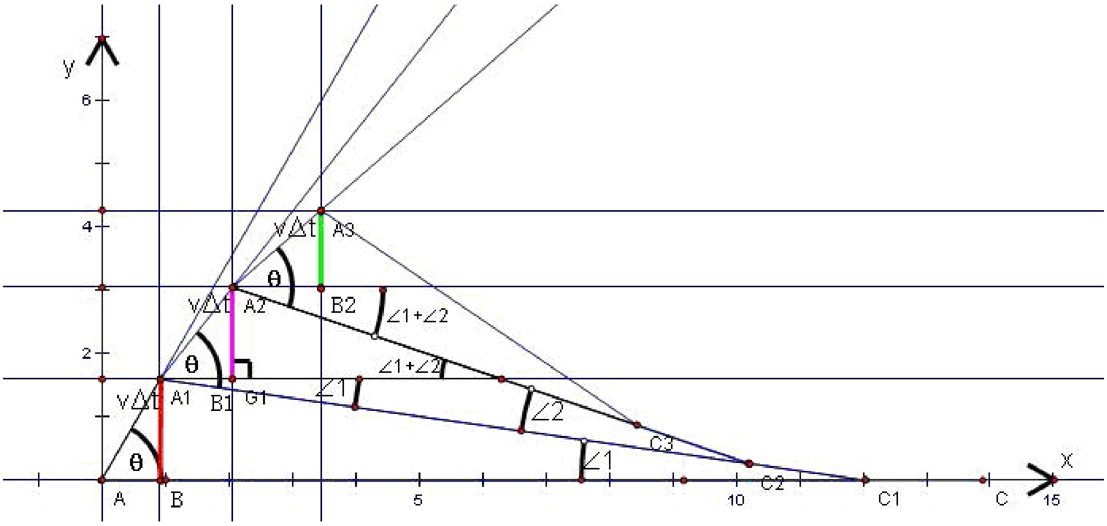


Figure 2-2-4

In figure 2-2-4, the length of Segment A1B, A2B1, and A3B2 can be presented as:

$$v\Delta t\sin\theta$$

$$v\Delta t\sin(\theta-\angle 1)$$

$$v\Delta t\sin(\theta-\angle 1-\angle 2)$$

The rest may be deduced by analogy:

$$v\Delta t\sin[\theta-\angle 1-\angle 2-\cdots-\angle(k-1)]$$

Since the y-coordinate of A is 0, we know that the y-coordinate of A1 and A2 are

$$v\Delta t\sin\theta$$

$$v\Delta t\sin\theta+v\Delta t\sin(\theta-\angle 1)$$

Finally the y-coordinate of Ak is:

$$\lim_{\Delta t\to 0} v\Delta t\sin\theta+v\Delta t\sin(\theta-\angle 1)+v\Delta t\sin(\theta-\angle 1-\angle 2)+\cdots+v\Delta t\sin(\theta-\sum_{i=1}^{k-1}\angle i)$$

Then the equation of such a converging curve is:

$$\begin{cases} x=\lim\limits_{\Delta t\to 0} v\Delta t\cos\theta+v\Delta t\cos(\theta-\angle 1)+v\Delta t\cos(\theta-\angle 1-\angle 2)+\cdots+v\Delta t\cos(\theta-\sum\limits_{i=1}^{k-1}\angle i) \\ y=\lim\limits_{\Delta t\to 0} v\Delta t\sin\theta+v\Delta t\sin(\theta-\angle 1)+v\Delta t\sin(\theta-\angle 1-\angle 2)+\cdots+v\Delta t\sin(\theta-\sum\limits_{i=1}^{k-1}\angle i) \\ \Delta t=\dfrac{t}{k}(k\to\infty) \qquad k\in Z^{+} \\ \theta=\dfrac{(n-2)}{n}\pi \end{cases}$$

Now we have obtained the converging curve's equation in an n regular polygon. However, it still has a long way to go to the most general situation. This is because snails only move in a general regular polygon. In the following, we will discuss the patterns of a general rectangle and a general rhombus.

3. The Possibility of Same-time Converging in General Rectangles & General Rhombuses

3.1 The General Rectangle Pattern

With the converging curve equation of a general regular polygon, we tried to figure the equations of general rectangle and a general rhombus, but found the case different.

Enlightened by the original problem, we found that in a general rectangle, four snails would never gather at the same time if their velocities are the same. Two of them will gather beforehand, and form two groups to gather in a linear way as shown in figure 3-1-1.

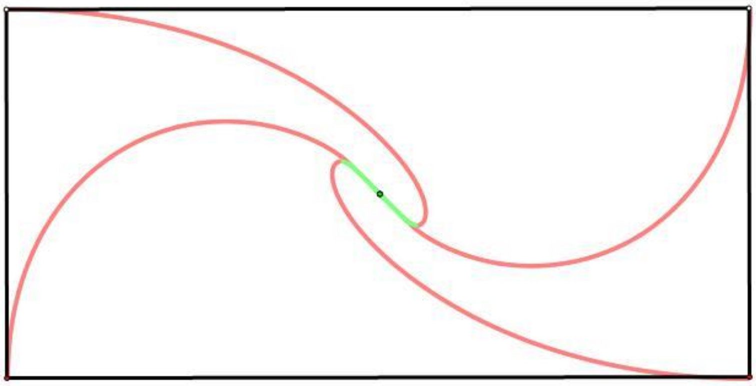
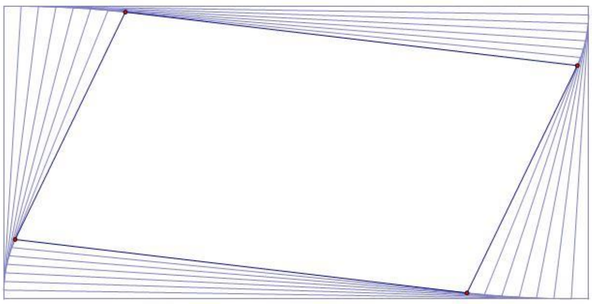


Figure 3-1-1

Then, by splitting the process of the whole motion, we found that, with the same velocity, the angle between the speeds of each pair of adjacent snails keep changing (Fig. 3-1-2) This means that their time of gathering can not be displayed by the formula obtained previously.



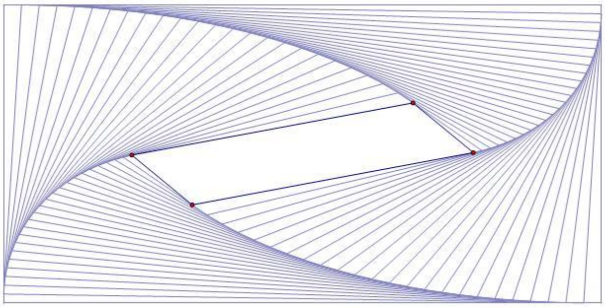


Figure 3-1-2

In this case, how to let the four snails get together at the same time is critical. Now that the key is to keep the angle a right angle, we suspect that when the speed can match along with the length of a rectangle, they can gather at the same time. For example in Fig. 3-1-3, A traces B, B traces C , C traces D, D traces A, then the ratio of their velocities is AB:BC:CD:DA.

Figure 3-1-3

Unexpectedly, the simulation revealed that the angles of their speeds were constantly changing. Such variation leads to the change of their relative speed. A simple result was not found. Some other ratios were also tried in order to find out some rules, but satisfactory results were not obtained. It seems that four points would never gather at the same time

Then we conjectured that it's impossible for the four points to gather at the same time if their ratio of velocities is invariant.

In the process of the motion, the slopes of the line joining two points are varying, but the angle will be a right angle sometime. That is to say, four points form a whole new rectangle which the ratio of the neighboring side is different from the original one. Through magnifying the new rectangle and continue the motion, we found that the four points would still form a new rectangle sometime. If a ratio of two velocities which allows the points gathering at the same time, the ratio must fit the new situation. But the new rectangle has different ratio of the neighboring side, and one ratio of velocities cannot be adopted in various new rectangles.

The key is to prove that in the situation of constant velocity, the angle will not be always a right angle. Because the angle determines their relative speed, which can keep the rate of the shrinking of distance invariant.

The following is a verification of our conjecture.

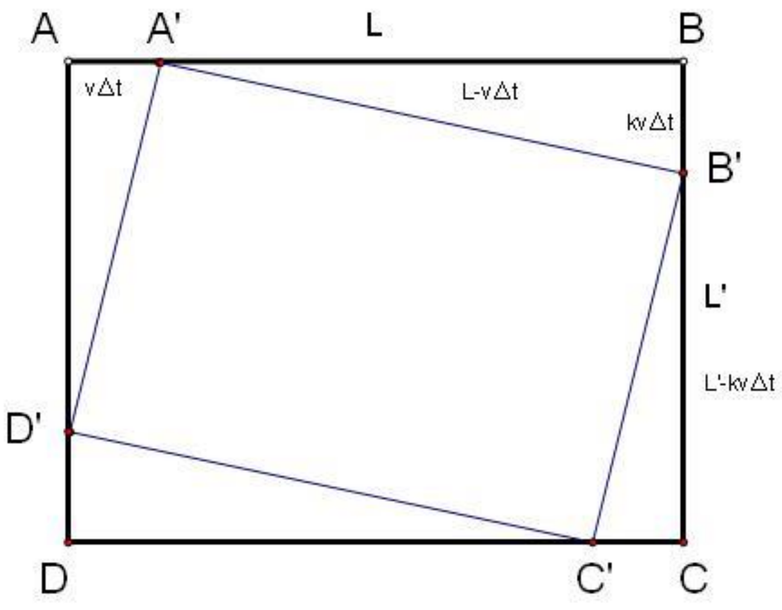


Figure 3-1-4

As figure 3-1-4 shows, in rectangle ABCD, there's another rectangle A′B′C′D′ whose vertices are located at the side of the bigger rectangle. Consider the smaller one at the moment when four points are just about to move. Let the speed of B be *k* times as that of A, AB is L, BC is L', then we have:

$$AA' = v\Delta t \qquad (3.1.1)$$

$$BB' = kv\Delta t \qquad (3.1.2)$$

Since Δt tends to 0, we have：

$$A'B' = L - v\Delta t \qquad (3.1.3)$$

$$B'C' = L' - kv\Delta t \qquad (3.1.4)$$

And then：

$$\angle BA'B' = \frac{kv\Delta t}{L - v\Delta t} \qquad (3.1.5)$$

$$\angle CB'C' = \frac{v\Delta t}{L - kv\Delta t} \qquad (3.1.6)$$

Since ∠A′ B′ C′ should be kept as a right angle, the rotating angle between the four snails' speeds should be equal：

$$\frac{kv\Delta t}{L - v\Delta t} = \frac{v\Delta t}{L' - kv\Delta t} \qquad (3.1.7)$$

At the next moment, there is：

$$A''B'' = L - 2v\Delta t \qquad (3.1.8)$$

$$B''C'' = L' - 2kv\Delta t \qquad (3.1.9)$$

Then：

$$\frac{kv\Delta t}{L - 2v\Delta t} = \frac{v\Delta t}{L' - 2kv\Delta t} \qquad (3.1.10)$$

Combine these two equations, and the following conclusion can be drawn：

$$k^2 v\Delta t = v\Delta t \qquad (3.1.11)$$

The equation presents that if the velocities are equal, the angle can always be right angle. Put 3.1.11 into 3.1.10, there is：

$$L = L' \qquad (3.1.12)$$

That is to say, only when the rectangle is a square and the velocities are the same, can the angle be a right angle. That means in a general rectangle, if the velocity is invariant, then it's impossible for the four points to converge at a same time.

3.2 General Rhombus Pattern

We have discussed the general rectangle case in the previous section, and will study the general rhombus case in this section. The four sides of a rhombus are the same in length， and it seems to us that it is not necessary to adjust the velocities. So we conjectured that in the process of the converging movement, the angle of their speeds would be invariant. And hence the same method can still be used to work out the time needed.

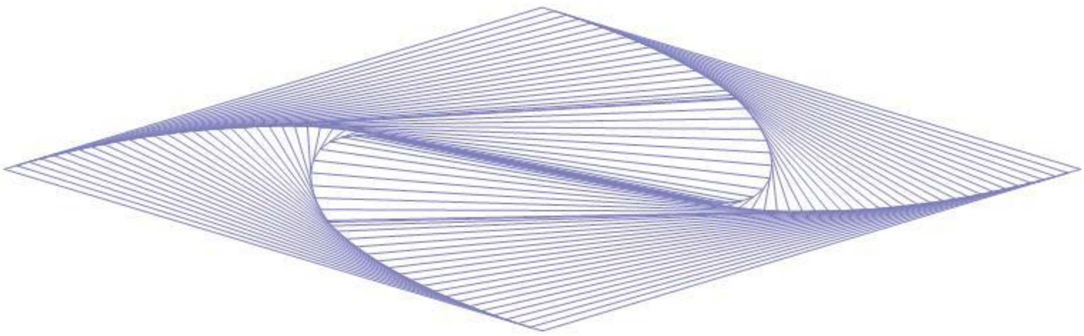


Figure 3-2-1

However，the computer modeling failed to produce results to support our conjecture. During the moving process, the angle is varying all the time (Fig. 3-2-1). In fact, during the converging process of rectangle, there is a time when the four points approximately form a rhombus pattern. (Fig. 3-2-2)

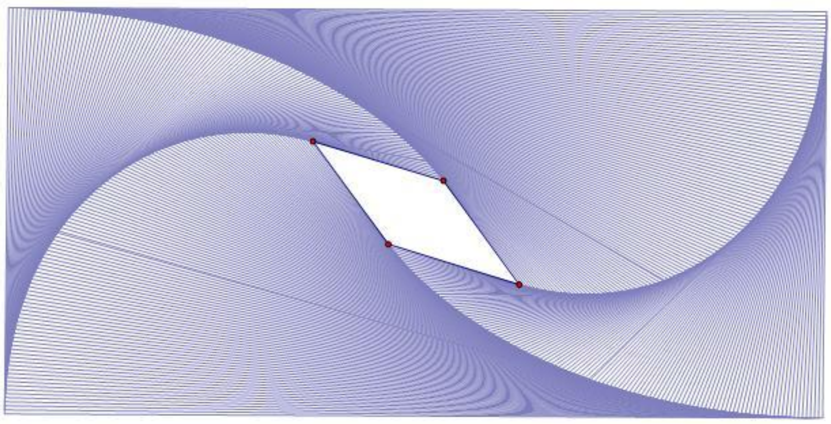


Figure 3-2-2

In this case, is it also impossible for the four points to converge spontaneously if their velocities are invariant? The following is the proof.

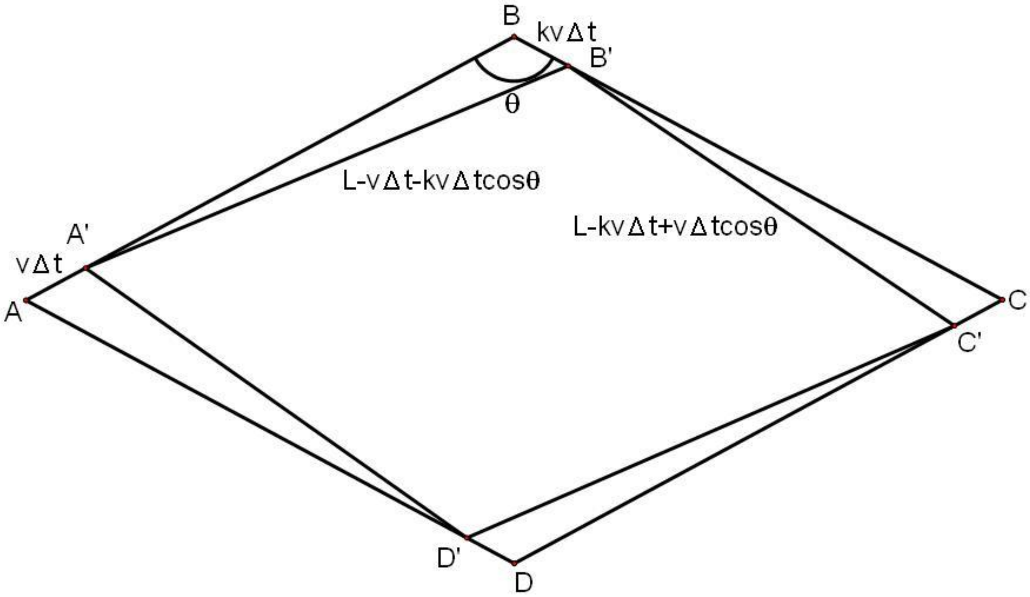


Figure 3-2-3

As displayed in the figure 3-2-3, in rhombus ABCD, let ∠ABC be θ. There is another rhombus A′ B′ C′ D′, whose vertices are located on the sides of the bigger rhombus. Let them be the positions of the four points for an extremely short period of time after they start. We suppose that the speed of Snail B is *k* times as fast as that of Snail A, and then we have:

$$AA' = v\Delta t \qquad (3.2.1)$$

$$BB' = kv\Delta t \qquad (3.2.2)$$

Assume the length of the rhombus is L. Since the angle of their speeds is not a right angle and considering the effect of the angle, we have:

$$A'B' = L - v\Delta t - kv\Delta t\cos\theta \qquad (3.2.3)$$

$$B'C' = L - kv\Delta t + v\Delta t\cos\theta \qquad (3.2.4)$$

Then:

$$\angle BA'B' = \frac{kv\Delta t\sin\theta}{L - v\Delta t - kv\Delta t\cos\theta} \qquad (3.2.5)$$

$$\angle CB'C' = \frac{v\Delta t\sin\theta}{L - kv\Delta t + v\Delta t\cos\theta} \qquad (3.2.6)$$

Since we require that the four points remain the shape of a rhombus during the movement, and the angle of their speeds cannot vary, we have:

$$A'B' = B'C' \qquad (3.2.7)$$

$$\angle BA'B' = \angle CBC' \qquad (3.2.8)$$

Then the result is：

$$k^2 v\Delta t = v\Delta t \qquad (3.2.9)$$

$$L - v\Delta t - kv\Delta t\cos\theta = L - kv\Delta t + v\Delta t\cos\theta \qquad (3.2.10)$$

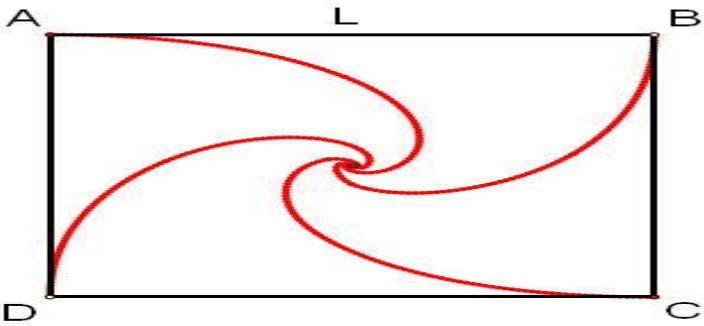
Put formula 3.2.9 into formula 3.2.10, there is：

$$\cos\theta = 0 \qquad (3.2.11)$$

That is to say, the angle must be 90°, and hence the rhombus must be a square, which makes it possible for the four snails to get together at a same time.

The example of rectangle and rhombus shows that the difference of both the side and the angle will make our equation inapplicable. The equation can just be used at the regular polygon situation.

However, our probing did not stop at this point. A question came to our mind. Is there a function of the velocities, which can make the points gather at the same time in general rectangle or rhombus? We found that if being viewed from a certain viewing angle and direction, the square will be like figure 3-2-4, which is a rectangle, or a rhombus. And the trace of converging curve would be like this.



3-2-4

In this case, the speed of every point at each time can be divided into horizontal direction and vertical direction. Since the length of the rectangle is the same as the side of the original square, the speeds at the horizontal direction is the same as before; and the wide of the rectangle comes from the side of the square shrinking at a certain ratio, the speeds in the vertical direction also need to shrink in a certain ratio. In this way, by adjusting the velocities according to the direction, it is possible to find out the function of the velocities which can make the points converge at the same time.

4. Equation of Tracing Curve

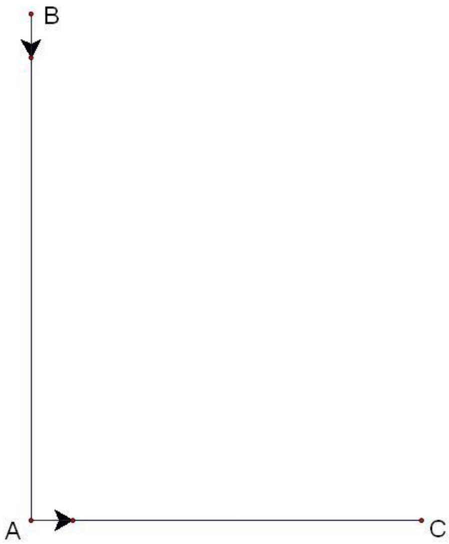


Figure 4-1

Our thoughts brought us further to some special cases. As trying to study the scatter-case, we separated the following case from it:

Assume that Snail A is moving towards the vegetable C. Snail B, knowing that the vegetables contain pesticides, tracks A with the same velocity (Fig. 4-1). Then, what will the trace of Snail B be like?

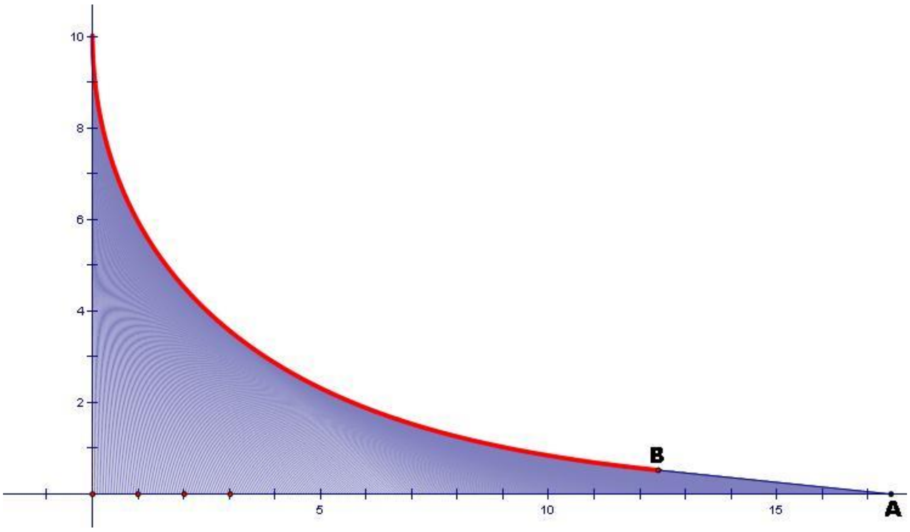


Figure 4-2

By modeling the situation above (Fig.4-2), we got the trail of Snail B, which is called the "tracing curve".

In this section we will give the equation of tracing curve in some special cases.

A simple solution appeared in our mind at the first glimpse. If the vegetable is not infinite far away from Snail A, Snail A will not be caught up by Snail B before it reaches the vegetable and will be poisoned, because their velocities are the same. Compared with the trace of Snail A, the speed direction of Snail B will be gradually parallel to the path of Snail A. That means no matter how far the vegetable is from Snail A, the speed of Snail B will finally be almost the same as that of Snail A, hence in this case, it is impossible for Snail B to catch up with Snail A.

We still tried to depict Snail B's moving path, which we named as the "tracing curve". By figuring out the equation, we might know how and when Snail B could catch up with Snail A.

However, the differential methods for tracing curves are much more complicated than those for converging curves. For converging curves, the invariant angle is the key to figure out the converging equation. But for tracing curves, as we can see in figure 4-3, the angles of their speeds keep varying ($\angle 1 > \angle 2$). It is very difficult to find out the regulation. We've tried several methods, for instance the method of similar triangles, but eventually what we've got are a huge stack of formulas which are extremely difficult to express clearly.

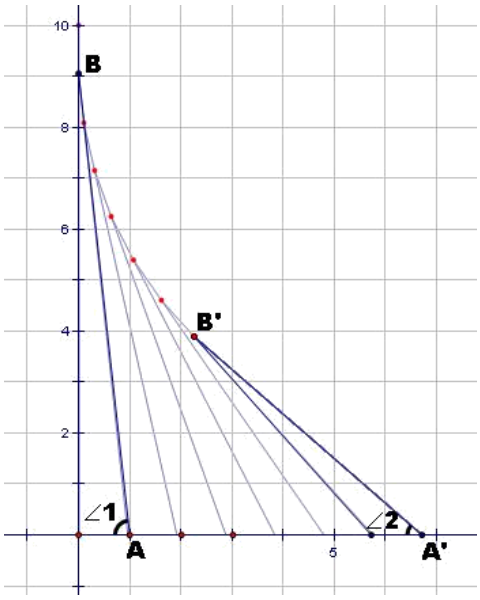


Figure 4-3

As our mathematic tools are so limited, we decided to model and figure out the approximate equation of the path. We start from a special case, assuming that B is 10 units far from A, and figure out the approximate equation in this case. Then we tried to add the original distance in order to make our equation more general.

Nevertheless, the tracing curve is very strange. We failed to find a function whose argument can be 0, with its slope not existing at that point, and the function tending to 0 as the increase of its argument, like figure 4-2.

We could only try some other way. The experiments showed that the differential equation of the tracing curve is easier to figure out, it tends to negative infinity when the argument tends to 0, and it is 0 when the argument tends to positive infinity. We could easily associate it with the inverse proportion function. So we conjecture that, the equation of the differential equation is:

$$y' = -\frac{10}{x}(x > 0)$$

From figure 4-4, we can see that the estimated function clings to the real path of the differential equation of the tracing curve. But the function begins to disengage from the after part. Compared with the real path, it seems too slow to press close to the x-axis.

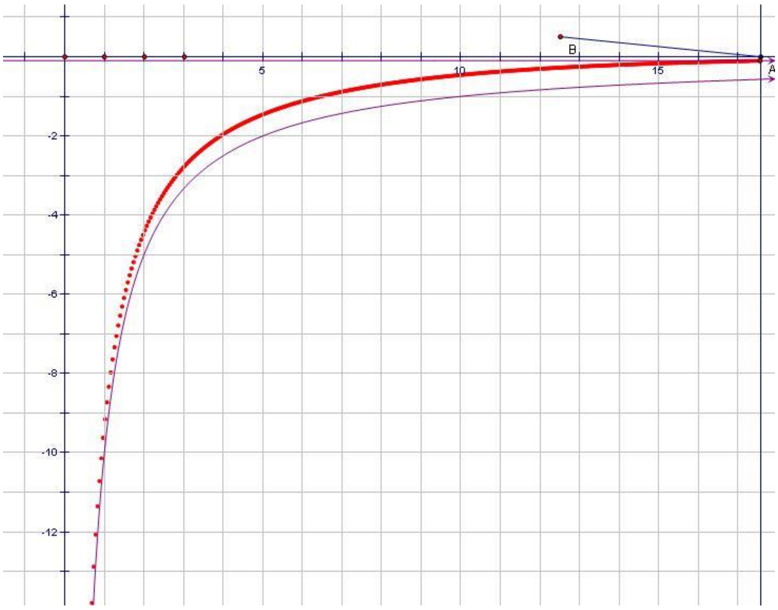


Figure 4-4

Now, we should find out the regulation, and then correct our estimated function.

As figure 4-4 shows, our estimated function must "turn up" at the latter part, and it demands us to add a function which tends to be 0. And in the more latter part, it demands that the added function has less and less effect. The estimated equation can keep close to the real one in this way. We assume that the estimated function must be added with a function like the figure 4-5.

Figure 4-5

This function seems familiar to us, which are the normal distribution curve. However, the right part of it has changed a bit, so we rebuilt the parameter in the normal distribution curve, and finally got a better result. The function is:

$$y' = \frac{10}{x} + \frac{1}{x} 10^{\frac{-(x-\sqrt{10})^2}{10^x}}$$

From the improved curve (Fig. 4-6), we could see that the estimated function did improve a lot, but it is still not good enough. It starts to break from the path when x is larger than 2, which is not what we want. So it still needs some more modification. We tried to change the function at some points, and then we found that it is the coefficient of $\frac{1}{x}$ in the second part that controls the degree that the function "turns up". That is, if the coefficient of $\frac{1}{x}$ is less than 1, the function will "turn up", otherwise, if the coefficient is more than 1, the function will "turn down". In order to let the function "turn up", we changed the coefficient to a constant less than 1.

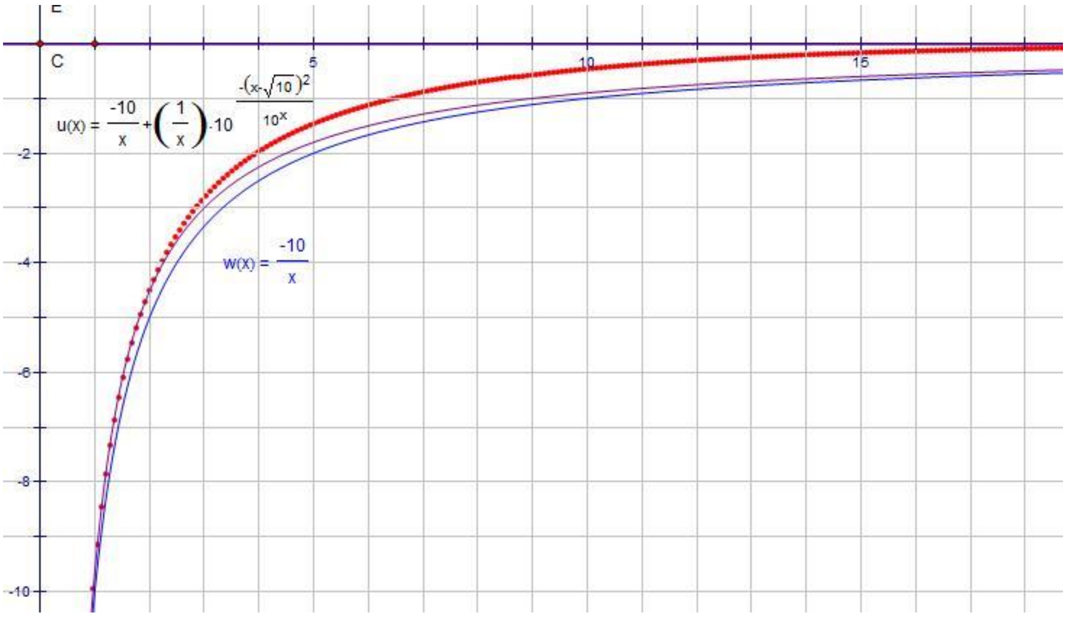


Figure 4-6

As we can see, the new function is closer to the real one. But it still has a problem which cannot be ignored, i.e., its value is not 0. This contradicts with our real function. So we could change the estimated function like:

$$y' = -\frac{10}{x} + \frac{1}{ax} 10^{\frac{-(x-\sqrt{10})^2}{10^x}}$$

Obviously, our main working changes from whole to local, that is a.

By trying various constants between 0 and 1, we found that every different estimated function will has a crossover point with the real one, though it is not so close to the real function. (see Fig. 4-7).

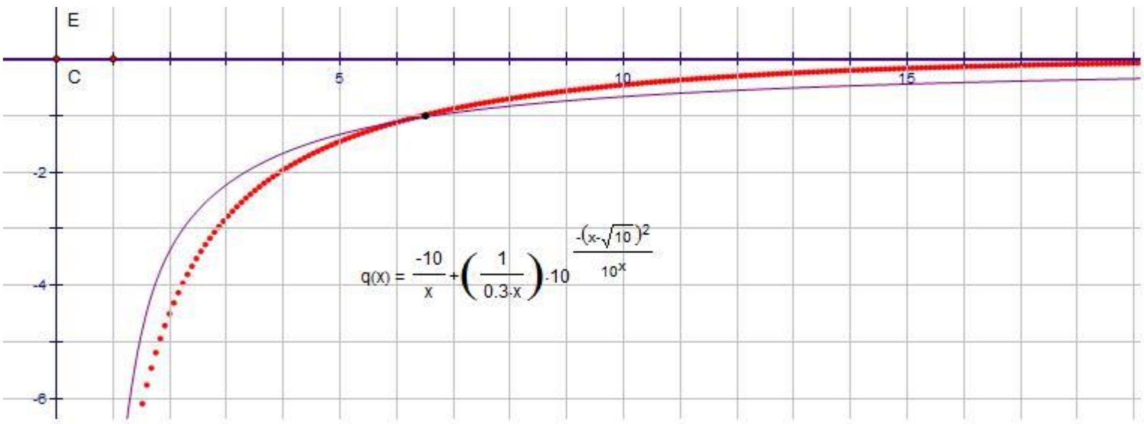


Figure 4-7

In that case, we experimented many times, and got the crossover points of the estimated function and the real one, then drew (x, a) in the coordinates. We could estimate the relation between a and x in this way (see Fig. 4-8).

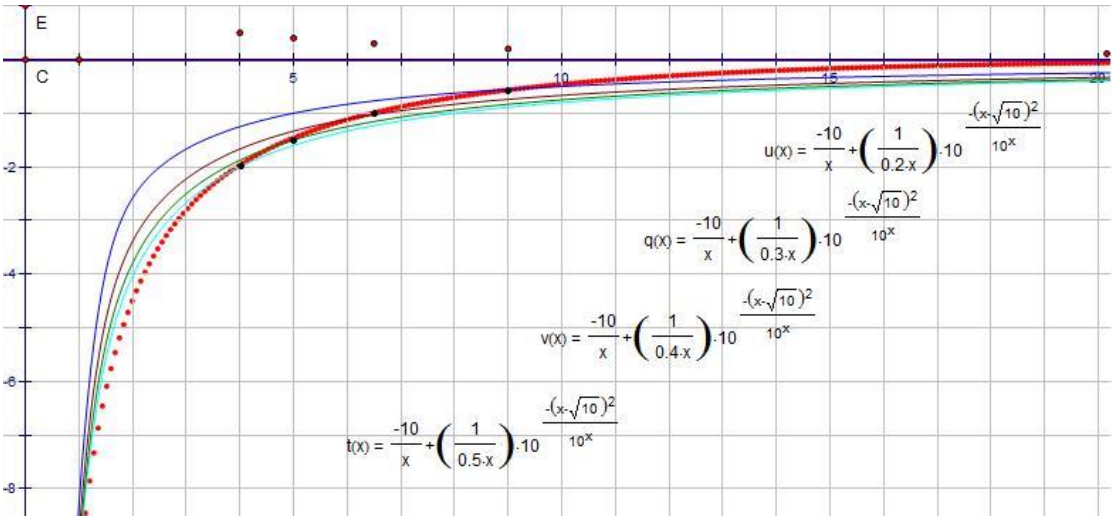


Figure 4-8

The four points on the x-axis seem very regular, we can quickly figure out its approximate function which is:

$$g(x)=\frac{1}{0.15x^2}+0.1$$

As figure 4-9 shows:

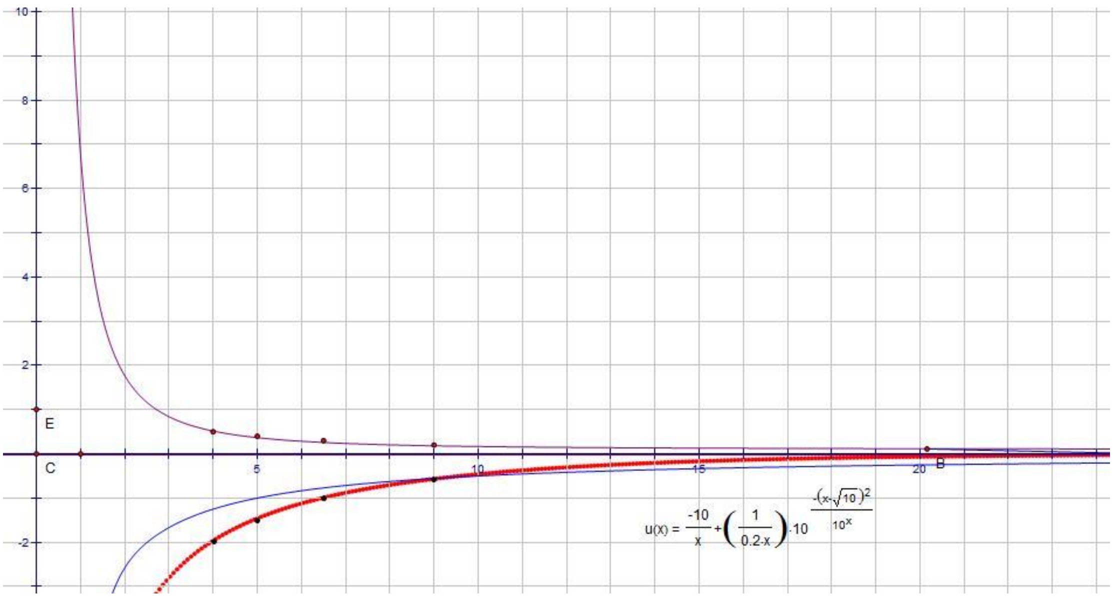
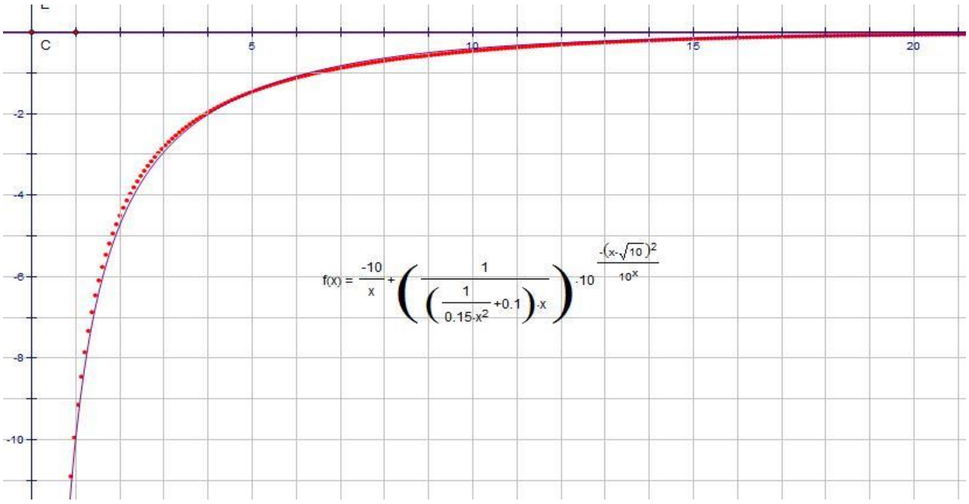


Figure 4-9

This expression is applicable to a, because it tends to be 0.1. When a equals to 0.1, the estimation function tends to be 0. We then changed the estimation function, and obtained the differential coefficient expression for the tracing curves as:

$$y'=-\frac{10}{x}+\frac{1}{(\frac{1}{0.15x^2}+0.1)x}10^{\frac{-(x-\sqrt{10})^2}{10^x}}$$

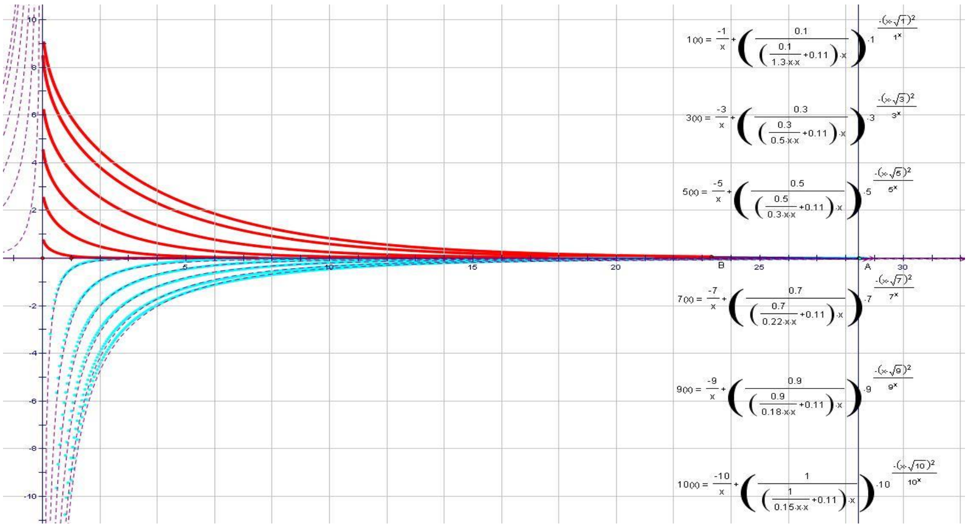
From figure 4-10 we could see, it really matches our demands.



4-10

Therefore, we can study the equation by using this method, try more origin distance, get different values of a, and then relate it with L in order to form functions. In this way, our formula can have better generality.

So we conducted more experiments. Letting L be denote different integers, we observed how a would change (see Fig. 4-11).



4-11

Then, we used the same method to find out the equation of the tracing curve whose original distance was 10, marked the point (L, a) (see Fig. 4-12). We discovered that the points were approximately on an inverse proportion function, and then we estimated that the function is:

$$a = \frac{\sqrt{2}}{L}$$

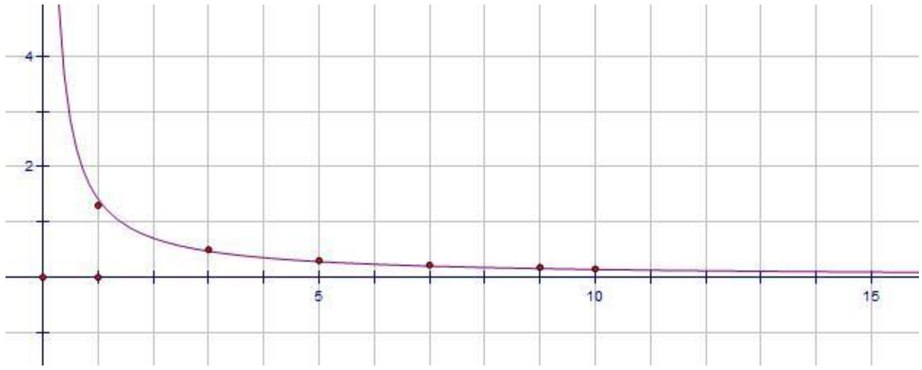


Figure 4-12

Next, we added this parameter, and finally got the function of the differential equation of the tracing curve which is:

$$y' = -\frac{L}{x} + \left(\frac{10\sqrt{2}Lx}{10L^2 + 11\sqrt{2}x^2}\right)L^{\frac{-(X-\sqrt{L})^2}{L^x}}$$

The parameter L in this formula means original distance.

We give L different value and experimented many times. We have to admit that as L becomes bigger, the error becomes obvious, but in the area from 1 to 20, the estimated function is proper. Then, if integrate the differential function, we could obtain the function of the tracing curve, and could add the parameter of time in order to study the location of the tracing point when its target reaches the destination.

5. Conclusions

This research explored two types of movements, “converging curve” and “tracing curve”. In this study, we found that:

1. by extending the original question to the case of right polygon, we can figure out the equation of the converging curve;
2. the four points cannot converge at the same time in the cases of rectangle and rhombus;
3. and despite the failure to get a precise equation for the tracing curve, by using the method of modeling, we could find out a proper function of its differential equation.

In this paper, we discussed about movements, and in the real world, everything is moving. We believe that the findings of this research would be helpful to solve some problems. For example, in modern military and some other industries, similar tracing technology might be applied. The function we got might not be very precise, but might be applicable in certain areas. For the converging curve, we think it could play an important role in studying the movements of irregular colony. So it can be used to study the distribution of population density, and to discuss the relevant best scheme.

This research was also embedded with some mathematical thoughts and methods which, during the exploration, widened our ways of thinking and deepened our understanding of skills in mathematic research such as using the invariant elements, argument by contradiction, differential methods, and so on. Mathematics is not only a subject to solve, but also one to discover problems. During the whole process of this research, we fully enjoyed the happiness of discovering and the splendor of nature. The questions we discovered are more than those we solved, and this added to our enthusiasm with the mathematics knowledge we are going to learn. Due to the limitation of our time and knowledge, this research is just a preliminary exploration. We believe in the future, when time and knowledge permit, we will bring out more inspiring findings.

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