正 N 边形形内对角线交点的计数问题

The Question of Count for Intersection Points of Inner Diagonal Lines of Regular N Polygon

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完成时间：**2008** 年 **8** 月 **23** 日星期六

The Question of Counting for Intersection Points of Inner Diagonal Lines of Regular N Polygon

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【Abstract】

In the early 1980s of twenty century, Professor Zhang Zhongfu, an expert in graph theory, raised a question in his research[1]: how many points of intersection of diagonal lines are there inside a regular N polygon, hoping to find out a formula of count. It has been more than 20 years since the question was raised, which has aroused the interest of quite a lot of experts, scholars and those who love mathematics, but still remains unsolved.

When N is an odd number, proposition 1 can be derived from formula of counting based on proposition 2, which can also be verified by the programme the author has made.

Proposition 1: when N is an odd number, there is no concurrence of 3 or more than 3 diagonal lines of regular N polygon.

Proposition 2: when N is an odd number, the number of intersection points of inner diagonal lines of regular N polygon is:

$$a_n = C_n^4 = \frac{1}{24}(n-1)(n-2)(n-3)$$

But when N is an even number, it becomes quite complicated. When N are some special even numbers referred to by Chang Jianming[3], there are some laws: the research is made from the perspective of prime factorization of figures.

The writer primarily studies the issue by making use of geometrical drawing and imitation by computer programme, so that the programme is improved and the imitating solutions of the numbers of intersection points of inner diagonal lines of regular N polygon with specific programmes can be solved. The solution is popularizable. The writer has worked out the numbers of intersection points of inner diagonal lines of regular 2N polygon when N$\leq$50, if conditions of computers in schools allow.

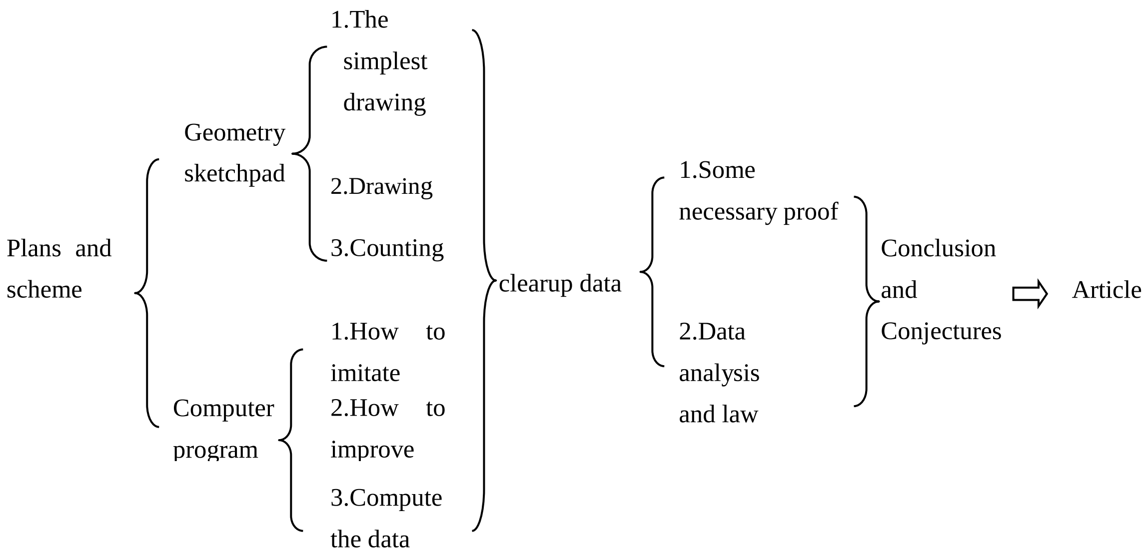
By comparing the counting between drawing and computer imitation, the writer has also found out some interesting laws about the numbers of intersection points of inner diagonal lines of regular N polygon, and has put forward some hypotheses.

【**Key words**】

Regular N polygon, intersection points of diagonal line, formula of count for intersection points, imitation programme

【Article】

1. The outline of the article



1.1: By using drawing software such as geometry sketchpad and so on to carry on the actual cartography.

1.1.1 By using the related mathematics knowledge to find out the most superior drawing technique.
1.1.2 By using this drawing method to draw regular even polygon’s figure which can be drawn in certain range, and count the total number of points.
1.1.3 To classify the special points, tag in the figure and record data.
1.1.4 Data processing.

1.2: Using the Pascal programming to calculate the total points

1.2.1 Design corresponding computer program by using the analytical method and simulate the process of mapping for point.
1.2.2 Making use of the advantages of computer in dealing with large amounts of data quickly and accurately to calculate the total points.
1.2.3 Continuously improve and optimize the program in processing data.
1.2.4 Data processing.

1.3: Compare and analyze data in order to solve the limitations.

Because there is limitation in calculating total points by geometry sketchpad and computer program, the author therefore unifies the data resulting from the two ways to make up the limitation, i.e. to prove the data resulting in program calculation by the test of geometry sketchpad, but to figure out chart spots with program which geometry sketchpad is unable to draw. Figure out another kind of data from the existing ones for further research in order to collect more complete data..

1.4: Obtain conclusions and conjectures

In comprehensive analysis of the data, the author repeatedly provides evidences to the data's accuracy, thus through the analysis and study of these data, discovers some rules, and proposes some suspicions and the conclusion.

2. The simplest method to plot the graph

2.1: Mapping tool

Chinese version 4.60 of the geometry sketchpad

2.2: Mapping goal

To prove whether the total points are correct by programming and to observe the rules of graph and so on.

2.3: Mapping limitation

2.2.1 Define "inner center angle": Regular n polygon's two adjacent vertices and the central point of connection into the angle which is the N polygon's inner angle. Degrees of the N polygon's inner center angle are $\frac{360^0}{N}$.

2.2.2 We can plot the graph, when 360 MOD N=0. At this time the degree of inner angle is Integer.

2.2.3 Because in the Geometer's Sketchpad the point of accuracy only accurates to 0.1^0, their inner angles are Finite decimal, such as N=14,22 and so on. If we take the approximate value the number of intersection point will be inaccurate. This will lose the geometry sketchpad's significance, so they cannot be drawn by geometry sketchpad.

2.2.4 But the Regular 32 polygon can be drawn. Because at this time the inner angles is 11.25^0, it can be obtained through making the internal bisector of the Regular 32 polygon. Therefore the graph of this kind can be drawn, such as when N=96, 64 and so on.

2.4 Steps of draw graph and count points and the skill

2.3.1 Drawing.

Draw the regular N polygon with the circle and connect the diagonal line, only connect the diagonal line which is through goal region. The linear number least needed to be connected is(not including green dashed line):

$$\frac{N}{2}+\left[\frac{N}{4}\right]-1+\{(2-1)+(3-1)+\cdots+(\frac{N-2}{2}-1)\}\times 2=\frac{N^2}{4}-N+\left[\frac{N}{4}\right]+1$$

where [N] denotes the greatest integer which is not more than N.

The steps of drawing:

1. Makes a circle in the coordinate axis and hides the Y-axis.
2. Make N/2 diagonal line which is of the same length with the diameter from the angle.
3. Draw green dashed line L(Internal bisector of goal region).
4. Connect points A_i and B_i which are symmetrical about L.(i=2,3,…, $\left[\frac{N}{4}\right]$)

5. Connect points A_i and B_j. (i=2,3,… $\frac{N-2}{2}$; j=1,2,…i-1)

6. Connect points B_i. and A_j (i=2,3,… $\frac{N-2}{2}$; j=1,2,…i-1)

Namely,we get the Figure 1.

We call this drawing method: The simplest drawing method

Figure 1 is drawn by the simplest drawing method, where N=24.

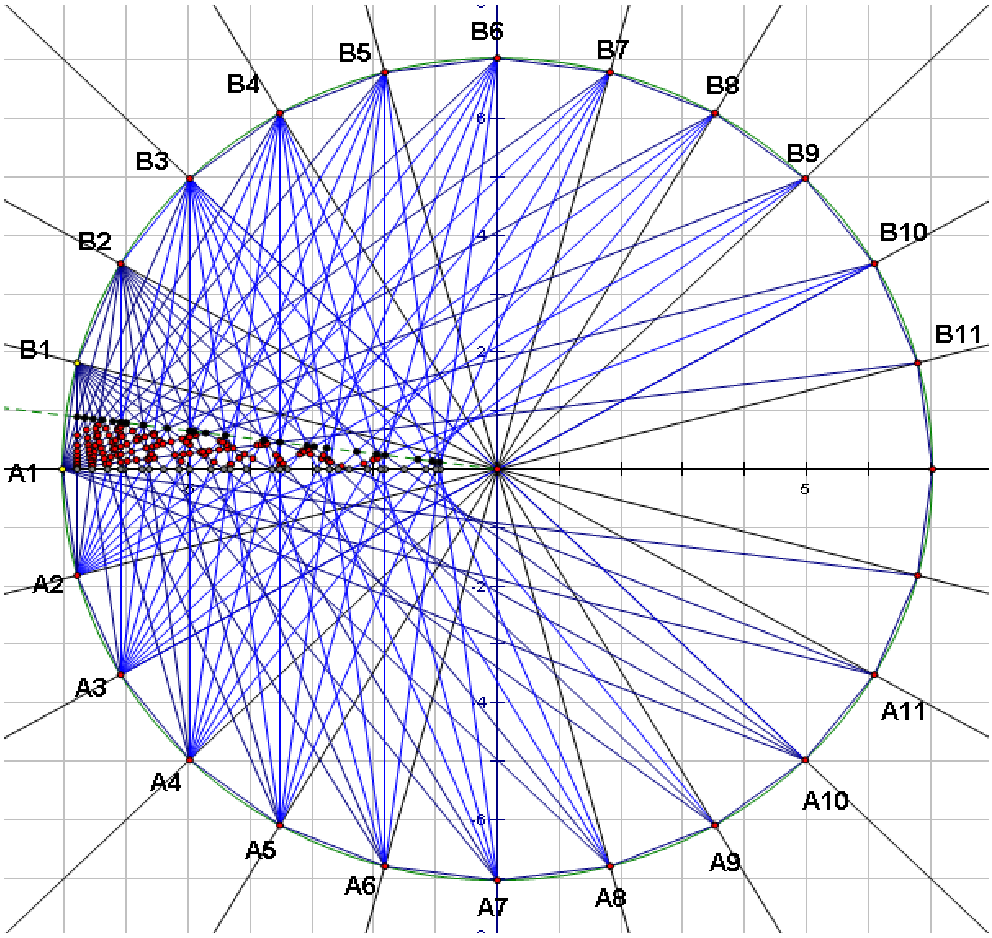


Figure 1

2.3.2 Tagging

Magnifying the figure and intercepting only $\frac{1}{N}$ of the whole figure, we get the figure 2, calling it target area figure. Magnifying target area figure again, we get the figure 3. Then we count the points in figure 3. According to symmetry of regular even polygon, we can compute the total number of points. We use black points to denote points on L, use grey points to denote points on x-axis and use red points to denote other points.

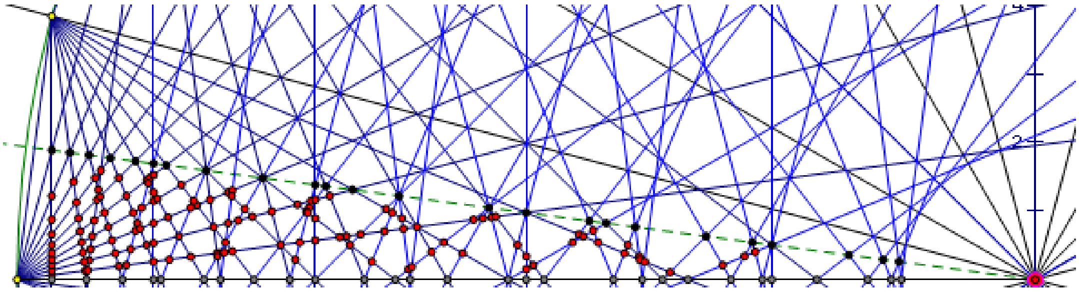


Figure 2. Target area figure

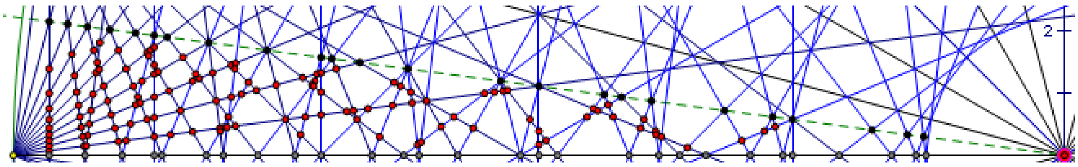


Figure 3. The half figure of figure 2

2.3.2 Counting

Count different colour points respectively. If tow points are so close that we can not judge whether they are overlapping or not, we magnify the figure again until it is big enough to count. Figure 4 is an example

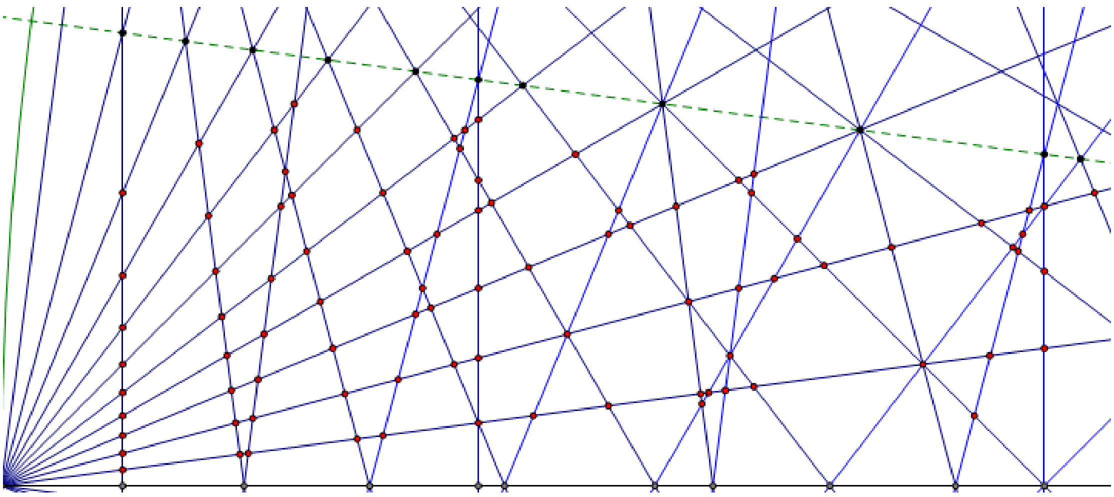


Figure 4. Some area of figure 3 after magnify

2.3.2 Record the data in the table

Explanation: The author has drawn figures with N=4,6,8,10,12,16,18,20,24,30,32,36. In consideration of the large space taken up by figures, we show the figures in appendix.

2.5: The Notion of Layer

According to the symmetry of regular N polygon when N is a even number, every point in target area graph (2.3.2 in Figure 2)has corresponding point in other N-1 areas. The distance between origin and these N points in N areas is equal. So we can get a secondary regular N polygon when N points are connected orderly. We call this secondary regular N polygon “a layer”. Obviously, the points of every two secondary regular N polygons are not coincident, so we only count how many layers there are in regular N polygon to figure out the total points. This shows that the importance of “layer”.

Observing the target area graph (2.3.2 in Figure 2), we can find out :layers=2× red points+grey points+black points, namely C=2R+G+B.

3. The idea of computer programme

Explaination: The author uses the Pascal to make program. Version: Free Pascal 2.0.4.

3.1 How to imitate the program

This is a question about intersection points number of regular N polygon diagonal, and vertex, diagonal, intersection points are related to this problem. These geometry elements can not be represented by computer program, however, they can be represented by analysis method.

We can use (x,y) to represent intersection points and vertexes. According to the line y=kx+b, every (k,b) and the line in Cartesian coordinate system are 1 to 1 map. We can use computer imitation program to find out the intersection points number. The steps are as follow:

①Draw a regular N polygon in Cartesian coordinate system.

②Connect the diagonal.

③Calculate the coordinate of intersection.

④omit unnecessary points and repeated intersection points

⑤output the total points T

3.2 Specific operations of 1-5

3.2.1 ① when N becomes larger, regular N polygon gradually looks more like a circle. So the author draws a circle with the origin as centre and 1as radius, the perigon of the central angle is equally divided into N portions , every angular bisector and the circle intersection is the vertex of regular N polygon, then calculate sine and cosine value of every angle through the angle. They respectively correspond vertical coordinate and abscissa of that point . The author provides X to positive axis as initiation .So the angle of every point is i× $\frac{360°}{N}$ （i=1,2,3,…， N）。

The program uses two dimension array a to deposit coordinate of these vertexes.

3.2.2 ②Take two different points (x_1,y_1)，(x_2,y_2) for example and calculate the line equation

$$\because \begin{cases} y_1 = x_1 \times k + b \\ y_2 = x_2 \times k + b \end{cases} \qquad \therefore \begin{cases} k = \dfrac{y_1 - y_2}{x_1 - x_2} \\ b = y_1 - x_1 \times k \end{cases}$$

The premise of the above equation is $x_1 \neq x_2$, when $x_1 = x_2$, the two points are in the same vertical, the line equation is $x=x_1$ or $x=x_2$, but it can not be represented by k and b, the author will use it in the later steps rather than here.

The program uses two dimension array b to deposit these line equation.

3.2.3 ③ Take two different lines (k_1,b_1)，$(k_2,b_2)$for example and calculate the coordinate of intersection point of this two straight lines.

$$\because \begin{cases} y = x \times k_1 + b_1 \\ y = x \times k_2 + b_2 \end{cases} \qquad \therefore \begin{cases} y = k_1 \times x + b_1 \\ x = \dfrac{b_1 - b_2}{k_1 - k_2} \end{cases}$$

The premise of the above equation is $k_1 \neq k_2$, when $k_1 = k_2$, the slope of two lines is equal, so two lines are parallel(no intersection).

Moreover , now we research the line x=c which has not been discussed in 3.2.2 .It is easy to know that the number of this kind of lines is n/2-1 , they are x=x(1), x=x(2),x=x(3),…， x=x(n/2).(x(i) is the abscissa of the ith point).These lines are parallel, so there is no intersection point. We can only enumerate the intersection points between this kind of lines and other kinds.

$$\because \begin{cases} y = x \times k_1 + b_1 \\ x = c \end{cases} \qquad \therefore \begin{cases} x = c \\ y = k_1 \times x + b_1 \end{cases}$$

To sum up, we find out the total intersection points of all diagonals.

The program uses two dimension array c to deposit coordinate of these intersection points.

3.2.4 ④In the following part, we firstly prove the proposition: there is no intersection point stated above in the area between the regular N polygon and its circumcircle.

Proof: Take regular 6 polygon for example, according to its symmetry property, to prove that no intersection point is in gray area of figure 3.2.4.1, we can just figure out that no intersection point is in gray area of figure 3.2.4.2.

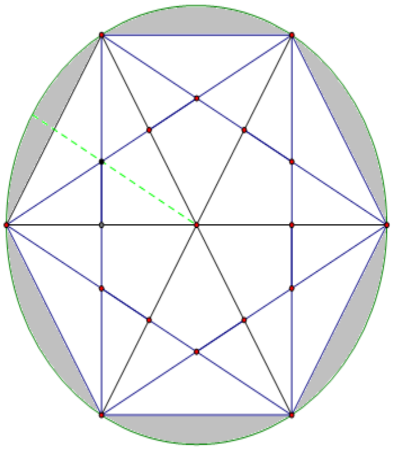


Figure 3.2.4.1

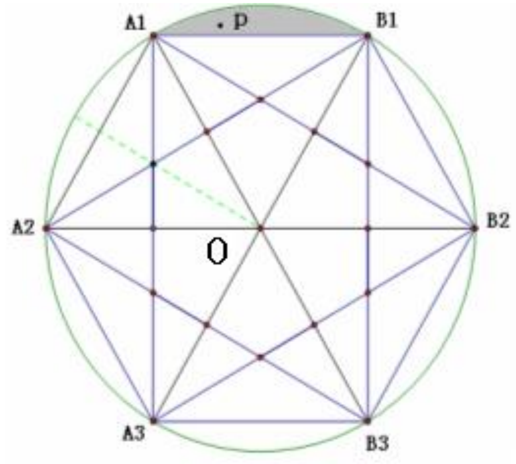


Figure 3.2.4.2

Because A_1,B_1 are two adjacent vertexes, inferior arc between any two vertexes is longer than inferior arc A_1B_1 . We suppose that there exists two diagonals L_1,L_2 and a point P, where $L_1 \cap L_2$=P, and P is in the gray area. If L1 is via point A_i (i=1,2,3), let L_1 is via anther vertex C, namely $L_1 \cap \square_O$=C, so that C is on inferior arc A_1B_1 , and inferior arc $CB_1 <$ inferior arc A_1B_1 . But this conclusion contradicts the above-mentioned one: inferior arc between any two vertexes is longer than inferior arc A_1B_1 .

So L_1 is not via A_i. In the same way, L_1 is not via B_i (i=1,2,3)

Further more, because regular N polygon is inscribed in a circle whose radius is 1, to judge whether or not a point is in circle is the standard to judge whether or not this point is in the regular N polygon. Namely, to judge whether or not the distance between the point and origin is less than 1.

The program uses two dimensions array d to deposit these intersection points coordinates which are in the regular N polygon.

Most of the coordinates of the points are infinite decimal fraction. Because of the limitation of program when it deals with the real form numbers, the 15^{th} number after radix point exists error (example, in Free Pascal,0=0.000000000000001 is possible). So two real number a and b, if $a-b<1\times10^{-14}$, we consider a and b are equal. $P(x_1,y_1)$, $Q(x_2,y_2)$ are two intersection points. If $x_1-x_2<1\times10^{-14}$ and $y_1-y_2<1\times10^{-14}$, P and Q overlap each other. We eliminate this point.

In the program, we assign 1 as the distance from repeated points to origin. This denotes the point is eliminated.

3.2.5 ⑤ Any four vertexes of regular N polygon can determine a convex 4 polygon, and the intersection points of diagonal of this convex 4 polygon are the intersection points of diagonal of regular N polygon. So there are C_N^4 intersection points in total, including repeated points. For the repeated points, we reserve only one, the others are eliminated. From the formula

$$\text{Total}=C_N^4\text{-all repeated points}$$

We can find out the total intersection points.

3.3 The results of the program and the unresolved problems (the limitation of the program)

The program has calculated the number of intersection points of diagonal and gray points(2.3.2) with N⩽50 and N is an even.

But the result can be received not very easily. We get the result with N=50 after we improve and modify the program again and again, primarily improve precision when the program deals with real form number. We can make sure that the result is correct when N⩽36(geometry sketchpad has verified the result). When N>50, the result is incorrect as T is an even. But T=C(layers）×N+1(origin) , obviously it is an odd number.

The writer thinks that when N becomes larger , the distance between two points is so close that the program can not calculate the precise result. In Free Pascal, the result is correct until 15^{th} number after radix point. This problem will be discussed in 7.

4. Make computer program with PASCAL.

The content in the bracket "{ }" is commentary.

```
program jihe; { the intersection points number of the regular even polygon diagonal }
type
arr=array[1..999999,1..2]of real;
label
lab;{define goto sentence}
var
a,b,c:arr;
{a:the abscissa and ordinate of the regular N polygon；b:the slope and intercept of line；c: the abscissa and ordinate of the intersection point}
d:array[1..999999,1..3]of real;
{d: the abscissa and ordinate of the intersection point of inner diagonals, and the distance between the points and origin}
total,grey,i,i1,j,j1,n,n1,s1,s2,m,l,t,h,k:longint;
```

```
{total:the total number of points; grey: the number of grey point，in order to improve the program, the other letters are added. }
m1,x,z:real;

begin
assign(input,'jihe.in');
reset(input);
assign(output,'jihe.out');
rewrite(output);

{4.1.1 according to symmetry of even N polygon and trigonometric functions, calculate a}
    readln(n);
    m:=n div 2;
    m1:=pi/m;
    for i:=1 to m do
    begin
    a[i,1]:=cos((i-1)*m1);
    a[i+m,1]:=-a[i,1];
    a[i,2]:=sin((i-1)*m1);
    a[i+m,2]:=-a[i,2];
    end;

{4.1.2 because of the limitation of real form number，the initialized value is assigned again for points on x-axis }
    a[1,2]:=0;
    a[1,1]:=1;
    a[1+m,2]:=0;
    a[1+m,1]:=-1;

{4.1.3 if point a is on the y-axis too, the initialized value is assigned again}
    if n mod 4=0 then
    begin
    t:=(n div 4)+1;
    a[t,1]:=0;
    a[t,2]:=1;
    a[t+m,1]:=0;
    a[t+m,2]:=-1;
    end;

{4.2.1 In order to avoid 0 being divisor , we do not discuss the vertical lines here}
    k:=0;
    n1:=n+2;
    for i:=n downto 2 do
    for j:=1 to i-1 do
```

```
if i+j<>n1 then
begin
inc(k);
b[k,1]:=(a[j,2]-a[i,2])/(a[j,1]-a[i,1]);
b[k,2]:=a[i,2]-b[k,1]*a[i,1];
end;
```
{**4.2.2** calculate the intersection point c of unparallel lines b}

```
l:=0;
for i:= k downto 2 do
for j:=1 to i-1 do
if b[i,1]<>b[j,1] then
begin
inc(l);
c[l,1]:=(b[j,2]-b[i,2])/(b[i,1]-b[j,1]);
c[l,2]:=b[i,2]+c[l,1]*b[i,1];
end;
```
{**4.2.3** figure out the intersection point c of vertical lines b and the other lines b}

```
for i:= 2 to m do
for j:=1 to k do
begin
inc(l);
c[l,1]:=a[i,1];
c[l,2]:=c[l,1]*b[j,1]+b[j,2];
end;
```
{**4.3.1**make use of the distance from point to origin to eliminate the points which are out of the circumcircle of N polygon，d[h,3] is used for depositing the distance}

```
h:=0;
for i:=1 to l do
if sqrt(sqr(c[i,1])+sqr(c[i,2]))<1 then
begin
inc(h);
d[h,1]:=c[i,1];
d[h,2]:=c[i,2];
d[h,3]:=sqrt(sqr(c[i,1])+sqr(c[i,2]));
end;
```
{**4.3.2** On the basis of d[h,3], we use word ‘goto’ to implement quick-sort. Due to great volume of data, recursion calling will display ‘runtime error 202: stack over flow error’ on condition that quick-sort adopts procedure and function, i.e. over calling or calling too much.}

```
s1:=1;
s2:=h;
lab:
begin
i1:=s1;
```

```
j1:=s2;
x:=d[i1,3];
repeat
  while(d[j1,3]>=x)and(j1>i1)do dec(j1);
  if j1>i1 then begin
                z:=d[i1,3];d[i1,3]:=d[j1,3];d[j1,3]:=z;
                z:=d[i1,1];d[i1,1]:=d[j1,1];d[j1,1]:=z;
                z:=d[i1,2];d[i1,2]:=d[j1,2];d[j1,2]:=z;
                end;
  while(d[i1,3]<=x)and(i1<j1)do inc(i1);
  if i1<j1 then begin
                z:=d[i1,3];d[i1,3]:=d[j1,3];d[j1,3]:=z;
                z:=d[i1,1];d[i1,1]:=d[j1,1];d[j1,1]:=z;
                z:=d[i1,2];d[i1,2]:=d[j1,2];d[j1,2]:=z;
                end;
until i1=j1;
d[i1,3]:=x;
inc(i1);
dec(j1);
if s1<j1 then begin
              s2:=j1;
              goto lab;
              end;
if i1<s2 then begin
              s1:=i1;
              goto lab;
              end;
end;
```

{**4.3.3** according to the distance from points to origin and the proof of 3.2.4, we take the first C_N^4 points, and the rest are in N polygon}

{**4.3.4** eliminate the repeated point. C_N^4 －repeated points＝Total}

```
total:=n*(n-1)*(n-2)*(n-3)div 24;
h:=total;
for i:= h downto 1 do
for j:=1 to i-1 do
if
(d[j,3]<>1)and(d[i,3]<>1)and(abs(d[i,1]-d[j,1])<1e-14)and(abs(d[i,2]-d[j,2])<1e-14)th
en
begin
d[j,3]:=1;
dec(total);
end;
```

```
{4.4.1 find out the grey points which are on negative x-axis，the criterion is that ordinates are 0}
    grey:=0;
    for i:= 1 to h do
    if (d[i,3]<>1)and(abs(d[i,2])<1e-14) then inc(grey);
    grey:=(grey-1)div 2;
{4.4.2 output the total points T and the grey points G}
    writeln(total,'    ',grey);
    close(input);
    close(output);
    end.
```

5.The results computed by program and geometry sketchpad respectively, and the results comparison

5.1 Results

	计算机程序得出的数据			几何画板得出的数据				
N	Ceng	Total	Grey	Total	Ceng	Red	Grey	Black
边数	层数	总点数	灰色点	总点数	层数	红色点	灰色点	黑色点
4	0	1	0	1	0	0	0	0
6	2	13	1	13	2	0	1	1
8	6	49	2	49	6	1	2	2
10	16	161	4	161	16	4	4	4
12	25	301	6	301	25	7	6	5
14	54	757	9					
16	86	1377	12	1377	86	31	12	12
18	102	1837	13	1837	102	38	13	13
20	192	3841	20	3841	192	76	20	20
22	270	5941	25					
24	304	7297	28	7297	304	126	28	24
26	480	12481	36					
28	616	17249	42					
30	560	16801	41	16801	560	242	41	35
32	966	30913	56	30913	966	427	56	56
34	1184	40257	64					
36	1305	46981	66	46981	1305	587	66	65
38	1710	64981	81					
40	2022	80881	90					
42	2020	84841	91					
44	2760	121441	110					
46	3190	146741	121					
48	3420	164161	124					
50	4176	208801	144					

Explanation: The layers in blue and yellow background table and the total points in yellow background table are derived from other data in the same background.

5.2 The relationship between the two data and the awareness on accuracy of data computed by program

On one hand the data computed by program can make up the shortage of geometry sketchpad. On the other hand it is easy to find that the two data in the table are consistent, so the results affirm the validity of the program.

The limitation of the program lies in accuracy of the computing. When the two different points become enough close , the program judges they are the same one.

If N=K, the data received by program is correct, meanwhile it can be validated by geometry sketchpad, it shows our program doesn't have problem of accuracy. Then N≤K, the result is correct too. Therefore, at least when N≤K, the results are credible.

6.The analysis and processing of the data, conclusion and conjecture

6.1 The formula for counting the total points

According to the analysis of the tables and figures, we can get the formula which calculate the total points easily:

$$\left.\begin{matrix} T = C \times N + 1 \\ C = 2R + G + B \end{matrix}\right\} T = (2R + G + B) \times N + 1$$

6.2 Some interesting laws

From the table, we find that the total points depend on the number of layers. But the layers, red points, black points and grey points have no obvious relationship with N. whereas, we find some surprising laws:

6.2.1 The relationship between the black and grey points

Let's observe the data in “”. The number of grey points G>the number of black points B for the same N.

The data not in “” satisfy the equation G=B for the same N.

6.2.2 The total points doesn't increase by degrees

Let’s observe the data in “ ”. We find that the total points increase by degrees from the overall N. But when N=28 and N=30, following inequalities are satisfied:

$$T28>T30$$
$$C28>C30$$
$$G28>G30$$

When N=40 and N=42, the following two inequalities and one equality are satisfied:

$$T42>T40$$
$$C42<C40$$
$$G42-G40=1$$

6.3 The analysis and conjectures

For 6.2.1, data in “ ” are corresponding to the value 12, 24, 30, 36 of N respectively. Whereas

$$12=2\times2\times3=2\times6$$
$$24=2\times2\times2\times3=4\times6$$
$$30=2\times3\times5=5\times6$$
$$36=2\times2\times3\times3=6\times6$$

It is a law that all above equalities are related to the number 6. So the writer has following conjectures.

Conjecture 1: For regular N polygon, when N MOD 6=0, $G\geqslant B$, where G denotes the number of intersection points on diagonals which are via the centre, B denotes the number of intersection points on angle bisector of inner center angle. (vision 2.2.1).

Conjecture 2: For regular N polygon, when N MOD $6\neq0$, G=B, where G denotes the number of intersection points on diagonals which are via the centre, B denotes the number of intersection points on angle bisector of inner center angle. (vision 2.2.1).

Conjecture 3: For T, the number of the intersection points of inner diagonals, T is not strictly monotone increasing as the N increases. Namely $\exists i, j \in N^*$ and $i<j$, such that Ti>Tj, where Ti denotes the number of intersection points of inner diagonals of regular i polygon.

Because N and C are directly related to T in conjecture 3, the “unwonted” evidence of T is affected by C. But C does not always affect T. For example, when N=40 and 42, although T42>T40, $C42<C42$. Therefore, the “unwonted” evidence of C is a necessary but not sufficient condition of T.

Conjecture 4: For the layers of intersection points of inner diagonals of regular N polygon, $\exists i, j \in N^*$ and $i<j$, such that Ci>Cj, where Ci denotes the number of layers of intersection points of regular i polygon diagonals.

7.The continuation of the problem

Due to the writer's level of making program and the limitation of computer hardware, the program has not resolved the problem perfectly when N>50. The writer thinks that more data should be received, and they are used to research the law when N becomes larger. This depends on the computing accuracy of program.

If we use a large-scale computer to deal with the problem, the accuracy will be improved much higher and more accurate decimal place can be preserved. Then we use the algorithms in 4 or another better algorithms to find out the number of intersection points of inner diagonals of regular N polygon, and the result will be more accurate. Meanwhile N can be much larger. After that, the results can be used for testing the conjectures and law proposed above. And new law may be found, and even the enumeration formula of intersection points of inner diagonals of regular even polygon may be derived.

【Acknowledgements】

I am most grateful to my teacher He Hangfei for providing me thesis material and valuable suggestions, and my classmate Liu Xuhong for his help. Besides, I also owe my thanks to my sister Fu Lei and my teacher Wang Shaofeng for helping me to translate this paper into English.

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【Appendix】The whole and part figures of regular N polygon

Figure1: the whole figure of regular 12 polygon
Figure2: the whole figure of regular 18 polygon
Figure3: the whole figure of regular 30 polygon
Figure4: the whole figure of regular 36 polygon
Figure5:a part of target area figure of regular 12 polygon
Figure6: a part of target area figure of regular 18 polygon
Figure7: a part of target area figure of regular 18 polygon
Figure8: a part of target area figure of regular 18 polygon

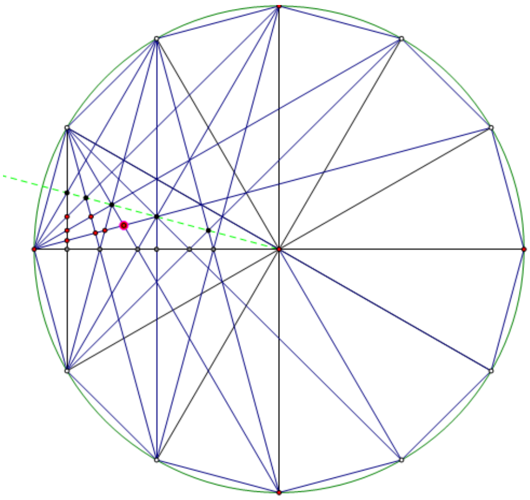


Figure 1

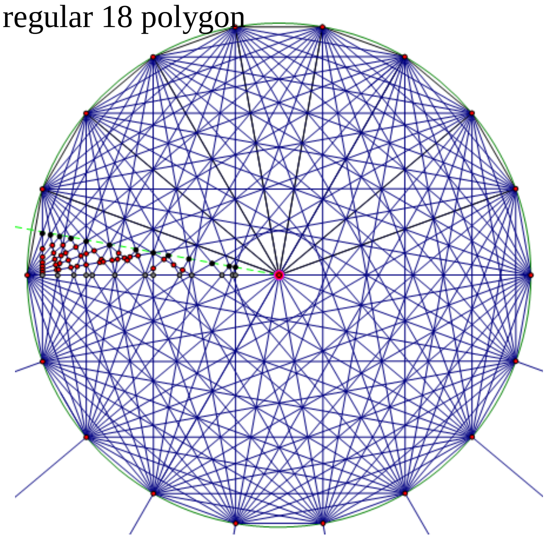
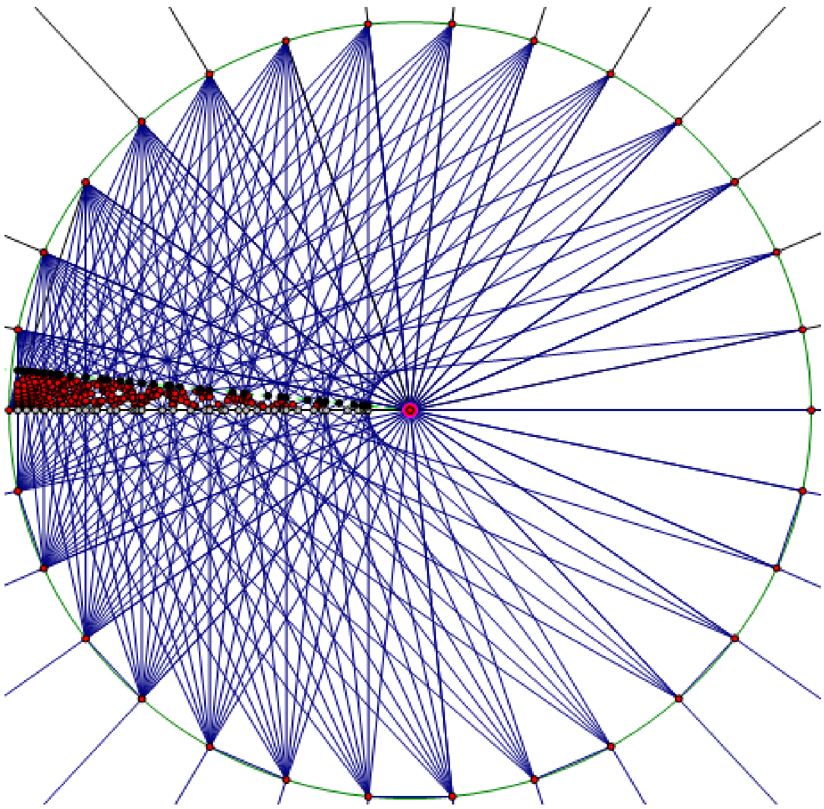


Figure2



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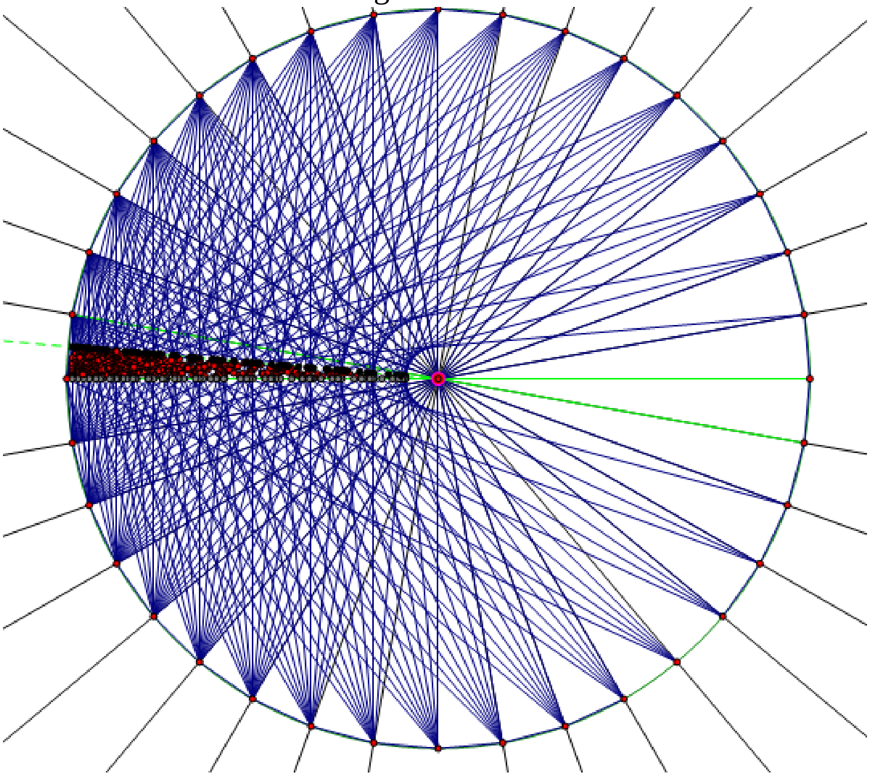

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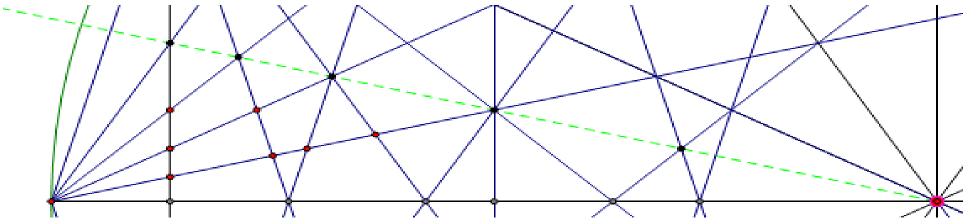

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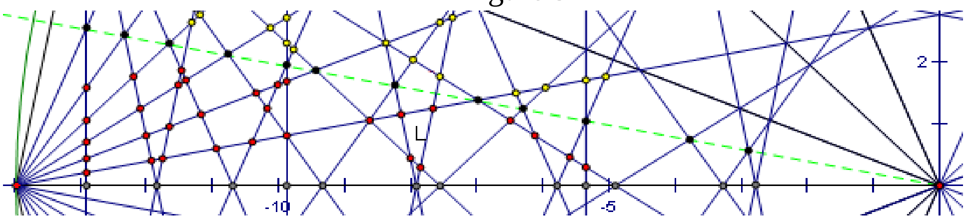

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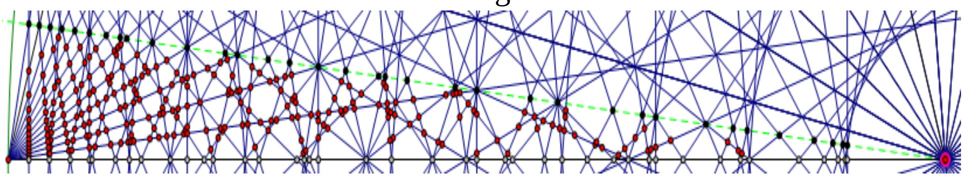

Figure 7

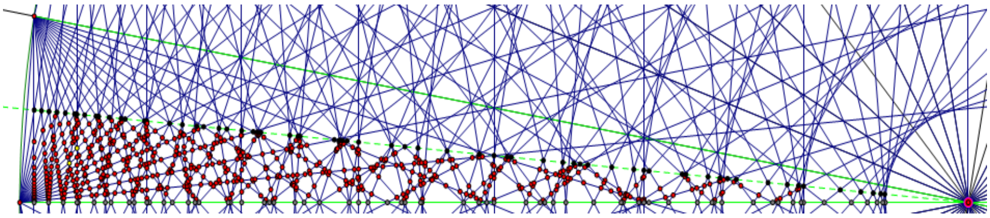

Figure 8